DETERMINATION OF AIRPLANE AERODYNAMIC PARAMETERS FROM FLIGHT DATA AT HIGH ANGLES OF ATTACK

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Abstract

The problem of determining airplane aerodynamic model equations and estimating the associated parameters from flight data taken at high angles of attack is addressed. Two representations of the aerodynamic function based on the polynomial and spline representations are given. Then the technique of building an adequate model using a stepwise regression is presented with examples demonstrating the construction of the model and various approaches to model verification.

I. Introduction

As documented in numerous papers and reports, the estimation of stability and control parameters from flight data has become a standard procedure for airplanes in flight conditions where the aerodynamic characteristics can be described in linear terms. Only recently the estimation of airplane parameters has been extended into pre- and post-stall flight regimes(1),(2),(3). At high angles of attack the aerodynamic forces and moments could very well be nonlinear in the airplane response and input variables. This introduces a problem of determining how complex the aerodynamic model equations should be. If too many parameters are sought from a limited amount of data, a reduced accuracy in evaluated parameters can be expected or an attempt to identify all parameters might fail. The inaccurate estimates are also obtained if an incomplete model is postulated. Therefore, before the set of parameters can be estimated an adequate model structure must be determined.

The present paper addresses the problem of determining a model structure and estimating the associated parameters from flight data taken at high angles of attack. Following this introduction, the form of the aerodynamic model equations is introduced. The relative merits of polynomial versus spline representation of the aerodynamic function are discussed. The section is completed with a definition of a two dimensional spline for application to the lateral motion of an airplane. The next section presents the stepwise regression technique and its practical application to flight data. The technique of data partitioning is presented and a discussion of the elements that constitute an adequate model is given. The fourth section contains two examples, involving longitudinal and lateral models, in which the techniques presented in previous sections are applied to flight data. A section of concluding remarks completes the paper.

II. Aerodynamic Model Equations

When an airplane is treated as a rigid body, then the general form of equations of motion is very well known. The only problem may be in formulating adequate expressions for aerodynamic forces and moments acting upon the airplane. The mathematical form of aerodynamic model equations was first introduced in 1911 by Bryan(4) and it has been commonly applied since. The aerodynamic model equations can be expressed as

$$y(t) = \theta_o + \theta_1 x_1(t) + \ldots + \theta_{n-1} x_{n-1}(t) \qquad (1)$$

In this equation $y(t)$ represents the resultant coefficient of aerodynamic force or moment (the dependent variable), θ_1 to θ_{n-1} are the aerodynamic parameters (stability and control derivatives), θ_o is the value of any particular coefficient corresponding to the initial steady-state (trim) conditions, and x_1 to x_{n-1} are the airplane response and control variables and their combinations (the independent variables).

The polynomial representation of aerodynamic forces and moments given by eq.(1) results from the Taylor series expansion of these quantities around the values of independent variables at trim conditions. As shown in(2), the polynomial representation is a good approximation of an aerodynamic model when determined from small amplitude flight maneuvers around the initial trim values. However, the polynomial representation may be inadequate for approximating some aerodynamic nonlinearities commonly encountered in large amplitude and high angle-of-attack, α, maneuvers. In these maneuvers the behavior of aerodynamic functions in one region of α may be totally unrelated to their behavior in another region. Due to the differentiable nature of polynomials, a high order polynomial could be required to represent such behavior. Unfortunately the increase in the number of terms in the polynomials often leads to large covariances on the estimated coefficients and poor prediction properties for the model.

To avoid these disadvantages spline functions can be used for approximating aerodynamic force and moment. Splines do not suffer the handicaps of polynomials because they are nonzero only on preselected intervals and because the low order terms may approximate various nonlinearities quite well. A polynomial spline of degree m with continuous derivatives up to degree m-1 approximating a function f(x), $x \in [a,b]$, can be defined as

$$S_m(x) = \sum_{r=0}^{m} C_r x^r + \sum_{i=1}^{k} D_i (x-x_i)_+^m \tag{2}$$

where $(x-x_i)_+^m = (x-x_i)^m, \; x \geq x_i$
$= 0, \; x < x_i$

C_r and D_i are constants, and the knots x_i obey the condition, $a < x_1 < x_2 < \ldots < x_k < b$.

If the vertical force coefficient $C_z(\alpha, q, \delta_e)$ is taken as an example, its polynomial representation has the form

$$C_Z = C_{Z,0} + C_{Z_\alpha}\Delta\alpha + C_{Z_q}(\bar{c}/2V)\Delta q + C_{Z_{\delta_e}}\Delta\delta_e + C_{Z_{\alpha^2}}\Delta\alpha^2 + C_{Z_{q\alpha}}(\bar{c}/2V)\Delta q\Delta\alpha + \text{other higher degree terms} \tag{3}$$

where q is the pitch rate, δ_e is the elevator deflection, $\Delta\alpha = \alpha - \alpha_o$, $\Delta q = q - q_o$, $\Delta\delta_e = \delta_e - \delta_{e,o}$

$C_{Z_\alpha} = \partial C_Z/\partial\alpha$, $C_{Z_q} = \partial C_Z/\partial(q\bar{c}/2V)$,

$C_{Z_{\delta_e}} = \partial C_Z/\partial\delta_e$, etc.

The form based on spline functions can be written as

$$C_Z = C_Z(\alpha)_{q=0,\;\delta_e=0} + C_{Z_q}(\alpha)q\bar{c}/2V + C_{Z_{\delta_e}}(\alpha)\delta_e \tag{4}$$

where

$$C_Z(\alpha) = C_Z(0) + C_{Z_\alpha}\alpha + \Sigma_i A_i(\alpha - \alpha_i)_+$$
$$C_{Z_q}(\alpha) = C_{Z_q} + \Sigma_i B_i(\alpha - \alpha_i)_+^o$$
$$C_{Z_{\delta_e}}(\alpha) = C_{Z_{\delta_e}} + \Sigma_i D_i(\alpha - \alpha_i)_+^o \tag{5}$$

Eqns. (5) indicate that $C_Z(\alpha)$ is approximated by a "broken line" function (the first-degree spline) whereas the remaining two functions are approximated by "staircase functions" (the zero-degree splines). It can be seen by comparing equations (4) and (5) with equation (3) that the spline representation used preserves the concept of stability and control derivatives while providing a representation of the aerodynamic function (C_Z in this example) over an extended range of α.

Splines in the single variable α seem to be sufficient for the approximation of all longitudinal coefficients, i.e. the longitudinal and vertical force coefficient C_X and C_Z, and pitching moment coefficient C_m, even if the aerodynamic coupling (for example due to sideslip angle β) may be present. In the general lateral case, however, the spline in two variables α and β must be considered in approximating the lateral force coefficient C_Y, and rolling and yawing moment coefficients C_ℓ and C_n. The yawing moment coefficient $C_n(\alpha, \beta, p, r, \delta_a, \delta_r)$ taken as an example can be expressed as

$$C_n = C_n(\alpha,\beta)_{\substack{\delta_a=\delta_r=0\\ p=r=0}} + C_{n_p}(\alpha)pb/2V + C_{n_r}(\alpha)rb/2V + C_{n_{\delta_a}}(\alpha)\delta_a + C_{n_{\delta_r}}(\alpha)\delta_r \tag{6}$$

where p and r are the rolling and yawing velocity, and δ_a and δ_r are the aileron and rudder deflection.

To introduce a spline in two variables a rectangle $a \leq x \leq b$, $c \leq y \leq d$ in the (x,y) plane is defined. Then the two ranges [a,b] and [c,d] are subdivided by sets of knots x_i and y_i, where $a < x_1 < x_2 \ldots \; x_k < b$ and $c < y_1 < y_2 \ldots < y_\ell < d$. The points (x_i,y_i) partition the rectangle into rectangular panels. The resulting two dimensional spline for the whole rectangle of degree m for x and n for y with continuous derivatives up to degree m-1 and n-1 can be formulated as

$$S_{mn}(x,y) = \sum_{r=0}^{m}\sum_{s=0}^{n} C_{rs}x^r y^s + \sum_{i=1}^{k} P_i(y)(x-x_i)_+^m + \sum_{j=1}^{\ell} Q_j(x)(y-y_j)_+^n + \sum_{i=1}^{k}\sum_{j=1}^{\ell} D_{ij}(x-x_i)_+^m(y-y_j)_+^n \tag{7}$$

where $P_i(y)$ and $Q_j(x)$ are polynomials of degrees n and m respectively.

Using eq.(7) for $x=\alpha$ and $y=\beta$, and m=0 and n=1, the function $C_n(\alpha,\beta)$ will be approximated as

$$C_n(\alpha,\beta) = C_o + C_1\beta + \sum_{i=1}^{k}(A_{0i} + A_{1i}\beta)(\alpha - \alpha_i)_+^o + \sum_{j=1}^{\ell} B_{0_j}(\beta - \beta_j)_+ + \sum_{i=1}^{k}\sum_{j=1}^{\ell} D_{ij}(\beta - \beta_j)_+(\alpha - \alpha_i)_+^o \tag{8}$$

The remaining functions in (6) are then approximated by splines in α. Regardless of the form of spline approximation, the general expression for the aerodynamic force and moment coefficients remain the same as indicated by eq.(1) which means that y(t) is a linear function of aerodynamic parameters.

III. Model Structure Determination and Parameter Estimation

As indicated in the previous chapter, the general form of the aerodynamic model equations is given by eq.(1) for both types of approximation mentioned. When these equations are postulated, the determination of significant terms among the candidates variables (determination of model structure) and estimation of corresponding parameters (coefficients) can follow. The dependent variables y(t) are obtained from the measured linear and angular accelerations, angular rates and airspeed. The independent variables x(t) are obtained from the measurement of airplane response variables and control surface deflections (β, α, p, q, r, δ_a, δ_e, δ_r).

When a sequence of N observations on both y and x has been made at times t_1, t_2, ..., t_N, and the measured data denoted by y(i) and x(i), i=1,2, ..., N, then these data can be related by the following set of N linear equations (regression equations)

$$y(i) = \theta_o + \theta_1 x_1(i) + \ldots + \theta_{n-1} x_{n-1}(i) + \varepsilon(i) \tag{9}$$

Because eq.(1) is only an approximation of the actual aerodynamic relations, the right-hand side of equation (9) includes an additional term $\varepsilon(i)$, often referred to as the equation error. For N>n and the given form of eq.(1), the unknown parameters can be estimated from the measurement by the method of least squares.

If the model structure is not known, a stepwise regression technique described in (2) and (5) can be applied. The stepwise regression is a procedure which inserts independent variables into the regression model, one at a time. The order of insertion is determined by the partial correlation coefficient which is a measure of the importance of variables not yet in the regression equation.

At every step of the regression, the variables incorporated into the model in previous stages and a new variable entering the model are reexamined using a statistical criterion. This provides a judgment on the contribution made by each variable. The process of selecting and checking variables continues until no more variables will be admitted to the equation and no more are rejected. The complete computing scheme for the stepwise regression can be found in (5).

An adequate model is considered as one which sufficiently fits the data, facilitates the successful estimation of unknown parameters, and has good prediction capabilities. Experience with many test runs showed that the model based only on the statistical significance of individual parameters in the regression equation can include too many terms thus degrading its prediction capability. It is, therefore, recommended in (2) that several quantities be examined as possible criteria for selection of an adequate model.

The regression analysis for model structure determination and parameter estimation provides an opportunity to use subsets of measured data rather then the whole data set. These subsets can be obtained by partitioning the data as a function of one variable, e.g. α. The partitioned data in α can give a better resolution of the structure as a function of α, see (6) and (2). This approach, however, must be used with care. For some aerodynamic coefficients, especially the longitudinal and vertical force, the partitioning of the data into small intervals may introduce limited variation of independent variables within these intervals and thus a degradation in the accuracy of estimated parameters.

The last step in model structure determination and parameter estimation is model verification. The parameter estimates must have realistic values and should be compared with wind tunnel results and theoretical predictions. Whenever possible the least squares estimates should be compared with the estimates using different techniques, e.g. the maximum likelihood estimation method presented in (7) and (8). Finally, the model should be a good predictor within the region of its assumed validity.

IV. Examples

In the following examples the technique for model structure determination and parameter estimation was applied to measured data of a single-engine, low-wing research airplane. This airplane had undergone certain wing leading edge modifications which allowed the airplane to be trimmed at angles of attack up to approximately 24 degrees. The data were available in the form of input and response time histories sampled at .05 sec. The measured data included basically two different sets of maneuvers. For the first set small amplitude longitudinal and lateral maneuvers were excited by control surface deflections at different trimmed conditions within the range $4 < \alpha < 24$ degrees. From these maneuvers local models of aerodynamic coefficients were determined. The second set of data consisted of large amplitude longitudinal, and combined longitudinal and lateral maneuvers with the α variation between 0 and 30 degrees for each maneuver. The large amplitude maneuvers were analyzed for determining an extended model, i.e. a model valid over an extended range of α. The combined maneuvers were intended for determining global model which would be valid within the whole flight envelope.

In Figure 1 some of the results obtained from small amplitude maneuvers are presented. In this figure only the parameters corresponding to the linear terms in eq.(3) are plotted against the α-values corresponding to the trimmed conditions. The resulting relationships may be considered as an extended model for C_Z over the range of α from 4 to 24 degrees. This approach of finding an extended model is time consuming and may be limited by the ability of an airplane to maintain steady-state regimes around and beyond the stall. Therefore in the second approach of obtaining an extended model, a large amplitude longitudinal maneuver was analyzed using the spline representation of the aerodynamic force and moment coefficients. The spline terms in the approximation of C_Z expressed by eq.(4) and (5) are presented in Figure 1 for comparison with the previous results. The 17 knots for splines were postulated as α_1 = 6 deg., α_2 = 7 deg., ..., α_{17} = 22 deg.

The zero-degree spline for the function $C_{Z_q}(\alpha)$ represents rather coarse approximation. In the next step this spline was therefore replaced by the second-degree spline. The new approximation is also plotted in Figure 1 as a dotted line. The refinement in $C_{Z_q}(\alpha)$ approximation did not change the remaining splines significantly. In Figure 2 the spline term of the $C_Z(\alpha)$ function is compared with the measurement of this relationship in the quasi-steady flight represented by a slow deceleration-acceleration maneuver. The quasi-steady measurement resulted in a double-value function $C_Z(\alpha)$ for α between 14 to 22 degrees depending on increasing or decreasing values of α. This phenomenon can be caused by the aerodynamic hysteresis. Because of the relatively small differences in both branches of the $C_Z(\alpha)$ curve, the hysteresis was not modeled in eq.(4).

The development of an adequate model by the stepwise regression is demonstrated in Figure 3 in which the measured values of the coefficient $C_Z(\alpha, q, \delta_e)$ are plotted aginst α for a large amplitude longitudinal maneuver. In Figure 3a, the line represents the model with one term in addition to the constant $C_{Z,O}$. Since α was selected as that term, the model at this point is $C_Z = C_{Z,O} + C_{Z_\alpha}\alpha$. With this two-term-model accounted for, the next most important term selected by the algorithm is $(\alpha - \alpha_7)_+$ with coefficient A_7. This model is represented by the broken line in Figure 3b. Figure 3c reflects the model after the entry of the next most important term $(\alpha - \alpha_6)_+^\circ\, q\bar{c}/2V$ with its coefficient B_6.

Even if the agreement between the results from small amplitude maneuvers and large amplitude maneuver are very good, the resulting model is further verified by simulating the airplane longitudinal responses, using the extended model approximated by splines and the elevator deflection time history from a selected maneuver. In Figure 4 the time histories of input and response variables are presented and the response variables V, α, and q are compared with those predicted by the model. The comparison reveals the good prediction capabilities of the model determined.

The lateral parameters $C_{\ell_p} = \partial C_\ell/\partial(pb/2V)$, C_{n_β} and $C_{n_r} = \partial C_n/\partial(rb/2V)$ from twenty small amplitude maneuvers are plotted against α in Figure 5. The three parameters selected exhibit different degrees of accuracy of their estimates. The estimates of C_{ℓ_p} are very consistent, whereas the values of the remaining two parameters are scattered. The inaccuracy in C_{n_β} and C_{n_r} can be caused by a nonlinear variation of C_n with β and C_{n_β} with α, and/or by a small excitation of the airplane yawing motion. Also in Figure 5, the parameters from small amplitude maneuvers are compared with those obtained from three large amplitude combined maneuvers excited over different range of α. The agreement between the paramater values of C_{ℓ_p} is very good thus confirming the high accuracy of these estimates. The agreement in other two parameters is much worse due to the reasons mentioned above.

For obtaining more accurate estimates of lateral parameters, the measurements from twelve large amplitude maneuvers were joined together into one set of data. The resulting ensemble of about 13,000 data points was then partitioned into 22 subsets according to the values of α. The modeling of the lateral parameters was conducted on one degree subspaces of the 0 to 30 degree α-space. As an example, the model for C_n was postulated as

$$C_n(\alpha=\bar{\alpha}, \beta, p, r, \delta_a, \delta_r) = C_{n_\beta}\beta + \sum_{i=1}^{5} C_{n_{\beta i}} (\beta-\beta_i)_+ + C_{n_p} pb/2V + C_{n_r} rb/2V + C_{n_{\delta_a}}\delta_a + C_{n_{\delta_r}}\delta_r \quad (10)$$

where

$$\begin{aligned}(\beta-\beta_i)_+ &= 0, \; |\beta| < \beta_i \\ &= \beta-\beta_i, \; \beta \geq \beta_i \\ &= \beta+\beta_i, \; \beta \leq -\beta_i\end{aligned}$$

and where $\bar{\alpha}$ denotes the midpoint of an α-interval. The knots of the spline in β were selected at 4, 8, 12, 16 and 20 degrees.

The estimates of the three parameters from partitioned data are presented in Figure 6. The new estimates of C_{n_β} and C_{n_p} are more consistent than those from individual maneuvers and are closer to the results from small amplitude maneuvers. The nonlinear variation of $C_n(\beta)$ for $\alpha = \bar{\alpha}$ and remaining lateral variables equal to zero is demonstrated in Figure 7 for four different values of $\bar{\alpha}$.

As with the longitudinal case, the final step of the verification process is in the predictive abilities of the model. The lateral equations of motion with the aerodynamic model estimated from the combined large maneuvers for $\bar{\alpha} = 22.5$ deg were integrated using the initial conditions and control time histories from a flight trimmed at $\alpha \approx 20$ deg. The control input for this maneuver consisted of a doublet in aileron followed by a doublet in rudder. The predicted time histories are plotted against the actual flight data in Figure 8.

V. Concluding Remarks

It has been shown that the original general form of the aerodynamic force and moment coefficients as presented by Bryan to describe the forces and moments associated with small departures from trimmed flight have a much more global applicability. To estimate a model at high angles of attack, two representations of the Bryan form were given. In the first form, several non-linear polynomial terms are added to the linear model. In the second, the aerodynamic coefficients are represented by splines with knots in angle of attack (longitudinal), and angle of attack and sideslip angle (lateral). The technique of building an adequate model using a stepwise regression was presented with examples demonstrating the construction of the model and various approaches to model verification. It is felt by the authors that the above techniques, when carefully applied to good flight data, can estimate an aerodynamic model for an airplane throughout its flight regimes.

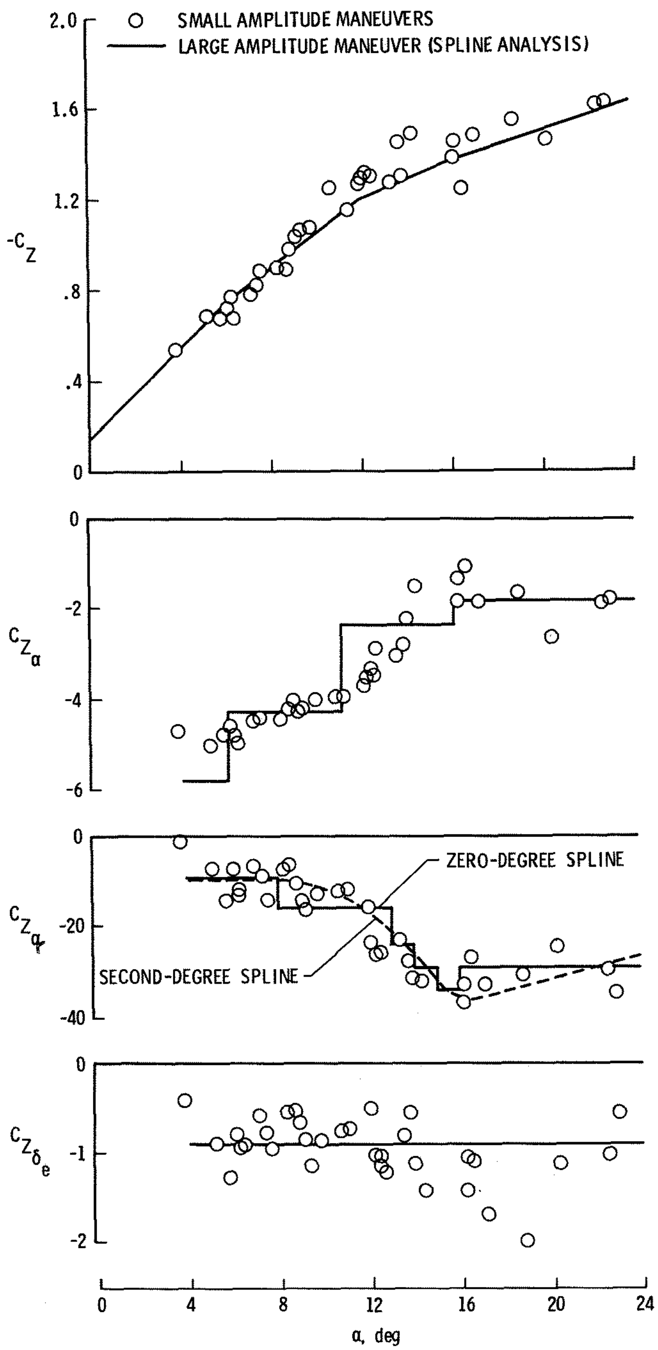

Figure 1. Comparison of Longitudinal Parameters Estimated From Different Maneuvers

VI. References

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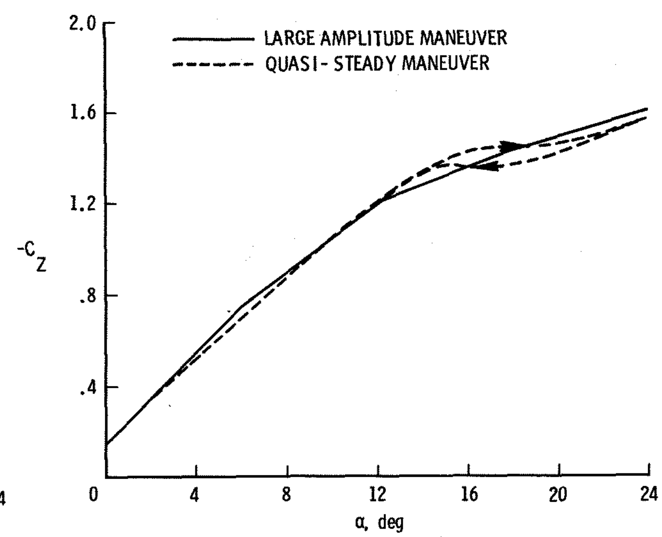

Figure 2. Comparison of Vertical Force Coefficient in Steady State Conditions Estimated From Different Maneuvers

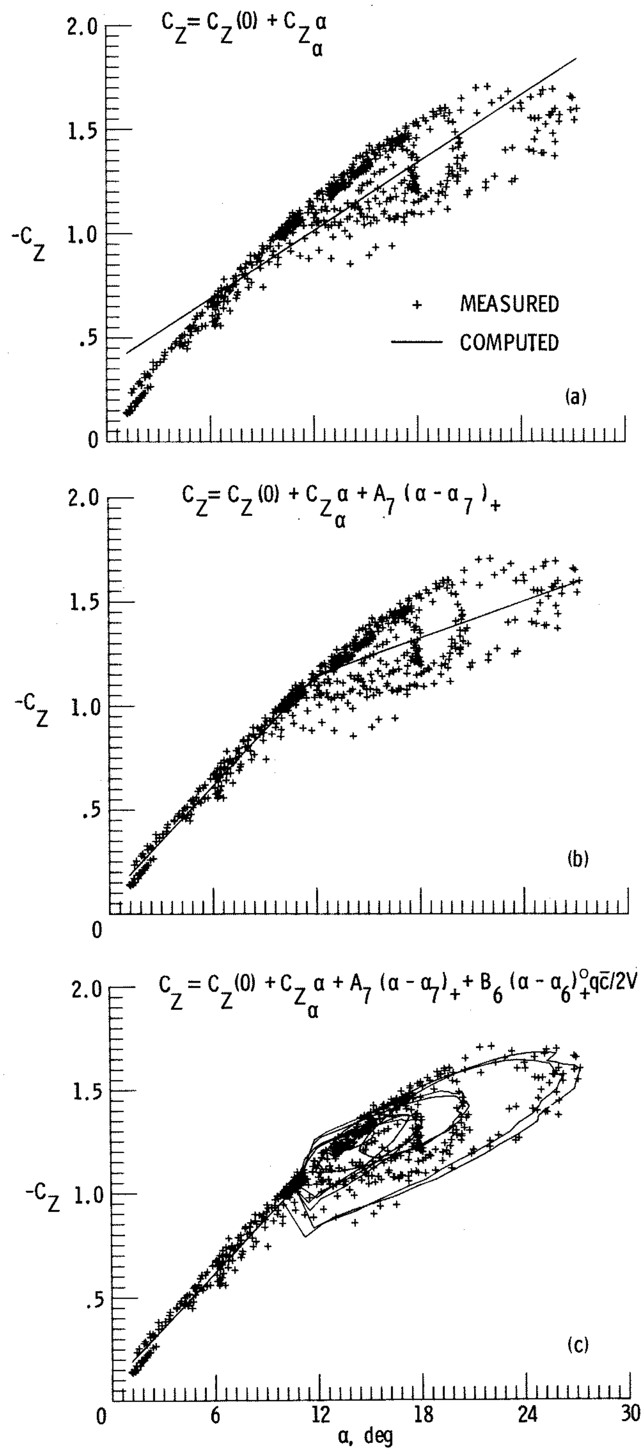

Figure 3. Model Structure and Calculated Vertical Force Coefficient at Different Entries of Stepwise Regression Algorithm Compared with Measured Values

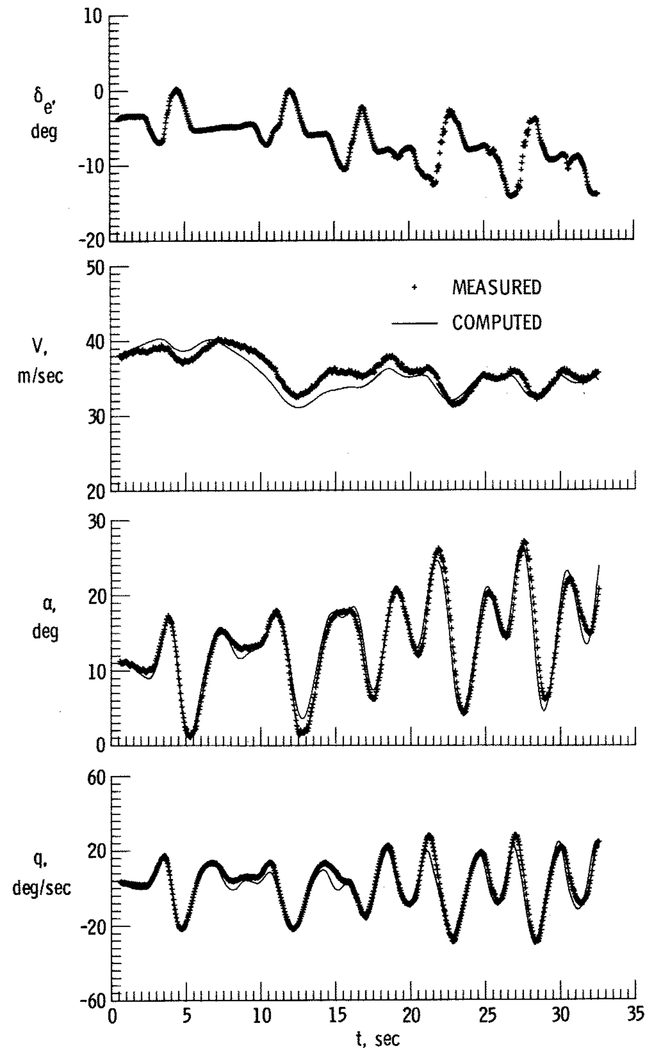

Figure 4. Time Histories of Measured Longitudinal Flight Data and Those Computed by Using Parameters Obtained by Stepwise Regression

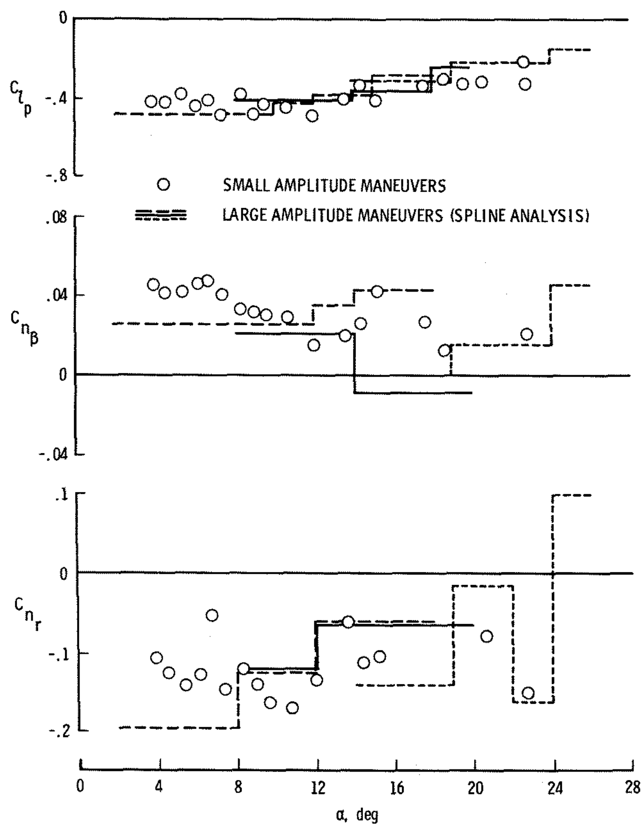

Figure 5. Comparison of Lateral Parameters Estimated From Different Maneuvers

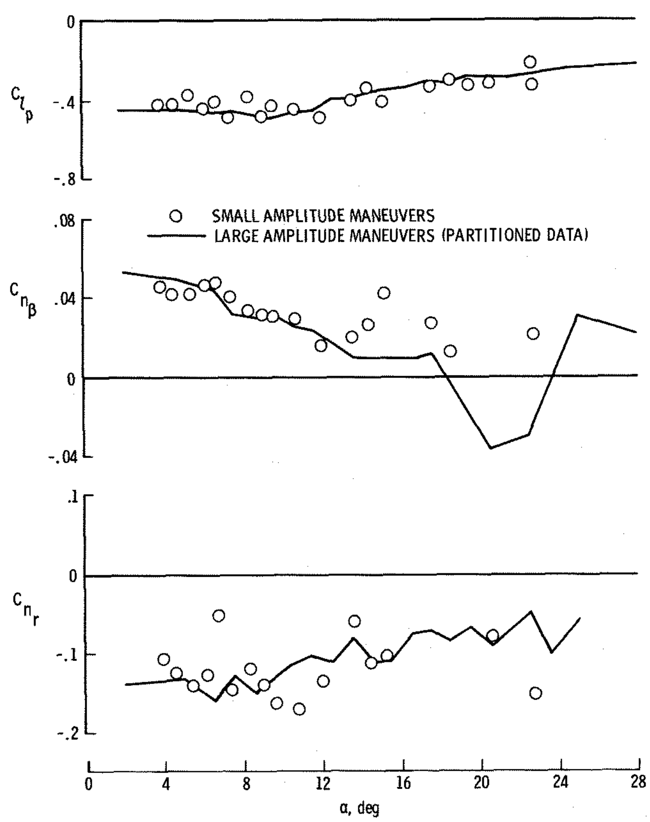

Figure 6. Comparison of Lateral Parameters Estimated From Different Maneuvers

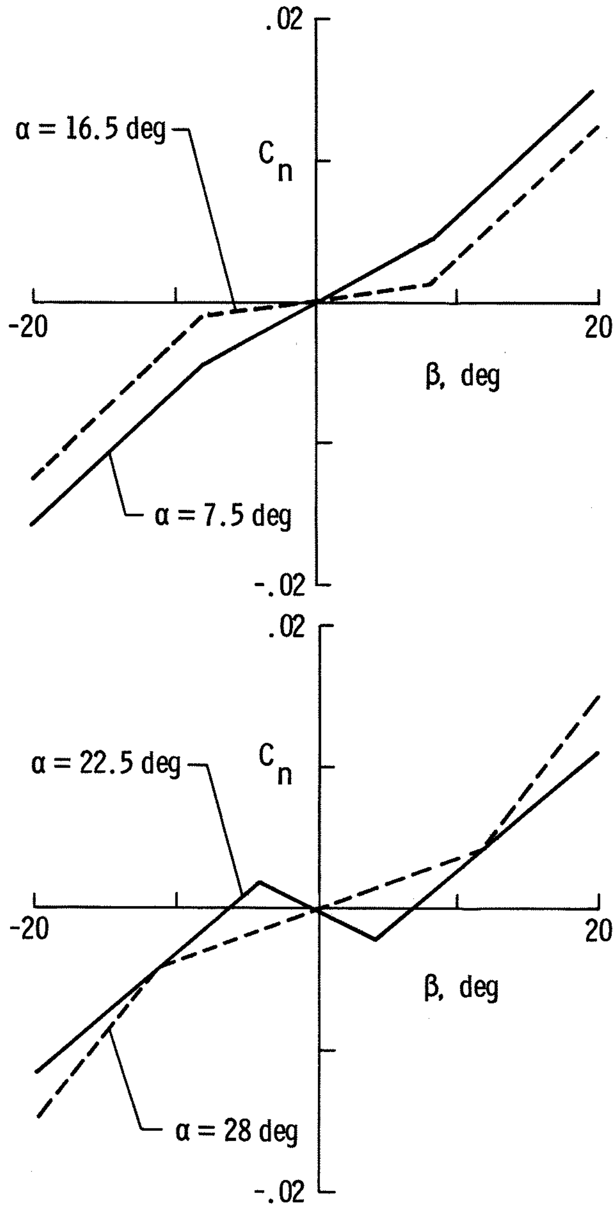

Figure 7. Steady State Values of Yawing Moment Coefficient Estimated From Large Amplitude Maneuvers. Partitioned Data

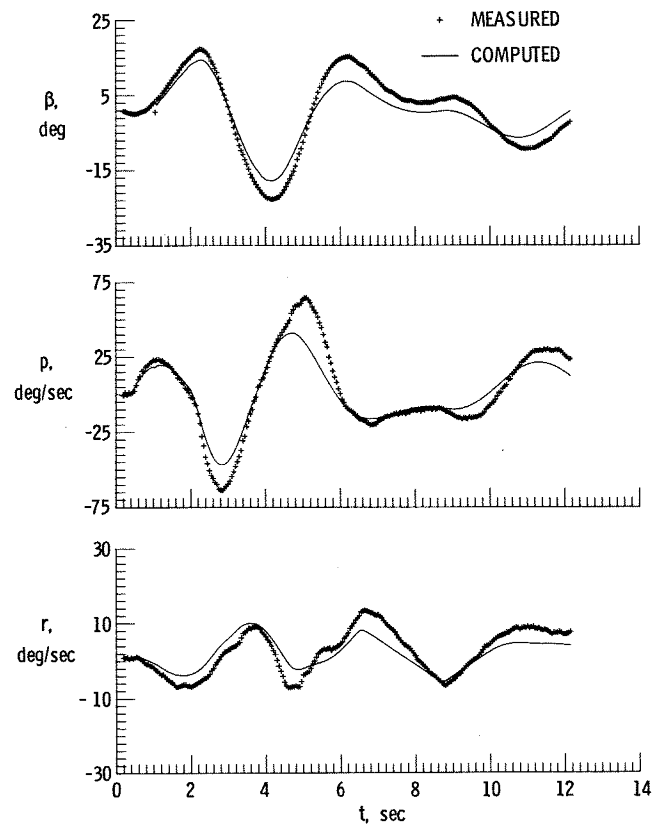

Figure 8. Time Histories of Measured Lateral Flight Data and Those Computed by Using Parameters Obtained by Stepwise Regression From Partitioned Data