UHSH1: AN EFFICIENT THREE DIMENSIONAL SHAPE OPTIMAL DESIGN PROGRAM

Liu Yingwei and Huang Jianmin
Nanchang Aircraft Company
Nanchang, People's Republic of China

ICAS-84-3.7.3

Abstract

In this paper an efficient design program for the elastic solids (UHSH1) is developed. The program can efficiently design complex three dimensional structure under multiple loading cases. There are three different objective functions which may be selected by users. They are minimum weight, minimum maximum stress, and weighted objective function. The program can deal with multiple constraints, such as static stress, displacement, the prohibited band of frequency and side constraints. A family of rectangular type of elements such as 8-noded isoparametric element, 20-noded isoparametric element and 9-19 variable noded isoparametric elements are used, in order to consider the distortion of the elements during the optimizing process, a triangular type of elements--15 noded isoparametric element is included. The efficiency of the program is enhanced by different ways, such as 1) by improving the efficiency of the FEM subroutines; 2) by deriving an analytical sensitivity technique for these elements; 3) by developing some approximate reanalysis techniques as well. The optimization techniques used here are with improved movelimit methods of sequential linear programming and sequential quadric programming. The constraint deletion techniques are involved also. The techniques for numerical shape representation are super curve technique and superposition of shape technique. Examples of the application of the program to a number of three-dimensional structures demonstrate its efficiency and accuracy.

Introduction

The shape optimum design is a new branch of optimal structural design. In 1973, Zienkewicz and Campbell presented the first paper[1] in this field. Several authors have investigated the problem since then, References 2--14 list some of the published works. All the research work in these publications was limited to two-dimensional problems. In 1982, M. Hasan Imam published the first paper [15] on three dimensional shape optimization. It investigated the fundamental problems associated with shape optimization of elastic solids. The basic concepts and techniques of numerical shape representation suitable for shape optimization are developed.

But the techniques developed in this paper were only demonstrated on simple cantilever beam problems. The structural member was only modeled by the 20-noded three- dimensional isoparametric element. Thus it could not threat the distortion of the elements during the optimizing process. The approximation methods were not developed for three dimensional isoparametric elements yet, the responses were evaluated every time by full finite element analysis and the derivatives were evaluated by finite difference using the results of finite element analysis. Therefore its efficiency was rather low.

In order to design some complex elastic solid aircraft comporents efficiently, we have developed three dimensional shape optimization Program UHSH1.

The program can design efficiently complex three dimensional structure under the multiple loading cases.

In this program, there are three different objective functions, which may be selected by users. They are:
Minimum weight;
Minimum maximum stress;
Weighted objective function;

The multiple constraints are stress constraints, displacment constraints, frequency-prohibited bands and the upper and lower bounds of each variable.

In order to avoid the distorsion of the elements during optimization, a triangular type of elements--15 noded isoparametric elements are included, besides rectangular type of elements such as 8-noded isoparametric elements, 20-noded isoparametric elements and 9-19 variable noded isoparametric elements.

To enhance the efficiency of the program, we have done the following research work:

1. Improve the methods of analysis to reduce the time taken by each finite element analysis. The analysis efficiency of the Program UHSH1 is evidently higher than that of the Program SAP5.
2. Enhance the efficiency of the sensitivity analysis by developing analytical sensitivity technique for the three dimensional isoparametric elements.
3. Reduce the number of the finite element analyses by developing an approximate reanalysis techniques for the three dimensional isoparametric elements.

In the Program UHSH1, there are three reanalysis techniques:

1) the sub-structuring technique
2) the combined Taylor series-Iterative technique
3) the reduced dimensional technique

In the Program, the following two efficient optimization techniques are inculded:

1) Sequential linear Programming technique with improved movelimit;
2) Sequential quadric programming technique with improved movelimit;

The constraint deletion technique is also included.

In the program, the techniques of numerical shape representation are super curve technique and superposition of shape technique, which were developed in Reference[15].

Some three dimensional shape optimization examples were solved by the program and the program has already been used in practical design.

Formulation

The shape optimization problems for three dimensional structures may be represented as variational problems with variable domain mathematically. Due to the complexity of the three dimensional structures in practical engineering, Finite Element Method is usually applied to solve these problems. Thus, the domain shape optimization problems are turned into the optimization boundary problems for the finite element models.

In the Program UHSH1, the techniques of numerical shape representation used are super curves and superposition of shapes. Thus, the design variables (shape variables) can be either the coordinates of finite element model nodes or the some parametric variables describing the shape of the three dimensional structures.

A three dimensional shape optimization problem can be formulated as:

To determine X

$$\text{Min } F(x) \tag{1}$$

$$\text{s.t.} \quad g_j(X) \leqslant 0, \quad j=1,2,\cdots\cdots m$$

where X is a (nx1) design vector.

The constraints here involved are stress constraints, displacment constraints under multiple loading cases, frequency-prohibited bands, and the upper and lower bounds of each variable. So, equation (1) may be expressed as follows:

To determine X

$$\text{Min } F(X) \tag{2}$$

$$\text{S.T.} \quad \sigma_{k,kp} \leqslant \sigma_b \qquad k=1,2.\cdots\cdots KZ(kp),\ kp=1,2,\cdots\cdots MP \tag{3}$$

$$U_{j,kp} \leqslant \overline{U}_{j,kp} \qquad j=1,2,\cdots\cdots J(kp),\ kp=1,2,\cdots\cdots MP \tag{4}$$

$$\omega_{i^*} \leqslant \underline{\omega} \tag{5}$$

$$\omega_{j^*} \geqslant \overline{\omega} \tag{6}$$

$$\underline{X}_i \leqslant X_i \leqslant \overline{X}_i \qquad i=1,2,\cdots\cdots n \tag{7}$$

$$\underset{\sim}{K}\,\underset{\sim}{U}=\underset{\sim}{P} \tag{8}$$

$$\underset{\sim}{K}\,\underset{\sim}{V}=\omega^2\,\underset{\sim}{M}\,\underset{\sim}{V} \tag{9}$$

$$j^*=i^*+1 \tag{10}$$

where $\underset{\sim}{K}$ is the structural stiffness matrix, $\underset{\sim}{M}$ is the structural mass matrix, $\underset{\sim}{U}$ is nodal displacement matrix of finite element model, $\underset{\sim}{V}$ is the natural mode,

ω is the natural angular frequency, $\omega^2_{i^*}$, $\omega^2_{j^*}$ is the ith and jth eigenvalues of eqution (9), $\underline{\omega}$, $\overline{\omega}$ is the lower and upper bounds of the frequency-prohibited band, $\sigma_{k,kp}$ is the kth controling point stress of three dimensional structures under kpth loading case. The stress is the principle stress σ_4:

$$\sigma_4=\sqrt{(\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_z-\sigma_x)^2+6(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2)} \tag{11}$$

$\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{zx}$ are the six stress components.

σ_b is the stress limit. $U_{j,kp}$ is the jth displacement under kpth loading case. $\overline{U}_{j,kp}$ is the jth displacement limit under kpth loading case.

X_i is the ith design variables, $\underline{X}_i$, $\overline{X}_i$ is the lower and upper bounds of X_i, n is the number of independent design variables. KZ(kp) is the number of stress controling points under the kpth loading case. J(kp) is the number of displacement controling points under the kpth loading case. Mp is the number of the loading cases.

The specification of the frequency prohibited band is accord to the demand of avoiding resonance. Once $\underline{\omega}$ and $\overline{\omega}$ has been determined, an experienced engineer is usually able to judge the orders i* and j* resonably. If the judgement is difficult under some cases, they may be deternined by the program also.

In the Program UHSH1, there are three different objective functions which may be selected by users.

(1) Minimum weight

The objective function is

$$F(x)=W(x) \tag{12}$$

W(x) is the weight of the three dimensional structure.

(2) Minimum maximum stress

The objective function is

$$F(x)=\max_{\substack{i=1,2,\cdots\cdots KZ(kp)\\ kp=1,2,\cdots\cdots Mp}} \sigma_{i,kp}(x) \tag{13}$$

It is easy to show that eqution (1) can be transformed into the following minimum problem:

To determine X

$$\text{Min } \beta$$

$$\text{S.T.} \quad \sigma_{i,kp} \leqslant \beta \qquad i=1,2,\cdots\cdots KZ(kp),\ kp=1,2,\cdots\cdots Mp$$

$$g_j'(x) \leqslant 0 \qquad j=1,2,\cdots\cdots m \tag{14}$$

where $g_j'(x) \leqslant 0$ $(j=1,2,;\cdots\cdots m)$ are displacement, weight, frequency and side constraints.

(3) Weight objective function

The objective function is

$$F(x)=c_1\frac{W}{W_o}+c_2\frac{\oint(\sigma-\sigma_a)^2 ds}{\oint(\sigma_o-\sigma_a)^2 ds} \tag{15}$$

Where W is the weight of three dimensional structure. Wo is the weight of the three dimensional structure at initial shape. $\oint(\sigma_o-\sigma_a)^2 ds$ is stress leveling term for the initial shape. σ is the maximum principal stress and σ_a is average stress at initial shape. σ_o corresponds to stresses at initial shape. The integration is carried out numerically using stresses at the controling points.

c_1, c_2 are the weighted coefficients, and

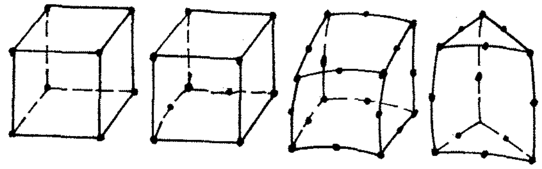
$$c_2=1-c_1 \tag{16}$$

If $c_1=c_2=0.5$, then equation (15) is transformd into a weighted objective function having equal weight-age for volume minimization and stress leveling, which was presented in reference(2).

Finite element analysis

In the program UHSH1, the stress, displacement and frequency analysis of the three dimensional structures is carried out by finite element analysis with three dimensional isoparametric elements.

In order to consider extreme distortion of the elements during the optimization, in the program, the triangular type of elements—15-noded isoparametric elements are included, besids rectangular type of elements such as 8-noded isoparametric elements, 20-noded isoparametric elements and 9-19 variable noded isoparametric elements. They are showed in Fig. 1.



a. 8-noded element b. 9-19 variable noded element c. 20-noded element d. 15-noded element

Fig. 1 Finite elements

The program UHSH1 is developed from the program FEM3 which is an efficient three dimensional finite analysis program. Its efficiency is higher than SAP5 evidently. For example, for a same problem, the computer time taken by the Program SAP5 is about three times over that of the Program FEM3.

Therefore, the Program UHSH1 has high efficiency for behavior analysis also, so that this reduces the time taken by each finite element analysis during the optimization.

Besids, the stress smoothing technique is used in the program, thus the accuracy of the stress at the nodes of the elements is high.

The sensitivity analysis

For the three dimensional shape optimization, the efficiency of the sensitivity analysis is of great importance. In order to enhance the efficiency of the sensitivity analysis in the Program UHSH1, an analytical sensitivity technique for the three dimensional isoparametric elements is developed.

1. Displacement sensitivity analysis

For the displacement sensitivity analysis, both pseudo-load technique and virtual load technique are used in the Program UHSH1.

(1) Pseudo-load technique

The displacement derivative is formulated as following by the pseudo-load technique:

$$\left\{\frac{\partial U}{\partial Ak}\right\} = [K]^{-1}\left(\frac{\partial \{P\}}{\partial Ak} - \sum_{i \in k} \frac{\partial [k]_i^e}{\partial Ak}\{U\}_i^e\right) \qquad (17)$$

where Ak is kth design variable.

$i \in k$ is ith element associated with kth design variables.

$[k]_i^e$ is ith element stiffness matrix.

$\{U\}_i^e$ is the nodal displacement associated with ith element.

(2) Virtual load technique

The displacement derivative is formulated as following by the virtual load technique:

$$\frac{\partial U_j}{\partial Ak} = \{V\}_j^e\left(\frac{\partial\{P\}}{\partial Ak} - \sum_{i \in k}\frac{\partial [k]_i^e}{\partial Ak}\{U\}_i^e\right) \qquad (18)$$

where U_j is the jth displacement component, $\{V\}_j$ is the virtual displacement vector due to the virtual load $\{q\}_j$ which is associated with U_j. $\{V\}_j$ can be written as:

$$\{V\}_j = [K]^{-1}\{q\}_j \qquad (19)$$

The virtual load technique is recommended to use when MQ is less than Mpxn, otherwise pseudo-load technique should be used. MQ is the number of displacement constraints, Mp is the number of the loading cases and n is the number of design variables.

2. The Derivation of the Element Stiffness matrix

The three dimensional isoparametric element stiffness matrix can be written as:

$$[k]^e = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} [B]^T[D][B]|J|\, d\xi\, d\eta\, d\zeta \qquad (20)$$

where $[B]$ is the strain matrix. It can be written as:

$$[B] = [\,[B]_1,\ [B]_2, \cdots\cdots [B]_N\,]_{6\times 3N} \qquad (21)$$

$$[B]_i^T = \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 & 0 & \frac{\partial N_i}{\partial y} & 0 & \frac{\partial N_i}{\partial z} \\ 0 & \frac{\partial N_i}{\partial y} & 0 & \frac{\partial N_i}{\partial x} & \frac{\partial N_i}{\partial z} & 0 \\ 0 & 0 & \frac{\partial N_i}{\partial z} & 0 & \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} \end{bmatrix}_{3\times 6} \qquad (22)$$

N is the node number of a three dimemisional isoparametric element, $N_i (i=1,2,\cdots\cdots N)$ are the shape functions.

$$\begin{Bmatrix} \frac{\partial N_i}{\partial x} \\ \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial z} \end{Bmatrix} = [J]^{-1} \begin{Bmatrix} \frac{\partial N_i}{\partial \xi} \\ \frac{\partial N_i}{\partial \eta} \\ \frac{\partial N_i}{\partial \zeta} \end{Bmatrix} \qquad (23)$$

where $[J]$ is the Jacobian matrix, it can be written as:

$$[J] = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix} \qquad (24)$$

in which

$$J_{11} = \sum_{i=1}^{N} \frac{\partial N_i}{\partial \xi} x_i,\quad J_{12} = \sum_{i=1}^{N} \frac{\partial N_i}{\partial \xi} y_i,\quad J_{13} = \sum_{i=1}^{N} \frac{\partial N_i}{\partial \xi} z_i$$

$$J_{21} = \sum_{i=1}^{N} \frac{\partial N_i}{\partial \eta} x_i,\quad J_{22} = \sum_{i=1}^{N} \frac{\partial N_i}{\partial \eta} y_i,\quad J_{23} = \sum_{i=1}^{N} \frac{\partial N_i}{\partial \eta} z_i$$

$$J_{31} = \sum_{i=1}^{N} \frac{\partial N_i}{\partial \zeta} x_i,\quad J_{32} = \sum_{i=1}^{N} \frac{\partial N_i}{\partial \zeta} y_i,\quad J_{33} = \sum_{i=1}^{N} \frac{\partial N_i}{\partial \zeta} z_i \qquad (25)$$

$$[D] = \frac{E(1-\mu)}{(1+\mu)(1-2\mu)} \begin{bmatrix} 1 & \frac{\mu}{1-\mu} & \frac{\mu}{1-\mu} & & & \\ \frac{\mu}{1-\mu} & 1 & \frac{\mu}{1-\mu} & & & \\ \frac{\mu}{1-\mu} & \frac{\mu}{1-\mu} & 1 & & & \\ & & & \frac{1-2\mu}{2(1-\mu)} & & \\ & & & & \frac{1-2\mu}{2(1-\mu)} & \\ & & & & & \frac{1-2\mu}{2(1-\mu)} \end{bmatrix} \qquad (26)$$

$|J|$ is Jacobian matrix determinant, it can be written as:

$$|J| = \begin{vmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{vmatrix} = J_{11}(J_{22}J_{33}-J_{23}J_{32})+J_{21}(J_{32}J_{13}-J_{33}J_{12})+J_{31}(J_{23}J_{12}-J_{13}J_{22}) \quad (27)$$

Differentiating eqn (20) with respect to Ak gives

$$\frac{\partial [k]^e}{\partial A_k} = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} \frac{\partial [B]^T}{\partial A_k}[D][B]|J|\,d\xi\, d\eta\, d\zeta + \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}[B]^T[D]\frac{\partial [B]}{\partial A_k}|J|\,d\xi\, d\eta\, d\zeta + \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}[B]^T[D][B]\frac{\partial |J|}{\partial A_k}d\xi\, d\eta\, d\zeta \quad (28)$$

Using the notations

$$\frac{\partial [k_1]^e}{\partial A_k} = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}\frac{\partial [B]^T}{\partial A_k}[D][B]|J|\,d\xi\, d\eta\, d\zeta$$

$$\frac{\partial [k_2]^e}{\partial A_k} = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}[B]^T[D][B]\frac{\partial |J|}{\partial A_k}d\xi\, d\eta\, d\zeta \quad (29)$$

We get

$$\frac{\partial [k]^e}{\partial A_k} = \left[\frac{\partial k_1}{\partial A_k}\right] + \left[\frac{\partial k_1}{\partial A_k}\right]^T + \left[\frac{\partial k_2}{\partial A_k}\right] \quad (30)$$

where

$$\frac{\partial [B]}{\partial A_k} = \left[\frac{\partial [B]_1}{\partial A_k}, \frac{\partial [B]_2}{\partial A_k}, \cdots\cdots \frac{\partial [B]_N}{\partial A_k}\right]_{6\times 3N} \quad (31)$$

To obtain $\frac{\partial [B]_l}{\partial A_k}$ $(l=1,2,\cdots\cdots N)$, we differentiate both sides of eqn (22):

$$\frac{\partial [B]_l}{\partial A_k} = \frac{\partial}{\partial A_k}\begin{bmatrix} \frac{\partial N_l}{\partial x} & 0 & 0 \\ 0 & \frac{\partial N_l}{\partial y} & 0 \\ 0 & 0 & \frac{\partial N_l}{\partial z} \\ \frac{\partial N_l}{\partial y} & \frac{\partial N_l}{\partial x} & 0 \\ 0 & \frac{\partial N_l}{\partial z} & \frac{\partial N_l}{\partial y} \\ \frac{\partial N_l}{\partial z} & 0 & \frac{\partial N_l}{\partial x} \end{bmatrix} \quad \begin{pmatrix} l=1,2,\cdots\cdots N \\ k=1,2,\cdots\cdots n \end{pmatrix} \quad (32)$$

From eqn (23), we obtain

$$\frac{\partial}{\partial A_k}\begin{bmatrix} \frac{\partial N_l}{\partial x} \\ \frac{\partial N_l}{\partial y} \\ \frac{\partial N_l}{\partial z} \end{bmatrix} = \frac{\partial [J]^{-1}}{\partial A_k}\begin{bmatrix} \frac{\partial N_l}{\partial \xi} \\ \frac{\partial N_l}{\partial \eta} \\ \frac{\partial N_l}{\partial \zeta} \end{bmatrix} \quad (33)$$

where

$$\frac{\partial [J]^{-1}}{\partial A_k} = -[J]^{-1}\frac{\partial [J]}{\partial A_k}[J]^{-1} \quad (34)$$

From eqns (33), (34), (23), we obtain

$$\frac{\partial}{\partial A_k}\begin{bmatrix} \frac{\partial N_l}{\partial x} \\ \frac{\partial N_l}{\partial y} \\ \frac{\partial N_l}{\partial z} \end{bmatrix} = -\left[[J]^{-1}\frac{\partial [J]}{\partial A_k}\right]\begin{bmatrix} \frac{\partial N_l}{\partial x} \\ \frac{\partial N_l}{\partial y} \\ \frac{\partial N_l}{\partial z} \end{bmatrix} \quad (35)$$

From eqn (24), we obtain

$$\frac{\partial [J]}{\partial A_k} = \begin{bmatrix} \frac{\partial J_{11}}{\partial A_k} & \frac{\partial J_{12}}{\partial A_k} & \frac{\partial J_{13}}{\partial A_k} \\ \frac{\partial J_{21}}{\partial A_k} & \frac{\partial J_{22}}{\partial A_k} & \frac{\partial J_{23}}{\partial A_k} \\ \frac{\partial J_{31}}{\partial A_k} & \frac{\partial J_{32}}{\partial A_k} & \frac{\partial J_{33}}{\partial A_k} \end{bmatrix} \quad (36)$$

From eqn (25), we obtain:

$$\begin{aligned} &\frac{\partial J_{11}}{\partial A_k} = \sum_{i=1}^{N}\frac{\partial N_i}{\partial \xi}\frac{\partial x_i}{\partial A_k}, \quad \frac{\partial J_{12}}{\partial A_k} = \sum_{i=1}^{N}\frac{\partial N_i}{\partial \xi}\frac{\partial y_i}{\partial A_k}, \quad \frac{\partial J_{13}}{\partial A_k} = \sum_{i=1}^{N}\frac{\partial N_i}{\partial \xi}\frac{\partial z_i}{\partial A_k} \\ &\frac{\partial J_{21}}{\partial A_k} = \sum_{i=1}^{N}\frac{\partial N_i}{\partial \eta}\frac{\partial x_i}{\partial A_k}, \quad \frac{\partial J_{22}}{\partial A_k} = \sum_{i=1}^{N}\frac{\partial N_i}{\partial \eta}\frac{\partial y_i}{\partial A_k}, \quad \frac{\partial J_{23}}{\partial A_k} = \sum_{i=1}^{N}\frac{\partial N_i}{\partial \eta}\frac{\partial z_i}{\partial A_k} \\ &\frac{\partial J_{31}}{\partial A_k} = \sum_{i=1}^{N}\frac{\partial N_i}{\partial \zeta}\frac{\partial x_i}{\partial A_k}, \quad \frac{\partial J_{32}}{\partial A_k} = \sum_{i=1}^{N}\frac{\partial N_i}{\partial \zeta}\frac{\partial y_i}{\partial A_k}, \quad \frac{\partial J_{33}}{\partial A_k} = \sum_{i=1}^{N}\frac{\partial N_i}{\partial \zeta}\frac{\partial z_i}{\partial A_k} \end{aligned} \quad (37)$$

where $\frac{\partial x_i}{\partial A_k}, \frac{\partial y_i}{\partial A_k}, \frac{\partial z_i}{\partial A_k}$ can be given by the shape algorithm easily.

To obtain $\frac{\partial |J|}{\partial Ak}$, we differentiate both sides of eqn (27):

$$\begin{aligned} \frac{\partial |J|}{\partial Ak} = {} & \frac{\partial J_{11}}{\partial A_k}(J_{22}J_{33}-J_{23}J_{32}) + \frac{\partial J_{12}}{\partial A_k}(J_{31}J_{23}-J_{21}J_{33}) + \frac{\partial J_{13}}{\partial A_k}(J_{21}J_{32}-J_{31}J_{22}) + \frac{\partial J_{21}}{\partial A_k}(J_{32}J_{13}-J_{33}J_{12}) + \frac{\partial J_{22}}{\partial A_k}(J_{11}J_{33}-J_{31}J_{13}) \\ & + \frac{\partial J_{23}}{\partial A_k}(J_{31}J_{12}-J_{11}J_{32}) + \frac{\partial J_{31}}{\partial A_k}(J_{23}J_{12}-J_{13}J_{22}) + \frac{\partial J_{32}}{\partial A_k}(J_{21}J_{13}-J_{11}J_{23}) + \frac{\partial J_{33}}{\partial A_k}(J_{11}J_{22}-J_{21}J_{12}) \end{aligned} \quad (38)$$

3. Stress sensitivity analysis

To obtain $\frac{\partial \sigma_4}{\partial A_k}$, we differentiate both sides of eqn (11):

$$\begin{aligned} \frac{\partial \sigma_{4i,KP}}{\partial A_k} = {} & \Big(\Big(\sigma_{x_{i,KP}}-\sigma_{y_{i,KP}}\Big)\Big(\frac{\partial \sigma_{x_{i,KP}}}{\partial A_k}-\frac{\partial \sigma_{y_{i,KP}}}{\partial A_k}\Big) + \Big(\sigma_{y_{i,KP}}-\sigma_{z_{i,KP}}\Big)\Big(\frac{\partial \sigma_{y_{i,KP}}}{\partial A_k}-\frac{\partial \sigma_{z_{i,KP}}}{\partial A_k}\Big) \\ & + \Big(\sigma_{z_{i,KP}}-\sigma_{x_{i,KP}}\Big)\Big(\frac{\partial \sigma_{z_{i,KP}}}{\partial A_k}-\frac{\partial \sigma_{x_{i,KP}}}{\partial A_k}\Big) + 6\Big(\tau_{xy_{i,KP}}\frac{\partial \tau_{xy_{i,KP}}}{\partial A_k} + \tau_{yz_{i,KP}}\frac{\partial \tau_{yz_{i,KP}}}{\partial A_k} + \tau_{zx_{i,KP}}\frac{\partial \tau_{zx_{i,KP}}}{\partial A_k}\Big)\Big)\Big/2\Big/\sigma_{4_{i,KP}} \quad \begin{pmatrix} k=1,2,\cdots n \\ i=1,2,\cdots\cdots K\xi(KP) \\ KP=1,2,\cdots\cdots MP \end{pmatrix} \end{aligned} \quad (39)$$

where $\sigma_{4_{i,KP}}$ is the ith principle stress under the kpth loading case.

$\sigma_{x_{i,KP}}, \sigma_{y_{i,KP}}, \sigma_{z_{i,KP}}, \tau_{xy_{i,KP}}, \tau_{yz_{i,KP}}, \tau_{zx_{i,KP}}$ are six stress components at the ith node of the element under the kpth loading case.

They are given by the following equations:

$$\begin{aligned} &\sigma_{x_{i,KP}} = \Big(\sum_{j=1}^{IGB_i}\tilde{\sigma}^{e}_{x_{j,KP}}\Big)\Big/IGB_i, \quad \sigma_{y_{i,KP}} = \Big(\sum_{j=1}^{IGB_i}\tilde{\sigma}^{e}_{y_{j,KP}}\Big)\Big/IGB_i \\ &\sigma_{z_{i,KP}} = \Big(\sum_{j=1}^{IGB_i}\tilde{\sigma}^{e}_{z_{j,KP}}\Big)\Big/IGB_i, \quad \tau_{xy_{i,KP}} = \Big(\sum_{j=1}^{IGB_i}\tilde{\tau}^{e}_{xy_{j,KP}}\Big)\Big/IGB_i \\ &\tau_{yz_{i,KP}} = \Big(\sum_{j=1}^{IGB_i}\tilde{\tau}^{e}_{yz_{j,KP}}\Big)\Big/IGB_i, \quad \tau_{zx_{i,KP}} = \Big(\sum_{j=1}^{IGB_i}\tilde{\tau}^{e}_{zx_{j,KP}}\Big)\Big/IGB_i \end{aligned} \quad (40)$$

where IGB_i is the number of the elements associated with ith node.

$\tilde{\sigma}_{x_{j,KP}}, \tilde{\sigma}_{y_{j,KP}}, \cdots\cdots \tilde{\tau}_{xz_{j,KP}}$ are the stress components smoothed of the jth element at ith node under kpth loading case.

The smoothing stress components $\tilde{\sigma}_{lpj}$ $(l=1,2,\cdots\cdots 8,\ p=1,2,\cdots\cdots 6)$ at 8 corners of the jth hexahadron type of elements can be given by the following equations:

$$\{\tilde{\sigma}\} = [A]\{\sigma\} \qquad (41)$$

in which

$$[A] = \begin{bmatrix} a & b & c & b & b & c & d & c \\ b & a & b & c & c & b & c & d \\ c & b & a & b & d & c & b & c \\ b & c & b & a & c & d & c & b \\ b & c & d & c & a & b & c & b \\ c & b & c & d & b & a & b & c \\ d & c & b & c & c & b & a & b \\ c & d & c & b & b & c & b & a \end{bmatrix} \qquad (41a)$$

$$\text{where } a=\frac{5+3\sqrt{3}}{4}, \quad b=\frac{-\sqrt{3}+1}{4}, \quad c=\frac{\sqrt{3}-1}{4} \qquad (42)$$

$\{\tilde{\sigma}\}^T=[\tilde{\sigma}^e_{1pj}, \tilde{\sigma}^e_{2pj}, \qquad \tilde{\sigma}^e_{8pj}]$

$\{\sigma\}^T=\{\sigma^e_{1j}, \sigma^e_{2j}, \cdots\cdots \sigma^e_{8j}$ are the stresses at Gauss points of integration for jth element.

To obtain $\frac{\partial\sigma_{xikp}}{\partial Ak}, \cdots\cdots \frac{\partial\tau_{zxikp}}{\partial Ak}$, we differentiate both sides of eqn (40):

$$\frac{\partial\sigma_{x_{i,kp}}}{\partial A_k}=\left(\sum_{j=1}^{IGB_i}\frac{\partial\tilde{\sigma}^e_{x_{j,kp}}}{\partial A_k}\right)\Big/IGB_i, \quad \frac{\partial\sigma_{y_{i,kp}}}{\partial A_k}=\left(\sum_{j=1}^{IGB_i}\frac{\partial\tilde{\sigma}^e_{y_{j,kp}}}{\partial A_k}\right)\Big/IGB_i$$

$$\frac{\partial\sigma_{z_{i,kp}}}{\partial A_k}=\left(\sum_{j=1}^{IGB_i}\frac{\partial\tilde{\sigma}^e_{z_{j,kp}}}{\partial A_k}\right)\Big/IGB_i, \quad \frac{\partial\tau_{xy_{i,kp}}}{\partial A_k}=\left(\sum_{j=1}^{IGB_i}\frac{\partial\tilde{\tau}_{xy_{j,kp}}}{\partial A_k}\right)\Big/IGB_i$$

$$\frac{\partial\tau_{yz_{i,kp}}}{\partial A_k}=\left(\sum_{j=1}^{IGB_i}\frac{\partial\tilde{\tau}^e_{yz_{j,kp}}}{\partial A_k}\right)\Big/IGB_i, \quad \frac{\partial\tau_{zx_{i,kp}}}{\partial A_k}=\left(\sum_{j=1}^{IGB_i}\frac{\partial\tilde{\tau}^e_{zx_{j,kp}}}{\partial A_k}\right)\Big/IGB_i \qquad (43)$$

From eqn (43), we obtain

$$\frac{\partial\{\tilde{\sigma}\}}{\partial Ak} = [A]\,\frac{\partial\{\sigma\}}{\partial Ak} \qquad (44)$$

$$\text{where } \frac{\partial\{\tilde{\sigma}\}^T}{\partial Ak} = \left[\frac{\partial\tilde{\sigma}^e_{1pj}}{\partial Ak}, \quad \frac{\partial\tilde{\sigma}^e_{2pj}}{\partial Ak} \cdots\cdots, \frac{\partial\tilde{\sigma}^e_{8pj}}{Ak}\right]^T$$

$$\frac{\partial\{\sigma\}^T}{\partial Ak} = \left[\frac{\partial\sigma^e_{1j}}{\partial Ak}, \quad \frac{\partial\sigma^e_{2j}}{\partial Ak}, \cdots\cdots, \frac{\partial\sigma^e_{8j}}{\partial Ak}\right]^T$$

From the relation of stress-strain, the stress components at any point for the element can be given by the following equation:

$$\{\sigma\}_{6x1} = [S]^e \quad \{U\}^e \qquad (45)$$

where $\{\sigma\}=[\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{zx}]^T$

$\{u\}^e$ is the node displacment vector of the element.

$[S]^e$ is the stress matrix of the element, it can be written:

$$[S]^e_{6x3N} = [D]_{6x6} \quad [B]^e_{6x3N} \qquad (46)$$

Thus, From eqns (45), (46), we obtain

$$\frac{\partial\{\sigma\}}{\partial Ak} = [D]\left(\frac{\partial[B]^e}{Ak}\{U\}^e + [B]^e\,\frac{\partial\{U\}^e}{\partial Ak}\right) \qquad (47)$$

where

$\frac{\partial[B]^e}{\partial Ak}$ can be giben from eqns (31)---(37)

$\frac{\partial\{U\}^e}{\partial Ak}$ can be given from eqn (17)

4. Frequency sensitivity analysis

From eqn (9), we can obtain

$$\frac{\partial\omega_i^2}{\partial A_k} = \frac{v_i^T\frac{\partial(K)}{\partial A_k}v_i - \omega_i^2 v_i^T\frac{\partial(M)}{\partial A_k}v_i}{v_i^T(M)\,v_i} \qquad (48)$$

$$\text{where } \frac{\partial(K)}{\partial Ak}V_i=\sum_{j\in k}\frac{\partial[K]^e_j}{\partial Ak}V^e_i \qquad (49)$$

$$\frac{\partial(M)}{\partial A_k}v_i=\sum_{j\in k}\frac{\partial(M)_j}{\partial A_k}v^e_i \qquad (50)$$

$\frac{\partial[K]^e_j}{\partial Ak}$ can be given from eqns (28)--(38).

$[M]^e_j$ is the lump mass matrix of jth element. It can be expressed as

$$[M]^e_j = m^e_j\begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & \ddots & & \\ & & & & 1 \end{bmatrix}_{3Nx3N} \qquad (51)$$

where N is the node number of the jth element.

$$m^e_j = \frac{M^e_j}{N} \qquad (52)$$

M^e_j is the mass of the jth element. It can be expressed as:

$$M^e_j=\rho V^e_j=\rho\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}|J|d\xi d\eta d\zeta \qquad (53)$$

From eqns (50)--(53), we can obtain

$$\frac{\partial(M)}{\partial Ak}V_i = \sum_{j\in k}\frac{\partial m^e_j}{\partial Ak}V_i \qquad (54)$$

$$\text{and } \frac{\partial m^e_j}{\partial A_k}=\frac{1}{N}\rho\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}\frac{\partial|J|}{\partial A_k}d\xi d\eta d\zeta \qquad (55)$$

where ρ is the density.

$\frac{\partial|J|}{\partial Ak}$ can be obtained from eqn (38).

Optimization technique

In order to enhance the efficiency of the program UHSH1, the constraint deletion technique is included.

Besides, there are two kinds of different optimization techniques suitable for the different shape optimization problem are adopted. These are

(1) Sequential linear programming technique improved movelimit.
(2) Sequential quadric programming technique with improved movelimit.

1. Sequential linear programming technique with improved movelimit

At the current trial design $X^{(k)}$, using the first approximation of the Taylor expansion for the objective function and the surviving constraints after deleting redundant ant unimportant constraints, we can obtain the following linear program problem:

To determine $X_i \; (= X_i^{(k+1)})$

$$\text{Min} \quad \tilde{F}(x) = F(x^{(k)})+\sum_{i=1}^{n}\left(\frac{\partial F}{\partial x_i}\right)^{(k)}(x_i-x_i^{(k)})$$

$$\text{S.T.} \quad \tilde{G}_r(x)=G_r(x^{(k)})+\sum_{i=1}^{n}\left(\frac{\partial G_r}{\partial x_i}\right)^{(k)}(x_i-x_i^{(k)})\leqslant\bar{G}_r$$

$$(r=1,2,\cdots\cdots mr)$$

$$\underline{X_i} \leqslant X_i \leqslant \overline{X_i} \quad (i=1,2,;\cdots\cdots n) \qquad (56)$$

$$-s_i^{(k)} \leq \Delta x_i^{(k+1)} \leq s_i^{(k)} \quad (i=1,2,\cdots\cdots n) \qquad (57)$$

where $s_i^{(k)}$ is the movelimit of the ith design variable at the kth iteration.

mr is the number of the constraints retained. In the program, the movelimit is computed by the following formula[22]:

$$s_i^{(k)} = s_i^{(k-1)} \sqrt{\frac{\bar{\delta}^{(k)}}{d^{(k-1)}}} \qquad (58)$$

$s_i^{(k-1)}$ is the movelimit of the ith design variable at the (k-1) the iteration.

$s_i^{(k)}$ is the movelimit of the ith design variable at the kth iteration.

$\bar{\delta}^{(k)}$ is the nonlinear diviation to be controled.

$$d^{(k-1)} = \max_r \left| \frac{\tilde{G}r^{(k)} - Gr^{(k)}}{Gr^{(k)}} \right| \qquad (59)$$

$Gr^{(k)}$ is the accurate value of the rth surviving constraint, which is computed by the finite element analysis at the kth iteration.

$\tilde{G}r^{(k)}$ is the approximate value of the rth surviving constraint, which is computed by the linear terms of the Talor expansion at the (k-1)th iteration.

$$\bar{\delta}^{(k)} = 0.3 \sim 0.4 \qquad \text{if } k \leq 3 \qquad (60)$$
$$\bar{\delta}^{(k+1)} = 0.5\, \bar{\delta}^{(k)} \qquad \text{if } k > 3$$

2. Sequential quadric programming technique with improved movelimit

At the current trial design $X^{(k)}$, using the second approximation of Taylor expansion for the objective function and still using the first approximation of Taylor expansion for the surviving constraints, we can obtain the following quadric program problem:

To determine $X_i\ (=X_i^{(k+1)})$

$$\text{Min } \tilde{F}(x) = F(x^{(k)}) + \sum_{i=1}^{n} \left(\frac{\partial F}{\partial x_i}\right)^{(k)} (x_i - x_i^{(k)}) + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \left(\frac{\partial^2 F}{\partial x_i \partial x_j}\right)^{(k)} (x_i - x_i^{(k)})(x_j - x_j^{(k)})$$

$$\text{S.T.} \quad \tilde{G}r(x) = Gr(x^{(k)}) + \sum_{i=1}^{n} \left(\frac{\partial Gr}{\partial x_i}\right)^{(k)} (x_i - x_i^{(k)}) \leq \bar{G}r \quad (r=1,2,\cdots\cdots mr)$$

$$\underline{x}_i \leq x_i \leq \bar{x}_i \quad (i=1,2,\cdots\cdots n)$$

$$-s_i^{(k)} \leq \Delta x_i^{(k+1)} \leq s_i(k) \quad (i=1,2,\cdots\cdots n) \qquad (61)$$

where $s_i^{(k)}$ is the movelimit of the ith design variable at the kth iteration.

The movelimit is computed by eqns (58-60) also.

Approximate technique of three dimensional structural reanalysis

In order to reduce the number of the finite element analysis, some approximate techniques of three dimensional structural reanalysis are developed.

In the program UHSH1, the three approximate techniques are included. They are

1) the sub-structuring technique
2) the combined Taylor series-iterative technique
3) the reduced dimensional technique.

1. The sub-structuring technique (structure level)

One of the special features of the shape optimization problems is that only a part of the domain is changed at each iteration, the other part of the domain occupied by the structure remains the same, their stiffness properties do not alter. Keep it in mind, we subdivide the whole structure into the substructure Ω_f with fixed domain and some substructures Ω_R with varying boundary. Denote the nodal displacement unknowns of the substructures Ω_f and Ω_R by u_f and u_r, respectively, the nodal displacement unknowns on the interfaces between Ω_f and Ω_R by u_i.

Similar definitions are introduced for the nodal load vectors F_f, F_r and F_i, the equilibrium equations (8) in partitioning form are:

$$\begin{bmatrix} K_{ff} & K_{fi} & 0 \\ K_{if} & K_{ii} & K_{ir} \\ 0 & K_{ri} & K_{rr} \end{bmatrix} \begin{bmatrix} U_f \\ U_i \\ U_r \end{bmatrix} = \begin{bmatrix} F_f \\ F_i \\ F_r \end{bmatrix} \qquad (62)$$

Note the fact that K_{ii}, K_{if}, K_{ff}, K_{fi} and F_i, F_f are all independent on x, they are invariant during iterations. Eliminate U_f from the above equations leads to

$$\begin{bmatrix} \bar{K}_{ii} & K_{ir} \\ K_{ri} & K_{rr} \end{bmatrix} \begin{bmatrix} U_i \\ U_r \end{bmatrix} = \begin{bmatrix} \bar{F}_i \\ F_r \end{bmatrix} \qquad (63)$$

in which

$$\bar{K}_{ii} = K_{ii} - K_{if} K_{ff}^{-1} K_{fi} \qquad (64)$$

$$\bar{F}_i = F_i - K_{if} K_{ff}^{-1} F_f \qquad (65)$$

$\bar{K}_{ii}$ and $\bar{F}_i$ are computed only at the first iteration and stored in the memory of the computer.

In the later reanalysis they are called and assembled together with the contribution from the basic elements in Ω_R, which yields the equations (63).

The above sub-structuring technique tremendously reduces the computational effort spent for assembling and solving the global stiffness matrix. Thus, it becomes on indispensable tool for the numerical method in the shape optimization problems. Finally, the computation of the virtual displacements needed in the sensitivity analysis can be also carried out by the sub-structuring technique.

This technique was used successfully for two dimensional shape optimization[17].

2. The combined Taylor series-iterative technique

The other special feature of the shape optimization problems is that the changes of the domain, stresses, displacements and frequences are small at each iteration because the movelimits are always small. Keep it in mind, we have applied the following approximate reanalysis technique in the program UHSH1.

This is a combined Taylor series-iterative technique. In this technique an approximate value of $\{\tilde{U}\}$ is obtained using the following equation:

$$\{\tilde{U}\}^{t}=\{U(x^{(k)})\} + \sum_{i=1}^{n} \left\{\frac{\partial U(x^{(k)})}{\partial x_i}\right\} (x_i-x_i^{(k)}) \qquad (66)$$

And then using this approximate $\{\tilde{U}\}$ as initial estimate, i.e. $\{\tilde{U}\}^{(o)} = \{\tilde{U}\}^{t}$. An improved appoximation is obtained using the following equation:

$$[K]\{\tilde{U}\}^{(k)}= \{P\}- [\Delta K]\{\tilde{U}\}^{(k-1)} \qquad (67)$$

From eqn (45), we can obtain $\{\sigma\}^{(k+1)}$.

3. The reduced dimensional technique

The reduced dimensional technique is an approximate reanalysis technique of the sensitivity. To enhance the efficiency of the sensitivity analysis, the technique is included in the Program UHSH1.

In the technique, the displacement derivative vector $\left\{\frac{\partial U_j}{\partial A_i}\right\}^{(k+1)}_{(N_1 x1)}$ is approximated by a linear combination of S=n+1 linearly independent vectors $\{U_j\}^{(k)}, \left\{\frac{\partial U_j}{\partial A_1}\right\}^{(k)} \ldots\ldots \left\{\frac{\partial U_j}{\partial A_n}\right\}^{(k)}$, where S is usually much less than N_1 which is the order number of the displacement vector.

$$\left\{\frac{\partial U}{\partial A_i}\right\}^{(k+1)}_{N_1 x1} =[\psi]_{N_1 xS} \{C\}_{Sx1} \qquad (68)$$

where $[\psi]_{N_1 x\ S} =\left[\{U\}^{(k)}, \left\{\frac{\partial U}{\partial A_1}\right\}^{(k)}, \left\{\frac{\partial U}{\partial A_2}\right\}^{(k)}, \ldots\ldots \left\{\frac{\partial U}{\partial A_n}\right\}^{(k)} \right]$ (69)

$\{C\}_{Sx1}$ is a vector of undetermined coefficient that is obtained from the following eqns:

$$[\tilde{K}]_{sxs} \{C\}_{sx1} = \{\tilde{P}\}_{sx1} \qquad (70)$$

with $[\tilde{K}] =[\psi]^T [K]^{(k+1)} [\psi]$ (71)

$$\{\tilde{P}\} =[\psi]^T\left(\left\{\frac{\partial P}{\partial A_i}\right\}^{(k+1)} - \sum_{i\in j} \left(\frac{\partial [K]^e_j}{\partial A_i}\right)^{(k+1)} \{U\}^{(k+1)}\right) \qquad (72)$$

where superscript **T** denotes transposition. $\{U\}^{(k+1)}$ is obtained from eqn (67).

Example problems

A number of examples have been successfully calculated by the Program UHSH1. Because of length limitations to the paper, only three of them are given here.

Example 1. Optimum tapering of a cantilever beam

The problem of finding the optimum tapering of a cantilever beam with rectangular cross-section of given uniform width has been a subject of theoretical as well as of practical interest. For the case of a cantilever beam subjected to a force acting at the free end, this optimization problem with stress constraints has a simple closed- form salution. Based on the assumption of constant bending stress, the minimum mass shape is given by

$$y = \sqrt{\frac{6px}{b\sigma_b}} \qquad (74)$$

where y is the depth of the beam at any cross-section at a distance x from the free end of the beam, b is the width of the beam, and σ_b is the value of the maximum allowable bending stress. The problem was evaluated by M. Hasan Imam [15].

The problem has been evaluated by the Program UHSH1. The finite element model is shown in Fig.2.

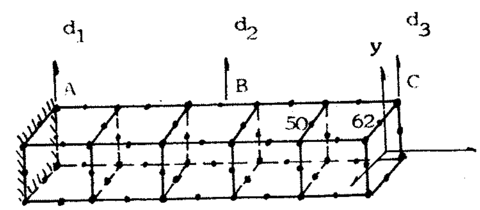


Fig. 2 Model of cantilever bean

Using symmetry and constant width (Z dimension) conditions, the shape of curve ABC determines the shape of the whole beam. The super curve technique is used allowing the shape of curve ABC to remain quadric only. The three nodes shown with pointer vectors d_1, d_2 and d_3 in Fig. 2 were the only nodes moving independently.

The width of the beam is b=25.4 cm, the length of the beam is L=254.0 cm, the load is P=10000 kg σ_b=1500 kg/cm^2.

The optimum shape obtained by this program is shown superimposed on the theoretical optimum shape in Fig. 3. The agreement is excellent. The process and result are shown in Table 1.

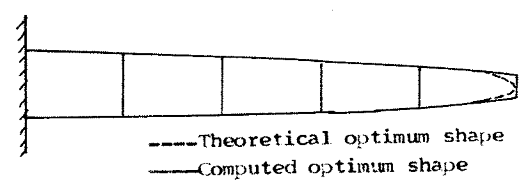


Fig. 3 Optimum shape of rectangular cross-section beam

Table 1. Optimization Process of the Cantilever Beam

Itera. No.	Weight	d_1	d_2	d_3
Initial	1278.2	12.7	12.7	12.7
1	1076.2	11.43	10.8	9.525
2	914.6	10.41	9.271	6.985
3	791.3	9.967	8.052	4.953
4	698.9	10.00	7.076	3.327
Theoretical		10.00	7.071	0.0

Example 2. Optimum tapering of a cantilever beam with stress and frequency constraints

The problem designed is still shown in Fig. 2, but the constraints considered are the stress constraints under two load cases and the frequency-prohibited bonds. Table 2 gives the details of the two load conditions to which the structure is subjected.

Table: 2 Load condition for the cantilever beam

Load condition	Node	direction of loads		
		X	Y	Z
1	50	0	15000 kg	0
2	62	0	10000 kg	0

The allowable stress σ_b=1500 kg/cm^2
$\underline{\omega}$=250 , $\overline{\omega}$ =700

The initial values of the design variables are:

$d_1^{(0)}$= 12.0 cm, $d_2^{(0)}$=9.0 cm,

$d_3^{(0)}$=8.0 cm.

The process and result are shown in Table 3.

Table 3. Optimization process of the cantilever beam with the stress and frequency constraints

Itera. No.	Weight	d_1	d_2	d_3	σ_{max}	ω_i	ω_j
Initial	939.4	12.0	9.0	8.0	1269	226.0	931.0
1	907.4	11.68	8.682	7.682	1342	227.0	903.0
2	877.0	11.38	8.381	7.381	1418	228.0	877.0
3	848.2	11.09	8.094	7.094	1496	229.0	851.0
4	825.9	11.12	7.822	6.822	1498	232.0	833.0
5	807.7	11.34	7.564	6.564	1499	237.0	820.0
6	792.7	11.58	7.339	6.318	1499	241.0	809.0
7	774.6	11.70	7.164	5.818	1499	247.9	798.0
8	767.8	11.82	7.058	5.724	1500	250.0	792.5
9	767.8	11.82	7.058	5.725	1500	250.0	792.4

Example 3. Optimizing a lug

The initial shape of the lug, the load case and the finite element model are shown in Fig. 4 (a), (b).

The shape variables are the eccentricity e, the radius of the lug R1, the radius of the hole R2 and the thickness of the lug t. The optimization problem has four stress constraints and three displacement constraints. The stresses at the 56th, 61th, 93th and 101th nodes should be all lower than the maximum allowable stress σ_b=17000 kg/cm^2. The displacements at the 89th, 94th and 97th node should be all under 0.025 cm.

For the optimum problem, the following technological constraint must been considered: the boundary curves can be only straight lines and circular arc.

The optimum shape obtained by the program is shown in Fig. 5. The process and results are shown in Table 4.

Table 4. Optimization process of the lug

Itera. No.	Weight	e	R_1	R_2	t
Initial	2.353	1.0	4.5	2.0	2.5
1	2.334	1.011	4.612	2.006	2.451
2	2.215	0.999	4.506	2.053	2.356
3	2.099	0.988	4.404	2.098	2.266
4	1.994	0.977	4.308	2.141	2.180
5	1.896	0.967	4.216	2.182	2.098
6	1.873	0.965	4.194	2.191	2.279
7	1.853	0.962	4.174	2.184	2.061
8	1.845	0.965	4.193	2.175	2.046
9	1.837	0.967	4.212	2.167	2.032
10	1.829	0.969	4.230	2.159	2.019

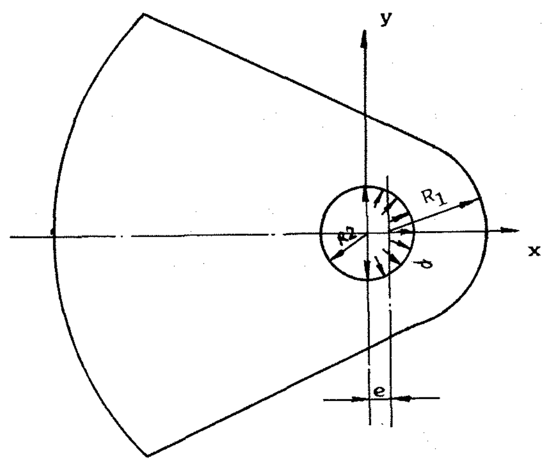


Fig. 4 (a) the initial shape of the lug and load case

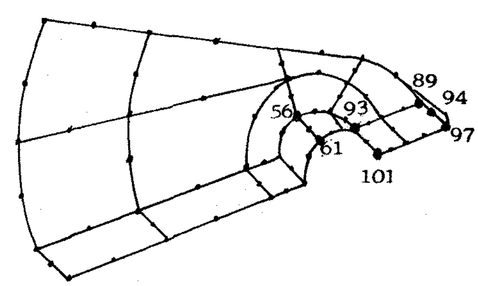


Fig. 4 (b) the finite element model of the lug

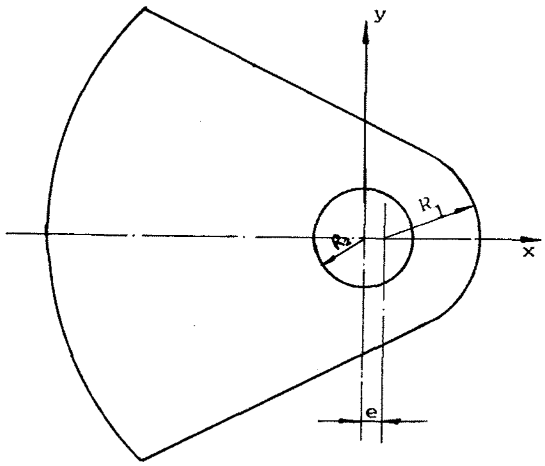


Fig. 5 the optimum shape of the lug

Conclusion

The program UHSH1 is an efficient design program for the three-dimensional shape. It can be applied to design different complex elastic solid structures with multiple constraints under multiple loading cases efficiently.

The program has applied successfully in the design of aircraft structures and will be developed more effectively during it's further application.

Acknowledgements---we wish to express our sincere gratitude to Prof. Qian Lingxi for his guidance and support, to Associate Prof. Cheng Kengtung for his helpful suggestion as well as to Associate Prof. Qiao Xin and our colleague Lee Xianda for their assistance in preparing this report.

REFERENCES

1. O.C. Zienkiewicz and J.S. Campbell,"optimum structural design", chap. 7 in Shape Optimization and Sequential Linear Programming (Ed. R.H. Gallagher and O.C. Zienkiweicz), Wiley, London, 1973.

2. S.S Bhavikatti and C.V. Ramakrishnan"optimum shape design of rotating disks", Int.J. Computer & Structures 11, 397-410 (1980)

3. K.Dems and Z.Mroz "Multiparameter structure shape optimization by the finite element method", Int.J.Num. Meth. Engng, 13, 247-263 (1978)

4. Y.W. Chun and E.J. Hang "Two dimensional shape optimal design", Int.J.Num. Meth. Engng 13, 311-336 (1978)

5. E.Vitiello, "shape optimization using mathematical programming and modelling techniques", 2nd AGRED Conf. on Optimization of structures (1973)

6. R.M. Hicks, E.M. Murman and G.N. Vanderplaats, "An assessment of aerofoil design by numerical optimization", NASA TM X-3092 (July 1974)

7. C.V. Ramakrishnan and A. Francavilla, "Structural shape optimization using penalty functions", J Struct. Mech. 3(4), 403-422 (1974)

8. A. Francavilla, C.V. Ramakrishnan and O.C. Zienkiewicz, "Optimization of shape to minimize stress concentration", J. Strain Anal. 10(2)(1975)

9. E.S. Kristensen and N.F. Madyea, "On the optimum shape of fillets in plates subjected to multiple in-plane loading cases", Int. J. Num. Meth. Engng, 10, 1007-1019 (1976)

10. G. Vanderplaats, "Approximation concepts for numerical airfoil optimization", NASA Tech. Paper No.1370 (1979)

11. J. Oda "On a technique to obtain an optimum strength shape by the finite element method", Bull JSME, 20 (140), (1977).

12. J.Oda and K. Yamazaki, "On a technique to obtain an optimum strength shape of an axisymmetric body by the finite element method", Bull. JSME, 20 (150), (1977)

13. Y. Umetani and S. Hiral, "shape optimization for beams subject to displacement restrictions on the basis of growing performance procedure", Bull. JSME, 21 (157), (1978)

14. J. Oda and K. Yamazaki, "On a technique to obtain optimum strength shape by the finite element method---applications to problems under body force", Bull. JSME, 22 (164), (1979)

15. M. Hasan Imam, "Three-dimensional shape optimization", Int. J. Num. Meth. Engng, 18, 661-673 (1982)

16. Liu Yingwei, "FEM3: An efficient three dimensional finite analysis program" (in Chinese)(1981)

17. Chen Kengtung, "Some developments of numerical methods for shape optimization", Proc. US-China workshop on Advance in Computational Engineering Mechanics. (1983)

18. Xue Xichao and Zhu Piqi, "An optimal algorithm for mesh node label", (in Chinese) (1981)

19. G. Akhras and G. Dhatt, "An automatic node relabeling scheme for minimizing a matrix or network bandwidth", Int. J. Num. Meth. Engng., V. 10:4 ((1976)

20. Qian Lingxi, "Engineering structure optimization" (in Chinese) (1983)

21. Cheng Kengtung, "Engineering structure optimization basis" (in Chinese) (1984)

22. Qian Lingxi, Zhong Wangxie, Cheng Kengtung and Sui Yunkang, "An approach to structural optimization---Sequential Quadratic Programming SQP", Proc. US-China workshop on Advance in Computational Engineering Mechanics (1983)