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MATHEMATICAL MODEL OF LINEAR UNSTEADY AERODYNAMICS OF WHOLE AIRCRAFT

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Summary

The paper gives a brief description of a method which enables linear non-stationary aerodynamic characteristics of an aircraft as a whole to be computed. The method has been derived under the assumption that the aircraft is in an ideal gas flow in the subsonic region.

The actual aircraft is simplified in the computation - it is replaced by the so-called basic shape. The basic shape is then divided into panels and the continuous load of the panels is replaced by a discrete force. The computation, conceived in this manner, contains no integration and has no particular demands on the computer; also the computer times required are sufficiently small so that the method may be used for parameteric studies.

The basic computer program for computing aerodynamic loads was developed to enable the direct use of intermediate results, or of the results of the basic computation, as inputs for other computer programs, for example, the computation of the generalized aerodynamic force coefficient, the induced drag coefficient, and for simulating aero--elastic properties. Further applications were then developed by applying the reversed flow theorem. Some examples of the computations are also given.

List of symbols used

c_p .... pressure coefficient

k .... reduced frequency

L_i^q .... discrete force acting on panel

l_i .... band width (3.7)

m .... number of computation step

M .... Mach number

p .... pressure

$P_q(N,t)$ ... value of aerodynamic transient function at time t

$P_q^*(N,\infty)$ ... steady-state value of transient aerodynamic function

R_{ij} .... distance between point where discrete force acts and point where boundary conditions are satisfied

Δt .... time step

$\vec{U}$ .... velocity of undisturbed flow

$\vec{v}$ .... vector of the absolute disturbance velocity of gas

v_i .... boundary condition on panel

φ .... matrix of normal shapes of oscillations

φ^T .... transposed matrix of normal shapes of oscillations

Δ .... Laplace´s operator

ψ .... angle of band with plain xz

1. Introduction

In recent years, methods of computing the aerodynamic characteristics of whole aircraft are being published more and more frequently. The methods are more or less general and mostly concentrate on solving concrete problems of a particular class of aircraft of a particular manufacturer. The methods also reflect the traditions of the manufacturer and the experience of the authors.

From the very beginning, our intention was to develop a method of computing the basic aerodynamic characteristics, which substantially affect the performance and properties of aircraft of the classes on which research in our institute is concentrated. In developing the method, emphasis was put from the beginning on the usability of the method in practical engineering and minimum requirements as regards computer equipment. The latter and simple preparation of the input data are a prerequisite if the method is to be used for parametric studies in the initial stages of the design. With this object in mind, certain simplifications may be introduced as regards the gas flow, on the one hand, and as regards the aircraft, on the other. The method has been developed for Mach numbers smaller than the critical value. The solution is based on the linear theory, i.e. on the assumption of continuous flow. This assumption is acceptable, because the shapes of aircraft in the principal flight conditions conform to linear dependences of lift , side force and moments on kinematic parameters.

2. Basic formulation of the problem

This method is based on the direct solution of the mathematical formulation of the flow problem of solving a system of linear partial differential equations with the prescribed boundary conditions. Under the assumption that the body in the flow may be considered to be a thin body, and this is possible in practically all the cases of the aircraft class involved, linearized equations of motion of an ideal gas are obtained, using the method of small perturbations, in the absolute coordinate system and dimensionless form:

$$\Delta p - M^2 \frac{\partial^2 p}{\partial t^2} = 0 \qquad (2.1)$$
$$\frac{\partial \vec{v}}{\partial t} = -\operatorname{grad} p$$

Inspite of the considerable variety in design, the aircraft of this particular class have one feature in common, i.e. the small value of the ratio of the thickness measured in the plane perpendicular to the longitudinal axis to the length along this axis. They may thus be considered an thin bodies, which enables the method of small perturbations to be applied in basic flight conditions.

The method of small perturbations assumes that the changes relative to some basic condition are small. Small changes in absolute gas velocity are caused not only by small changes in the kinematic parameters, but also by small geometric changes of the basic shape, which yields a zero absolute disturbance velocity of the gas under non-zero velocity of the undisturbed flow. This indicates that the basic shape may be composed of a system of infinitely thin surfaces parallel with the longitudinal axis of the aircraft. The thickness of the sections of the aircraft in planes perpendicular to the planes of this system and the deviations of the controls are considered to be small perturbations of the basic shape.

Under this choice of the basic shape, the normal velocities on both sides of the surfaces forming the basic shape are the same, and this is sufficient for calculating the aerodynamic derivatives. This concept of the basic shape also enables the values of some other coefficients to be obtained, such as the lift coefficient and the pitching moment coefficient under zero angle of incidence, values of the coefficients due to controls deviations, using the reversed flow theorem. It is known that in the first approximation, i.e. if the problem is linearized, the distribution of thickness has no effect on the aerodynamic derivatives. This is the reason why the effect of the thickness of the aircraft parts relative to the basic shape will not be considered further.

In very much the same way as in the theory of the thin profile, the boundary conditions here are also satisfied in their basic form, which conforms to the adopted accuracy in the case of bodies with a low aspect ratio.

Therefore, not only small values of the normal component of the absolute disturbance velocity of the gas flow are assumed, but also small values of the derivative in the direction of the normal. The method of small perturbations allows this assumption to be upset locally. Here this applies to the neighbourhood of the leading edges of the surfaces forming the basic shape. The singularities in these places will have no substantial effect on the final result of the linear approximation. As in the classical linearized theory of a wing of finite span, the induced drag coefficient of the aircraft as a whole can also be obtained in this case. Figure 1 shows the comparison of the calculation of the induced drag coefficient with the experiment for the jet trainer L 39. The comparison of the computation of the basic characteristics and of the experiment for a rectangular wing with an aspect ratio of 6 is shown in Fig. 2.

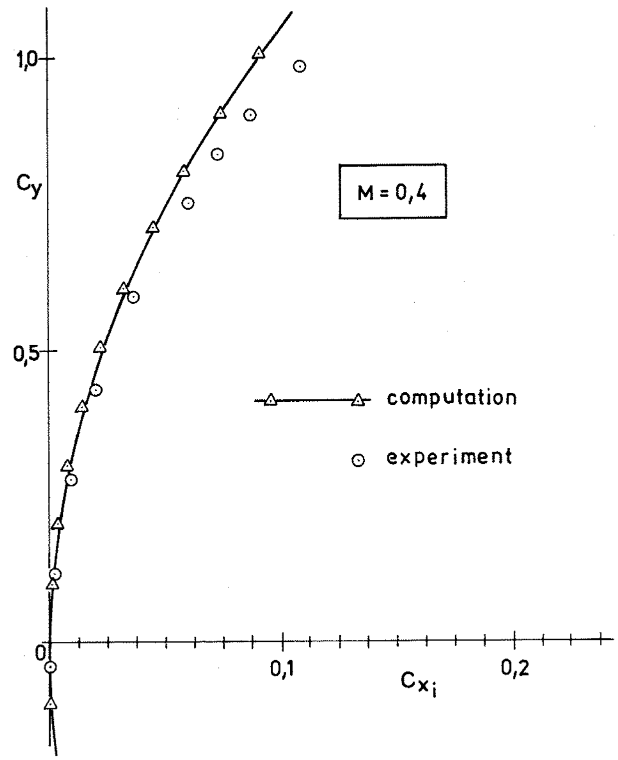

Fig. 1

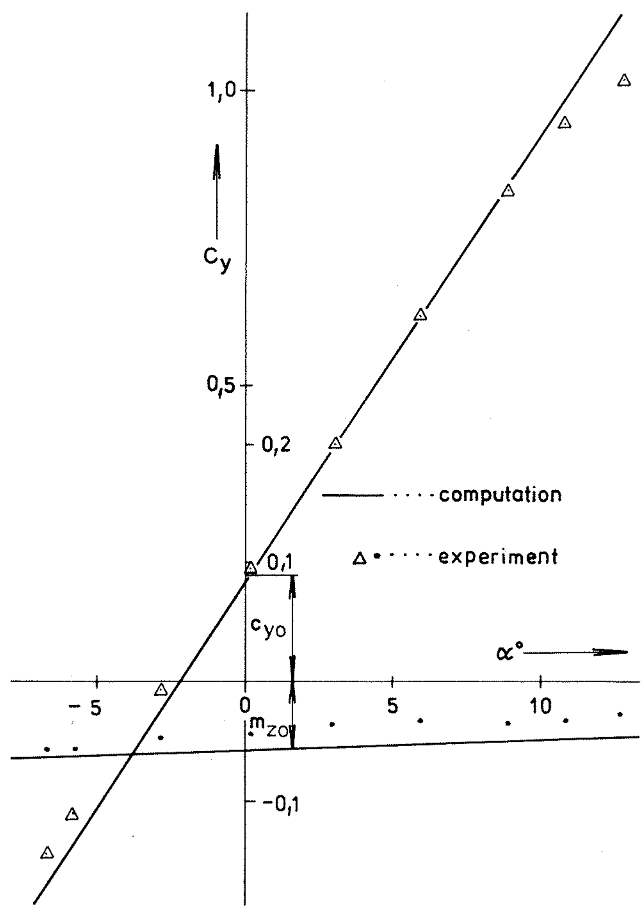

Fig. 2

3. Solution of the general formulation of the problem

The general formulation of the problem reduces to solving system (2.1) for the given values of the normal component of the flow velocity along basic shapes and to solving the problem of deriving the relations between the prescribed normal velocity component and the loading of the basic shapes.

The continuous motion of the basic shapes can be expressed as a continuous sequence of pressure pulses applied to the basic shape at every instant of time. By using the equation for a double layer and subsequent time integration, Eq. (2.1) will yield the following relation between the normal velocity component and the loading of the shape:

$$v_{n(N,t)} = \frac{1}{4\pi}\frac{\partial}{\partial n_N}\left\{\frac{\partial}{\partial n_p}\int_0^t d\tau_0 \times \right. \tag{3.1}$$

$$\left. \times \iint_{S(\tau_0)} \frac{c_{p(P,\tau_0)} H_{(t-\tau_0-MR)}}{R} dS_p \right\}$$

To compute the time variation of the load, the time integration in (3.1) is approximated in the following manner:

$$v_{n(N,t)} = \frac{1}{4\pi}\frac{\partial}{\partial n_N}\left\{\frac{\partial}{\partial n_p}\sum_{q=1}^{m}\iint_{S(t_q)} c_{p(P,t_q)} dS_p \times \right. \tag{3.2}$$

$$\left. \times \int_{t_q-\Delta t}^{t_q} \frac{H_{(t-\tau_0-MR)}}{R} d\tau_0 \right\}$$

$$H = \begin{cases} 1 \ldots (t-\tau_0-MR) \geq 0 \\ 0 \ldots (t-\tau_0-MR) < 0 \end{cases} \tag{3.3}$$

The indices m and q denote the individual time instant. After dividing the basic shape into N^* panels in the way described in Section 4 and replacing the load on each panel by the discrete force L_i^q, we may formally write the system of algebraic equations for computing the panel loads, based on Eq. (3.2):

$$\sum_{q=1}^{m}\sum_{i=1}^{N^*} L_i^q H_{ij}^{m-q} = 4\pi V_j \tag{3.4}$$

$$i = 1,2\ldots N^*$$
$$j = 1,2\ldots N^*$$

In computing the elements of the matrix of system (3.4) H_{ij}^{m-q}, the element is considred to be the difference of two parts,

$$H_{ij}^{m-q} = \left(H_{ij}^{m-q}\right)_{I} - \left(H_{ij}^{m-q}\right)_{II} \tag{3.5}$$

where

$$\left(H_{ij}^{m-q}\right)_{I} = \frac{\partial}{\partial n_N}\left\{\frac{\partial}{\partial n_p}\int_{-\infty}^{t_q}\frac{H_{(t_m-\tau_0-R_{ij})}}{R_{ij}} d\tau_0\right\}$$

$$\left(H_{ij}^{m-q}\right)_{II} = \frac{\partial}{\partial n_N}\left\{\frac{\partial}{\partial n_p}\int_{-\infty}^{t_q-\Delta t}\frac{H_{(t_m-\tau_0-R_{ij})}}{R_{ij}} d\tau_0\right\} \tag{3.6}$$

$$R_{ij} = \sqrt{(x_j-x_i+t_m-\tau_0)^2+(y_j-y_i)^2+(z_j-z_i)^2}$$

These expressions are already given in the frame of reference of the basic shape. After calculating the integrals in (3.6) for the elements of the matrix of system (3.4) we arrive at

$$\left(H_{ij}^{m-q}\right)_{I} = \begin{cases} \dfrac{1}{r_{ji}^2}\left[c_{ji}d_{ji}\left(\dfrac{2a_{ji}^{m-q}}{r_{ji}^2} - b_{ji}^{m-q}\right) - \right. \\ \left. -a_{ji}^{m-q} e_{ji}\right] \\ \\ \dfrac{\pi^2}{3L_i^2}\left[\operatorname{sign} u_{ji}^{m-q} - \operatorname{sign}(x_j-x_i)\right] - \\ -\dfrac{1}{2}\left[\dfrac{(1-M^2)\operatorname{sign}(x_j-x_i)}{(x_j-x_i)^2} - \dfrac{\operatorname{sign} u_{ji}^{m-q}}{(u_{ji}^{m-q})^2}\right] \\ \qquad z_i = z_j \\ \qquad y_i = y_j \end{cases}$$

$$a_{ji}^{m-q} = \frac{u_{ji}^{m-q}}{v_{ji}^{m-q}} - \frac{x_j - x_i}{w_{ji}} \tag{3.7}$$

$$u_{ji}^{m-q} = x_j - x_i + (m-q)\Delta t$$

$$v_{ji}^{m-q} = \sqrt{(u_{ji}^{m-q})^2 + r_{ji}^2}$$

$$w_{ji} = \sqrt{(x_j - x_i)^2 + (1-M^2)\, r_{ji}^2}$$

$$b_{ji}^{m-q} = \frac{(1-M^2)(x_j - x_i)}{w_{ji}^3} - \frac{u_{ji}^{m-q}}{(v_{ji}^{m-q})^3}$$

$$c_{ji} = (y_j - y_i)\cos\psi_i - (z_j - z_i)\sin\psi_i$$

$$d_{ji} = (y_j - y_i)\cos\psi_j - (z_j - z_i)\sin\psi_i$$

$$e_{ji} = \cos\psi_j \cos\psi_i + \sin\psi_j \sin\psi_i$$

$$r_{ji}^2 = (y_j - y_i)^2 + (z_j - z_i)^2$$

provided

$$(m-q)\Delta t \geq F$$

$$F = \frac{M^2}{1-M^2}(x_j - x_i) + \tag{3.8}$$

$$+ \frac{M}{1-M^2}\sqrt{(x_j-x_i)^2 + (1-M^2)\left[(y_j-y_i)^2 + (z_j-z_i)^2\right]}$$

If

$$(m-q)\Delta t < F, \; (H_{ij}^{m-q})_I = 0 .$$

The same relations also hold for $(H_{ij}^{m-q})_{II}$, the only difference being that in Eqs (3.7) and (3.8) $(t_m - t_q)$ is replaced by $(t_m - t_q + \Delta t)$ and $(m - q)$ by $(m - q + 1)$.

4. Basic shapes

The basic shapes should be chosen as simple as possible but sufficiently representative of the aircraft involved. The basic shape must, in the first instance, reflect the effect of the parameter which is most important in the problem being treated.

In general, it may be said that the basic shape is given by substituting the actual shape by the mean areas of the aircraft elements. These areas are then considered to be parts of the planes parallel with the longitudinal axis (the x-axis). Depending on the configuration of the aircraft and possibly on the solution of a partial problem, one then arrives at one of the types of the basic shapes shown in Figs 3 to 7. The basic shape is marked by the bold contour. Experience with basic shapes chosen in this manner indicate some interesting properties, e.g. in computing the characteristics of the forward motion of the aircraft, the basic shape in Fig. 3 may be replaced by the part of the xz-plane bounded by the aircraft ground plan, provided the wing does not have excessive coning and the seperelevation of the tail-plane relative to the wing is small. Experience also indicates that the ground plan can be adopted as the basic shape also in computing the characteristics of the longitudinal motion of thin rotational bodies; the same conclusion was also drawn in /2/.

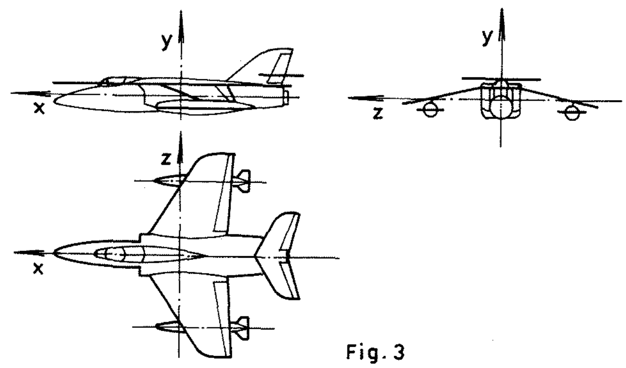

Fig. 3

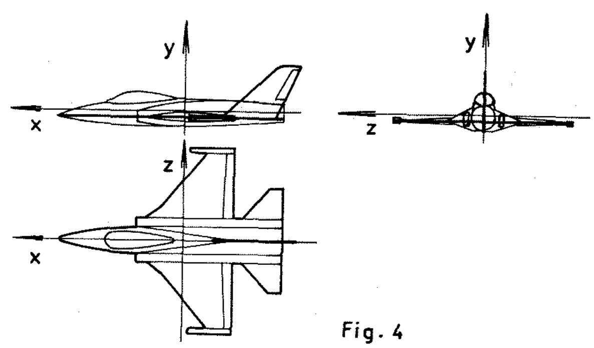

Fig. 4

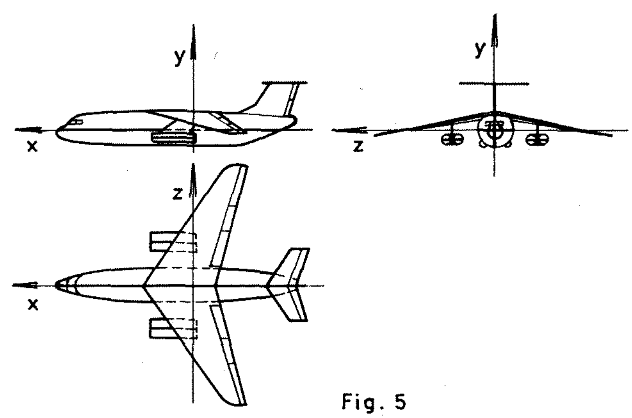

Fig. 5

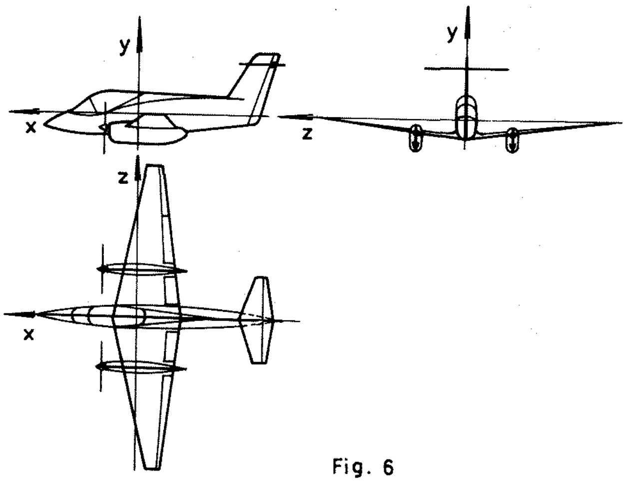

Fig. 6

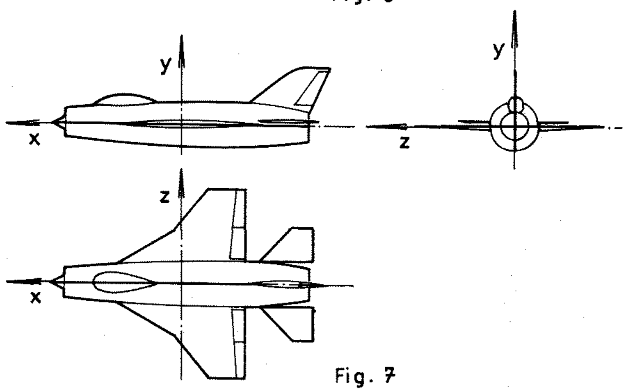

Fig. 7

In computing rockets, it was found that the result was the same whether one adopted the part of the xz-plane bounded by the sections defined by the axis of the fuselage and the stabilizer chord rotated into the xz-plane as the basic shape, or the basic shape shown in Fig. 7.

If the solution is to be effected by the method of discrete forces, the basic shape is divided into small elements - panels. The panels are arranged in sections of constant width, parallel with the longitudinal axis. The edges of the sections are chosen to reflect as best as possible the changes of shape given by coordinates x and z, and also identical with the edges of ailerons, flaps, spoilers, etc. The sections are further divided into bands; the latter are chosen to provide the possibility of computing the wing itself, the aircraft without the tail-planes, etc., with minimum intervention in the input data. The band is also considered to be a rigid part. In developing the method, the opinion was reached that it was sufficient to choose panels in the shape of parallelograms. Experience has shown that, given a sufficient number of panels, the directions of the leading edges of the sections could, to a certain extent, be chosen arbitrarily, e.g. parallel with the tangent to the contour of the basic shape at the point where the section axis intersects the leading or trailing edge of the basic shape, parallel with the z-axis, or with the axis of rotation of one of the deflected parts. A similar deliberation may be carried out in placing the sections on the basic shape, because their lengths are equal to multiples of the chord of the panel. In practical computaions, the leading edge of the band is located along the leading edge of the basic shape, or at the point of intersection of the section axis with the contour of the basic shape. In some cases it has been found advantageous to locate the sections so that the leading edges of the panels lie along the axis of rotation of a deflected part. Locating the sections on the basic shape so that the leading edges of the sections are identical with the leading edge of the basic shape has become established mainly with a view to to the behaviour of the load along the longitudinal axis and to computing the moment characteristics. The division of part of the basic shape into panels is shown in Fig. 8.

The load on each panel is approximated by a discrete force acting in 1/4 of the panel chord, and the boundary condition is satisfied in 3/4 of the panel chord where the axis is located.

5. Calculation of the derivatives of aerodynamic coefficients and transient curves

The division of the basic shape and the computation procedure, described in the previous two sections, enable the derivatives of aerodynamic coefficients to be calculated in two ways.

In the first, classical method, the appropriate boundary condition from Eq. (3,4) is satisfied simultaneously on all panels. In this way the loading of all panels is determined directly and the calculation of the derivatives of the coefficients is then very simple. The second method makes use of the assumption that every band is rigid, i.e. it may be displaced along its normal and rotated about its axis perpendicular to the aircraft axis. If one also considers that it is the linear region we are working in, any deforma-

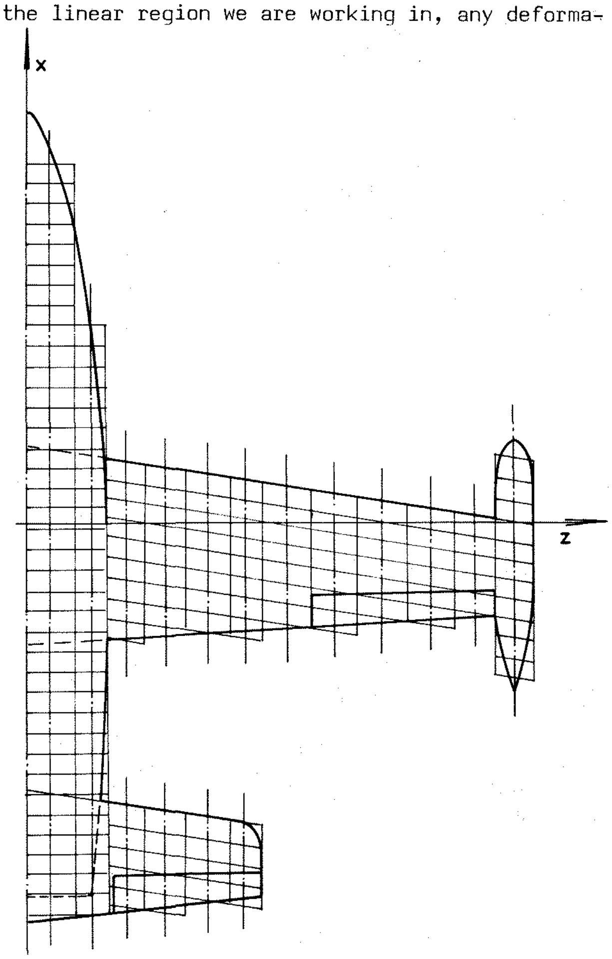

Fig. 8

tion can be compounded of these two deformation created artificially. It is, therefore, advantageous to carry out the computation by considering gradually a unit displacement or rotation of only one band. In this way we obtain the matrix of influence coefficients of the panel loads. These matrices are stored and used to calculate the derivatives of the coefficients for any deformations the basic shapes within the scope of validity of the theory. The actual panel loads are obtained from the load values for the unit displacements by multiplying by the actual deformations, and the derivatives of the aerodynamic coefficients are then calculated in the same way as in the first case. This second method is particularly advantageous, e.g. in computing the coefficients of generalized aerodynamic forces, i.e. at that stage of the project when the external shapes of the aircraft are no longer changed, but the rigidity characteristics may still be subject to considerable changes.

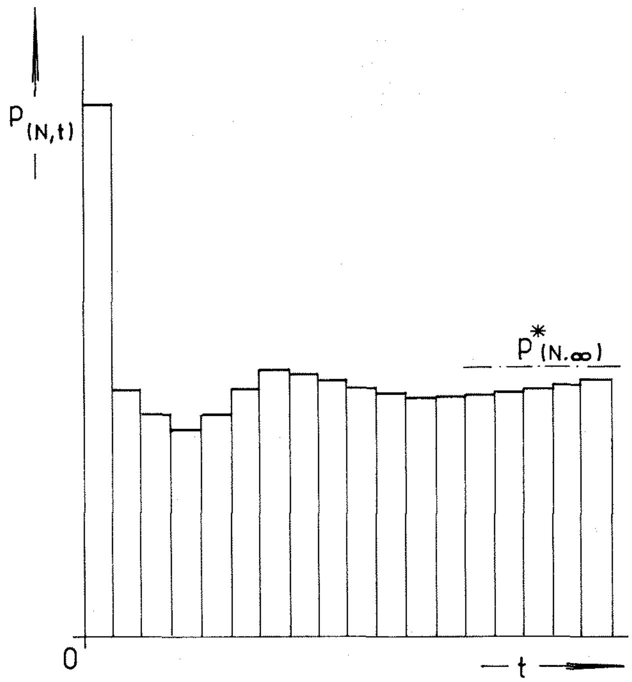

Fig. 9

Equations (3.4) and (3.7) indicate that two procedures may again be adopted in considering unit changes of deformation at a jump in the initial stages of motion. If the time step is taken to be a very large number, steady-state values of the coefficients of aerodynamic derivatives are obtained. If the calculations are made at individual instants of time corresponding to the displacement of the basic shape by one panel length, the aerodynamic transient curves are obtained. The computation is terminated as soon as the increments in the successive steps are small and the values are close to the steady-state value of the derivative, which is also part of the computation. It has been found that in some cases of computing the coefficients of generalized forces of a wing or a T-tail, it is sufficient to carry out the computation of the aerodynamic transient curves in the first few steps and to compute the steady-state value. Increasing the number of steps has only very little influence on the result. This has been verified by computing various cases and also compared with the results of other authors, e.g. /4/, /9/. This can again be applied to both methods of computation mentioned. In this way one obtains the aerodynamic transient curves for the aircraft as a whole, schematically shown in Fig. 9, and for an isolated wing, Fig. 10. This procedure also enables the computation to be carried out for Mach number equal to zero without any modification to the computation procedure at the beginning of the motion as is the case, e.g. in /1/.

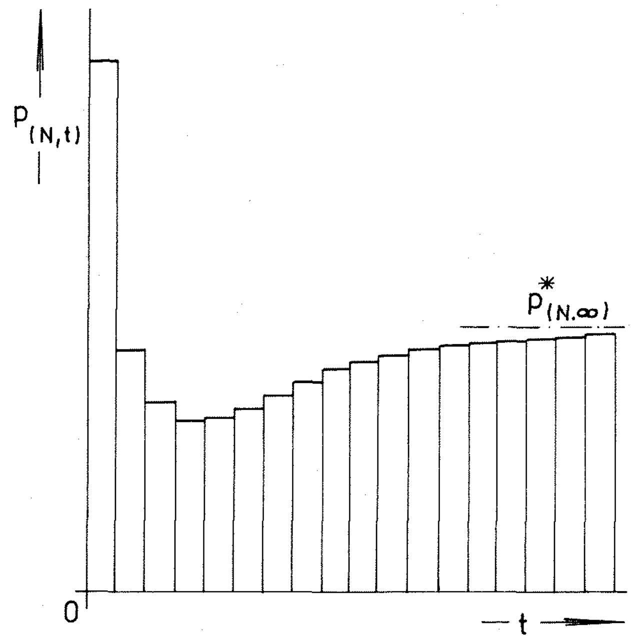

Fig. 10

6. Computation of the general load distribution by means of transient aerodynamic functions

After computing the aerodynamic transient curves, the general time variation of the aerodynamic loading is determined with the aid of Duhamel´s integral. The following relation for the time variation of the pressure function as a function of parameter q(t) is used in the computation:

$$P_{q(N,t)} = P^{*}_{q(N,\infty)}\, q_t + I_{q(N,t)}\, q_{(0)} + \int_0^t I_{q(N,t)} \frac{dq_{(t-\tau)}}{d_{(t-\tau)}}\, d\tau \qquad (6.1)$$

where

$$I_{q(N,t)} = P^{*}_{q(N,t)} - P^{*}_{q(N,\infty)} \qquad (6.2)$$

If the time variation of parameter q(t) is considered to be

$$q_{(t)} = \begin{cases} 0 \ldots\ldots t < 0 \\ \\ e^{-ikt} \ldots\ldots t \geq 0 \end{cases} \tag{6.3}$$

we arrive at the following equation for the quasistationary harmonic motion:

$$P_{q(N,k)} = \left[P_{q(N,k)} e^{-ikt} \right]_{t \to \infty} =$$

$$= q_o \left\{ P^{*}_{(N,\infty)} + ik \int_0^{\infty} I_{q(N,\tau)} e^{-ik\tau} d\tau \right\} \tag{6.4}$$

Let us now take a more detailed look at the case of quasistationary harmonic oscillations under deformation of the aircraft; if we again consider the possibility of resolving a general deformation into two independednt parts (translation and rotation), Eq. (6.4) will yield that part of the load due to translation and rotation. The total load may then be expressed as

$$\bar{P}_{(N,k)} = \frac{P_q + P_{\dot{q}}}{q_o} = P_{q(N,\infty)} + iP_{\dot{q}(N,\infty)} +$$

$$+ ik \int_0^{\infty} I_{q(N,\tau)} e^{-ik\tau} d\tau - k^2 \int_0^{\infty} I_{\dot{q}(N,\tau)} e^{-ik\tau} d\tau \tag{6.5}$$

This indicates that, having solved the fundamental problem, i.e. the computation of the aerodynamic transient curves, the basic conditions of flight mechanics, dynamics and aeroelasticity, as regards knowledge of the time variable loading of the aircraft, can be satisfied with the aid of Eq. (6.1).

7. Simulation of aeroelastic properties of aircraft

The simulation of aeroelastic properties of aircraft can be given as a new example of exploiting aerodynamic computations based on calculating the aerodynamic transient curves and matrices of aerodynamic influence coefficients.

Several mathematical models of aircraft are known, /2/, /5/. The individual models differ namely in the way aerodynamic data have been used. In developing the mathematical model for simulating aeroelastic properties, we started with the combination of modelling the properties of a non--rigid aircraft by means of discrete points of the construction /3/ and of the method of free oscillations /3/, and we used our own computation of the aerodynamic data.

The formula for the dynamic equilibrium of all mass points (panels), which schematically replace the non-rigid aircraft, was taken as the basis; in matrix form this can be expressed as

$$m \Delta \ddot{r} = \Delta L - \Delta F \tag{7.1}$$

where m is the diagonal matrix of weights, Δr the column vector of small changes of position perpendicularly to the surface of the panel relative to the initial state, ΔL the column vector of discrete aerodynamic forces acting along the normals to the panels, and ΔF the column vector of discrete rigidity forces of the construction, acting against the changes of position Δr .

After some algebra using the matrix of normal shapes φ and a coordinate system whose axes are identical with the principal axes of inertia, we arrived at a matrix model which can be expressed as

$$\widetilde{M}(\ddot{q} + 2\varkappa\Omega^2\dot{q} + \Omega^2 q) = \varphi^T \Delta L \tag{7.2}$$

where $\widetilde{M}$ is the matrix of generalized masses, $\varkappa$ construction damping, Ω natural frequency and q the output parameter. The r.h.s. of the equation is in dimensionless form.

The solution of this equation in time is complicated because the r.h.s. depends of the input and output parameters. The input parameters are the deflections of the control elements, wind gusts, atmospheric turbulence, etc.

After resolving the r.h.s. of Eq. (7.2) into two parts, which correspond to the input and output parameters, and considering the computation at instants spaced at Δt, this equation becomes

$$\widetilde{M}(\ddot{q}_m + 2\varkappa\Omega^2\dot{q}_m + \Omega^2 q_m) = \varphi^T \Delta L q_m + \varphi \Delta L_{\varepsilon m} \tag{7.3}$$

$$m = 1, 2 \ldots\ldots$$

q_m is the vector of output parameters, ΔL_{qm} the vector of aerodynamic loading due to output parameters at time t_m , $\Delta L_{\varepsilon m}$ the loading due to the input parameters at time t_m .

To treat the r.h.s., it is advantageous to consider Eq. (4.5) for time t_m in the following matrix form:

$$H_o \Delta L_m + H_1 \Delta L_{m-1} + \ldots\ldots + H_k \Delta L_{m-k} + \ldots\ldots = V_m \tag{7.4}$$

$$m = 1, 2 \ldots\ldots$$

The solution

$$\Delta L_m = A_o V_m + A_1 V_{m-1} + \ldots\ldots \tag{7.5}$$

is used for simulation; here

$$A_0 = H_0^{-1}$$
$$A_1 = -H_0^{-1} H_1 A_0 \qquad (7.6)$$
$$\cdots\cdots\cdots$$

Since the linear theory is involved, the boundary condition for computing aerodynamic loads due to the output parameters may be expressed as

$$V_m = \varphi \dot{q}_m - \frac{\partial \varphi}{\partial x} q_m \qquad (7.7)$$

The same applies to the input parameters, expressed in terms of matrices φ_ϵ which are functions of the generalized coordinates ϵ :

$$V_m = \varphi_\epsilon \dot{\epsilon}_m - \frac{\partial \varphi_\epsilon}{\partial_x} \epsilon_m \qquad (7.8)$$

Based on these relations, the aerodynamic loading due to the output parameters may now be written as

$$\varphi^T \Delta L_{qm} = (\varphi^T A_0 \varphi)\dot{q}_m + (\varphi^T A_1 \varphi)\dot{q}_{m-1} + \cdots$$
$$+ (\varphi^T A_0 \varphi_x) q_m + (\varphi^T A_1 \varphi_x) q_{m-1} + \cdots \qquad (7.9)$$

and the aerodynamic loading due to the input parameters as

$$\varphi^T \Delta L_{\epsilon m} = (\varphi^T A_{0\epsilon} \varphi_\epsilon)\dot{\epsilon}_m + (\varphi^T A_{1\epsilon} \varphi_\epsilon)\dot{\epsilon}_{m-1} + \cdots$$
$$+ (\varphi^T A_{0\epsilon} \varphi_{\epsilon x})\epsilon_m + (\varphi^T A_{1\epsilon} \varphi_{\epsilon x})\epsilon_{m-1} + \cdots \qquad (7.10)$$

where A_k are the matrices computed for the input parameters. Transient functions can be used to an advantage to compute matrices of the types $(\varphi^T A_k \varphi)$ and $(\varphi^T A_k \varphi_x)$ instead of Eq. (7.6) The following holds for the transient function of the j-th generalized coordinate if $t \geq 0$:

$$\dot{q} = (\overbrace{0,0\ldots,1}^{j},\ldots 0)^T = \dot{q}_j^* \qquad (7.11)$$

$$q = (0,0\ldots 0)^T = q_j^*$$

If we denote by ΔL_{jk}^* the vector of aerodynamic loading of the transient function for the j-th generalized coordinate at time t_k , Eq. (7.9) will yield the following for the individual time steps:

$$\varphi^T \Delta L_{j1}^* = (\varphi^T A_0 \varphi)\dot{q}_j^*$$
$$\varphi^T \Delta L_{j2}^* = (\varphi^T A_0 \varphi)\dot{q}_j^* + (\varphi^T A_1 \varphi)\dot{q}_j^* =$$
$$= \varphi^T \Delta L_{j1}^* + (\varphi^T A_1 \varphi)\dot{q}_i^* \qquad (7.12)$$
$$\cdots\cdots\cdots$$
$$\varphi^T \Delta L_{jk+1}^* = (\varphi^T A_k \varphi)\dot{q}_j^* + \varphi^T \Delta L_{jk}^*$$
$$\cdots\cdots\cdots$$

Since $(\varphi^T A_k \varphi)\dot{q}_j^*$ is the j-th column of matrix $(\varphi^T A_k \varphi)$, the whole of matrix $(\varphi^T A_k \varphi)$ can be determined on the basis of Eq. (7.12) from equation

$$\varphi^T A_k \varphi = \varphi^T(\Delta L_{1,k+1}^* - \Delta L_{1,k}^*, \Delta L_{2,k+1}^* - \Delta L_{2,k}^*, \ldots$$
$$\ldots, \Delta L_{j,k+1}^* - \Delta L_{j,k}^*, \ldots) \qquad (7.13)$$

$$\Delta L_{j,0}^* = 0 \qquad k = 1, 2 \ldots$$

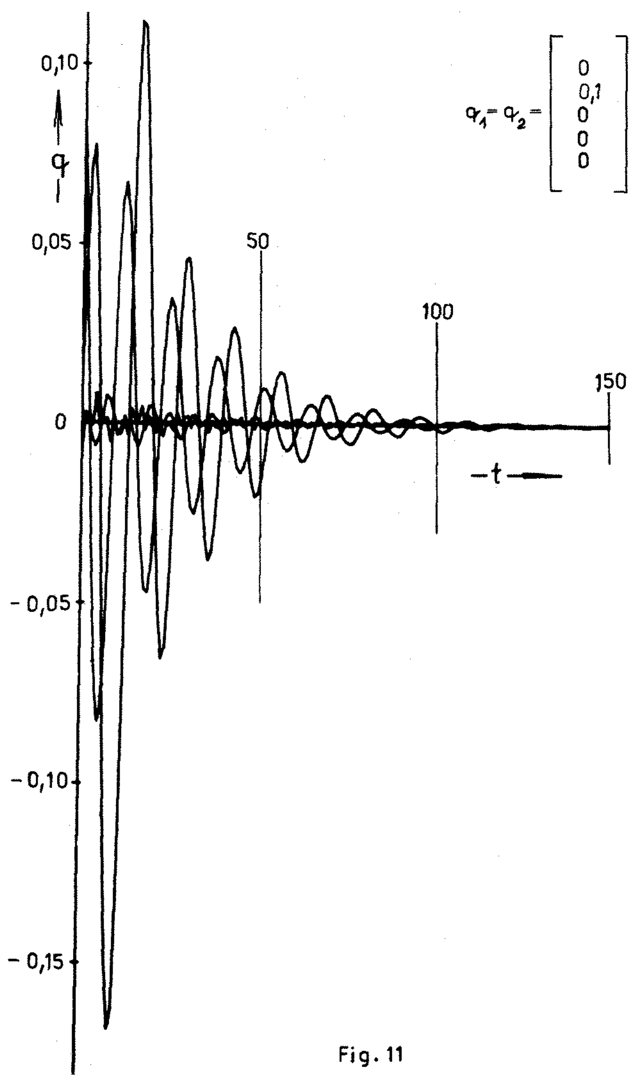

Fig. 11

If the aerodynamic transient functions are known, we are able to determine all the matrices required for modelling aerodynamic loading.Equation (7.13) clearly shows the physical meaning of the elements of matrix $(\varphi^T A_k \varphi)$. For a particular time these elements are proportional to the increment of aerodynamic loading of the transient curve between a particular and the preceding instant of time. It also follows that the corresponding number of matrices of type (7.13) can be used to simulate aerodynamic loading with the required accuracy, because the differences between the individual steps approach zero asymptotically with increasing number of steps, as can be seen from the transient curves. The required accuracy and, consequently, also the number of matrices to be used can be determined from the difference of the time-step value and the steady-state value of the transient function. One can see that it is convenient to take the value of the steady-state transient function as the last loading value.

Figure 11 shows an example of simulating the properties of a trapezoidal wing with an aspect ratio of 10,5.

8. Conclusion

The paper presentsa description of the method of computing non-stationary aerodynamic characteristics of an aircraft, based consistently on computing the aerodynamic transient curves and matrices of aerodynamic influence coefficients.

The computer programs were written for an EC 1040 computer, which is comparable with an IBM 360. The computer time required is relatively short and data preparation does not require very much time either.

The method has already been used for practical purposes and there are possibilities of developing it further.

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