DESIGN OF DIGITAL FLIGHT-MODE CONTROL SYSTEMS FOR HELICOPTERS WITH NON-LINEAR ACTUATORS

B Porter
Department of Aeronautical and Mechanical Engineering
University of Salford
Salford M5 4WT
England

Abstract

The synthesis of discrete-time tracking systems incorporating Lur'e plants with multiple non-linearities is illustrated by the design of a fast-sampling digital controller and associated transducers for the automatic control of the longitudinal motions of the CH-47 helicopter with both gang-collective and differential-collective non-linearities. In particular, it is demonstrated that non-interacting control of the vertical velocity and pitch attitude of the helicopter is readily achievable for large classes of non-linear actuator characteristics such as 'deadzone' provided that the controller and transducer parameters are chosen so as to ensure that state-bounded absolutely stable tracking occurs.

1. Introduction

In recent years, the design of digital flight-mode control systems for high-performance aircraft has been greatly facilitated by the careful elucidation by Porter [1] of those characteristics of complex multi-input multi-output dynamical systems which determine the achievability of non-interacting control of the various modes of such systems. This elucidation has led to the development by Porter [1] of powerful design methodologies for discrete-time tracking systems which indicate that non-interacting control is in general achievable by the implementation of fast-sampling error-actuated digital controllers only if extra plant output measurements are generated by the introduction of appropriate transducers and processed by inner-loop compensators.

These general results on discrete-time tracking systems have been illustrated by the design by Porter [2] of fast-sampling error-actuated digital controllers and associated transducers for the automatic control of the longitudinal motions of the CH-47 helicopter. In particular, it has been demonstrated [2] by the presentation of the results of extensive digital computer simulation studies that tight non-interacting control of the vertical velocity and pitch attitude of the helicopter is readily achievable by the implementation of an appropriate fast-sampling digital controller which nevertheless generates practically acceptable gang-collective and differential-collective rotor control inputs. Furthermore, it has also been demonstrated [2] that such a fast-sampling digital controller is extremely robust in the face of changes in stability derivatives.

However, the actuators of multi-input multi-output dynamical systems such as high-performance aircraft usually exhibit non-linear characteristics such as 'deadzone' or 'saturation'. The general results of Porter [1] for the design of linear discrete-time tracking systems have accordingly been extended [3] so as to embrace dynamical systems which can be modelled as Lur'e plants with multiple non-linearities by invoking the concept of state-bounded absolutely stable tracking [4]. These general results are illustrated in this paper by the design of a fast-sampling digital controller and associated transducers for the automatic control of the longitudinal motions of the CH-47 helicopter with both gang-collective and differential-collective non-linearities. In particular, it is demonstrated that non-interacting control of the vertical velocity and pitch attitude of the helicopter is still readily achievable for large classes of non-linear actuator characteristics such as 'deadzone' provided that the controller and transducer parameters are chosen so as to ensure that state-bounded absolutely stable tracking [3][4] occurs. Such fast-sampling error-actuated digital controllers are accordingly of great practical significance since multiple actuator non-linearities of arbitrary severity can be accommodated by appropriate 'tuning', thus enhancing the robustness of digital flight-mode control systems in the face of unmodelled multiple actuator non-linearities.

2. Discrete-Time Tracking Systems

2.1 System Configuration

The plants under consideration are governed on the continuous-time set $\mathcal{T} = [0,+\infty)$ by non-linear vector differential equations of the Lur'e form

$$\dot{x} = Ax + Bf(u) + Dd \quad , \qquad (1)$$

$$y = Cx \quad , \qquad (2)$$

and

$$w = Fx \quad , \qquad (3)$$

where the plant state vector $x \in \mathbb{R}^n$, the output vector $y \in \mathbb{R}^\ell$, the measurement vector $w \in \mathbb{R}^\ell$, the control input vector $u = [u_1, u_2, \ldots, u_\ell]^T \in \mathbb{R}^\ell$, and the disturbance input vector $d \in \mathbb{R}^m$. The matrices $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times\ell}$, $C \in \mathbb{R}^{\ell\times n}$, $D \in \mathbb{R}^{n\times m}$, and $F \in \mathbb{R}^{\ell\times n}$ are elementwise constant, the first Markov parameter $CB \in \mathbb{R}^{\ell\times\ell}$ is singular, the pairs (A,B) and (A,C) are respectively controllable and observable, and the inner-loop compensator generating the measurement vector $w \in \mathbb{R}^\ell$ is assumed to be such that (Porter [1])

$$\lim_{t\to+\infty} w(t) = \lim_{t\to+\infty} y(t) \quad . \tag{4}$$

Furthermore, the disturbance d is in the class

$$S_d = \{d : d(t) \in C^{(1)}(R,R^m), \dot{d}(t)\equiv 0\} \tag{5}$$

of admissible plant disturbances, and the vector non-linearity $f : R^\ell \to R^\ell$, $f(u) = [f_1(u_1), f_2(u_2),\ldots,f_\ell(u_\ell)]^T$, $f_i : R \to R$ $(i=1,2,\ldots,\ell)$ is in the class

$$S_f = \{f : f(u) \in C(R^\ell,R^\ell), f(0)=0,$$

$$\frac{f_i(u_i^1)-f_i(u_i^2)}{u_i^1-u_i^2} \in [\gamma_i,\kappa_i], \forall u^1,u^2 \in R^\ell,$$

$$u_i^1 \neq u_i^2 ; i=1,2,\ldots,\ell\} \tag{6}$$

of admissible non-linearities where γ_i and κ_i $(\kappa_i>\gamma_i; i=1,2,\ldots,\ell)$ are the positive real elements of the diagonal matrices

$$\Gamma = \text{diag}\{\gamma_1,\gamma_2,\ldots,\gamma_\ell\} \tag{7}$$

and

$$K = \text{diag}\{\kappa_1,\kappa_2,\ldots,\kappa_\ell\} \quad . \tag{8}$$

In order to reject any unmeasurable disturbance input vector $d \in S_d$ whilst causing the plant output vector $y \in R^\ell$ to track any unmeasurable command input vector $v \in R^\ell$ in the class

$$S_v = \{v : v(t) \in C^{(1)}(R,R^\ell), \dot{v}(t)\equiv 0\} \quad , \tag{9}$$

the effectiveness of fast-sampling error-actuated digital controllers and associated inner-loop compensators in the linear case $f(u) = Lu$ (Porter [1]) motivates the introduction in the non-linear case of fast-sampling error-actuated digital controllers governed on the discrete-time set $T_T = \{0,T,2T,\ldots\}$ by equations of the form

$$z_{k+1} = z_k + Te_k \quad , \tag{10}$$

$$e_k = v - w_k \quad , \tag{11}$$

and

$$u_k = f(K_1e_k+K_2z_k) \tag{12}$$

which generate the amplitudes $\{u_k\}$ of the piecewise-constant control input vectors $u(t) = u_k$, $t \in [kT,(k+1)T), kT \in T_T$, where $T \in R^+$ is the sampling period and $f = 1/T$. In equations (10), (11), and (12), the controller state vector $z_k \in R^\ell$, the error vector $e_k \in R^\ell$, and the non-singular controller matrices $K_1 \in R^{\ell\times\ell}$ and $K_2 \in R^{\ell\times\ell}$. Such error-actuated digital controllers and associated inner-loop compensators which ensure that

$$\lim_{k\to+\infty} e_k = 0 \tag{13}$$

necessarily ensure that

$$\lim_{k\to+\infty} y_k = v \tag{14}$$

as a consequence of the relationship between the steady-state values of the plant output measurement vectors expressed by equation (4).

2.2 System Synthesis

In the case of Lur'e plants with multiple non-linearities whose linear components have transfer function matrices with singular first Markov parameters it is desirable that tracking occurs for all admissible non-linearities and all admissible inputs, and therefore that closed-loop digital control systems incorporating such plants exhibit state-bounded absolutely stable tracking on T_T over $S_d \times S_v \times S_f$ (Grujic and Porter [4]). Indeed, in the case of fast-sampling error-actuated digital controllers and associated inner-loop compensators, the general result expressed by Theorem 2 of Grujic and Porter [4] provides a sufficient frequency-domain condition for such plants to exhibit such tracking characteristics. However, in view of the computational complexities of frequency-domain positivity tests for the absolute-stability properties of Lur'e plants with multiple non-linearities, the explicit characterisation of the class of Lur'e plants with multiple non-linearities which are amenable to fast-sampling error-actuated digital control and associated inner-loop compensation is clearly of crucial importance. Such a characterisation is provided by the following fundamental result:

Theorem (Porter [3])

In the case of plants (1), (2), (3) for which the measurement matrix F is chosen such that the set of transmission zeros $Z_t(A,B,F) \subset C^-$ where C^- is the open left half-plane and the matrix FB is non-singular, and for which the non-singular controller matrices K_1 and K_2 are chosen such that

(i) the matrix K_1FB is positive diagonal

and

(ii) $Z(K_1,K_2) = \{\lambda \in C : \det(\lambda K_1+K_2)=0 \subset C^- \quad ,$

then the closed-loop control system (1), (2), (3), (10), (11), (12) exhibits state-bounded absolutely stable tracking on T_T over $S_d \times S_v \times S_f$ as $T \to 0$ where S_f is determined by any matrix sector $[\Gamma\ ,\ K] \subset (0_\ell, 2(K_1FB)^{-1})$.

It is evident that this theorem greatly simplifies the synthesis of high-performance closed-loop digital control systems incorporating Lur'e plants with multiple non-linearities which are amenable to fast-sampling error-actuated digital control and associated inner-loop compensation by obviating the necessity for applying frequency-domain positivity tests. Indeed, the design problem is reduced to that of choosing measurement and controller matrices F, K_1, and K_2 which in effect compensate for the rank defect of the first Markov parameter CB and the presence of any 'infinite' zeros in the set $Z_t(A,B,C)$ of transmission zeros due to this rank defect.

Furthermore, since S_f is determined by any matrix sector $[\Gamma, K] \subset (0_\ell, 2(K_1FB)^{-1})$ as $T \to 0$, admissible multiple non-linearities $f \in S_f$ of arbitrary severity are tolerable in such Lur'e plants by the incorporation of fast-sampling controllers with sufficiently small sampling periods and with controller matrices K_1 which determine sufficiently large positive diagonal sector boundary matrices $2(K_1FB)^{-1}$ provided that appropriate associated inner-loop compensators are also introduced. However, it is important to note that no choice of controller and measurement matrices exists such that $Z_t(A,B,F) \subset C^-$ in the case of nonminimum-phase plants for which $Z_t(A,B,C) \not\subset C^-$ since $Z_t(A,B,C) \subseteq Z_t(A,B,F)$ in the case of 'finite' zeros. Therefore, before embarking upon the synthesis of high-performance closed-loop digital control systems incorporating Lur'e plants with multiple non-linearities, it is clearly imperative to compute [5] the set $Z_t(A,B,C)$ of transmission zeros of the transfer function matrices of the uncompensated linear components of such plants.

3. Helicopter Digital Flight-Mode Control System

The linearised longitudinal dynamics of the CH-47 helicopter at an airspeed of 40 knots are governed on the continuous-time set $T = [0,+\infty)$ by state, output, and measurement equations of the respective forms [6][7]

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 , & 0 , & 0 , & 1 \\ -32.0 , & -0.02 , & 0.005 , & 2.4 \\ -30.0 , & -0.14 , & 0.44 , & -1.3 \\ 1.2 , & 0 , & 0.018 , & -1.6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 , & 0 \\ 0.14 , & -0.12 \\ 0.36 , & -8.6 \\ 0.3 , & 0.009 \end{bmatrix} \begin{bmatrix} f_1(u_1) \\ f_2(u_2) \end{bmatrix} + \begin{bmatrix} 0 , & 0 , & 0 \\ 1 , & 0 , & 0 \\ 0 , & 1 , & 0 \\ 0 , & 0 , & 1 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} , \qquad (15)$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 , & 0 , & 1 , & 0 \\ 1 , & 0 , & 0 , & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} , \qquad (16)$$

and

$$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 0 , & 0 , & 1 , & 0 \\ 1 , & 0 , & 0 , & 0.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} , \qquad (17)$$

where the disturbance vector $d = [d_1,d_2,d_3]^T \in S_d$, the vector actuator non-linearity $f = [f_1,f_2]^T \in S_f$, the functions f_1 and f_2 are respectively the gang-collective and differential-collective non-linearities, and the class S_f of admissible actuator non-linearities is determined by the matrices $\Gamma = \text{diag}\{\gamma_1,\gamma_2\}$ and $K = \text{diag}\{\kappa_1,\kappa_2\}$. The fast-sampling error-actuated digital controller governed on the discrete-time set $T_T = \{0,T,2T,...\}$ by equations (10), (11), and (12) is to be synthesized by a suitable choice of $T \in R^+$, $K_1 \in R^{2x2}$. amd $K_2 \in R^{2x2}$ such that the resulting closed-loop digital control system exhibits state-bounded absolutely stable tracking on T_T over $S_d \times S_v \times S_f$ where the command input vector $v = [v_1,v_2]^T \in S_f$. It is evident from equations (15), (16), and (17) that

$$CB = \begin{bmatrix} 0.3600 , & -8.600 \\ 0 , & 0 \end{bmatrix} \qquad (18)$$

is singular,

$$FB = \begin{bmatrix} 0.3600 , & -8.6000 \\ 0.1750 , & 0.0045 \end{bmatrix} \qquad (19)$$

is non-singular,

$$Z_t(A,B,C) = \{-0.018\} \subset C^- \qquad (20)$$

and that

$$Z_t(A,B,F) = \{-0.018,-2.000\} \subset C^- \quad . \qquad (21)$$

Furthermore, the non-singular controller matrices

$$K_1 = \begin{bmatrix} 0.0015 , & 2.8541 \\ -0.0581 , & 0.1195 \end{bmatrix} \qquad (22)$$

and

$$K_2 = \begin{bmatrix} 0.0060 , & 11.4163 \\ -0.2323 , & 0.4779 \end{bmatrix} \qquad (23)$$

are clearly such that

$$K_1FB = \begin{bmatrix} 0.5000 , & 0 \\ 0 , & 0.5000 \end{bmatrix} \qquad (24)$$

is positive diagonal and

$$Z(K_1,K_2) = \{-4.000,-4.000\} \subset C^- \quad . \qquad (25)$$

Hence, all the conditions of the theorem are satisfied and the closed-loop helicopter digital flight-mode control system therefore exhibits state-bounded absolutely stable tracking on T_T over $S_d \times S_v \times S_f$ as $T \to 0$ where S_f is determined by any matrix sector $[\Gamma, K] \subset (0_2, \mathrm{diag}\{4,4\})$.

In the case of the vector actuator non-linearity $f = [f_1, f_2]^T \in S_f$ where f_1 and f_2 are respectively the 'deadzone' gang-collective and differential-collective non-linearities $n_1(u_1)$ and $n_2(u_2)$ shown in Figs 1 and 2, the excellent tracking behaviour of the initially quiescent controlled helicopter when the sampling period $T = 0.01$ sec, $[d_1, d_2, d_3]^T = [1,1,1]^T$ and $[v_1, v_2]^T = [10 \text{ ft/sec}, 0 \text{ rad}]^T$ is shown in Fig 3. The similarly excellent tracking behaviour of the helicopter in the case of the vector actuator non-linearity $f = [f_1, f_2]^T \in S_f$, where f_1 and f_2 are respectively the 'deadzone' gang-collective and differential-collective non-linearities $n_2(u_1)$ and $n_1(u_2)$ shown in Figs 2 and 1, is shown in Fig 4. The corresponding tracking behaviour of the initially quiescent controlled helicopter when the sampling period $T = 0.01$ sec, $[d_1, d_2, d_3]^T = [1,1,1]^T$, but $[v_1, v_2]^T = [0 \text{ ft/sec}, 0.1 \text{ rad}]^T$ is shown in Figs 5 and 6.

It is apparent from Figs 3, 4, 5, and 6 that non-interacting control of the vertical velocity and pitch attitude is satisfactorily effected in each case. Indeed, since the closed-loop helicopter digital flight-mode control system exhibits state-bounded absolutely stable control on T_T over $S_d \times S_v \times S_f$, these manoeuvres will be effected for any vector actuator non-linearity $f \in S_f$.

4. Conclusion

The general results of Porter [3] have been illustrated by the design of a fast-sampling digital controller and associated transducers for the automatic control of the longitudinal motions of the CH-47 helicopter with both gang-collective and differential-collective non-linearities. In particular, it has been demonstrated that non-interacting control of the vertical velocity and pitch attitude of the helicopter is readily achievable for large classes of non-linear actuator characteristics such as 'deadzone' provided that the controller and transducer parameters are chosen so as to ensure that state-bounded absolutely stable tracking occurs.

References

1. B Porter, "Design of high-performance tracking systems", Technical Report AFWAL-TR-82-3032, US Air Force Wright Aeronautical Laboratories, Wright-Patterson AFB, Ohio, USA, July 1982.

2. B Porter, "Design of direct digital flight control systems for helicopters", Proc 26th Israel Annual Conference on Aviation and Astronautics, Tel-Aviv/Haifa, Israel, February 1984.

3. B Porter, "Design of tracking systems incorporating inner-loop compensators and fast-sampling error-actuated controllers for Lur'e plants with multiple non-linearities", Report USAME/DC/124/83, University of Salford, 1983.

4. Lj T Grujic and B Porter, "Discrete-time tracking systems incorporating Lur'e plants with multiple non-linearities", Int J Systems Sci, Vol 11, pp 1505-1520, 1980.

5. B Porter, "Computation of the zeros of linear multivariable systems", Int J Systems Sci, Vol 10, pp 1427-1432, 1979.

6. G Stein and J C Doyle, "Singular values and feedback: design examples", Proc 16th Allerton Conference on Communication, Control, and Computing, Urbana, USA, October 1978.

7. J C Doyle and G Stein, "Multivariable feedback design: concepts for a classical/modern synthesis", IEEE Trans Automatic Control, Vol AC-26, pp 4-16, 1981.

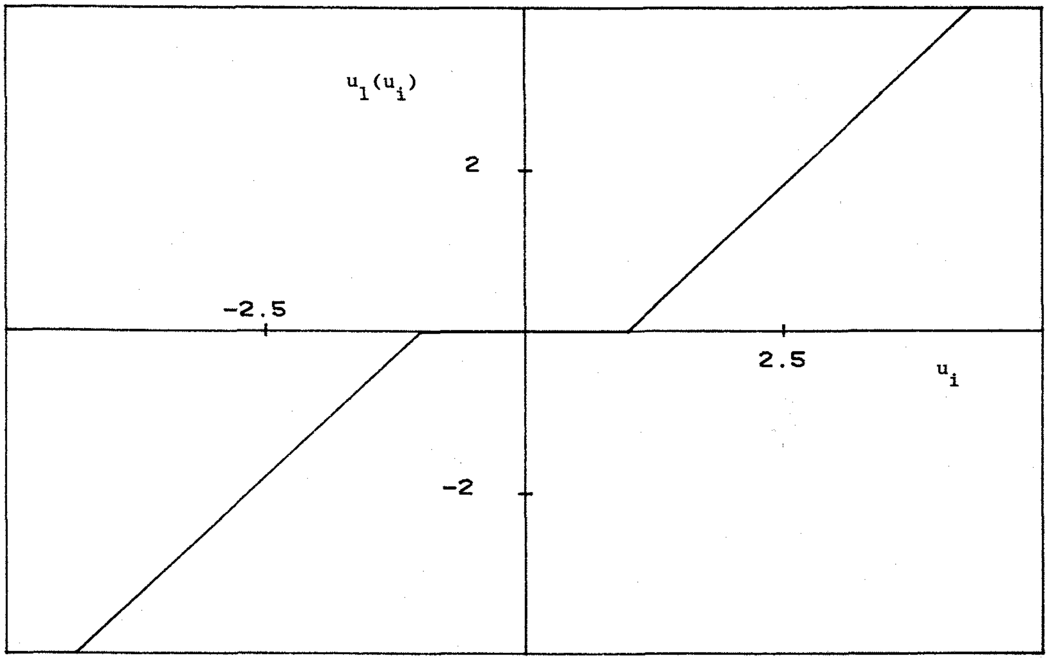

Figure 1

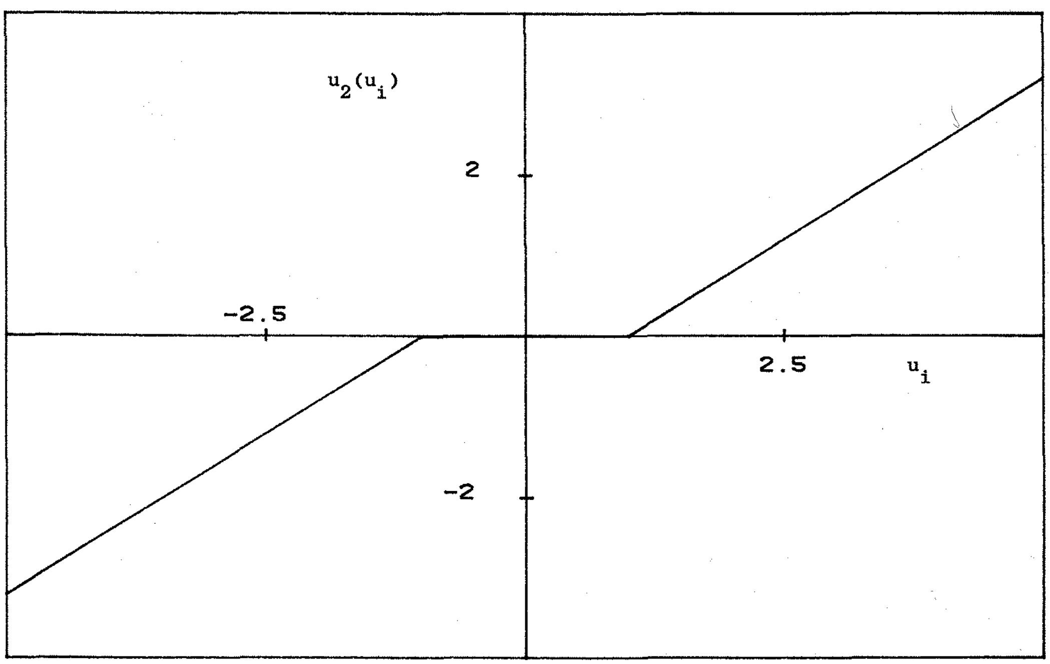

Figure 2

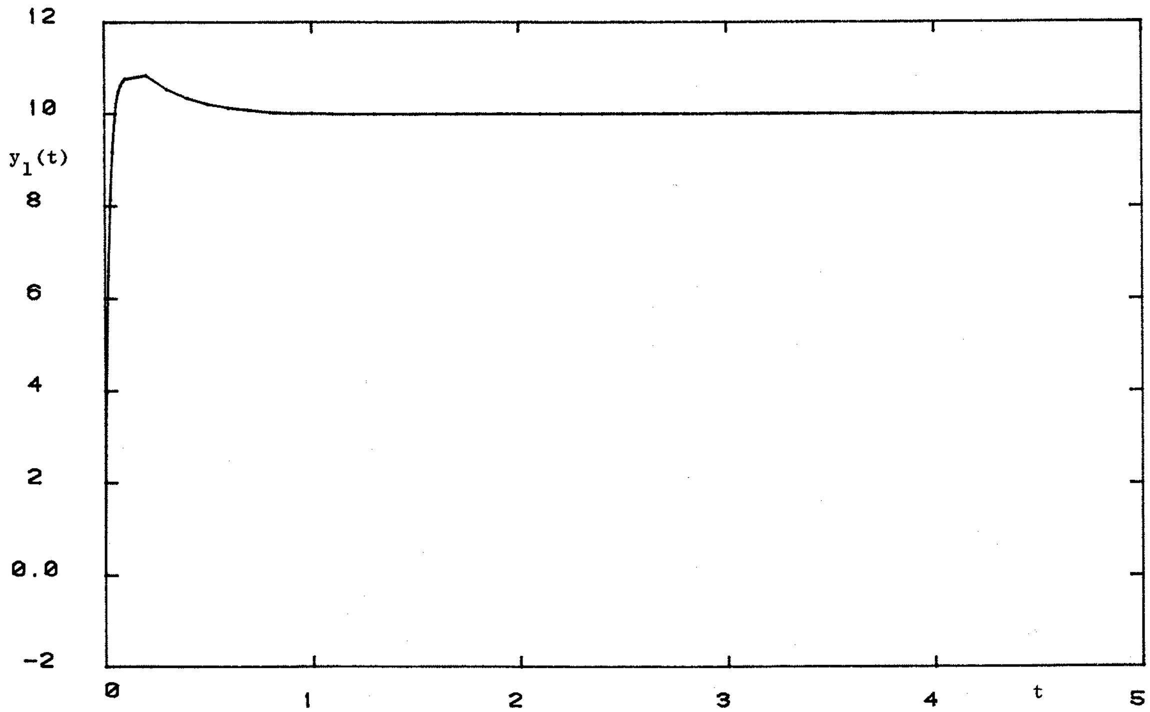

Figure 3(a)

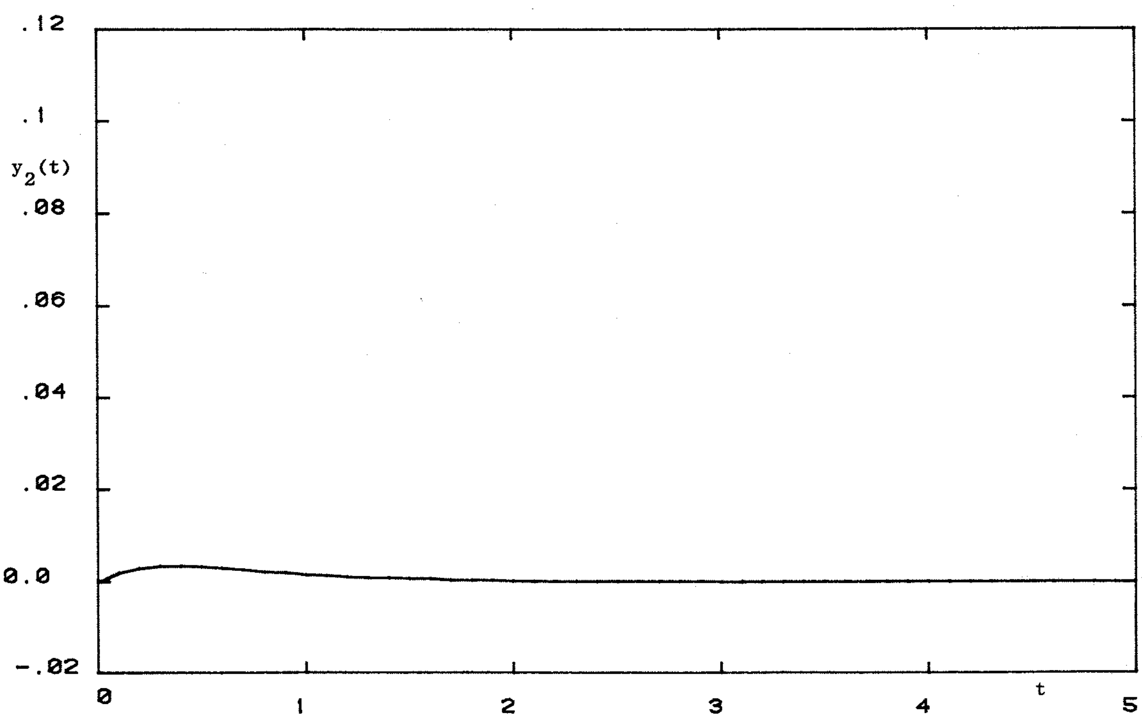

Figure 3(b)

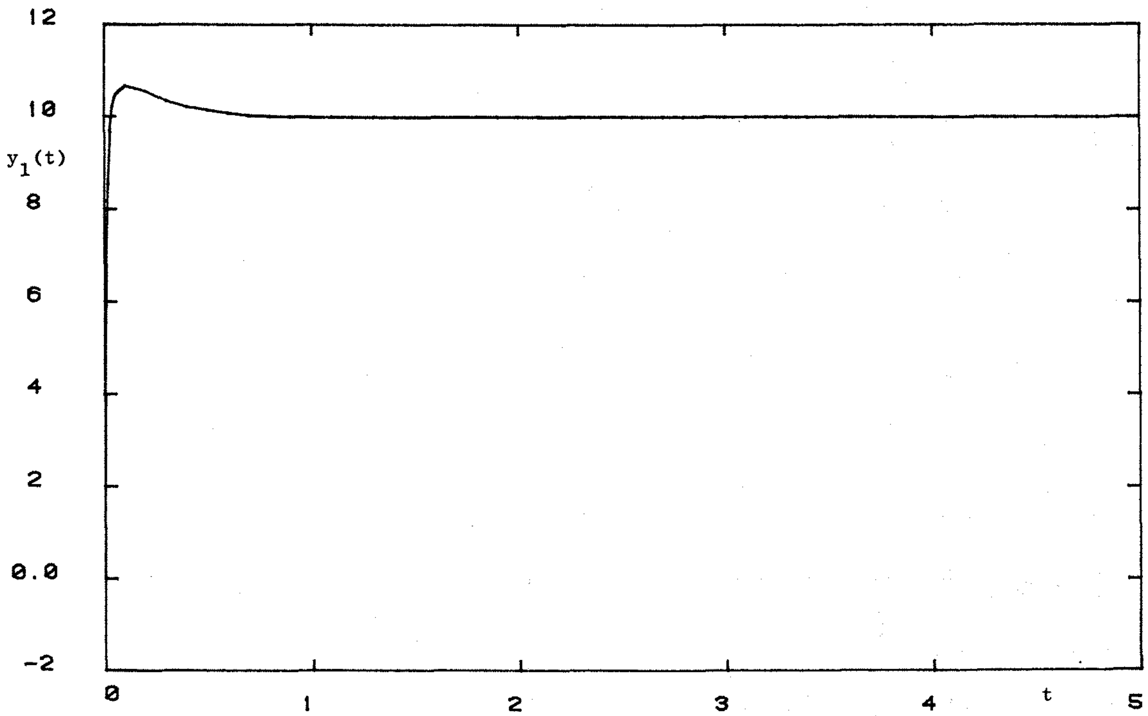

Figure 4(a)

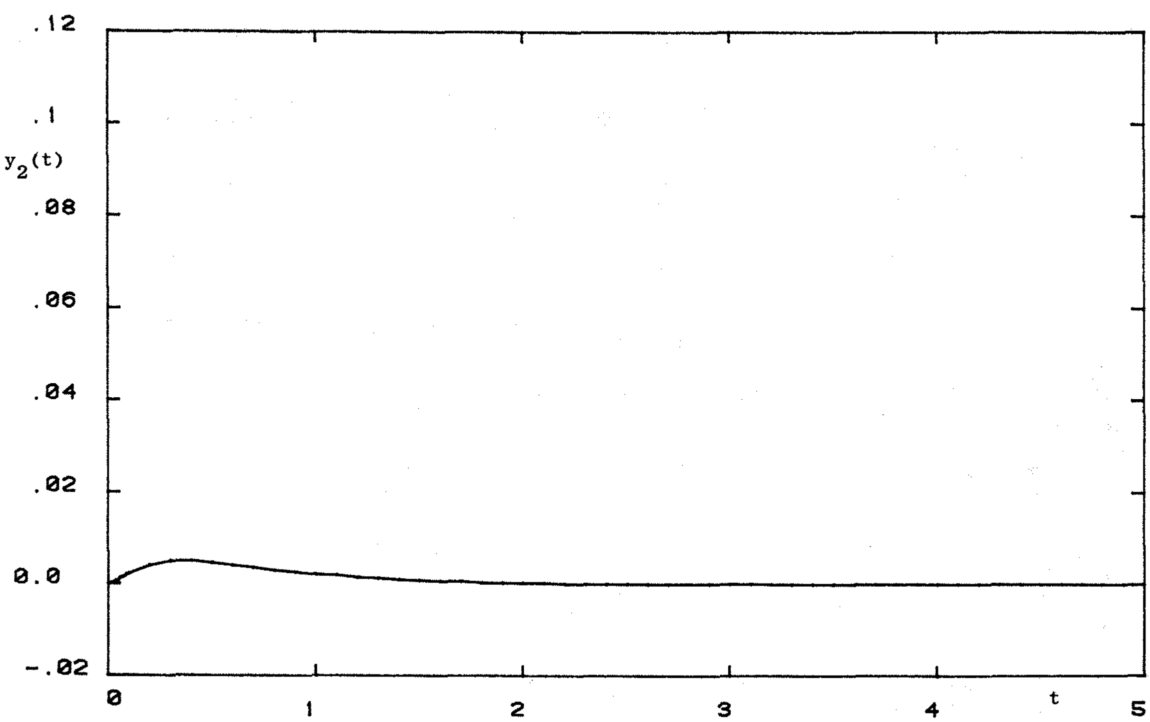

Figure 4(b)

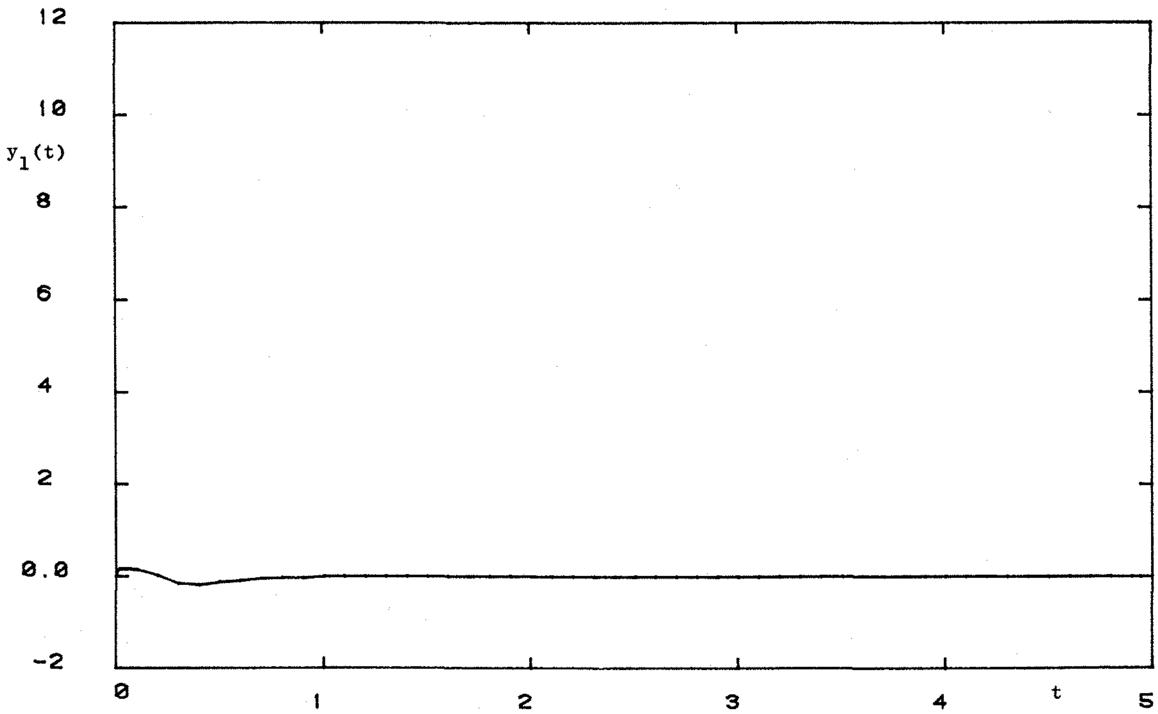

Figure 5(a)

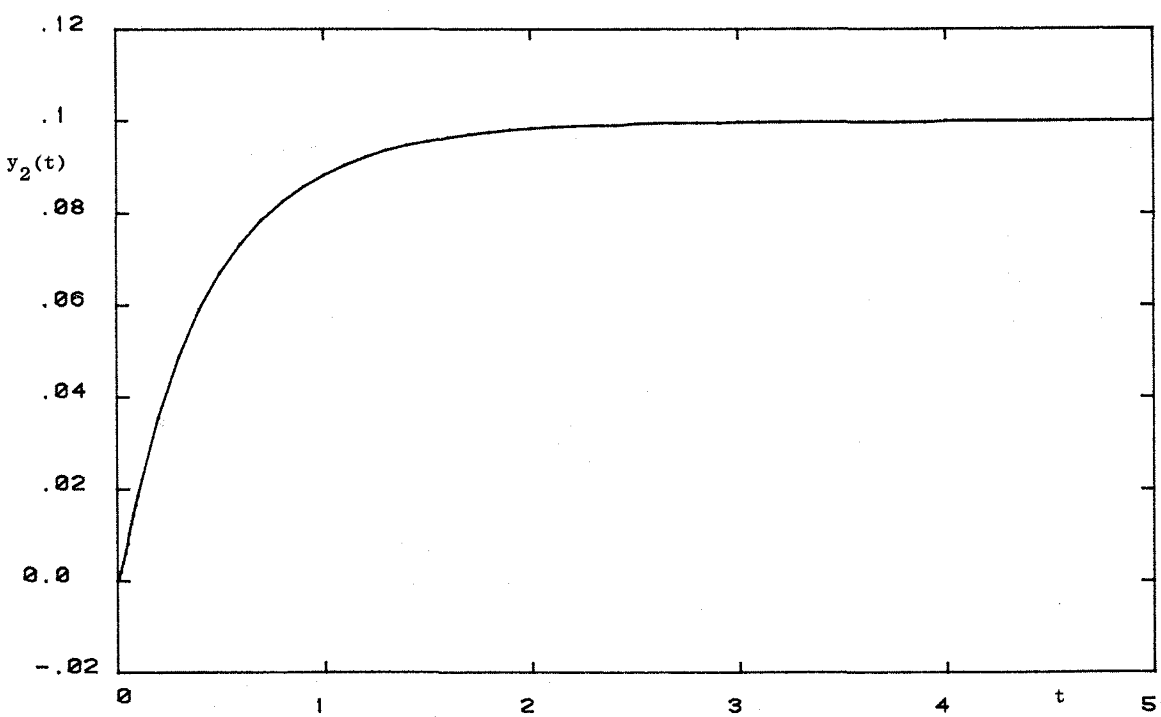

Figure 5(b)

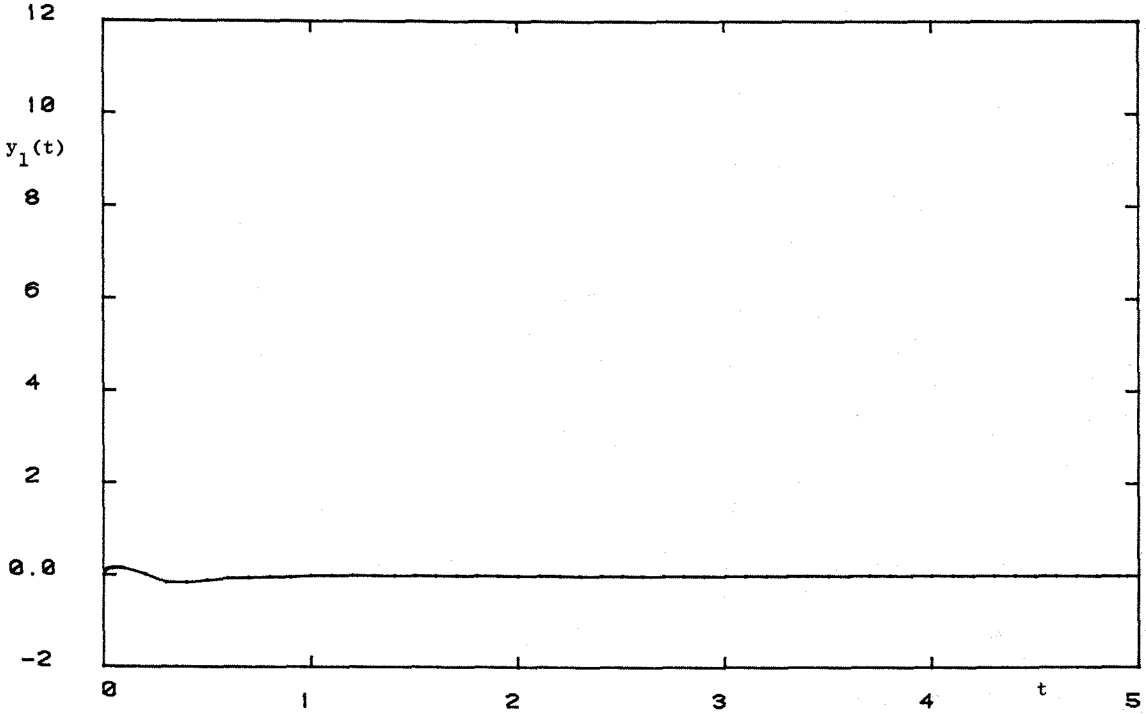

Figure 6(a)

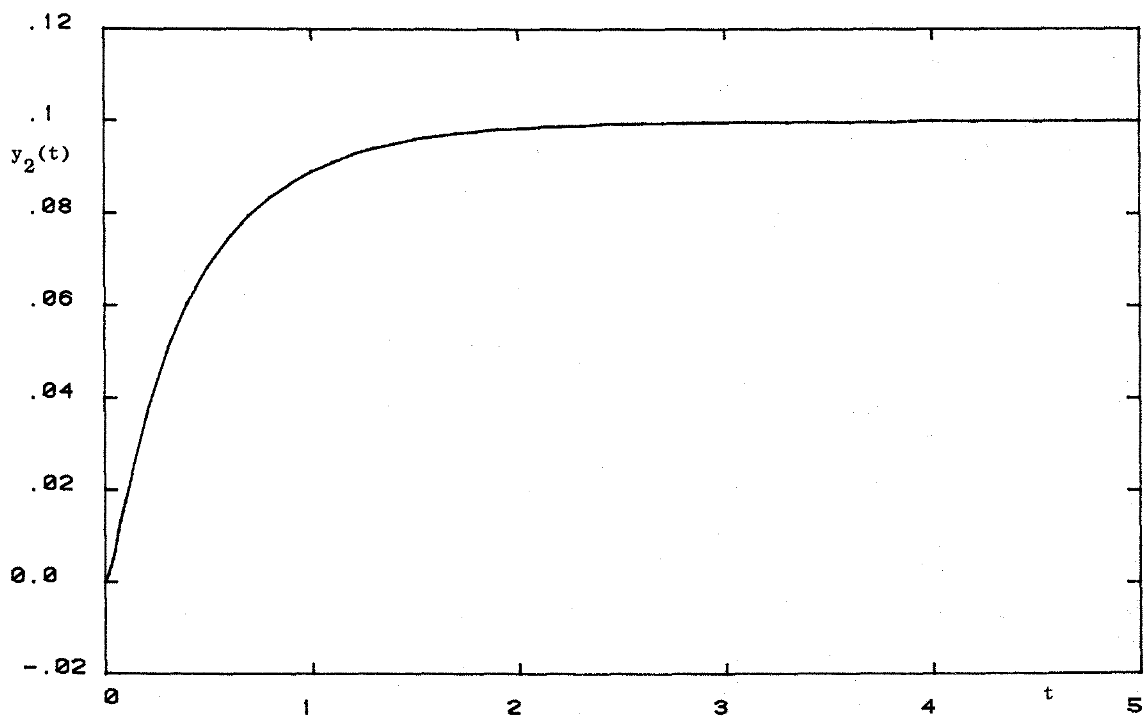

Figure 6(b)