A DIRECT APPROACH FOR THE PANEL METHOD IN COMPRESSIBLE FLOWS

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Abstract

Based on the kinematics of disturbance propagation from moving singularities the influence functions arising from the motion are derived for field points in an unbounded homogeneous medium. For generality, volume elements and surface elements of singularities are considered having arbitrary orientations in space and to the trajectory. Furthermore different translatory motions of the singularity elements, the field points and of the medium are admitted. The spatial and temporal influence functions show some universal relations and characteristic properties in the radiation field. Hence, for calculating the disturbance fields of moving bodies having subsonic or supersonic velocities the method can be applied directly both in the steady and unsteady cases. The solution of the field equation is obtained in the usual way by resorting to integral methods and fulfilling the kinematic boundary conditions on the actual body surface, the surface being subdivided into panel elements. The method then follows the same line as the classical panel method. For Mach number tending to zero all the expressions reduce exactly to the classical expressions for incompressible flow.

Notations

Geometric Quantities

F_o	control surface or singularity surface in the disturbance field
h_S	radial distance of a singularity from a given trajectory
h_o; h_ν	radial distance of a field point from the trajectory of a singularity ($h_o = h_\nu$)
H	height of a field point normal to a panel surface
l_o, l_ν	length of a singularity element and of the corresponding emission segment
l_n	lengths along panel boundary
P, P_o, P_ν	location of field point, singularity and corresponding emission point
r	radial distance in spherical polar coordinate
r_o	radial distance between singularity and field point at the time instant t_o
r_ν	radiation radius or emission radius of a spherical wave
r^*_ν	effective radiation radius
r_n	radial distance between panel-corner points and a field point
$R, R_1, \tilde{R}$	radial distance from field point projection on panel surface to singularity elements and panel contour (Fig. 8)
x,y,z	cartesian coordinates
x_1,y_1,z_1	local coordinates of a panel
$\hat{\varepsilon}_o$	spatial compatibility parameter for fulfilling the kinematic of disturbance propagation
$\vartheta_o, \bar{\vartheta}_o$	inclination angle of r_o to the trajectory of the singularity and of the field point
$\vartheta_\nu, \bar{\vartheta}_\nu$	inclination angle of r_ν to the trajectory of the singularity and of the field point
ϑ^*_ν	inclination of r_ν to the actual emitting surface l^*_ν for signals reaching a field point at time t_o
θ	angular position of surface element of a panel from a reference line
χ_S	inclination of a source element or panel surface to the trajectory
χ	sweep angle of a source line
ψ_o	inclination of the P_oPP_ν-plane to the XY-surface
ψ_S	inclination of the singularity surface to the XY-surface
λ_n	inclination of the panel boundaries to a reference axis (Fig. 8)

Aerodynamic Quantities

a	local sound velocity
a_∞	sound velocity in a homogeneous medium at rest
D	doublet strength
F_i	reduced disturbance force per unit volume
G_{ij}	reduced disturbance force due to momentum exchange per unit volume
I	inducing function comprising resultant influence function at a field point
I_{ij}	inducing function of the ith panel on a field point at panel j
k	wave number (ω/a_∞)
Ma_S	Mach number of the singularity

Ma_E Mach number of a moving field point

$\bar{M}a_E$ Mach number component in the P_oPP_ν -plane of a moving field point

p static pressure

p_∞ static pressure in the undisturbed medium

Δp perturbation pressure $(p - p_\infty)$

q_∞ nominal dynamic head as reference quantity $(\kappa p_\infty Ma_S^2/2)$

Q reduced source strength per unit volume

$\bar{s}$ nondimensional perturbation quantity for density and pressure

S source strength

t time

t_o, t_ν momentary time for signal reaching a field point and the corresponding emission time of the signals

u,v,w perturbation velocities in the medium

V_S velocity of the singularity relative to the medium

V_E velocity of a field point relative to the medium

V total velocity of medium elements based on a moving reference system

β Mach number parameter $(\sqrt{|1-Ma_S^2|})$

β_e, β_T Mach number parameter based on the tangential component along panel surface

κ ratio of the specific heats

$\hat{\kappa}_o$ temporal compatibility parameter to fulfill the kinematics of disturbance propagation

ρ local medium density in the disturbance field

ρ_∞ density of the undisturbed medium

$\Delta\rho$ density perturbation $(\rho-\rho_\infty)$

$\sigma_S ; \sigma_R$ spatial influence functions due to effective stretching of the emission elements or effective shifting of source-sink-elements during emission

$\sigma_D ; \sigma_D^*$ temporal influence functions yielding the Doppler factors at fixed or moving field points

ω_ν circular frequency of the disturbance source

ω_o circular frequency of the signal at the field point

Ω, Ω^* singularity functions

Index

o notation for momentary time

o coordinates relating singularity and field point at a time t_o

ν notation for radiation or emission quantities

i,j direction vector and numbering of panels

n emission locations for signals reaching a field point simultaneously

n, n_1, n_2 numbering of panel edges and corner points

I. Introduction

The propagation of disturbances from space fixed or moving singularities is described by the wave equation. The classical wave equation was first formulated by J.L. d'Alembert [1] for treating the one dimensional case of string vibrations. Thereafter the wave equation was applied extensively to various fields concerning propagation and vibration problems. The solution of the wave equation for spherical radiation of sound waves was first given by S.D. Poisson [2] . Following this result one can derive the two dimensional solution of cylindrical wave motions, as was shown by T. Levi Civita [3] , H. Lamb [4] and J. Hadamard [5]. The most general solutions of wave propagations from spatial distributions of singularities have been given by A. Cauchy [6] , H.v. Helmholtz [7] and G. Kirchhoff [8] , which are very useful for extensive application in the field of acoustics and aerodynamics.

Propagation of waves from moving singularities was first investigated in the field of electromagnetic radiation and propagation of light as is well known from the contribution of C. Doppler [9] . The actual mathematical theory on this topic was established later on by W. Voigt [10], H. Lorentz [11] and H. Poincarê[12] . Some lucid expositions of the physical phenomena due to wave radiation from moving sources could be given after the theory of relativity was postulated in the contributions of A. Einstein [13] , H. Minkowski [14] and some corresponding works.

The perturbation fields of moving singularities in aerodynamics were first formulated when the effect of Mach number or the concept of compressible flows were introduced. Thus, steady flows were treated by O. Janzen [15] , Lord Rayleigh [16] , H. Glauert [17] , L. Prandtl [18] , J. Ackeret [19] , Th. v. Kârmân and N.B. Moore [20] , while unsteady flows were analysed by H. Küssner [21] , C. Possio [22] , and I.E. Garrick [23] . The treatment of wave propagation from moving singularities in the field of aeroacoustics was initiated through the contributions of H. Hönl [24], H. Küssner [25] , N. Rott [26] , H. Billing [27] , H.L. Oestreicher [28] , M.J. Lighthill [23] and I.E. Garrick [30] .

For disturbance propagation from moving sources the linearized wave equation in the moving reference frame is equivalent to the linearized field equation of unsteady aerodynamics. The usual solution procedure for these equations, as is commonly followed, is the application of integral methods using integral transforms or Green's theorem with a suitable basic function. In both these methods the field equation is usually converted to the classical form by resorting to some mathematical transformations analogous to the Prandtl-Glauert-transformation, or using Lorentz-transformation.

In the integral methods for steady or unsteady flow fields the solution procedure involves the use of aerodynamic inducing functions, depending on the nature of the singularities and their locations relative to the field points. With the evolution of the computational fluid dynamics a very flexible and well suited method has been extensively developed and is being classified as panel method. The subsonic panel method in its initial form is developed for incompressible flow as described by J.L. Hess and A.M.O. Smith [31], F.A. Woodward [32] , P.E. Rubbert and G.R. Saaris [33] , Th.E. Labrujere, W. Loeve and J.W. Sloof [34] . Further extensions and applications of this methods are given in the contributions of W. Kraus [35], S.R. Ahmed [36] , F.A. Woodward [37] and J.L. Hess [38] . The treatment of the problem for unsteady subsonic flows were developed by E. Albano and W.P. Rodden [39] , W.P. Jones and J.A. Moore [40] and W. Geißler [41] , while for steady and unsteady supersonic flows the method was worked out by D.L. Woodcock [42] and W.P. Jones and K. Appa [43] . A unified treatment for steady and unsteady subsonic and supersonic flow fields has been formulated by L. Morino, L.T. Chen and E.O. Sucio [44] .

Based on the kinematics of wave propagation as described by N. Rott [26] the influence functions arising from the motion of the disturbance sources have been derived in detail in two papers by A. Das [45][46] . For complete generality the volume elements and surface elements of the moving sources are allowed to have arbitrary orientations in space and with regard to the trajectory.

In the present paper the treatment of flow fields of moving bodies is described, where the inducing functions arising from the singularity panels of the body surface are derived directly to fulfill the kinematic boundary conditions at their pivotal points. The treatment follows similar lines as in the panel methods for incompressible flows, but without resorting to the Prandtl -Glauert-transformation to account for the Mach number effect. Some useful reference text books are cited in [47] to [52] .

II. Basic Equations for Disturbance Propagation from Singularities in Motion

The basic field equation of disturbance propagation from space fixed or moving singularities in an unbounded homogeneous medium is derived from the laws of conservation of mass, momentum and energy. If friction and heat conduction is neglected the field of small disturbance can be assumed isentropic, thus enabling the introduction of a perturbation potential. The field equation in its most general formulation appears as a wave equation and is equivalent to the classical potential equation of unsteady aerodynamics. The equation reduces to a simpler expression if the negative x-axis is made to coincide with the trajectory and the assumption of small disturbances is introduced.

2.1 The linearized field equation and the perturbation quantities

The linearized wave equation in a moving reference frame can be described as follows:

$$\nabla^2\varphi - \frac{1}{a_\infty^2}\frac{D_o^2\varphi}{Dt^2} = \Omega\delta\left(t_o - t_\nu - \frac{r_\nu}{a_\infty}\right)\,\delta(x_\nu - x_o - Ma_S r_\nu) \tag{2.1}$$

with

$$\frac{D_o^2}{Dt^2} = \left[\frac{\partial}{\partial t} + V_S\frac{\partial}{\partial x_o}\right]^2 \tag{2.2}$$

$\vec{\nabla}^2$ as Laplace operator and δ as Dirac delta functions, defining the location of the moving disturbance source Ω on the trajectory at a corresponding time. The disturbance function is given in the most general form

$$\Omega = Q(t) + (\nabla\cdot F_i)(t) + (\nabla\cdot\nabla\cdot G_{ij})(t) \tag{2.3}$$

where the three terms in their respective order are defined as source,dipol, and quadrupole singularities, with Q as the reduced mass flux and F_i and G_{ij} as the reduced force functions per unit volume of the medium.

It is often advantageous to express eq. (2.1) in a space fixed coordinate system with the medium at rest, containing the complete propagating wave system in it as shown in Fig.1. This is achieved by the Galilean transformation

$$\hat{x} = x - V_S t \;;\quad \hat{y} = y \;;\quad \hat{z} = z \tag{2.4}$$

Eq.(2.1) then reduces to the classical wave equation

$$\nabla^2\varphi - \frac{1}{a_\infty^2}\varphi_{tt} = \Omega\delta\left(t_o - t_\nu - \frac{r_\nu}{a_\infty}\right)\,\delta(x_\nu - x_o - Ma_S r_\nu) \tag{2.5}$$

Although the left hand side of eq.(2.5) is identical to the wave equation of a space fixed singularity in a medium at rest, the kinematics of radiation from moving singularities are still retained in the delta functions, through which the propagation field is essentially modified. The solution of eq.(2.1) or eq.(2.5) yields the perturbation potential from which the other field quantities can easily be derived.

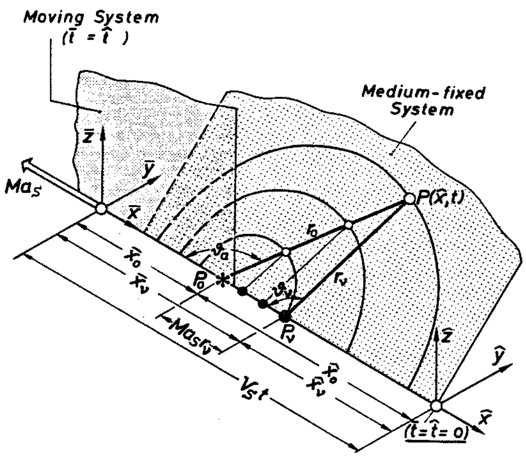

Fig.1 Wave propagation from a singularity in uniform motion presented in a moving and a medium-fixed reference system interrelated by the Galilean transformation.

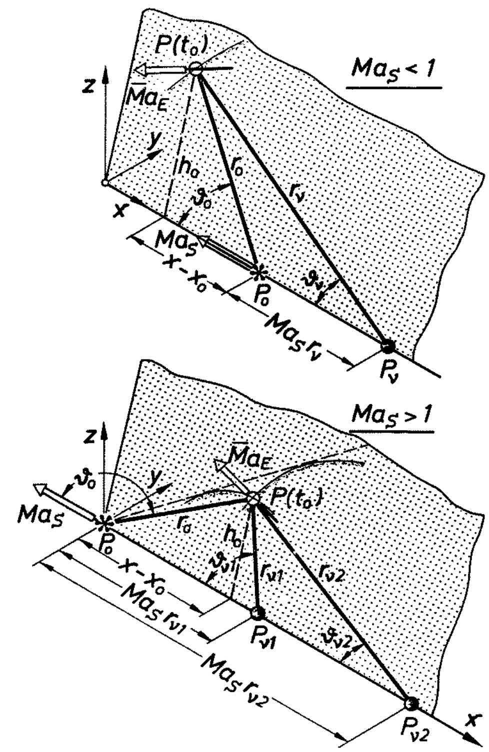

Fig.2 The kinematic relations in the disturbance fields of singularities in uniform motion at subsonic and supersonic velocities.

The pressure and density perturbations in the disturbance field are derived from the generalized Bernoulli equation yielding in the moving reference system

$$\bar{s} = \frac{\Delta\rho}{\rho_\infty} = \frac{\Delta p}{\kappa p_\infty} = -\frac{1}{a_\infty^2}\left[\frac{\partial\varphi}{\partial t} + V_S\frac{\partial\varphi}{\partial x} + \frac{(\nabla\varphi)^2}{2}\right] \quad (2.6a)$$

If a medium fixed reference system is chosen, eq.(2.6) simplifies to:

$$\bar{s} = \frac{\Delta\rho}{\rho_\infty} = \frac{\Delta p}{\kappa p_\infty} = -\frac{1}{a_\infty^2}\left[\frac{\partial\varphi}{\partial t} + \frac{(\nabla\varphi)^2}{2}\right] \quad (2.6b)$$

Due to the linearization of the propagation problem the perturbation quantity $\bar{s}$ also fulfills the wave equation with a new reduced disturbance function Ω^*.

2.2 The kinematics of disturbance propagation for singularities in motion

Let P_o be the momentary position of the singularity Ω moving with a constant velocity V_S, while a disturbance signal propagating with a velocity a_∞ meets the field point P at a time instant t_o, then the location of the emission point P_ν on the trajectory and the retarded time t_ν of signal emission can be determinated from elementary kinematic relations.

The momentary relative orientation between $P(t_o)$ and $P_o(t_o)$ being uniquely defined in space through the quantities r_o and ϑ_o as depicted in Fig.2, the radiation quantities r_ν, ϑ_ν and t_ν are obtained from the following expressions:

$$(1-Ma_S^2)r_\nu^2 - 2r_o Ma_S \cos\vartheta_o r_\nu - r_o^2 = 0$$

$$r_\nu \cos\vartheta_\nu - Ma_S r_\nu - r_o \cos\vartheta_o = 0 \quad (2.7)$$

$$r_\nu = a_\infty(t_o - t_\nu)$$

The solution of eq.(2.7) yields the following relations:

$$\frac{r_{\nu n}}{r_o} = \frac{1}{1-Ma_S^2}\left\{Ma_S\cos\vartheta_o + (-1)^{n+1}\sqrt{1-Ma_S^2\sin^2\vartheta_o}\right\} \quad (2.8)$$

$$\cos\vartheta_{\nu n} = Ma_S + \frac{(1-Ma_S^2)\cos\vartheta_o}{Ma_S\cos\vartheta_o + (-1)^{n+1}\sqrt{1-Ma_S^2\sin^2\vartheta_o}} \quad (2.9)$$

$$t_{\nu n} = t_o - \frac{r_{\nu n}}{a_\infty} \quad (2.10)$$

$$x_{\nu n} = x_o + Ma_S r_{\nu n} \quad (2.11)$$

with

$$n = 1 \qquad \text{for } Ma_S < 1$$

$$n = 1;2 \qquad \text{for } Ma_S > 1$$

Further relations connecting r_o, ϑ_o to r_ν and ϑ_ν are:

$$r_{\nu n}(\cos\vartheta_{\nu n} - Ma_S) = r_o \cos\vartheta_o$$

$$r_{\nu n}\sin\vartheta_{\nu n} = r_o \sin\vartheta_o$$

$$r_{\nu n}(1-Ma_S\cos\vartheta_{\nu n}) = (-1)^{n+1} r_o (1-Ma_S^2\sin^2\vartheta_o)^{1/2} \tag{2.12}$$

$$\sin\varepsilon = Ma_S \sin\vartheta_o$$

For an arbitrary volume- or surface-distribution of singularities the elements will be shifted from the centroid along and normal to its trajectory. In order to determine all the emitting positions in space, such that their signals reach the field point simultaneously at a time instant t_o the following derivatives are considered next.

For elements shifted along the trajectory:

$$\frac{\partial x_\nu}{\partial x_o} = \frac{1}{1 - Ma_S\cos\vartheta_\nu}$$

$$\frac{\partial h_\nu}{\partial x_o} = 0 \tag{2.13}$$

$$\frac{\partial r_\nu}{\partial x_o} = \frac{\cos\vartheta_\nu}{1 - Ma_S\cos\vartheta_\nu}$$

$$r_\nu \frac{\partial\vartheta_\nu}{\partial x_o} = -\frac{\sin\vartheta_\nu}{1 - Ma_S\cos\vartheta_\nu}$$

For elements shifted normal to the trajectory:

$$\frac{\partial x_\nu}{\partial h_S} = -\frac{Ma_S\sin\vartheta_\nu\cos(\psi-\psi_S)}{1 - Ma_S\cos\vartheta_\nu}$$

$$\frac{\partial h_\nu}{\partial h_S} = \cos(\psi-\psi_S) \tag{2.14}$$

$$\frac{\partial r_\nu}{\partial h_S} = -\frac{\sin\vartheta_\nu\cos(\psi-\psi_S)}{1 - Ma_S\cos\vartheta_\nu}$$

$$r_\nu \frac{\partial\vartheta_\nu}{\partial h_S} = \frac{(\cos\vartheta_\nu - Ma_S)\cos(\psi-\psi_S)}{1 - Ma_S\cos\vartheta_\nu}$$

Using these relations the spatial influence functions arising from the motion of the singularities are derived below.

2.3. The spatial and temporal influence functions arising from the motion of the singularities

The perturbation quantities at a field point P in space at the time instant t_o are determined by the singularity strengths at the time of emission t_ν, the radiation distance r_ν and the influence functions arising from the motion of the singularities. The kinematic relations established in section 2.2 show that the emission position may be such that the surface and volume elements of the singularities undergo stretching and shifting effects and the time sequences of signal emission and signal reception are influenced essentially by the Doppler effect. In the disturbance field of moving singularities one encounters four distinct influence functions.

The spatial influence functions

$\sigma_S = \frac{dl_\nu}{dl_o}$ Influence factor due to an effective stretching of the emitting surface- or volume-elements

$\sigma_R = \frac{dr_\nu}{dr_{\nu o}}$ Influence factor due to an effective shift of the emitting source -sink combinations $[dr_{\nu o} \equiv dr_\nu(Ma_S=0)]$

The temporal influence functions

$\sigma_D = \frac{dt_\nu}{dt_o}$ Doppler factor for signals passing through a medium fixed field point being emitted from a moving source.

$\sigma_D = \frac{dt_\nu}{dt_o^*}$ Doppler factor for signals passing through a moving field point being emitted from a moving source.

The effective stretching factor σ_S

For a source distribution on an elementary segment dl_o at P_o being arbitrarily orientated to the trajectory, the derivation of the emission positions for all the signals reaching the field point simultaneously at a time instant t_o yields the effective emission length dl_ν at the location P_ν . This has been derived in detail in [46] . Based on the location of the endpoints of the emission element, its effective stretching is given by

$$\sigma_S = \frac{dl_\nu}{dl_o} = \frac{dx_\nu}{dl_o}\cos\chi_S + \frac{dh_{S\nu}}{dl_o}\sin\chi_S \tag{2.15}$$

Using the partial derivatives already compiled in the eqs.(2.13) and (2.14) one obtains the exact relations:

$$\sigma_S = 1 + \frac{Ma_e\cos\vartheta_{\nu e}}{1 - Ma_S\cos\vartheta_\nu} \tag{2.16}$$

or alternatively

$$\sigma_S = \frac{1}{1 - Ma_e^* \cos\vartheta_\nu^*} \qquad (2.17)$$

where $Ma_e = Ma_S \cos\chi_S$ and $Ma_e^* = Ma_e/\cos(\chi_S - \chi_\nu)$ denote the Mach number components along dl_ν and dl_ν^*. For $\chi_S = 0$ the source elements lie parallel to the trajectory and hence

$$\sigma_S = \frac{dl_\nu}{dl_o} \equiv \frac{dx_\nu}{dx_o} = \frac{1}{1 - Ma_S \cos\vartheta_\nu} \quad \text{für } \chi_S = 0 \qquad (2.18)$$

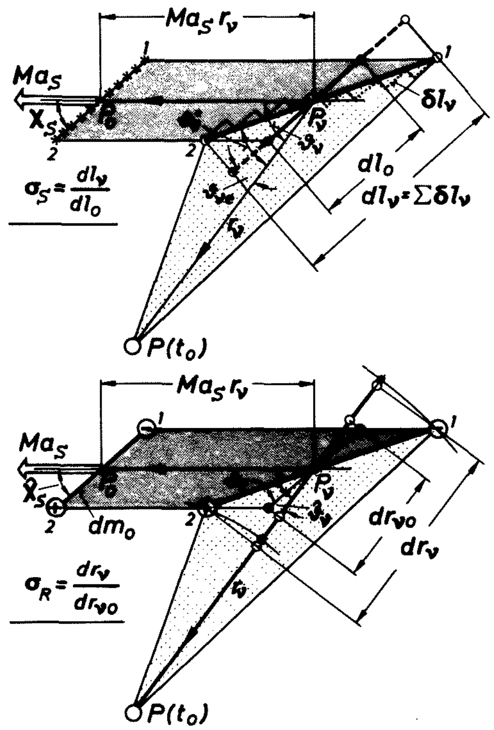

Fig.3 The spatial influence factors σ_S and σ_R due to the effective stretching and shifting of the emission elements for singularities in motion.

For source elements placed normal to the trajectory one obtains

$$\sigma_S = \frac{dl_\nu}{dl_o} \equiv \frac{dh_\nu}{dh_o} = 1 \quad \text{for } \chi_S = \frac{\pi}{2} \qquad (2.19)$$

For the arbitrary sweep angle χ of a two dimensional source line, where the effect of the tangential Mach number $Ma_S \sin\chi$ is completely ignored, the effective motion is $Ma_e = Ma_S \cos\chi$ with $\vartheta_{\nu e}$ as emission angle to the field point. Hence eq.(2.16) yields:

$$\sigma_S = \frac{dl_\nu}{dl_o} = \frac{1}{1 - Ma_e \cos\vartheta_{\nu e}} \quad \text{for } 0 < \chi > \frac{\pi}{2} \qquad (2.20)$$

This is in complete agreement with the classical concept of sweep effects. The stretched emission length dl_ν of a moving element dl_o having an arbitrary orientation to the trajectory is illustrated in Fig.3.

In the formal solution of disturbance fields commonly given in the literature, the stretching factor σ_S is incorporated in the emission radius in the following way

$$r_\nu^* = \frac{r_\nu}{\sigma_S} = r_\nu (1 - Ma_e^* \cos\vartheta_\nu^*) \qquad (2.21)$$

For singularity elements lying in the plane of the trajectory i.e. for $\chi_S = 0$ one obtains

$$r_\nu^* = r_\nu (1 - Ma_S \cos\vartheta_\nu) \qquad (2.22)$$

Using an equivalent rule derived from the kinematic relations this yields:

$$r_\nu^* = r_o (1 - Ma_S^2 \sin^2\vartheta_o)^{1/2} \qquad (2.23)$$

or

$$r_\nu^* = \left[(x-x_o)^2 + (1-Ma_S^2) \left\{ (y-y_o)^2 + (z-z_o)^2 \right\} \right]^{1/2} \qquad (2.24)$$

In the real physical process of the radiation field, σ_S is a stretching factor for the surface or volume of the emitting elements arising solely from the kinematic effect, and not from the compressibility effect as is commonly assumed.

The effective shifting factor σ_R

If a source-sink combination is in motion, the singularity elements are displaced from the trajectory of their centroid. Thus, their relative orientation with respect to the field point differ from each other. Hence for the emitted signals reaching the field point at a time t_o the corresponding emitting points, as derived from the kinematic relations, undergo relative shifts compared to their original displacement from the centroid. If dm_o is the displacement in space of the source-sink elements at P_o, the difference in their emission radius for the signals reaching the field point P at a time instant t_o can be derived easily by using the partial derivatives compiled in eq.(2.13) and eq. (2.14) and following the same line as in the previous section. This yields

$$\frac{dr_\nu}{dm_o} = \frac{\cos\hat{\vartheta}_\nu}{1 - Ma_S \cos\vartheta_\nu} \qquad (2.25)$$

where $\hat{\vartheta}_\nu$ is the angle between the emission radius r_ν and the source-sink axis dm_o at the emission point P_ν. The shifting effect of the source-sink combination is clearly displayed in Fig.3. The prescribed source-sink arrangemant placed at P_ν would have the following difference in emission radius

$$\left.\frac{dr_{\nu o}}{dm_o}\right|_{Ma_S=0} = \cos\vartheta_\nu \qquad (2.26)$$

Hence the shifting factor σ_R due to the motion of the singularity amounts to:

$$\sigma_R = \frac{dr_\nu}{dr_{\nu o}} = \frac{1}{1-Ma_S\cos\vartheta_\nu} \qquad (2.27)$$

This is a purely kinematic effect and is formally accounted for in the classical treatment where partial derivatives of the equivalent expression for r^*_ν as given in eq.(2.24) is carried out for the singularity locations at P_o.

The Doppler-Factor σ_D and σ^*_D

When signals are emitted from moving singularities the time sequence for their passing through a field point in space usually differs from the time sequence of emission of the signals. The ratio of the time sequences dt_ν of emission and dt_o of reception can easily be derived from the partial derivative of eq.(2.10) with respect to dt_o and is generally valid for a fixed or moving field point. As already derived in detail in [45] one obtains for a medium fixed field point:

$$\sigma_D = \left.\frac{dt_\nu}{dt_o}\right|_{\bar{Ma}_E=0} = \frac{1}{1-Ma_S\cos\vartheta_\nu} \qquad (2.28)$$

For a moving field point, with its velocity component being $\bar{Ma}_E$ in the P_oPP_ν-plane, the time derivative of eq.(2.10) yields the expression

$$\sigma^*_D = \frac{dt_\nu}{dt^*_o} = \frac{1}{1-Ma_S^2} + \frac{Ma_S\bar{Ma}_E\cos(\vartheta_\nu+\bar{\vartheta}_\nu)}{1-Ma_S^2} + \frac{Ma_S(\cos\vartheta_\nu-Ma_S)+\bar{Ma}_E\{\cos\bar{\vartheta}_\nu-Ma_S\cos(\vartheta_\nu+\bar{\vartheta}_\nu)\}}{(1-Ma_S^2)\,(1-Ma_S\cos\vartheta_\nu)} - \frac{Ma_S^2\bar{Ma}_E\sin\vartheta_\nu\,\sin(\vartheta_\nu+\bar{\vartheta}_\nu)}{(1-Ma_S^2)\,(1-Ma_S\cos\vartheta_\nu)} \qquad (2.29)$$

The physical process involving the influence factors σ_D and σ^*_D is depicted in Fig.4. So for the frequency relation at reception and emission the simple relation holds:

$$\omega_F = \omega_S \cdot \sigma^*_D \qquad (2.30)$$

For $\bar{Ma}_E = 0$ eq.(2.29) reduces to $\sigma^*_D = \sigma_D$. For field points accompanying the moving reference system eq.(2.29) yields $\sigma^*_D = 1$, for which $\omega_F = \omega_S$.

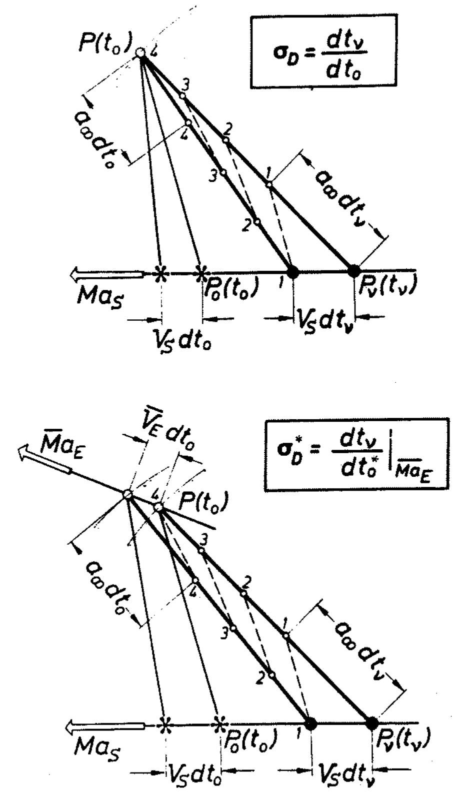

Fig.4 The temporal influence factors σ_D and σ^*_D due to the changed time sequence of signals passing through a fixed or moving field point in relation to the time sequence of the emitted signals from singularities in motion.

III. The Inducing Functions and Perturbation Quantities in the Radiation Field of Two- and Three-Dimensional Singularities in Motion.

The inducing functions in the radiation field of space fixed unsteady singularities in an unbounded homogeneous medium at rest are known from the solution of the classical wave equation. For unsteady disturbance sources moving slowly at a low Mach number, one can neglect the effect of the motion and treat the problem as a quasi -steady case accounting for the corresponding relative positions of the sources and the field points at different time instants. With Mach number increasing the spatial and temporal influence functions arising from the motion of the singularities become increasingly significant. The resulting induction at a field point then depends on the Mach number of the sources and the relative positions in space of the emitting elements, whose signals arrive simulatanously at the time instant t_o. The radiation fields of some basic distributions of moving singularities is illustrated below, first for two and three-dimensional cases. The treatment will then be extended to moving bodies of arbitrary shapes.

3.1 Radiation process and inducing functions due to moving source lines and source surfaces.

If a source line of infinite length set at an arbitrary angle χ to the trajectory is in uniform motion with a Mach number Ma_S, subsonic or supersonic, then the emission points of the signals arriving at a field point P simultaneously at a time instant t_o can be determined from eq.(2.11) as shown in Fig.5. The position of the source line being known at time t_o, the following simple guide line relations define the emission lines L_ν looked for:

$$\frac{r_\nu}{x_\nu - x_o} = \frac{r_\nu}{Ma_S r_\nu} = \frac{1}{Ma_S} \qquad (3.1)$$

This equation represents a hyperbola for $Ma_S<1$, a parabola for $Ma_S=1$ and an ellipse for $Ma_S>1$, with the field point as the focal point of these curves, situated in the plane of the source line. For arbitrary field points in space the emission lines are the envelopes of these basic curves.

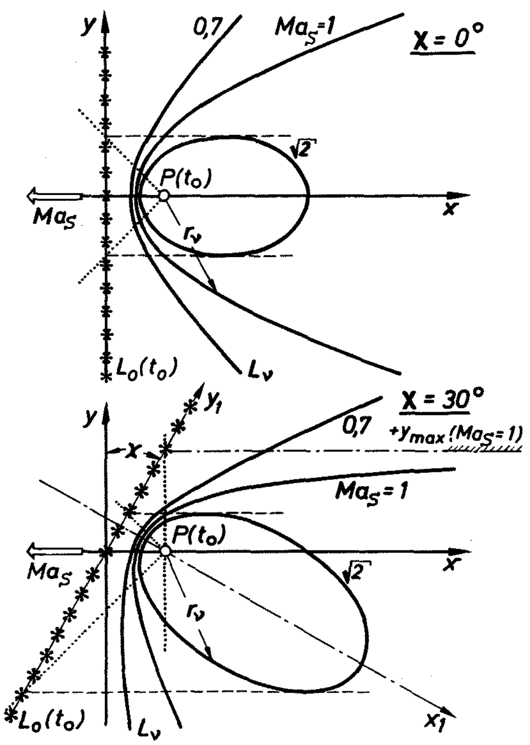

Fig.5 The emission lines L_ν for signals arriving at a field point P(x,o,o) at the time instant t_o, originating from infinite line sources of different sweep angles χ and Mach numbers Ma_S.

For source lines set at an angle χ to the trajectory the emission lines L_ν are no longer symmetrical about the x-axis through the field point, and for $Ma_S>1$ the length of the effective source line lying within the Mach fore cones of the field point will be altered essentially. If a x_1-axis is chosen passing through P and normal to the source line, then all the emission lines reveal complete symmetry about it. The solution procedure in the classical literature resorts to the use of this property.

In the sonic and supersonic case with $Ma_S \geq 1$ a similar procedure would violate the physics, as the signals reaching P from the emission lines L_ν would then be attributed to source elements lying outside the fore-cone as can be seen from Fig.5. Thus, the resolution of the motion Ma_S in normal and tangential components to the source line, as is done in classical treatment is no more than an artifical means to arrive at the correct solution valid only for infinite source lines. For finite source lines with varying source intensity such a procedure will violate extremely the physics of disturbance propagation. The radiation phenomena from moving sources as already mentioned in section 2, will give rise to the spatial and temporal influence functions. As the relative positions in space of the source line, of the field point, and of the emission lines are known all the influence factors σ_S, σ_R and σ_D are completely defined. According to eqs.(2.20), (2.27) and (2.28)

$$\sigma_S = \frac{1}{1 - Ma_e \cos\vartheta_{\nu e}} ,$$
$$\sigma_R = \frac{1}{1 - Ma_S \cos\vartheta_\nu} , \qquad (3.2)$$
$$\sigma_D = \frac{1}{1 - Ma_S \cos\vartheta_\nu} ,$$

where $Ma_e = Ma_S \cos\chi_S$ and $\chi_S = \chi$, the sweep angle ranging from 0 to $\pi/2$. For an infinite source line moving longitudinally but with the source elements placed perpendicularly to the trajectory ($\chi_S=\chi=\pi/2$) their stretching factor reduces to $\sigma_S=1$ for signals meeting any field point in space. If in contrast, a source line moves in the longitudinal direction but with the source elements aligned to it ($\chi_S=0$), as occurring in the axisymmetric case, then the propagation process exhibits some remarkable properties in the region close to the trajectory (x-axis). Eqs.(2.18) and (2.22) show that the effective emission radius for signals meeting a field point at P(x,0,0) simply becomes

$$r_\nu^* = \frac{r_\nu}{\sigma_S} = r_o \qquad (3.3)$$

with ϑ_ν = 0 or π . This means that the Mach number dependency of the inducing functions, contained essentially in r_ν and σ_S as given in eqs.(2.8) and (2.18) drops out exactly in this case. This is the true physical explanation of the slender body effect, as is exactly proven from the kinematics of the radiation process, without resorting to any transformations or assumptions as are commonly called on in the literature. The validity of eq.(3.3) is illustrated in the three examples shown in Fig.6 comprising subsonic and supersonic motion of the sources.

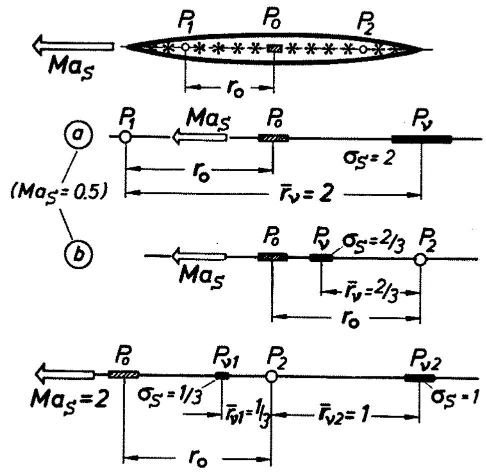

Fig.6 The radiation process for moving source elements and field points on a single trajectory leading to the exact cancellation of the Mach number effect. ($\bar{r}_\nu = r_\nu / r_o$)

In case of an arbitrary singularity line the perturbation quantities at a field point in space and at a time instant t_o can easily be derived, when the inducing function of the emitting singularities and their strengths at the emitting time t_ν in relation to the source elements that have actually passed through them along the true trajectory are known.

For an infinite singularity line, with arbitrary sweep angle χ and carrying a constant source distribution, their resultant inducing function is obtained simply by integrating the expression $1/r^*_\nu$ for all the emission elements, whose signals reach the field point at a time instant t_o. Accordingly for $Ma_S<1$

$$\bar{I}_o = -\ln\left[(\bar{x}-\bar{x}_o)^2 + \beta_e^2\bar{z}^2\right] + C \qquad (3.4a)$$

and for $Ma_S>1$

$$\bar{I}_o = \frac{1}{2\beta_e} \qquad (3.4b)$$

where $\bar{x},\bar{y},\bar{z}$ denote the coordinate system with the $\bar{x}$-axis normal to the source line L_o and $\beta_e^2 = |1-Ma_e^2|$. To perform the integration of the inducing functions over the emission lines L_ν , as shown in Fig.5 formal use is made of the relation

$$r^*_\nu = r_\nu(1-Ma_e\cos\vartheta_{\nu e}) \equiv r'_o(1-Ma_e^2\sin^2\vartheta'_o)^{1/2} \qquad (3.5)$$

For a line segment Δl containing infinite singularity lines with constant strengths all over, the inducing function at a field point is given by

$$\bar{I}_S = -\int_{-\Delta l/2}^{+\Delta l/2} \ln\left\{(\bar{x}-\bar{x}_o)^2 + \beta_e^2\bar{z}^2\right\} d\bar{x}_o \qquad (3.6)$$

for $Ma_S<1$ and

$$\bar{I}_S = -\frac{1}{2\beta_e}\int_{-\Delta l/2}^{+\Delta l/2} d\bar{x}_o \qquad \text{for } Ma_S>1 \qquad (3.7)$$

with $(x_o+\Delta x_o) \leq (x-\beta z)$ and $\Delta x_o = \Delta\bar{x}_o/\cos\chi$ affirming that the line segment and the source lines lie in the Mach fore cone of the field point $P(x,y,z)$, formed with the source line Mach number $Ma_S>1$. Hence for $Ma_S<1$ one obtains

$$\bar{I}_S = -\left[(\bar{x}-\bar{x}_o+\Delta\bar{x}_o)\ln\left\{(\bar{x}-\bar{x}_o+\Delta\bar{x}_o)^2+\beta_e^2\bar{z}^2\right\}\right]_{\Delta\bar{x}_o=-\frac{\Delta l}{2}}^{\frac{\Delta l}{2}}$$
$$-2\beta_e\bar{z}\left[\text{arc tg}\left\{\frac{\bar{x}-\bar{x}_o+\Delta\bar{x}_o}{\beta_e\bar{z}}\right\}\right]_{\Delta\bar{x}_o=-\frac{\Delta l}{2}}^{\frac{\Delta l}{2}} \qquad (3.8)$$

and for $Ma_S>1$

$$\bar{I}_S = -\frac{1}{2\beta_e}\left[\Delta\bar{x}_o\right]_{\Delta\bar{x}_o=-\frac{\Delta l}{2}}^{\frac{\Delta l}{2}} \qquad (3.9)$$

These expressions of the inducing functions given by the eqs.(3.8) and (3.9) are valid for two dimensional steady disturbance fields . Equivalent expressions for the unsteady cases can be derived in a similar way. For treating the disturbance fields of arbitrary singularity distributions in space the inducing functions of the individual surface elements are to be determined.

Now considering an element of the source surface in space moving uniformly with Mach number Ma_S, the trajectory being along the negative $\bar{x}$-axis and the surface inclination to it being χ , the kinematic relations of Sec.2 are used to define the relative locations of the source surface at $P_o(t_o)$ and the emission surface at $P_\nu(t_\nu)$ for signals reaching the field point P at a time instant t_o. The radiation process from the moving source surface is displayed in Fig.7.

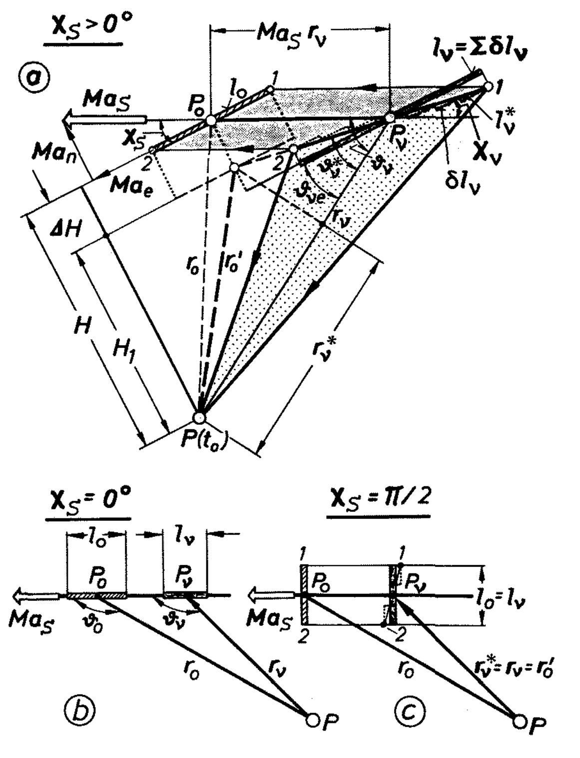

Fig.7 The spatial influence function σ_S of a moving surface singularity element having arbitrary orientations in space and with regard to the trajectory.

The stretching effect of the emission segment l_ν arises through the addition of all the partial elements δl_ν contributing their signals to the field point P. From eq.(2.17) one obtains

$$\sigma_S = \frac{l_\nu}{l_o} = \frac{1}{1 - Ma_e^* \cos\vartheta_\nu^*} \quad (3.10)$$

with Ma_e^* as the component Mach number along the surface l_ν^*, having an angle ϑ_ν^* to the emission radius r_ν. If one applies the equivalent rule using the source surface position at the time instant t_o, the stretching effect of the segment l_o can be incorporated into the effective emission radius:

$$r_\nu^* = \frac{r_\nu}{\sigma_S} = r_\nu(1-Ma_e^*\cos\vartheta_\nu^*) = r_o'(1-Ma_e^{*2}\sin^2\vartheta_o^*)^{1/2} \quad (3.11)$$

It is often advantageous to base the derivation of r_ν^* on the original surface element l_o. This is achieved by shifting the surface element l_o parallel to itself along the height H of P^o such that the centroid of the surface element yields the effective emission radius r_ν^* according to the following relation:

$$r_\nu^* = r_o'(1 - Ma_e^2 \sin^2\vartheta_o'')^{1/2} \quad (3.12)$$

The shift as illustrated in Fig.7 amounts to

$$\Delta H = Ma_S r_\nu \left\{\sin\chi_S - \cos\chi_S \ tg(\chi_S-\chi_\nu)\right\} \quad (3.13)$$

For surface elements with $\chi_S=0$, lying in the plane of the trajectory no shift ΔH of a panel is neccessary and the effective emission radius r_ν^* is identical with the values given by Prandtl-Glauert transformation. For surface element lying normal to the trajectory with $\chi_S = \pi/2$ the stretching of the emission element vanishes as is confirmed through eq.(3.11) and eq.(3.12) yielding $r_\nu^* = r_\nu = r_o'$. In this case the Prandtl-Glauert transformation would yield an effective emission radius r_ν^* differing largely from r_ν and r_o' and hence does not conform to the physical process.

3.2 Inducing Functions due to Moving Source and Doublet Panels

Having established the basic radiation process and the inducing functions of source surface elements it is a simple matter to derive the total inducing effect of a moving source or doublet-panel at a field point in space. A moving source panel 1-2-3-4 having arbitrary orientation to the trajectory induces a perturbation potential at a medium fixed field point P in space.

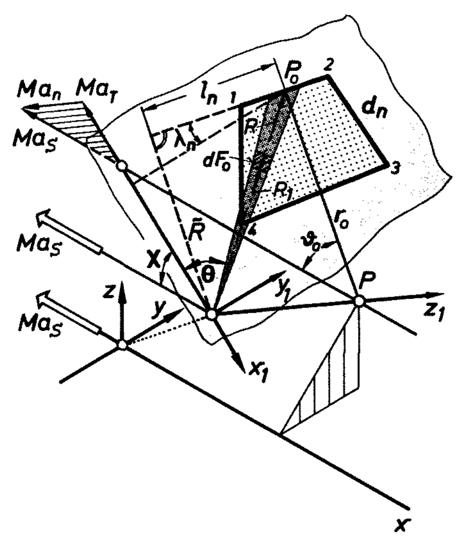

Fig.8 Geometrical relations used in the calculation of the inducing functions of a moving source panel for a given field point in space.

The shape of the panel and its geometric relations to the field point is completely defined and H is the vertical height of P from the plane of the panel surface. The local reference system of the panel surface will be denoted by the coordinates x_1, y_1, z_1 with the footpoint of H as the origin. $\tilde{R}_n$ is a normal to the corresponding panel edge as shown. An elementary surface of the panel is denoted as

$$dF_o = R_1 dR_1 d\theta \tag{3.14}$$

If the panel surface is displaced parallel to itself to the height H_1, so that the effective emission radius of the centroid is matched to the exact value given by the radiation process, then the radial distance of an element from the field point amounts to

$$r_o = \sqrt{R_1^2 + H_1^2} \tag{3.15}$$

and the effective emission radius of the element becomes

$$r_\nu^* = \left[R_1^2(1-Ma_T^2 \sin^2\theta) + \beta_T^2 H_1^2\right]^{1/2} \tag{3.16}$$

with $\beta_T = \sqrt{1-Ma_T^2} = \sqrt{1-Ma_S^2\cos^2\chi}$

For panel surface containing the x-axis ie. for $\chi=0$ one has $Ma_T = Ma_S$ and $H_1 = H$ and thus r_ν^* reduces to the usual expression with the panel surface on the body.

The above considerations suggest that the inducing function of a source panel for a given field point can be presented in the general form:

$$I_S = \oint \int_0^R \frac{R_1 dR_1 d\theta}{\left[R_1^2(1-Ma_T^2\sin^2\theta) + \beta_T^2 H_1^2\right]^{1/2}} \tag{3.17}$$

Carrying out the integrations with respect to R and Θ one obtains

$$I_S = \sum_n \left[\frac{R}{A} \ln\left\{(1-\frac{B}{A}) + \sqrt{(1^2 - \frac{2B}{A}1 + \frac{(C+D)}{A})}\right\}\right]_{n1}^{n2}$$

$$+\sum_n H_1\left[\operatorname{arc\,tg}\left\{\frac{\sqrt{D/A}}{\sqrt{(C/A)-(B/A)^2}} \frac{1-B/A}{\sqrt{1^2-\frac{2B}{A}1+\frac{C+D}{A}}}\right\}\right]_{n1}^{n2}$$

$$+\sum_n H_1\left[\operatorname{arc\,tg}(\beta_T \cdot tg\theta)\right]_{n1}^{n2} \tag{3.18}$$

where

$$A_n = 1 - Ma_T^2 \cos^2\lambda_n$$

$$B_n = Ma_T^2 \tilde{R}_n \cos\lambda_n \sin\lambda_n$$

$$C_n = \tilde{R}_n^2(1 - Ma_T^2 \sin^2\lambda_n) \tag{3.19}$$

$$D_n = \beta_T^2 H_1^2$$

Following the same procedure as in [31] the panel geometry can be defined in terms of the corner point coordinates (x_{n1}, y_{n1}), (x_{n2}, y_{n2}) and the intersection point $(x_o, y_o, 0)$ of H on the panel surface:

$$d_n = \sqrt{(x_{n2}-x_{n1})^2 + (y_{n2}-y_{n1})^2}$$

$$S_n = \sin\lambda_n = \frac{x_{n2}-x_{n1}}{d_n} \; ; \; C_n = \cos\lambda_n = \frac{y_{n2}-y_{n1}}{d_n}$$

$$\tilde{R}_n = \left[(x_o - x_{n1}) C_n - (y_o - y_{n1}) S_n\right]$$

$$tg\chi = \frac{r_S}{x_S} \tag{3.20}$$

$$l_{n1} = -(x_{n1} - x_o)S_n + (y_{n1} - y_o)C_n$$

$$l_{n2} = -(x_{n2} - x_o)S_n + (y_{n2} - y_o)C_n$$

For $Ma_T=0$; $A_n = 1$; $B_n=0$; $C_n = \tilde{R}^2$; $D_n = H^2$ and consequently eq.(3.18) reduces exactly to the expression given by Hess and Smith [31].

For a moving doublet panel the inducing function is derived in a similar way by resorting to a contour integration along the panel sides.

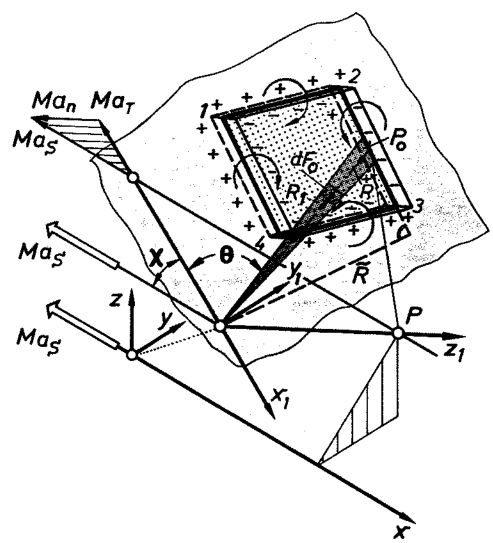

Fig.9 Geometrical relations used in the calculation of the inducing functions of a moving doublet panel for a given field point in space.

With the source-sink axis perpendicular to the panel surface the derivative of the source function of eq.(3.17) with respect to Z_1 or H_1 leads to

$$I_D = \oint \beta_T^2 \int_0^R \frac{H_1 R_1 dR_1 d\theta}{\left[R_1^2(1-Ma_T^2 \sin^2\theta) + \beta_T^2 H_1^2\right]^{3/2}} \tag{3.21}$$

The integration is straight forward, yielding for the inducing function of a doublet panel:

$$I_D = \sum_n \text{arc tg}\left[\frac{\sqrt{\frac{D}{A}}\,(1 - \frac{B}{A})}{\sqrt{\frac{C}{A} - (\frac{B}{A})^2}\sqrt{1^2 - \frac{2B}{A}1 + \frac{(C+D)}{A}}}\right]_{n1}^{n2}$$

$$+\sum_n \Big[\text{arc tg } (\beta_T \cdot \text{tg}\theta)\Big]_{n1}^{n2} \qquad (3.22)$$

with A,B,C and D being identical with the expressions of eq.(3.19).

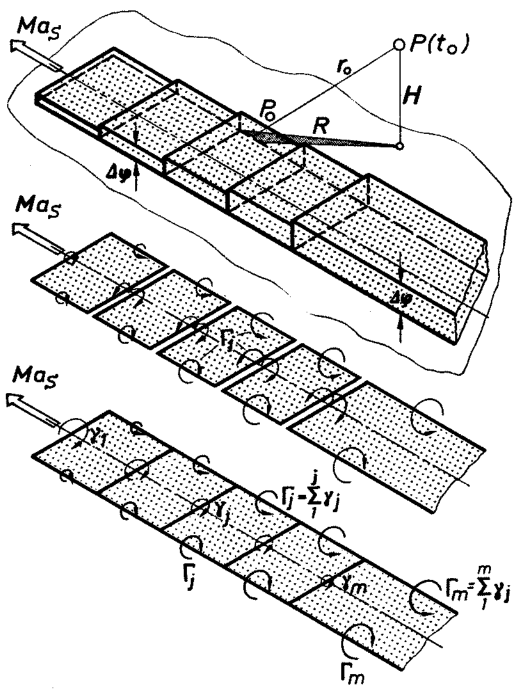

Fig.10 Doublet distribution and the equivalent horse-shoe vortex system for a lifting surface.

3.3 Method of Solution of Perturbation Fields due to the Motion of Arbitrary Bodies

The solution of the perturbation field of an arbitrary body in motion can be obtained by applying the integral method to the linearized wave equation in the moving reference frame or to the linearized field equation based on the perturbation potential. Using Greens theorem the solution is obtained in the following form for $Ma_S<1$

$$\varphi(P) = \frac{1}{4\pi}\iint_{F_o}\frac{S}{r^*_\nu}\,dF_o + \frac{1}{4\pi}\iint_{F_o} D\,\frac{\partial}{\partial n}\left(\frac{1}{r^*_\nu}\right)dF_o \qquad (3.23)$$

with S the source distribution and D the doublet distribution. The kinematic boundary condition at the field points on the surface is

$$\frac{\partial\varphi}{\partial n}(P) = -\vec{n}_i\cdot\vec{V}_\infty \qquad (3.24)$$

Dividing the body surface into panels of surface areas F_{oj} the inducing functions of the panels on a field point can now be introduced into eq.(3.23) in the following manner:

$$\sum_{j=1}^{N} -\vec{n}_i\cdot\left[\nabla\iint_j\left\{\left(\frac{1}{r^*_\nu}\right)\right\}_{ij} dF_o\right]\frac{S_j}{4\pi V_\infty} +$$

$$+\sum_{k=1}^{M} -\vec{n}_i\cdot\left[\nabla\iint_k\frac{\partial}{\partial n}\left\{\left(\frac{1}{r^*_\nu}\right)\right\}_{ik} dF_o\right]\frac{D_j}{4\pi V_\infty} = -\vec{n}_i\cdot\frac{\vec{V}_\infty}{V_\infty}$$

(3.25)

The integral expressions for each panel element has already been evaluated as I_S and I_D in eqs.(3.18) and (3.22).

The bracketed terms being purely functions of the body geometry i.e. of the relative locations of field points and the panels, they can be computed once and for all.

Denoted as inducing coefficients A_{ij} and B_{ij} they lead to a system of N + M equations for the same number of unknowns:

$$\sum_{j=1}^{N} -\vec{n}_i\cdot\vec{A}_{ij}\,X_i + \sum_{k=1}^{M}\vec{n}_i\cdot\vec{B}_{ik}Y_i = -\vec{n}_i\cdot\frac{\vec{V}_\infty}{V_\infty}$$

(3.26)

Here X_i and Y_i denote the unknown source and doublet singularities. The whole procedure then follows the same line as the classical panel method. Having determined X_i and Y_i all the aerodynamic coefficients on the body or in the field around the body can be determined without difficulty. The panel method for supersonic flows can be dealt with in a similar way.

IV. Conclusion

In the panel method outlined in this paper a direct treatment is formulated for arbitrary bodies in compressible flows - subsonic and supersonic . In order to incorporate the effect of the translatory motion of the panels the inducing coefficients are rederived including Mach number terms. The underlying physical principles comprising the basic kinematics of disturbance propagation are outlined extensively. The effects due to the Mach number originate from the spatial and temporal stretching effects in the process of emission and propagation of the disturbance signals. The resultant inducing functions of source- and doublet panels for field points in space are derived in closed form. The calculation of the perturbation field by using integral methods follows the same line as the panel method for incompressible flows. In the limit of Mach number tending to zero, all the influence coefficients reduce to the classical expressions known from literature, thus including the standard panel method as a special case.

V. Literature

[1] d'Alembert, Récherches sur la courbe que forme une corde tendue mise en vibration. Kgl.Akad.Wiss. Berlin 3 (1747), S. 214 ff.

[2] Poisson, S.D. Mémoire sur la théorie des ondes. Mém. de l'Acad.Roy. de Science (1816).

[3] Levi-Civitá, T. Charactéristiques des systemes différentiels et propagation des ondes. Alcan, Paris (1932), (vgl.Nuovo Cimento (4) VI (1897).

[4] Lamb, H. On Wave Propagation in Two Dimensions. Proc. London Math.Soc. (1) XXXV (1902) S. 147.

[5] Hadamard, J. Leçons sur la propagation des ondes et les equations de l'Hydrodynamique. A.Herman Verlag, Paris (1903).

[6] Cauchy, A. Mémoire sur la théorie des ondes .Mém. de l'Acad.Roy. de Science (1827).

[7] v.Helmholtz, H. Theorie der Luftschwingungen der Röhren mit offenen Enden. Crelle, Band LVII (1860), S. 1 ff.

[8] Kirchhoff, G. Zur Theorie der Lichtstrahlen. Berl.Ber. (1882), S.641 ff, Vorl. über Mechanik, Leipzig (1897).

[9] Doppler, C. Über den Einfluß der Bewegung auf die Erscheinungen der Äther-, Luft- und Wasserwellen. Abh. d. Böhmischen Ges.d.Wiss. V.Folge, Band 161 (1847).

[10] Voigt, W. Über das Dopplersche Prinzip. Nachrichten von der kgl. Ges. d. Wiss., 2 (1887), S.41-51.

[11] Lorentz, H.A. Electromagnetic Phenomena in a System Moving with any Velocity less than that of Light. Proc.Acad.Sci.Amsterdam 6, (1904), S. 809.

[12] Poincaré, H. Sur la dynamique de l'éléctron. S.R.Acad.Sci.Paris, 140 (1905), S. 1504.

[13] Einstein, A. Zur Elektrodynamik bewegter Körper. Ann.Phys.Leipzig, 17 (1905), S. 891.

[14] Minkowski, H. Raum und Zeit.Phys. Zeitschrift, 10 (1909), S. 104.

[15] Janzen, O. Beitrag zu einer Theorie der stationären Strömung kompressibler Flüssigkeiten. Phys.Zeitschr. 14 (1913), S. 639-643.

[16] Lord Rayleigh On the flow of compressible fluid past an obstacle. Phil. Mag.32 (1916), S.1-6.

[17] Glauert, H. The effect of compressibility on the lift of an airfoil. Proc.Roy.Soc. A 118 (1925), S. 113-119.

[18] Prandtl, L. Allgemeine Betrachtungen über die Strömung zusammendrückbarer Flüssigkeiten. Proc.5. Volta Kongress, Rom (1935), S. 169-197.

[19] Ackeret, J. Luftkräfte auf Flügel, die mit größer als Schallgeschwindigkeit bewegt werden. Zeitschr.f.Flugtech.u. Motorluftschiffahrt 16, (1925), S. 72-74.

[20] v.Kármán, Th. Moore, N.B. Resistance of Slender Bodies Moving with Supersonic Velocities, with Special Reference to Projectiles. Trans. ASME 54 (1932), S. 303-310.

[21] Küssner, H.G. Allgemeine Tragflächentheorie. Luftfahrtforschung 17 (1940), S. 370-378, NACA Tech. Mem. 979 (1941).

[22] Possio, C. Aerodynamica sul Profilo oscillanto in un fluido compressibile a velocita ipsonora. Aerotechnica, Vol.18 (1938) , S. 441-458.

[23] Garrick, I.E. Nonsteady Wing Characteristics. Aerodynamic Components of Aircraft at High Speeds, Section F (Editors A.F. Donovan and H.R. Lawrence).

[24] Hönl, H. Über das Schallfeld einer gleichförmig-translatorisch bewegten punktförmigen Schallquelle. Annalen der Physik. Leipzig, Bd.43 (1943) S. 437-464.

[25] Küssner, H.G. Lösungen der klassischen Wellengleichung für bewegte Quellen. ZAMM 25 (1945), S. 243-250.

[26] Rott, N. Das Feld einer raschbewegten Schallquelle. Mitt. Nr.9, Mitt.d.Inst.f.Aerodynamik, Eidg.Tech.Hochschule,Zürich (1945).

[27] Billing, H. Geradlinig bewegte Schallquellen. ZAMM Bd. 29 (1949) S. 267-274.

[28] Oestreicher, H.L. The Effect of Motion on the Acoustic Radiation of a Sound Source. Centr.Air. Docum.Off. (Washington). Techn. Data Digest 16, No. 9 (1951), S.16-19.

[29] Lighthill, M.J. On Sound Generated Aerodynamically. I.General Theory. Proc.Roy.Soc.A 211 (1952), S. 564-587.

[30] Garrick, I.E. Moving Sources in Nonsteady Aerodynamics and in Kirchhoffs Formula. Proc. First U.S. Natl.Congr.Appl. Mech., Chicago (1951). Trans.Am.Soc.Mech.Engrs.74 (1952), S.732-739.

[31] Hess, J.L. Smith, A.M.O. Calculation of Potential Flows about Arbitrary Bodies. (Progress in Aeronautical Sciences Vol.8). Pergamon Press (1967).

[32] Woodward, F.A. Analysis and Design of Wing-Body Combinations at Subsonic and Supersonic Speeds. Jour.of.Aircraft 5, pp 528-534 (1968).

[33] Rubbert, P.E. Saaris, G.R. A General Three-Dimensional Potential-Flow Method Applied to V/STOL Aerodynamics. SAE Air Transportation Meeting, New York (1968).

[34] Labrujere, Th.E. Loeve, W. Sloof, J.W. An Approximate Method for Calculation of the Pressure Distribution on Wing-Body Combinations at Sub-Critical Speeds. AGARD Conf. Proceedings Nr.71 (1970).

[35] Kraus, W. Das MBB Unterschall-Panel-Verfahren. Teil II: Das Auftriebsbehaftete Verdrängungs-Problem in kompressibler Strömung. MBB-Bericht Nr.UFE 633-70 (1970).

[36] Ahmed, S.R. Calculation of the Inviscid Flow Field Around Three-Dimensional Lifting Wings, Fuselages and Wing-Fuselage Combinations Using the Panel-Method. ESA-Translation, ESA TT-210, DLR-FB 73-102 (1973).

[37] Woodward, F.A. An Improved Method for the Aerodynamic Analysis of Wing-Body-Tail Configurations in Subsonic and Supersonic Flow. NASA CR-2228 (1973).

[38] Hess, J.L. The Problem of Three-Dimensional Lifting Potential Flow and its Solution by Means of Surface Singularity Distribution. Computer Methods in Applied Mechanics and Engineering 4, pp 283-319 (1974).

[39] Albano, E. Rodden, W.P. A Doublet-Lattice Method for Calculating the Lift Distribution on Oscillating Surfaces in Subsonic Flow. AIAA-Journal 7, pp 279-285 (1969).

[40] Jones, W.P. Moore, J.A. Simplified Aerodynamic Theory of Oscillating Thin Surfaces in Subsonic Flow. AIAA-Journal 11, pp .1305-1309 (1973).

[41] Geissler, W. Nonlinear Unsteady Potential Flow Calculations for Three-Dimensional Oscillating Wings. AIAA-Journ. 16, (1978).

[42] Woodcock, D.L. The Theoretical Prediction of Steady and Unsteady Aerodynaic Loading on Arbitrary Bodies in Supersonic Flow. AGARD Conf. Proceedings 204 (1976).

[43] Jones, W.P. Unsteady Supersonic Aerodynamic Theory by the Method of Potential Gradient. AIAA-Journal, 15 (1977).

[44] Morino, L. Chen, L.T. Sucio, E.O. Steady and Oscillatory Subsonic and Supersonic Aerodynamics Around Complex Configurations. AIAA-Jour.13 (1975).

[45] Das, A. Some Basic and New Aspect on the Disturbance Fields of Unsteady Singularities in Uniform Motion. AGARD CP No. 227 Unsteady Aerodynamics, Paper No. 6, (1977), S.1-31.

[46] Das, A. Kinematik, Einflußfunktionen und Feldgrößen bei Störungsausbreitung von bewegten Störquellen. Int. Ber., FB-SM 129 80/26 DFVLR.

[47] Lord Rayleigh The Theory of Sound. 2 Vol. (1878), 2nd Ed. Dover Publication New York (1945).

[48] Lamb, H. Hydrodynamics, 6th Edition. Cambridge Univ. Press, London (1932), Reprinted 1945, Dover Publication, New York.

[49] Tietjens, O. Strömungslehre, Erster Band Springer Verlag (1960).

[50] Courant, R. Hilbert, D. Methoden der mathematischen Physik, Bd. 1 und 2. Springer Verlag (1968).

[51] Bateman, H. Partial Differential Equations of Mathematical Physics. Dover Publication (1944)

[52] Morse, P.M. Ingard, K.V. Theoretical Acoustics McGraw-Hill Book & Co., (1968).