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PARAMETERS ESTIMATION OF A NONSTATIONARY AERODYNAMICS MODEL FOR LONGITUDINAL MOTION OF AEROPLANE FROM FLIGHT MEASUREMENTS

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Abstracts

A quantitative analysis was done of the significance of nonstationary aerodynamics effect in the mathematical model of an unsteady longitudinal aeroplane motion at a constant flight speed which was excited by a pulse deflection of the elevator in a slow steady straight flight. By using two sorts of the angle-of-attack changes, i.e. of the "attitude" and "path" changes, in terms of generalized coordinates, comparable analytical expressions were possible to be deduced for aerodynamic frequency transfers of the whole aeroplane. The expression for deviations of the moment equation of motion in the frequency domain involves both an experimental frequency transfer of moment of the inertial aeroplane forces and the total aerodynamic frequency transfer of the aeroplane. A quadratic loss function following from their difference may be exploited in three ways. From an analysis made for a light transport A 145 aeroplane interesting conclusions have followed on the transport lag of the downwash angle at tailplane and on the significant effect of the inertial part in the nonstationary aerodynamics model in the moment equation of aeroplane motion.

1. Introduction

When analysing the results of flight measurements carried out with A 145 light transport aeroplane and described in /1/, some differencies were found out at longitudinal motion in frequency transfer functions of the aeroplane responses to different time histories of elevator deflections, that were of triangular, step and sinusoidal shape. There were also observed differencies in aerodynamic derivatives values according to having been measured at steady or unsteady flights. At that time the opinion was pronounced that it might be owing to the fact that a more complicated model of nonstationary aerodynamics should be considered instead of the used quasi-stationary one. Similar conclusions have drawn authors of /2/ and /3/. The significance of the nonstationary aerodynamics effect was proved qualitatively in /4/ by means of "Weighted complex aerodynamic derivatives".

In this paper an attempt is realized to estimate parameters in an analytical model of nonstationary aerodynamics. It is started from the physical analysis done in /4/ and from the data given in /5/, /6/ and /7/ for a rigid wing and tailplane and for interactions of the wing on the tailplane.

A nonstationary aerodynamics model is proposed in the frequency domain and besides aerodynamic derivatives it comprises also nondimensional normalized aerodynamic frequency transfer functions. The benefits and disadvantages of the frequency domain application at parameters estimating were analyzed in /11/. As addition it must be asserted that for a linear system the frequency domain allows a possibility of averaging frequency transfer functions of aeroplane responses from a set of measurements at one class of input shape by the elevator. In the given case the triangular shape inputs are considered. By the averaging the precision of data measured in flight may be increased. At using triangular pulses with a time base shorter than 0.4 s, the frequency spectrum contents is grater then 0.8 for nondimensional angular frequences considered up to 0.33, as it can be seen from fig.11 in /4/. The expression of nonstationary aerodynamic forces and moments in the frequency domain makes possible also to avoid convolutory integrales and to simplify in this way numerical calculations which will be also physically clearer.

With respect to the features of methods for optimal parameters values estimation the number of parameters to be estimated in the nonstationary aerodynamics model should be as small as possible. This may be achieved by a "a priori" statement of some aerodynamic derivatives on the bases of measurements at steady flights, which are relatively precise and correct, see /1/. It is also convenient to estimate the transport lag of the wing trailing edge vortex moving to the tailplane. It is given by the aeroplane geometry and by the measured relative velocity of the airflow at the tailplane. This approach is justified by the fact that this transport lag estimation from measurements fails, see /1/. It remains then to estimate optimal values of parameters of nondimensional normalized aerodynamics frequency transfer functions from measurements at unsteady longitudinal flights having two degrees of motion freedom at a constant flight speed, which are excited by elevator pulse form deflections. Essentially certain methods of "motion equation" described e.g. in /8/ and /9/ are dealt here with. These are extended to estimation of parameters of nondimensional normalized transfer aerodynamic functions being nonlinear in parameters.

For a more detailed quantitative study of nonstationary aerodynamics phenomena, a quadratic loss function of individual motion equations for nonsteady longitudinal aeroplane motion at a constant flight speed may be used. The attention is aimed at the moment equation in which the nonstationary aerodynamics effect was seen to be the greatest one. The loss function may be utilized in different ways: a) to prove the correctness of aerodynamic frequency transfer functions determined by numerical methods on computers; b) to study the influence of the nonstationary aerodynamics model form and of the model parameters values on the loss function; c) to estimate optimal parameters values in nondimensional normalized aerodynamic frequency functions on a basis of flight measurements results.

2. Equations for the longitudinal motion of an aeroplane

It is assumed that a nonstationary longitudinal motion of a rigid aeroplane with the zero propulsion thrust at the steady straight gliding flight in the calm atmosphere is excited by a triangular pulse deflection of the elevator. The traced time interval of the motion is as small as the flight speed and state quantities of the atmosphere may be considered to be constant. A motion of an aeroplane with the defined mass and geometry characteristics is then dealt with which is controlled by means of an elevator deflection as the input quantity. In order to be possible to consider the controlled system defined in this way to be linear, maximum values of the elevator impulse must be limited to suitably small values. As this aeroplane motion is of two degreed of freedom, it is determined by four state quantities. For the given aim it is sufficient to consider two generalized coordinates: the aeroplane angle-of-attack and the aeroplane pitch angle or pitch angular velocity.

It is convenient to write the equations of motion in the air-path axis system and in the form of deviation equations from an initial steady flight condition. As the influence of nonstationary aerodynamic forces and moments is studied, the forces and moments acting on an aeroplane are put together into two groupes: the forces and moments of the aerodynamic origin and those of some other origin. This is also the reason why in the equations aerodynamic forces are not divided into the control and response ones and why the motion equations are not reproduced as state equations.

If considering real time, the motion equations are

$$\Delta C_A[\Delta\gamma(t), \Delta\dot{\gamma}(t); \Delta\Theta(t), \Delta\dot{\Theta}(t); \Delta\eta(t), \Delta\dot{\eta}(t); t] = \mu\tau_A \Delta\dot{\gamma}(t) + C_{WO}\Delta\gamma(t) \qquad (1)$$

$$\Delta C_m[\Delta\gamma(t), \Delta\dot{\gamma}(t); \Delta\Theta(t), \Delta\dot{\Theta}(t); \Delta\eta(t), \Delta\dot{\eta}(t); t] = \mu\, \tilde{i}_y^2\, \tau_A^2 \cdot \Delta\ddot{\Theta}(t) \qquad (2)$$

where

$$\Delta\Theta(t) = \Delta\gamma(t) + \Delta\alpha(t) \quad \text{and} \quad \Delta\dot{\Theta}(t) = \omega_y(t).$$

As they are deviation equations from a steady state, the initial conditions are zero. By the integral Fourier transformation and by dividing them by means of $\bar{\Delta\eta}$, eq.(1) and (2) are converted into the form:

$$\left[\frac{\bar{\Delta C_A}}{\bar{\Delta\eta}}(i\omega)\right]_T = \mu\tau_A\left[\frac{\bar{\omega_y}}{\bar{\Delta\eta}}(i\omega) - i\omega\frac{\bar{\Delta\alpha}}{\bar{\Delta\eta}}(i\omega)\right] + C_{WO}\left[\frac{1}{i\omega}\cdot\frac{\bar{\omega_y}}{\bar{\Delta\eta}}(i\omega) - \frac{\bar{\Delta\alpha}}{\bar{\Delta\eta}}(i\omega)\right] \qquad (3)$$

$$\left[\frac{\bar{\Delta C_m}}{\bar{\Delta\eta}}(i\omega)\right]_T = \mu\,\tilde{i}_y^2\,\tau_A^2\cdot i\omega\,\frac{\bar{\omega_y}}{\bar{\Delta\eta}}(i\omega) \qquad (4)$$

In the following analysis the moment equation only is considered, as in it the influence of nonstationary aerodynamics is expressed in the most outstanding way.

With respect to the assumption of constant speed of flight, the equations (3) and (4) are correct from angular frequencies only which are appropriately greater than the angular frequency of the phugoid oscillations, at which the flight speed changes with time.

In the moment equation is:

$$\mu\,\tilde{i}_y^2\,\tau_A^2\cdot i\omega\,\frac{\bar{\omega_y}}{\bar{\Delta\eta}}(i\omega) = Y_E(i\omega) \qquad (4a)$$

the experimental frequency transfer of the moment of inertial aeroplane forces and

$$\left[\frac{\bar{\Delta C_m}}{\bar{\Delta\eta}}(i\omega)\right]_T = Y_T(i\omega) \qquad (4b)$$

the total aerodynamic moment frequency transfer. These frequency transfer functions may be plotted in the complex plane as vectors

$$\vec{Y}_E \equiv Y_E(i\omega) = U_E(\omega) + iV_E(\omega) \qquad (5)$$

$$\vec{Y}_T \equiv Y_T(i\omega) = U_T(\omega) + iV_T(\omega) \qquad (6)$$

An analysis of these vectors (5) and (6) is given in chap.4.

3. Nonstationary aerodynamics model

3.1 Basic relations

A historical survey of the nonstationary aerodynamics research is given in /4/. There is also given a classification scheme of nonstationary aerodynamics phenomena in fig.5. As a nondimensional parameter for these phenomena, the Strouhal number is considered. In the time domain it is defined by $s = t/\tau_A$ and in the frequency domain by $\omega^x = \omega\tau_A$, where $\tau_A = l/v$ is the aerodynamic time unit.

A lift change for a unit span of a rectangular wing is defined:

a) for a step form of angle-of-attack by

$$\Delta A_j(s) = \frac{\rho V^2}{2}\cdot l.2\pi.k_j(s).\Delta\alpha \qquad (7)$$

where $k_j(s)$ are nondimensional normalized transition functions, called aerodynamic /indicial/ admittances. For j = 1, this is the Wagner function for an instantaneous angle-of-attack change all over the whole wing and for j = 2 it is the Küssner function for an gradual angle-of-attack change suceeding from the leading edge;

b) for a sinusoidal form change of angle-of-attack amplitude $\Delta\alpha_0$ by

$$\Delta A(s) = \frac{\rho V^2}{2}\cdot l.2\pi \left\langle \begin{matrix} C(i\omega^x) \\ H(i\omega^x) \end{matrix} \right\rangle \Delta\alpha_0 \cdot e^{i\omega^x s} \qquad (8)$$

where C and H are nondimensional normalized frequency transfer functions for lift. For an instantaneous angle-of-attack change on the whole wing, it is the Theodorsen function and for an angle-of-attack change succeeding from the leading edge it is the Sears function. For a wing with a final span, in all the mentioned cases the aerodynamic derivative $C_{A\alpha}$ must be used instead of 2π. Generally the Fourier integral transform of the admittance $A_{A.\alpha}(s)$ is related to the frequency transfer function $F_{A.\alpha}(i\omega^x)$ by

$$\mathcal{F}\left\{A_{A.\alpha}(s)\right\} = F_{A.\alpha}(i\omega^x)\cdot\frac{1}{i\omega^x} \qquad (9)$$

The index A used in the symbols for functions in eq.(9) denotes the lift A as a response to a change of the input quantity α, which has at the admittance a step form and at the frequency transfer function a sinusoidal form.

An arbitrary time change of the angle-of-attack excites a lift change which is given in the time domain by a convolutory integral and in the frequency domain by a simple relation

$$\bar{\Delta A}(i\omega^x) = F_{A.\alpha}(i\omega^x).\bar{\Delta\alpha}(i\omega^x) \qquad (10)$$

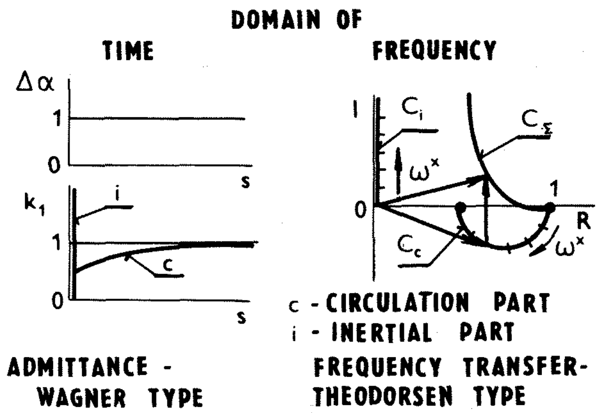

FIGURE 1 - SCHEME DIAGRAM OF AERODYNAMIC NORMALIZED NONDIMENSIONAL FUNCTIONS

Instead of the very complicated functions $k_1(s)$, $k_2(s)$ and $C(i\omega^x)$, $H(i\omega^x)$, approximate supplementary exponential functions and Fourier integral transforms of them may be used. So e.g. for $j = 1$ /Wagner function/ the following formulas are used

$$k_1(s) \doteq 1 - \sum_{i=1}^{n} {}^{\circ}c_{1i} \cdot e^{-b_{1i}^{x} s} \tag{11}$$

$$C(i\omega^x) = 1 + i\omega^x \int_0^\infty \left[k_1(s) - 1\right] . e^{-i\omega^x s} ds = 1 - i\omega^x \sum_{i=1}^{n} {}^{\circ}c_{1i} \frac{T_{1i}^x}{1 + i\omega^x T_{1i}^x} \tag{12}$$

where

$$\mathcal{F}\left\{ {}^{\circ}c_{1i} e^{-b_{1i}^x s} \right\} = {}^{\circ}c_{1i} \frac{1}{b_{1i}^x + i\omega^x}, \; b_{1i}^x = 1/T_{1i}^x \tag{13}$$

Original nondimensional normalized aerodynamic functions were extended to different wing shapes, to different wing aspect ratios and for a compressible fluid environment. The Wagner and Theodorsen functions, deduced on the basis of velocity circulation around the wing section, were completed by members expressing the influence of air moved by the wing. This lift component is here called the "inercial component" ΔA_i in contradiction to the "circulation component" ΔA_c.

The total aerodynamic nondimensional normalized admittance for an instantaneous angle-of--attack change $\Delta\alpha$ on the whole wing is given by

$$k_1(s)_\Sigma = k_1(s)_c + k_1(s)_i \; , \tag{14}$$

where the circulation component $k_1(s)_c$ is expressed in (11) and the inertial component is given by

$$k_1(s)_i = K_\lambda \cdot \delta(s), \tag{15}$$

where $K_\lambda = K_\lambda(\lambda)$,

λ is the wing aspect ratio and $\delta(s)$ is the unit impulse function. Analogically the total aerodynamic nondimensional normalized frequency transfer function is expressed by

$$C(i\omega^x)_\Sigma = C(i\omega^x)_c + C(i\omega^x)_i \; , \tag{16}$$

where the circulation component is given by (12) and (13) and the inertial component is determined by

$$C(i\omega^x)_i = i\omega^x . K_\lambda \cdot \mathcal{F}\{\delta(s)\} = i\omega^x . K_\lambda \tag{17}$$

A schematic illustration of the mentioned functions is given in fig. 1.

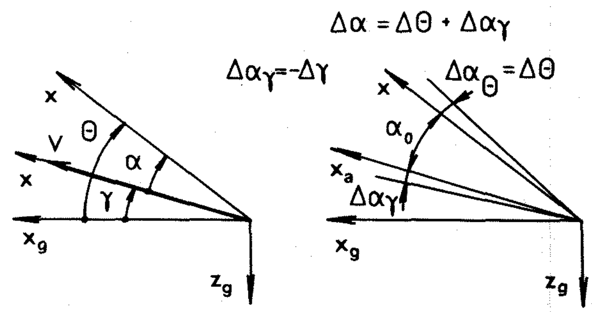

FIGURE 2 - SCHEME DIAGRAM OF ANGLES

The modern computation techniques has opened new possibilities for improving method for calculation of normalized nondimensional complex coefficients of generalized aerodynamic forces and moments for a rigid aeroplane of different geometric shapes. Inspite of it, there is sometimes more suitable in dynamics of flight and in identification of models of conventional aeroplane motions to use approximate analytical aerodynamic expressions, see /1/, /2/ and /3/.

3.2 Realization of angle-of-attack changes

In analytical expressions for aerodynamic admittances and for frequency transfer functions, an angle-of-attack takes place which is determined by two semi-straightlines, see fig. 2. One of them has the direction of the relative velocity vector of the aeroplane with respect to the air and the second one lies in the direction of a reference axis fixed to the wing or to the aeroplane. Angle-of-attack changes may be realized in two ways: by a rotation of the relative velocity vector and by a rotation of the reference axis of the aeroplane. Each of these both directions is of physically different nature. One can thus expect that each of the both possible modes of changes will show itself in an aerodynamically different way.

A change of the angle-of-attack due to the change of orientation of the aeroplane reference axis is called here the "attitude change" $\Delta\alpha_\Theta = \Delta\Theta$. An angle-of-attack change due to a rotation of the relative velocity vector of the aeroplane with respect to the environment air is called the "path change" $\Delta\alpha_\gamma = -\Delta\gamma$. To describe the total angle-of-attack change $\Delta\alpha = \Delta\Theta - \Delta\gamma = \Delta\alpha_\Theta + \Delta\alpha_\gamma$, two generalized coordinates $\Delta\Theta$ and $\Delta\alpha_\gamma$ are thus needed. In aeroelasticity and sometimes also in flight dynamics, a pair $\Delta\Theta$ and Δz_a is used instead of the mentioned two generalized coordinates. They are e.g. in aeroelasticity a rotation of the section reference axis and trenslation of the origin of this axis /twisting and bending of the wing/. A disadvantage of the pair $\Delta\Theta$ and Δz_a is that these coordinates are dimensionally and physically nonhomogeneous and expressions for aerodynamic forces and moments due to them are not comparable.

In the following aerodynamic considerations the use of the physically homogeneous pair of the angle-of-attack changes $\Delta\Theta$ and $\Delta\alpha_\gamma$ is prefered.

3.3 Interaction of a wing and tailplane

In /5/ R.T.Jones deals with a proposal of approximate relations for computation of an admittance of the down-wash angle at tailplane which was brought about by a step form change of angle--of-attack at the same time on the whole wing. He has used a model of four vortices: the lifting vortex A fixed to the wing; the trailing edge vortex B of a contrary orientation than the vortex A and fixed to the calm air in which the aeroplane moves; the free vortices C,D which are due to the final span of the wing.

For calculation of a velocity induced by the vortices A, B, C, D in a sufficiet distance from the wing, there is assumed that to the wing just one vortex A is fixed in the middle of the chord of a supplementary rectangular wing and that from the trailing edge of this wing just one vortex B separates at a step change of the angle-of-attack.

The induced velocity orientation at the tailplane leading edge depends on if the wing trailing vortex B is before or already behind tailplane leading edge. In the case that this vortex would be just in this leading edge, the induced velocity theoretically should change its value from $-\infty$ to $+\infty$. This case may be modelled in the frequency domain by a transcendent "integral exponential" function.

In the frequency domain the frequency transfer function of the down-wash angle may be expressed according to /5/ by:

$$F_{\alpha_a \cdot \alpha}(i\omega^x) \doteq F_{\alpha_a \cdot \Gamma}(i\omega^x) . F_{\Gamma . \alpha_F}(i\omega^x) \qquad (18)$$

The frequency transfer $F_{\Gamma . \alpha_F}$ describes the evolution of the circulation on a wing in dependency on the wing angle-of-attack. The frequency transfer $F_{\alpha_a . \Gamma}$ describes the effect of the vorties A, B, C, D which depends on the change of velocity circulation around the wing. According to /5/ the first transfer consist from a sum of three Fourier transforms of exponential functions with six parameters, the second transfer is formed by a sum of four Fourier transforms of exponential functions with eight parameters and one frequency transfer function due to the wing trailing vortex B.

$$F_B(i\omega^x) = i\omega^x . a_B^x . e^{-i\omega^x \tau_w^x} . Ei(i\omega^x \tau_w^x) \qquad (19)$$

where $Ei(x) = \int_{-\infty}^{x} \frac{e^{-u}}{u} \cdot du$ is the transcendent "integral exponential" function;

$a_B^x = 1/4\pi$; $e^{-i\omega^x \tau_w^x}$ is the Fourier transform of the transport lag of a step form change of the induced velocity at tailplane leading edge and

$\tau_w^x = \tau_W / \tau_A$ is the nondimensional transport lag.

The transport lag $\tau_W = \xi_H / V$ defined in this way does not respect the flow velocity reduction at tailplane leading edge. Also definitions $\tau_{WH} = \xi_H / V_H$ and $\tau_H = r_H / V_H$ are used that will be proved in chap. 5.

It follows from this analysis that the analytical model according to /5/ has 16 parameters which represent from the point of view of parameters estimation from flight measurements a too great number. Therefore in /4/ a simplified analytical model was proposed of the following nondimencional normalized form:

$$C_{\alpha_a}(i\omega^x) = \frac{1}{1 + i\omega^x T_1^x} \cdot e^{-i\omega^x \tau_w^x} \qquad (20)$$

In the quasi-stationary aerodynamics model the expression (20) with $T_1 = 0$ is considered and the transport lag is then expessed as follws:

$$\cos \omega^x \tau_w^x - i \sin \omega^x \tau^x \doteq 1 - i\omega^x \tau_w^x \qquad (21)$$

By means of this mathematically correct simplication the physical nature of the model is changed. The time lag of the circulation component of a tailplane lift is formally transfigurated in the complex domain into the form of the inertial component of the lift (17). A more detailed analysis is in chap. 5.

3.4. Total aerodynamic frequency transfers

Changes of aerodynamic coefficients ΔC_A and ΔC_m in (1) and (2) in the nonstationary aerodynamics model are functions of time as explicitly as even implicitly by means of time changes of the control quantity $\Delta\eta(t)$ and of the generalized coordinates $\Delta\alpha_\gamma(t)$ and $\Delta\Theta(t)$, see chap. 3.2. As deviations with zero initial conditions are dealt with, changes of the considered aerodynamic coefficients may be expressed after the Fourier integral transformation in the form:

$$\Delta C_k(i\omega^x) = F_{C_k . \eta}(i\omega^x) . \overline{\Delta\eta}(i\omega^x) + F_{C_k . \alpha_\gamma}(i\omega^x) . \overline{\Delta\alpha}_\gamma(i\omega^x) + F_{C_k . \Theta}(i\omega^x) . \overline{\Delta\Theta}(i\omega^x), \qquad (22)$$

where k = A, m.

The first frequency transfer $F_{C_k . \eta}$ represents a control frequency transfer which is determined by the aerodynamic lift due to a deflection of the elevator of tailplane. The second and third frequency transfers $F_{C_k . \alpha_\gamma}$ and $F_{C_k . \Theta}$ include the influence of aerodynamic forces of the whole aeroplane due to a "path" and "attitude" changes of angle-of-attack. For expressing these effects, an analysis was done in /4/ in which expressions from the appendix in the book /6/ by Scanlan and Rosenbaum were used. As a reference point a common point for the wing and for the tailplane in the aeroplane c.g. was considered. It is illustrated in fig 3 together with characteristic lengths defined on an aeroplane. The expressions from /6/ were transduced into the present notation according to the ISO 1151 standard. From the analysis af /4/ a very important relation follows:

$$F_{C_k . \Theta}(i\omega^x) = F_{C_k . \alpha_\gamma}(i\omega^x) + i\omega^x . F_{C_k . \dot\Theta^x}(i\omega^x), \qquad (23)$$

k = A, m.

When using (23), the relation $\overline{\Delta\Theta} + \overline{\Delta\alpha}_\gamma = \overline{\Delta\alpha}$ and by dividing them by $\overline{\Delta\eta}(i\omega^x)$, the relations (22) are transformed into the form which expresses the total aerodynamic frequency transfer

$$\left[\frac{\overline{\Delta C_k}}{\overline{\Delta\eta}}(i\omega^x)\right]_T = F_{C_k . \eta}(i\omega^x) + F_{C_k . \alpha_\gamma}(i\omega^x) \frac{\overline{\Delta\alpha}}{\overline{\Delta\eta}}(i\omega^x) + F_{C_k . \dot\Theta^x}(i\omega^x) . \frac{\overline{\omega_y^x}}{\overline{\Delta\eta}}(i\omega^x) \qquad (24)$$

where k = A, m .

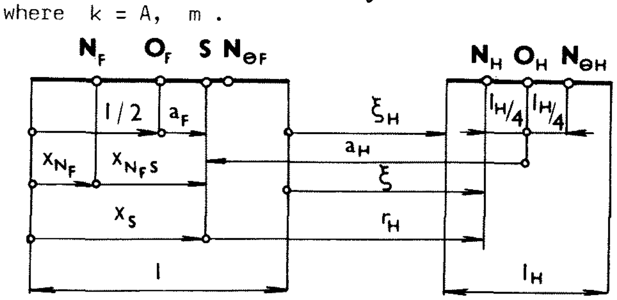

FIGURE 3 - SCHEME DIAGRAM OF LENGTHS

In this total transfer the aerodynamic control frequency transfer function $F_{C_k \cdot \eta}(i\omega^x)$ is not dependent on the aeroplane motion. But the effect of aerodynamic transfer functions $F_{C_k \cdot \alpha_\gamma}$ and $F_{C_k \cdot \dot{\Theta}^x}$ does depend on dynamic frequency properties of the aeroplane that are characterized by the responses transfers of the aeroplane to elevator deflections. The frequency transfers of aeroplane responses $F_{\alpha \cdot \eta}$ and $F_{\omega_y^x \cdot \eta}$ in the total aerodynamic transfer (24) are therefore considered as a measure of the utilization of the aerodynamic frequency transfers $F_{C_k \cdot \alpha_\gamma}$ and $F_{C_k \cdot \dot{\Theta}^x}$ at the given aeroplane motion. The total aerodynamic frequency transfer $\left[\frac{\Delta C_k(i\omega^x)}{\Delta \eta}\right]_T$ therefore is not only an aerodynamic function but also a frequency transfer function of flight dynamics. Frequency transfer functions of aeroplane responses may be measured in flight or they may be calculated from the system of motion equations.

The expression (24) for a nonstationary aerodynamics model may be transformed into a quasi-stationary aerodynamic model when instead of aerodynamic frequency transfers the complex aerodynamic derivatives are substituted in the form:

$$F_{C_k \cdot x}(i\omega^x) = U_{C_k \cdot x}(0) + i\omega^x \cdot V'_{C_k \cdot x}(0) = C_{k \cdot x} + i\omega^x C_{k \cdot \dot{x}} \qquad (25)$$

where $k = A, m$ and $x = \eta, \alpha_\gamma, \Theta$.

3.5 Analytical models of aerodynamic frequency transfer functions

Aerodynamic frequency transfer functions in the total aerodynamic transfer (24) may be determided by numerical methods by means of computers. But they may be also expressed analytically so that the influence of quasi-stationary aerodynamics may be separated by means of using aerodynamics derivatives independent on $i\omega^x$ from the effect of nonstationary aerodynamics by means of using nondimensional normalized aerodynamic frequency transfer functions.

For individual aerodynamic frequency transfer functions $F_{C_k \cdot x}$ ($k = A, m$; $x = \eta, \alpha_\gamma, \Theta$) analytical expressions may be proposed according to /4/ in the form:

a) for the lift coefficient:

$$F_{C_k \cdot \eta}(i\omega^x) = a_1 k_H \cdot \left(\frac{a_2}{a_1}\right) \cdot a'_{H1} C_H(i\omega^x) \qquad (26)$$

$$F_{C_A \cdot \alpha_\gamma}(i\omega^x) = \Big[a . C_F(i\omega^x) + a_1 k_H a'_{H1} C_H(i\omega^x) + - a_1 k_H a'_{H1} \frac{d\alpha_a}{d\alpha} \cdot C_{\alpha_a}(i\omega^x) . h_H(i\omega^x)\Big]_C + i\omega^x \left[a K_{\lambda F} + a_1 k_H K_{\lambda H} \cdot a'_{H2}\right]_i \qquad (27)$$

$$F_{C_A \cdot \dot{\Theta}^x}(i\omega^x) = \left[a . a' . C_F(i\omega^x) + a_1 k_H \cdot a'_{H3} . C_H(i\omega^x)\right]_C + i\omega^x \left[a K_{\lambda F} \cdot a'_{F4} + a_1 k_H K_{\lambda H} \cdot a'_{H4}\right]_i \qquad (28)$$

b) for the pitching moment coefficient:

$$F_{C_m \cdot \eta}(i\omega^x) = a_1 k_H \cdot \left(\frac{a_2}{a_1}\right) \cdot m'_{H1} \cdot C_H(i\omega^x) \qquad (29)$$

$$F_{C_m \cdot \alpha_\gamma}(i\omega^x) = \Big[a m'_{F1} C_F(i\omega^x) + a_1 k_H m'_{H1} C_H(i\omega^x) + - a_1 k_H m'_{H1} \frac{d\alpha_a}{d\alpha} C_{\alpha_a}(i\omega^x) . h_H(i\omega^x)\Big]_C + i\omega^x \left[a K_{\lambda F} m'_{F2} - a_1 k_H K_{\lambda H} m'_{H2}\right]_i \qquad (30)$$

	PARAMETER	WING: GEOMETRY x)	WING: AERODYNAMICS xx)	PARAMETER	TAILPLANE: GEOMETRY x)	TAILPLANE: AERODYNAMICS xx)
LIFT EQ (26,27,28)	a_{F1}	-	$a = \partial C_{AF}/\partial \alpha_F$	a_{H1}	$\tilde{S}_H$	$a_1 k_H = k_H \, \partial C_{AH}/\partial \alpha_H$
	a_{F2}	-	$a K_{\lambda F}$	a_{H2}	$\tilde{S}_H . \tilde{\tilde{l}}_H$	$a_1 k_H \, K_{\lambda H}$
	a_{F3}	$\tilde{X}_{SN_\Theta} = (0{,}75 - \tilde{X}_S)$	a	a_{H3}	$\tilde{S}_H \, (\tilde{\tilde{r}}_H + 0{,}50) . \tilde{\tilde{l}}_H$	$a_1 \, k_H$
	a_{F4}	$\tilde{X}_S - 0{,}50$	$a K_{\lambda F}$	a_{H4}	$\tilde{S}_H (\tilde{\tilde{r}}_H + 0{,}25) \, \tilde{\tilde{l}}_H$	$a_1 \, k_H \, K_{\lambda H}$
MOMENT EQ (29,30,31)	m_{F1}	$\tilde{X}_{N_F S}$	a	m_{H1}	$-\tilde{r}_H \, \tilde{S}_H$	$a_1 \, k_H$
	m_{F2}	$\tilde{X}_S - 0{,}50$	$a K_{\lambda F}$	m_{H2}	$+(\tilde{\tilde{r}}_H + 0{,}25) \tilde{S}_H \tilde{l}_H \, \tilde{\tilde{l}}_H$	$a_1 \, k_H \, K_{\lambda H}$
	m_{F3}	$\tilde{X}_{N_F S}(0{,}75 - \tilde{X}_S)$	a	m_{H3}	$-\tilde{r}_H \tilde{S}_H \, (\tilde{\tilde{r}}_H + 0{,}50) \, \tilde{\tilde{l}}_H$	$a_1 k_H$
	m_{F4}	$0{,}25$	$a K_{\lambda F}$	m_{H4}	$+0{,}25 \;\; \tilde{S}_H \, \tilde{\tilde{l}}_H$	$a_1 \, k_H \, K_{\lambda H}$
	m_{F5}	$m'_{F5} = (\tilde{X}_S - 0{,}50)^2$ $m''_{F5} = 1/128$	$a \, [K_{\lambda F} \cdot m'_{F5} + m''_{F5}]$	m_{H5}	$m'_{H5} = \tilde{S}_H . \tilde{l}_H \, [\tilde{\tilde{r}}_H + 0{,}25]^2 . \tilde{\tilde{l}}_H$ $m''_{H5} = \tilde{S}_H . \tilde{l}_H . \tilde{\tilde{l}}_H . 1/128$	$a_1 k_H [K_{\lambda H} \cdot m'_{H5} + m''_{H5}]$
$a_{Xi} = a'_{Xi} \cdot \hat{a}_{Xi}$, $m_{Xi} = m'_{Xi} \; \hat{m}_{Xi}$, $X = F, H$; $i = 1,2,3,4$				x) a'_{Xi}, m'_{Xi} ; xx) $\hat{a}_{Xi}, \hat{m}_{Xi}$		
$\tilde{\tilde{r}}_H = r_H/l_H = \tilde{r}_H/\tilde{l}_H$; $\tilde{\tilde{l}}_H = \tilde{l}_H/\sqrt{k_H}$			$K_{\lambda F}, K_{\lambda H}$ see (17)	$K_{A\dot{\alpha}^x} = +(a_{F2} + a_{H2})$; $K_{A\dot{\Theta}^x} = +(a_{F4} + a_{H4})$		
$K_{m\dot{\alpha}^x} = +(m_{F2} - m_{H2}) = b_8/\tau_A$			$K_{m\dot{\Theta}^x} = -(m_{F4} + m_{H4}) = b_9/\tau_A$		$K_{m\ddot{\Theta}^x} = -(m_{F5} + m_{H5}) = b_{10}/\tau_A^2$	

TABLE 1 - PARAMETERS OF AERODYNAMIC FREQUENCY TRANSFERS

$$F_{C_m.\dot{\Theta}^x}(i\omega^x) = [a m'_{F3}.C_F(i\omega^x) + a_1 k_H m'_{H3} C_H(i\omega^x)]_C + \\ - \{-[a K_{\lambda F} m'_{F4} + a_1 k_H K_{\lambda H} m'_{H4}] + \\ + i\omega^x [a(K_{\lambda F} m'_{F5} + m''_{F5}) + \\ + a_1 k_H (K_{\lambda H} m'_{H5} + m''_{H5})]\}_i \qquad (31)$$

In these expressions the following notation is used:
a) aerodynamic derivatives:

$$a = \frac{\partial C_{AF}}{\partial \alpha_F}, \quad a_1 = \frac{\partial C_{AH}}{\partial \alpha_H}, \quad \frac{a_2}{a_1} = \frac{\partial \alpha}{\partial \eta}$$

$\frac{d\alpha_a}{d\alpha}$ - the downwash angle derivative at tailplane.

b) coefficients:

$k_H = q_H/q$ - the airflow retarding coeficient;

$K_{\lambda F}$, $K_{\lambda H}$ - coefficients of aerodynamic inertial forces, see (17).

c) aerodynamic nondimensional normalized frequency transfer functions:
$C_F(i\omega^x)$ - for the wing, $C_H(i\omega^x)$, $h_H(i\omega^x)$ for the tailplane (Theodorsen or Sears type),
$C_{\alpha_a}(i\omega^x)$- for the effective downwash angle at tailplane.

d) geometrical parameters a'_{F1} up to a'_{H4} and m'_{F1} up to m'_{H5} are defined in table 1.

4. Loss function of the moment equation

4.1 Basic relations

The Fourier transform (4) of the moment equation (2) expresses in the complex plane a difference of two vectors (5) and (6): $\vec{Y}_E - \vec{Y}_T = 0$

The first vector defined in (4a) and the second one defined in (24) comprise the frequency transfers of aeroplane responses $F_{\alpha.\eta}$ and $F_{\omega_y^x.\eta}$. These frequency transfers may be computed from flight measurement results as mean values on individual levels of the angular frequency ω_j^x (j = 1,.. , k) from data got at repeated experiments with numbers of reperitions ν = 1,.. , n_j , see /1/ and /4/. The measured data with experimental variance s^2_{Ej} are loaded also by uncorrectable residua of systematic errors due as a rule by "freezing" of random errors from instruments graduations. Experimental variance with residua of systematic errors in the experimental vector $\vec{Y}_E$ are deformed by the derivation which is expressed in the form $i\omega^x. F_{\omega_y^x.\eta}(i\omega^x)$and in the second vector $\vec{Y}_T$ they are deformed by the aerodynamic frequency transfers $F_{C_m.\alpha_F}$and $F_{C_m.\dot{\Theta}^x}$. In consequence of it the equation (4) is of the form

$$\hat{Y}_E(i\omega_j^x) - \hat{Y}_T(i\omega_j^x) = e(i\omega_j^x) \qquad (32)$$

As an "a priori" estimation of the covariance matrix of the motion equation deviations $e(i\omega_j^x)$in the sense of the above analysis is difficult and untrustworthy, for the proposal of a loss function the most simple relation was used, for which as an "a priori" information a knowledge of the measured values of frequency transfers of aeroplane responses is sufficient. The loss function is then defined by the relation

$$S = \sum_{j=2}^{k} |\vec{e}_j|^2 = \sum_j [e_R^2(\omega_j^x) + e_I^2(\omega_j^x)], \qquad (33)$$

where e_R and e_I are the real and imaginary parts of the deviation $e(i\omega_j^x)$ from (32). The function S in (33) is called the "loss" function because it is a measure of the loss of information involved in both vectors.

The loss function according to (33) in connection with measured frequency transfers of aeroplane responses may be used in three ways: a/ for proving the correctness of the aerodynamic frequency transfers $F_{C_m.\eta}$, $F_{C_m.\alpha_F}$, $F_{C_m.\dot{\Theta}^x}$, given in a tabular or analytical form; b/ for investigation of the influence of individual aeroplane parameters on the loss function S ; c/ for parameters estimation in the aerodynamic nondimensional normalized frequency transfers C_F, C_H, C_{α_a} and for estimation of the coefficients $K_{\lambda F}$, $K_{\lambda H}$.

For the first purpose it is suitable to define the motion equation deviation in the form:

$$e(i\omega_j^x) = \mu\, \tilde{i}_y^2 . [i\omega_j^x . F_{\omega_y^x.\eta}(i\omega_j^x)]_E + \\ - \{F_{C_m.\eta}(i\omega_j^x) + F_{C_m.\alpha_F}(i\omega_j^x)[F_{\alpha.\eta}(i\omega_j^x)]_E + \\ + F_{C_m.\dot{\Theta}^x}(i\omega_j^x). [F_{\omega_y^x.\eta}(i\omega_j^x)]_E\} \qquad (34)$$

As a comparation measure for the loss function values, its maximum value S_{max} for $\vec{Y}_{Tj} \equiv 0$ suits, i.e. $S_{max} = \sum_j |\vec{Y}_{Ej}|^2$.
This method of proving the correctness of aerodynamic frequency transfers has the advantage that one need not know any analytical model for the aeroplane responses in the real time domain nor in the frequency domain.

For the second and third purpose the vector $\vec{Y}_T$ must be expressed analytically as a function of the investigated or estimated parameters β_l (l = 1, ..., p), i.e.

$$Y_{T_j} = Y_T(i\omega_j^x ; \beta_l) \qquad (35)$$

In the second case the parameters β_l influence is estimated by means of the loss function $S(\beta_l)$value. In the third case the loss function $S(\beta_l)$ is used for estimation of the optimal values parameters in nondimensional normalized aerodynamic transfers.

4.2 The loss function for estimation nonstationary aerodynamics parameters

As a basis the equations (29), (30), (31) are used, where for a simplification the coefficients in the sense of tab. 1 are introduced. The total aerodynamic moment frequency transfer Y_{Tj} in the loss function is in the sense of (35) espressed in the form

$$Y_{T_j} = Y_{T\eta}(i\omega_j^x; \beta_3, \beta_4) + Y_{Tc}(i\omega_j^x; \beta_{1,2,3,4,5,6,7}) + \\ + Y_{Ti}(i\omega_j^x; \beta_{8,9,10}) \qquad (36)$$

where

$$Y_{T\eta j} = m_{H1} \frac{a_2}{a_1} C_H(i\omega_j^x; \beta_3, \beta_4) \qquad (37)$$

is the control transfer of the circulation origin;

$$Y_{Tcj} = K_{C_F}(i\omega_j^x).C_F(i\omega_j^x; \beta_1, \beta_2) + \\ + K_{C_H}(i\omega_j^x).C_H(i\omega_j^x; \beta_3, \beta_4) + \\ + K_{C\alpha}(i\omega_j^x).C_{\alpha H}(i\omega_j^x; \beta_{5,6,7}) \qquad (38)$$

is a component of the circulation origin;

$$Y_{Tij} = i\omega_j^x \cdot [F_{\alpha\cdot\eta}(i\omega_j^x)]_E \cdot \beta_8 + [F_{\omega_y^x\cdot\eta}(i\omega_j^x)]_E \cdot \beta_9 + i\omega_j^x \cdot [F_{\omega_y^x\cdot\eta}(i\omega_j^x)]_E \cdot \beta_{10} \quad (39)$$

is a component of the inertial origin. In the transfer (38) the following functions are enclosed:

$$K_{C_F}(i\omega_j^x) = m_{F1}[F_{\alpha\cdot\eta}(i\omega_j^x)]_E + m_{F3}[F_{\omega_y^x\cdot\eta}(i\omega_j^x)]_E \quad (40a)$$

$$K_{C_H}(i\omega_j^x) = m_{H1}[F_{\alpha\cdot\eta}(i\omega_j^x)]_E + m_{H3}[F_{\omega_y^x\cdot\eta}(i\omega_j^x)]_E \quad (40b)$$

$$K_{C_\alpha}(i\omega_j^x) = m_{H1}\frac{d\alpha_a}{d\alpha}[F_{\alpha\cdot\eta}(i\omega_j^x)]_E \quad (40c)$$

$$C_{\alpha H}(i\omega_j^x) = C_{\alpha_a}(i\omega_j^x; \beta_5) \cdot h_H(i\omega_j^x; \beta_{6,7}) \quad (41a)$$

$$h_H(i\omega_j^x) = H_H(i\omega_j^x; \beta_{6,7}) \cdot e^{+i\omega_j^x T^x_{H0,25}} \quad (41b)$$

The functions (40a, b, c,) do not depend on the estimated parameters. In the function (41a) in the first approximation $h_H \doteq 1$ may be considered.

The loss function used for this purpose is not linear in parameters and is of the from:

$$S(\beta_\ell) = \sum_j [e_R^2(\omega_j^x; \beta_\ell) + e_J^2(\omega_j^x; \beta_\ell)], \quad (42)$$

$$\ell = 1, \dots, p$$

When estimating parameter optimal values by some gradient method one need still know: the column matrix (p x 1) of the loss function gradients with elements $\partial S/\partial\beta_\ell$ and the Hess square matrix (p x p) with elements $\partial^2 S/\partial\beta_r\,\partial\beta_\ell$, which has two components and should be positively semidefinite one at least.

5. The infuence of various parameters on the loss function.

For analyzing the influence of various parameters β_ℓ in (36) on the loss function (33), results of flight measurements of a light transport A 145 aeroplane described in /1/ and /4/ are used as are also theoretical values of nonstationary aerodynamics paremeters given in ref /4/. With respect to the form of the measurements

τ_W	ξ_H/V	ξ/V_H	–	r_H/V_H	–
[S]	0,065	0,0725	0,082	0,095	0,105
CASE	**LOSS FUNCTION S**				
Q - S ^{x}	0,544	0,524	0,505	0,488	0,485
1 $^{+}$	0,653	–	0,680	0,725	–
5 $^{+}$	0,429	–	0,496	0,571	–
10 $^{+}$	1,004	1,080	1,181	–	–
$^{x)}\ C_{\alpha_a} = 1 - i\omega\tau_W$	$^{+}\ C_{\alpha_a} = \cos\omega\tau_W - i\sin\omega\tau_W$				
CASE 1,5,10, see TAB 4 ;	$\xi_H/V_H = 0{,}068$ s				

TABLE 2 - EFFECT OF TRANSPORT LAG IN $C_{\alpha a}$ FUNCTION ON LOSS FUNCTION OF THE A 145 AEROPLANE

results in ref /1/, the analysis is done with using the dimensional form of circular frequency $\omega = \omega^x/\tau_A$ [s^{-1}] to which estimates of parameter values b_ℓ ($\ell = 1,..,10$) in tab. 4, case 10, correspond.

In the first place the question should be elucidated how to define the transport lag in the frequency transfer $C_{\alpha_a}(i\omega)$ of the downwash angle at tailplane. Further a global evaluation is dealt with of the effect of components of nondimensional normalized frequency transfers C_F, C_H, C_{α_a} due to the circulation and the effect of their inertial components.

5.1 Effect of transport lag

The transport lag τ_W occurs in all the considered expressions of the frequency transfer of the downwash angle at tailplane. In the literature above all four definition expressions are used which are given in tab. 2, where $\tau_W = \xi_H/V$, $\tau_{WH} = \xi_H/V_H$, $\tau_{W\xi} = \xi/V_H$, and $\tau_H = r_H/V_H$ and the definitions of the lengths enclosed are shown in fig.3. According to the theoretical analysis for a correct one the expression $\tau_{WH} = \xi_H/V_H$ should be regarded. The results of proving the given expressions for the four basic cases of parameter values are shown in tab.2. The first two cases correspond to the quasi-stationary model in which all the parameters are considered to be zero, i.e. $b_\ell = T_{H0,25} = 0$ for $\ell = 1,..,10$. These two cases differ just in the form of expressing the transport lag:

for the case Q-S :

$$e^{-i\omega\tau_W} \doteq 1 - i\omega\tau_W; \quad (43)$$

for the cases 1, 5, 10 from tab.4:

$$e^{-i\omega\tau_W} = \cos\omega\tau_W - i\sin\omega\tau_W \quad (44)$$

The cases 5 and 10 from tab.4 correspond to a nonstationary aerodynamic model. In the case 5 just the effect of the inertial components of aerodynamic forces is involved which are represented by the parameters b_8, b_9 and b_{10}. The case 10 corresponds to a complete set of nonzero parameters b_ℓ in a dimensional form.

The corresponding values of the loss function are shown both in tab.2 and in fig.5. In this figure a different character of the dependence (S, τ_W) is striking for the cases where the transport lag is expressed according to eq. (43) or to (44). For the (43) expression the optimal value is $\tau_W \equiv \tau_H = 0{,}095$ s, whilst for (44) this value is $\tau_W = 0{,}065$ s. This may be explained by the fact that expression (43) is formally the same as the expression of the inertial component according to (17), where K_λ has an analogical function like τ_W in (43). It follows then already from tab. 3 that to the decreasing negative values of the parameters b_8 and b_{10}, which correspond to the increasing values of $K_{\lambda F}$ and $K_{\lambda H}$, a diminishing of the loss function follows. The transport lag described by (43) expresses in fact, especially at greater circular frequencies, the other effect than is that of the transport lag, namely the effect of the inertial lift component of tailplane which increases with lengthening the arm in the expression of τ_W. By this the fact may be explained that a physically incorrect length of the arm r_H gives in

			CASE										
			1	2	3	4	5	6	7	8	9	10	11
CIRCULATION	COMPONENT	C_{Fc}	$^oc_F = b_1$ [1]	0	0	0	0	0	0	0	0	0	0,361	0,361
			$T_F = b_2$ [s]	0	0	0	0	0	0	0	0	0	0,0358	0,0358
		C_{Hc}	$^oc_H = b_3$ [1]	0	0	0	0	0	0	0	0	0	0,283	0,283
			$T_H = b_4$ [s]	0	0	0	0	0	0	0	0	0	0,0184	0,0184
		$C_{\alpha Hc}$	$T_1 = b_5$ [s]	0	0,075	0,075	0,075	0	0	0,075	0,075	0,075	0,075	0,075
			$^oh_H = b_6$ [1]	0	0	0,679	0,679	0	0	0	0,679	0,679	0,679	0
			$T_h = b_7$ [s]	0	0	0,0178	0,0178	0	0	0	0,0178	0,0178	0,0178	0
INERTIAL		C_i	$K_{m\dot{\alpha}} = b_8$ [s]	0	0	0	0	-0,0229	-0,0229	-0,0229	-0,0229	-0,0229	-0,0229	-0,0229
			$K_{m\dot{\vartheta}} = b_9$ [s]	0	0	0	0	0	-0,0096	0	0	-0,0096	-0,0096	0
			$K_{m\ddot{\vartheta}} = b_{10}$ [s^2]	0	0	0	0	-0,0020	-0,0020	-0,0020	-0,0020	-0,0020	-0,0020	-0,0020
x = 0,00497 [s]			$T_{H\,0,25}$	0	0	0	x	0	0	0	x	x	x	0
LOSS FUNCTION (j = 1÷25)			[1]	0,653	1,074	1,181	1,137	0,429	0,490	0,958	1,029	1,173	1,004	0,825
S_{max} = 19,069			%	3,43	5,63	6,19	5,96	2,25	2,57	5,03	5,40	6,15	5,27	4,32

TABLE 4 - PARAMETER INFLUENCE ON LOSS FUNCTION OF THE A 145 AEROPLANE. TRANSPORT LAG τ_W = 0,065 s

the quasi-stationary aerodynamics model better results than the physically justified length ξ_H.

5.2 Global influence of various components of nonstationary aerodynamics

In this paragraph by means of the moment loss function S the influence of nondimensional normaliz-

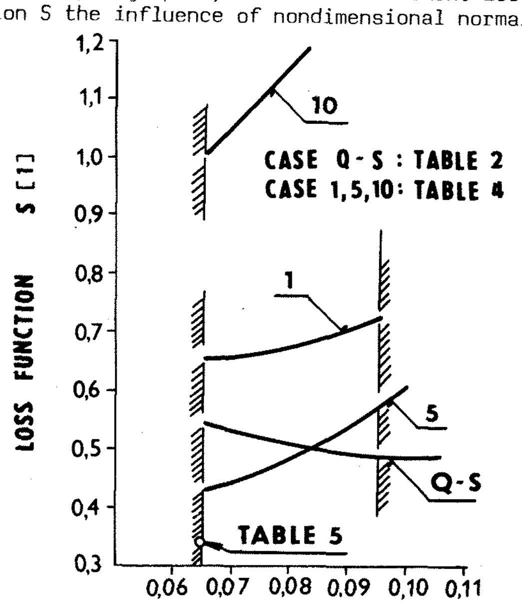

FIGURE 4 - INFLUENCE OF TRANSPORT LAG ON LOSS FUNCTION OF THE A 145 AEROPLANE

ed aerodynamic transfers of various physical origins is researched at the same optimal value of the transport lag τ_W = 0,065 s. The survey of the searched cases is given in tab. 4, where also the correlation of symbols used in the functions C_F, C_H, C_{α_a}, in table 1 and in (36) is shown. The parameters effect is compared according to the respective values of the loss function.

From comparisons of columns 1 and 5 and also of columns 2 and 7 a very favorable effect of inertial lift components is visible with exception of the b_9 parameter in the column 6. An unfavorable effect of using more complicated expression for the C_{α_a} transfer is seen in the columns 3 and 4. The addition of circulation lift components in the column 10 shows an improvement in comparison with the column 9. An improvement in the case 10 is achieved in the column 11 by a simplification of the expression C_{α_a} (b_6 = b_7 = 0) and by omission the b_9 parameter (b_9 = 0). As to this case the value S = 0,825 corresponds, it was taken as a basis for estimation of optimal values of parameters b_i. The case 5 gives the minimum value S = 0,429 which is minor then S = 0,488 for the quasi-stationary model with the transport lag τ_W = 0,095 that involves also the effect of the inertial lift component in the sens of par.5.1. A favorable effect of the inertial lift component is seen also from tab.3.

The analysis of values in tab.3 and 4 supports the statement that in the moment equation of motion the nonstationary aerodynamics effect is applied mainly by means of inertial lift components of the wing and of tailplane (in the real time domain by lift pulses at t = 0).

PARAMETER [s] or [s²]	CASE OF TABLE 4			
	5	-	-	-
$b_8 = K_{m\dot{\alpha}}x \cdot \tau_A$	-0,0229	-0,0270	-0,032	-0,037
$b_9 = K_{m\dot{\Theta}}x \cdot \tau_A$	0	0	0	0
$b_{10} = K_{m\ddot{\Theta}}x \cdot \tau_A^2$	-0,0020	-0,0025	-0,0030	-0,0035
S [1]	0,429	0,402	0,380	0,367
$b_i = T_{H0,25} = 0$; $i = 1,2,3,4,5,6,7$; $\tau_W = 0{,}065\,s$				

TABLE 3 - EFFECT OF INTERTIAL COMPONENT ON LOSS FUNCTION OF THE A 145 AEROPLANE

6. Estimates of parameters of a nonstationary aerodynamics model

As the loss function is not linear in parameters, for their estimation a set of minimum gradient method was used. From orientation calculations performed for initial values which are nearly corresponding to the quasi-stationary aerodynamics model (group A) and carried out for values which correspond to the theoretical values from /4/ (group B), see tab 4, case 11, it follows that in the loss function more local minima exist, some of them giving indeed parameter optimum values, but not always having a physical meaning. Therefore for parameter estimation a space of their values must be given with respect to physical analysis.

A parameter estimation in the limited space of their values was done by the method described in /12/. The survey of the considered parameter values limits, parameter initial values in the A and B groups and the resulting estimates of parameter optimal values are shown in tab. 5. One can see that estimates of parameters, of the loss function values and of the sums of real and imaginary components of the moment equation deviations are nearly the same for both the groups A and B of parameter initial values. The loss function values in tab. 5 in both groups are smaler than in the cases quoted in tab.2, 3 and 4 and they prove the existence of the nonstationary aerodynamics effect especially by means of the inertial components. To the parameters b_8 and b_{10} the nondimensional coefficients $K_{\lambda F}$ = 0.260 and $K_{\lambda H}$ = = 0.433 correspond. The nonzero sums of the moment equation deviations disclose the presence of the systematic errors remnants, to the origin of which any statement can be made for the present (measurement or model). The research is going to be continued also with regarding to the effect of variances of aerodynamic derivatives measured at steady flights (by means of bayesian approach)

7. Conclusions

The object of this paper is an quantitative analysis of the significance of nonstationary aerodynamics effects in a mathematical model of a longitudinal unsteady motion of an aeroplane at a constant flight velocity which was excited by a pulse deflection of the elevator in a slow steady straight flight.

For assuring the physical comparability of expressions it is considered that generalized coordinates are two sorts of physically different angle-of-attack changes: the "attitude" and "path" ones. It was proved that aerodynamic frequency transfers of the whole rigid aeroplane for the "attitude" changes of angle-of-attack are equal to the sum of transfers for the "path" changes of the angle-of-attack and of the transfers for time rates of the attitude angle. The expression for deviations of the moment equation of motion, expressed in the frequency domain, contains both the experimental frequency transfer of the moment of inertial aeroplane forces and the total aerodynamic moment transfer. Frequency transfers of aeroplane responses on an elevator input have in the total aerodynamic transfer the meaning of a measure of exploiting aerodynamic moments at the considered controlled aeroplane motion. With respect to difficulties at "a priori" estimation of a covariance matrix of the motion equation deviations, a loss function is proposed as a simple sum of squares of deviations composed by real and imaginary parts.

The loss function may be used in three ways:
a) for proving correctness of aerodynamic frequency transfers found out by means of numerical calculations or determined experimentally in a wind tunnel;
b) for stuying effects of the form and of values of an analytical model of nonstationary aerodynamics;
c) for estimating optimal values of parameters of aerodynamic nondimensional normalized frequency transfers.

For the last two purposes an analytical expression was proposed for the total aerodynamic transfer which starts from aerodynamic derivatives that were stated by measurements at steady flights. In their expressions three components may be distinguished: a control moments component and components

GROUP	ESTIMATED PARAMETERS			^oc_F	T_F	^oc_H	T_H	T_1	$K_{m\dot{\alpha}}$	$K_{m\dot{\Theta}}$	S (j=2÷25)		$\sum_i e_{R_j}$	$\sum_i e_{J_j}$
				b_1 [1]	b_2 [s]	b_3 [1]	b_4 [s]	b_5 [s]	b_8 [s]	b_{10} [s²]	1	%	1	1
-	PARAMETERS LIMITATION	min		0	0	0	0	0	-0,026	-0,0025	-	-	-	-
		max		0,45	0,04	0,40	0,02	0,08	-0,014	-0,0012	-	-	-	-
A	INITIAL		VALUES	0,45	0,015	0,35	0,010	0,020	-0,018	-0,0015	0,503	2,638	1,555	1,465
	OPTIMAL			0,450	0,0211	0,271	0,020	0	-0,026	-0,0025	0,339	1,780	1,340	0,833
B	INITIAL			0,361	0,0358	0,283	0,0184	0,075	-0,0229	-0,0020	0,823	4,314	1,013	1,864
	OPTIMAL			0,450	0,0211	0,271	0,020	0	-0,026	-0,0025	0,339	1,780	1,340	0,834

TABLE 5 - THE A 145 AEROPLANE AERODYNAMIC PARAMETERS ESTIMATION BY MEANS OF THE GRADIENT METHOD FROM REF. [12]

of aerodynamic moments one of which is of the circulation origin and the second one of the inertial origin. With respect to properties of methods for optimal parameters estimation, for the third mentioned purpose the number of parameters is decreased to the lowest possible value i.e. to seven parameters. Orientation computations have shown that for optimal parameters estimation it is moreover necessary to delimit a space of physically meaningful parameters values.

By an analysis of results of flight measurements done on a light transport A 145 aeroplane it succeeded to make clear a physical difference between the two basic expressions (43) and (44) for the transport lag in the frequency transfer of the downwash angle at tailplane. The aerodynamic inertial component has proved to be the most significant one in the nonstationary aerodynamics model applied in the moment equation of aeroplane motion.

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8. Appendix

The A 145 aeroplane data are taken from /1/ and are given in tab.6. The values of parameters of aerodynamic normalized nondimensional frequency transfer functions C_F, C_H, H_H, h_H and C_{α_a}, approximately expressed in (12), (13), (17), (20) and (41b), are taken from /4/ and are given in tab. 4, case 10 (to the parameters b_8 and b_{10} nondimensional coefficients $K_{\lambda F}$ = 0,267 and $K_{\lambda H}$ = 0,351 correspond).

Aerodynamic derivatives and coefficients from measurements at steady flights are given in tab. 7. Coefficients for computations of the loss function, see tab. 1 and expressions (34), (36) and (40a,b.c) are in tab. 8. The aeroplane responses frequency transfer functions $F_{\alpha.\eta}(i\omega)$ and $F_{\omega y.\eta}(i\omega)$ and the experimental frequency transfer of the pitching moment of inertial forces $Y_E(i\omega)$ for the A 145 aeroplane will be given in an ARTI Report. Mean experimental variances of frequency transfers of aeroplane responses are $(s_E^2)_{\alpha.\eta}$ = 0,004 56 and $(s_E^2)_{\omega y.\eta}$ = 0,015 00 s.

9. Symbols

A, A_F, A_H — Lift of aeroplane, wing an tailplane respectively

$A_{y.x}(s)$ — Unit step admittance- response of the y quantity on a unit step change of the x quantity

b_l — parameter β_l estimate

$C_A = A/qS$ — Lift coefficient of aeroplane

$C_m = M/qSl$ — Pitching moment coefficient

$C_{y.x} = \frac{\partial C_y}{\partial x}$ — Aerodynamic derivative, y = A, m or A_F, A_H ; x = $\alpha, \alpha_\gamma, \Theta, \eta$; $\dot{\alpha}^x, \dot{\Theta}^x, \dot{\eta}^x$; α_F, α_H.

C_W — Drag coefficient

m	kg	1530	v	m/s	54,20
$\tilde{i}_H^2$	1	0,859	H	m	1390
$\tilde{x}_S$	1	0,247	ρ	kg/m^3	1,070
μ	1	112,95	τ_A	s	0,027 31
$\tau = \mu\tau_A$	1	3,084	$1/\tau_A$	s^{-1}	36,622
$\tilde{x}_{NF}$	1	0,126	$\tilde{x}_{SN\Theta}$	1	0,503
$\tilde{x}_{N_FS}$	1	0,121	ξ_H	1	2,373
l	1	1,480	l_H	m	1,030
b	m	12,25	b_H	m	3,39
λ	1	8,78	λ_H	1	3,47
S	m^2	17,09	S_H	m^2	3,31
r	m	5,119	r_H	m	4,940
ξ	m	3,770	ξ_H	m	3,512
$\tilde{l}_H$	1	0,696	$\tilde{\tilde{l}}_H$	1	0,726
$\tilde{r}_H$	1	3,338	$\tilde{\tilde{r}}_H$	1	4,796
$\tilde{S}_H$	1	0,194	$\tilde{\tilde{r}}_H + 0,25$	1	5,046
$\tilde{S}_H \tilde{r}_H$	1	0,646	$\tilde{\tilde{r}}_H + 0,5$	1	5,296

TABLE 6 - CHARACTERISTICS OF THE A 145 SMALL TWIN ENGINED AEROPLANE AND OF STEADY FLIGHTS, REF. [4]

DERIVATIVE	VALUE	COEFFICENT	VALUE
$a = \partial C_{AF}/\partial\alpha_F$	4,735	C_{AO}	0,546
$a_1 = \partial C_{AH}/\partial\alpha_H$	3,261	$k_H = q_H/q$	0,920
a_2/a_1	0,587	$\sqrt{k_H}$	0,959
$d\alpha_a/d\alpha$	0,304		

TABLE 7 - DATA FROM STEADY FLIGHT MEASUREMENTS OF A 145 AEROPLANE

	VALUE		VALUE
m_{F1}	+0,572 935 [1]	m_{H1}	-1,939 427 [1]
$\tau_A m_{F3}$	+0,007 869 [s]	$\tau_A \cdot m_{H3}$	-0,203 503 [s]
m_E x)	+0,072 386 [s^2]	τ_A	0,027 306 [s]
x) $m_E = \mu\tau_A^2 \tilde{i}_y^2$; $Y_E = m_E.(i\omega).F_{\omega y\eta}(i\omega)$			

TABLE 8 - COEFFICIENTS OF THE A 145 AEROPLANE LOSS FUNCTION

$C(i\omega^x)$	Aerodynamic normalized nondimensional transfer function of the Theodorsen type
$C_{\alpha_a}(i\omega^x)$	Normalized nondimensional transfer function for the downwash angle α_a
$F_{y.x}(i\omega^x)=U_{y.x}(\omega^x)+i\,V_{y.x}(\omega^x)$	Frequency transfer function of the response y on the input x
$F_{Cy.x}(i\omega^x)=U_{Cy.x}(\omega^x)+i\omega^x V'_{Cy.x}(\omega^x)$	Aerodynamic transfer function, y = A, m, A_F, A_H; $x = \alpha, \alpha_\gamma, \Theta, \eta; \dot{\Theta}^x$.
$H(i\omega^x), h(i\omega^x)$	Aerodynamic normalized nondimensional transfer functions of the Sears type, related to the leading edge or to 0,25 l_H respectively
$\tilde{i}_y=\sqrt{J_y/ml^2}$	Nondimensional moment of inertia around the y - axis
$k_H = q_H/q$	
$k_1(s)$	Normalized nondimensional lift admittance of the Wagner type
l	Length of aerodynamic mean chord - - aeroplane reference length, [m]
m	Aeroplane mass, [kg]
$q=\varrho V^2/2$	Kinetic pressure, [N/m^2]
r_H	Distance between aeroplane c.g. and tail aerodynamic centre, [m]
$s = Vt/l$	Strouhal number
$S=\sum_j e_j^2$	Loss function, j = 2,...,k
S, S_H	Wing or tailplane area, [m^2]
V	True velocity of an aeroplane, [m/s]
$\alpha, \alpha_F, \alpha_H$	Angle of attack of aeroplane, wing, tailplane respectively
α_a	Downwash angle - positive in opposite sign of α.
$\Delta\alpha_\gamma, \Delta\alpha_\Theta$	"Path" or "attitude" change of angle of attack
β_l	Parameter in the total aerodynamic moment transfer; l = 1,...,p
γ	Flight path inclination angle
Θ	Aeroplane inclination angle
η	Elevator angle
λ	Aspect ratio
$\mu=2m/\varrho Sl$	Aeroplane normalized mass
ϱ	Air density, [kg/m^3]
$\tau_A = l/V$	Aerodynamic unit of time, [s]
ω	Circular frequency, [s^{-1}]
$\omega^x=\omega\tau_A$	Strouhal number - reduced frequency

Denominations

$\bar{x}(i\omega^x)$	Fourier transform of the x(t)
$\hat{x}$	Estimate of x
$\tilde{x} = x/x_{ref}$;	$\dot{x}^x = \tau_A . \dot{x}$
$^\circ x = x/x(0)$	Normalized quantity

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