OPTIMAL OPEN-LOOP AIRCRAFT CONTROL FOR GO-AROUND MANEUVERS UNDER WIND SHEAR INFLUENCE

H.G. Jacob

Institute of Flight Guidance
Technical University of Braunschweig, F.R.G.

Abstract

The optimization procedure is based on the representation of time dependent command inputs by analytical functions. The coefficients of these functions are iterated by a static search algorithm to values optimizing the behavior of the process.

The longitudinal motion in heavy tail wind shear of the Airbus A 300 aircraft is described by a system of nonlinear equations of 4th order bounded by numerous design, safety and comfort constraints. The quality criterion is defined in a way to maximize the minimal distance between aircraft and ground as well as the area between flight path and ground. For these studies a very simple optimization program has been used which allows to consider boundaries and which comprises less than 100 FORTRAN-statements.

The optimal input functions and other interesting variables are shown and discussed for the particular case that full power is available and for half power.

List of symbols

- a minimal distance to ground during a go-around
- A area between fligh path and ground
- c_i coefficients to be optimized
- C_D drag coefficient
- C_L lift coefficients
- D drag
- F quality criterion
- g gravity acceleration
- h altitude
- K number of coefficients to be optimized
- L lift
- m mass of the aircraft
- n load factor
- p weighting factor
- q impact pressure
- s range above which the area A is computed
- S reference wing area
- t time
- T thrust
- T_i Tschebyscheff-polynomials
- u range above which the controls may be manipulated
- V_a airspeed (see fig. 4)
- V_K flight-path velocity (see fig. 4)
- V_W horizontal wind velocity (see fig. 4)
- W weight of the aircraft
- x distance
- α angle of attack
- γ flight-path angle
- γ_a aerodynamic flight-path angle (see fig. 4)
- δ_F flap deflection
- ρ atmospheric density
- τ_c time constant
- θ pitch angle

I. Introduction

In recent years several aircraft accidents have been known which occured during landing approaches under heavy wind shear effects [1-6]. Taking the Airbus A 300 aircraft as an example and using a very simple and generally applicable numerical optimization procedure it will be shown in this paper what should be the optimal longitudinal open-loop time dependent controls to be applied to an aircraft for introducing go-around maneuvers under different wind shear and engine power conditions.

For the computation of optimal trajectories numerous analytical and numerical methods are available. Computer aided optimization concepts are advantageous if rather complex problems have to be solved. Such problems with highly nonlinear and bounded systems of differential equations are known to be quite common in the field of aerospace activities [7-11].

General findings and directions to be considered during a landing approach can be found in [12-17], where also the effects of wind shears and the possibilities of their partial compensation are demonstrated. Concerning the application of optimization techniques during landing procedures it can be referred also to the optimal design of closed-loop control structures [18-20]. Optimal closed-loop control laws for landings affected especially by wind shears are shown in [21].

However, if the landing process has to be aborted (e.g. failure of an engine under the simultaneous influence of wind shear, and lack of automatic landing control devices) then a go-around maneuver has to be introduced. In this case the question is of interest how this go-around maneuver should be performed to obtain the safest trajectory, assuming that the possible adverse atmospheric and power conditions are known. In [22] this question is treated in connection with several analytical and numerical optimization methods and a simple linearized mathematical model of an aircraft flying in an undisturbed atmosphere.

The study presented here for the determination of optimal go-around maneuvers is based on the use of a very simple and generally applicable optimization procedure together with a more realistic nonlinear model bounded by numerous constraints.

In the next section the optimization method is described. Afterwards the four independent program parts which are required for implementing the proposed optimization procedure are presented:

- the quality criterion associated with a go-around maneuver,
- the mathematical models allowing the simulation of an aircraft flying through a heavy tail wind shear,
- the structure of the control inputs, the coefficients of which have to be iterated to optimal values and
- the search algorithm used in this study.

In the last section the results of the investigation on optimal controls for introducing a go-around under wind shear will be explained. The optimal evolutions of the command inputs and other interesting variables are shown and discussed for the case that full power is available and for the circumstance that it is possible to operate only with half power in case of an engine failure. The contribution closes with a conclusion on the presented study.

II. The optimization method

The aim of every optimization task is to determine the free and manipulable inputs or parameters of the system to be optimized such that the optimum of the quality criterion adjoined to the system is achieved. Depending upon the type of the optimization problem these inputs may be represented by a vector point with constant coefficients (static system), by a time dependent multidimensional trajectory (dynamic system) or even by a hypersurface as a function of time and space (dynamic system with distributed parameters).

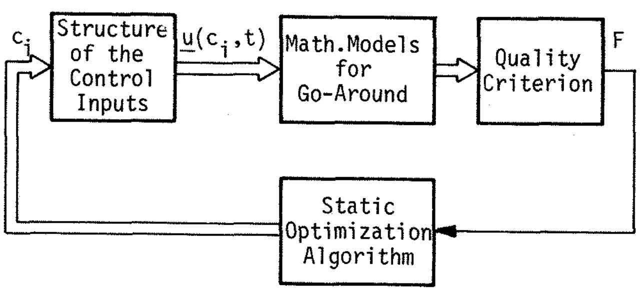

Fig. 1 The optimization procedure

The computer aided optimization procedure (24) applied in this study is neither elegant nor rigorous, but it is easy to use. The originally complicated dynamic optimization problem (even possibly based on a system with distributed parameters) is transformed to a easily solvable static parameter optimization task by expressing the time dependent (distributed) evolutions of the control inputs $\underline{u}(c_i,t)$ by a suitable system of functions (fig.1). The coefficients c_i of these functions are then iterated by a static search algorithm to values which optimize a performance index F associated with the process.

The following program parts are used for implementing the optimization procedure:

- With the program of the "quality criterion" it is possible to adjoin to every go-around a performance index F. The following criteria were considered: maximising the minimal distance during the flight between aircraft and ground, maximising the area between flight path and ground or a combination of both preceding criteria.
- The program part with the "mathematical models" allows the simulation of an aircraft go-around maneuver under the influence of wind shear. The longitudinal motion of the Airbus A 300 is described by a nonlinear system of 4th order bounded by numerous desing-, safety and comfort constraints. The highly nonlinear and time-varying dynamic response of the CF6 - 50C - engines is also computed fairly precisely by a description in the bounded phase plane. For the simulation of the wind shear it was assumed that a heavy tail wind shear acts on the motion of the aircraft.
- In the program part named "structure of the control inputs" the coefficients c_i delivered by the search algorithm are associated with either a Tschebyscheff polynomial system or with a Spline function system $\underline{u}(c_i,t)$ which represent the time dependent control for the flap setting and the pitch angle. Thus it is assumed that the pilot or an automatic system is able to adjust a desired pitch angle by a suitable action on the elevator. It is supposed moreover that the throttle will be pushed to its maximum position as soon as the decision for a go-around is given.
- The program part with the "optimization algorithm" can consist of any search procedure which determines the coefficients c_i iteratively in a way to find the optimal value F of the quality criterion. For the described study a very simple optimization program has been used, which allows also to consider boundaries and which comprises less than 100 FORTRAN-statements (24).

The main advantage of the described optimization strategy consists in its simple and flexible application. The method can be easily used because the originally dynamic optimization problem is solved by a given static search algorithm which needs not to be adapted to the task considered. The method is very flexible because the different program parts are independent from each other. This means that the mathematical model or the quality criterion or the structure of the input commands may be changed without the obligation to modify accordingly the other program parts, as it is often the case when using more difficult optimization methods.

III. Definition of optimal go-around maneuvers

The aim for defining a quality criterion is to get a scalar value F representing the performance of the system to be optimized in relation to the set of coefficients c_i applied to the system's model by the search algorithm (fig. 1).

As a first attempt the minimal distance during the flight between aircraft and ground was chosen to define the performance of the go-around maneuver:

$$F_1 = a_{min} \overset{!}{\rightarrow} \max. \qquad (1)$$

But this first performance index $F_1 = a_{min}$, a_{min} representing the minimal distance between aircraft and ground during a go-around maneuver, turned out to be useless. This was due to the remainder of the flight after reaching the minimal distance to ground, which did not have any influence on the value of the criterion.

To guarantee best possible circumstances during the whole maneuver the following second criterion was defined,

$$F_2 = A \overset{!}{\rightarrow} \max. \qquad (2),$$

expressing that the area A between flight-path and ground has to be maximized by the optimization algorithm. In general this quality criterion yielded reasonable results. However, in the case of go-around flights in undisturbed atmosphere with only half power available this criterion produced also useless outcomes. It showed up that in view to maximise the area defined above the aircraft had to absorb during the first part of the go-around supplementary kinetic energy to reach as soon as possible the best climb velocity. Because of the reduced power this was done by accelerating the sinking flight until crash at the ground, which certainly cannot be considered as an optimal go-around.

So as a third and last criterion the formula

$$F_3 = a_{min} + p \cdot A \overset{!}{\rightarrow} \max. \qquad (3),$$

describing a combination of the criteria (1) and (2), was finaly used to optimize also the go-around under the particular conditions mentioned above. Here F_3 is the value to be driven by the search algorithm to a maximum, a_{min} represents the minimal distance to ground and A is the area between flight-path and ground. The weighting parameter p has to be chosen in a way that mainly the minimal distance to ground is maximized.

IV. Mathematical models

To obtain the performance index F adjoined to a certain set of coefficients c_i delivered by the optimization algorithm a simulation model of the considered Airbus A 300 aircraft flying through a wind shear must be build up.

a) Mathematical model of a typical heavy tail wind shear

For the simulation of go around maneuvers under adverse conditions the following mathematical model of a dangerous wind shear will be described. The wind profile of the fig. 2 illustrates the approximate and idealized meteorological situation (1-3) during a landing approach of a Boeing-727-aircraft in June 1975 in direction to the Kennedy Airport, New York, which lead to the crash of the aircraft.

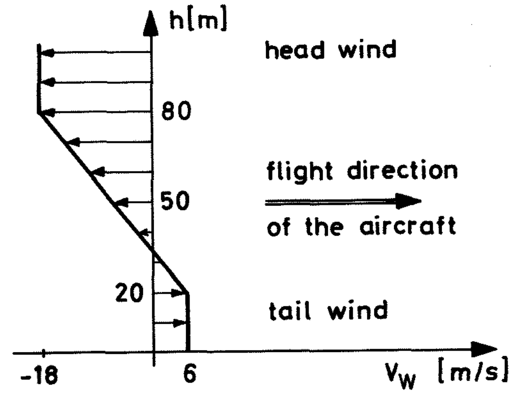

Fig. 2 Typical dangerous tail wind shear

For simplification purposes and in view to better understand the results of the study it was assumed that the wind shear appears only in function of altitude and that no vertical wind influences the motion of the aircraft.

With regard to fig. 2 the wind shear gradient

$$\frac{dV_W}{dh} = -\frac{0.4 \text{ m/s}}{\text{m}} \qquad (4)$$

delivers the horizontal wind velocity as noted below, depending upon the altitude:

$$\left.\begin{array}{ll} V_W = -18 \text{ m/s} & \text{for } h > 80 \text{ m} \\ V_W = 14 \text{ m/s} - (0.4\, h)/\text{s} & \text{for } 80 \text{ m} \leq h \leq 20 \text{ m} \\ V_W = 6 \text{ m/s} & \text{for } h < 20 \text{ m} \end{array}\right\} (5)$$

b) Mathematical model of the Airbus A 300 aircraft

Considering the fact that only the longitudinal motion of the aircraft affected by a horizontal wind shear had to be simulated the following 4th order nonlinear system of equations was chosen:

$$\left.\begin{array}{l} \dot{V} = (T \cdot \cos\alpha - D)/m - g \cdot \sin\gamma_a - \dot{V}_W \cdot \cos\gamma_a \\ \dot{\gamma}_a = [(L + T \cdot \sin\alpha)/m - g \cdot \cos\gamma_a]/V + \dot{V}_W \cdot \sin\gamma_a / V \\ \dot{h} = V \cdot \sin\gamma_a \\ \dot{x} = V \cdot \cos\gamma_a + V_W \end{array}\right\} (6)$$

where, with the wind shear defined in (4),

$$\dot{V}_W = \frac{dV_W}{dh} \cdot \frac{dh}{dt} = -\frac{0.4}{s} \cdot \dot{h} = -\frac{0.4}{s} \cdot V \cdot \sin\gamma_a \qquad (7).$$

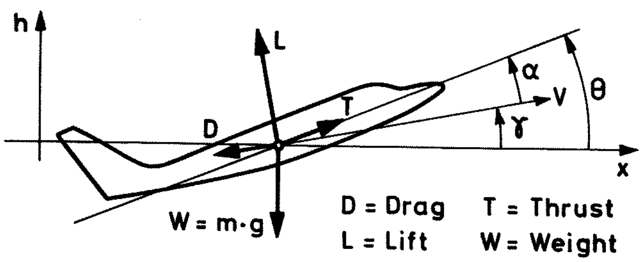

Fig. 3 Forces affecting the motion of the aircraft

With regard to fig. 3 and to fig. 4 the variables in the equations of motion (6) have the following meaning:

- V_a = airspeed, velocity along the airpath axis
- γ_a = aerodynamic flight-path angle (see fig.4)
- $\dot{h}$ = vertical geographical speed
- $\dot{x}$ = horizontal geographical speed
- α = angle of attack
- g = 9.81 m/s², gravity acceleration
- m = 130 000 kg, max. admissible mass of the Airbus A 300 aircraft in landing configuration.

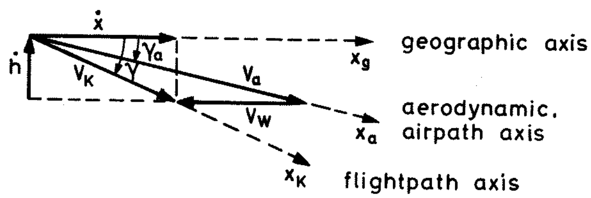

Fig. 4. Velocity vectors under wind

The forces L = Lift and D = Drag are given in terms of the lift and drag coefficients C_L and C_D and the parameters and variables indicated below:

$$L = C_L \cdot q \cdot S/m$$
$$D = C_D \cdot q \cdot S/m \qquad (8)$$

where $q = \rho \cdot V^2/2$, impact pressure (9)

and ρ = 1.225 kg/m³, atmospheric density

S = 260 m², reference wing area

The coefficients C_L and C_D are obtained from functions fitted to test data, as represented on fig.5, where the drag coefficient has to be augmented by ΔC_D = 0.021 if the undercarriage is extended.

With the above equations all state and output variables of the aircraft necessary for the evaluation of the performance index may be computed, e.g. with a 4th order Runge-Kutta-Gill integration routine as used for the simulated go-around maneuvers presented here.

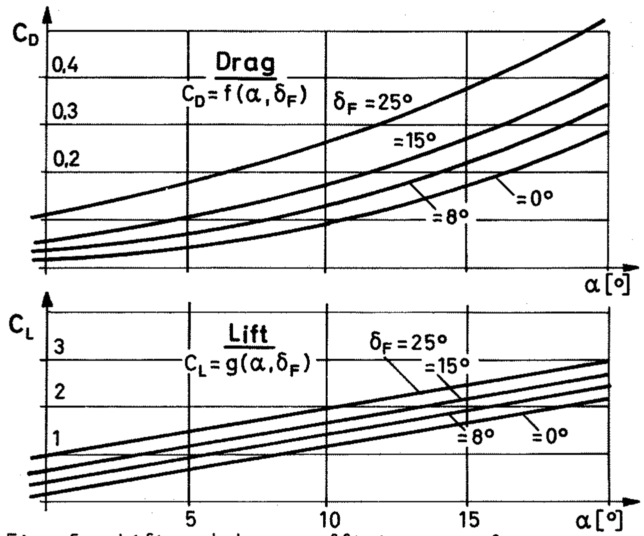

Fig. 5 Lift and drag coefficients in function of the angle of attack and the flap deflection

The control variables in equations (6) to (9), whose optimal time dependent values have to be determined, are the thrust T applied to the aircraft via the throttle angle and the mathematical model of the engines as described below, the pitch angle θ affecting the angle of attack α through the equation

$$\alpha = \theta - \gamma_a \qquad (10)$$

and the flap angle δ_F having an influence on lift and drag as indicated on fig. 5.

During a simulated go-around maneuver the following constraints have to be observed:

Design constraints:

- α_{max} = 20° (angle of attack)
- $0 \leq \delta_F \leq 25°$ (flap deflection)
- $25\% \leq T/T_{max} \leq 100\%$ (thrust range, T_{max} = 366 000 N)

Safety and comfort constraints:

- α_{max} = 12° (angle of attack)
- $n = 1+\ddot{h}/g \leq 1.15$ (load factor)
- θ_{max} = 20° (pitch angle)
- $\dot{\theta}_{max}$ = 8°/s (pitch angle variation)
- $h > 0$ (flight above ground)

For some investigations other limits had to be introduced (e.g. α_{max} = 15° for go-arounds under wind shear with only half power available) or supplementary boundaries have been considered (e.g. $\dot{h} > 0$ at the end of the go-around maneuver).

c) Mathematical model of the GE CF6-50C-engines

For introducing go-around maneuvers it was assumed that the throttle is pushed by the pilot to its maximum position. As shown on fig.6 it takes approximately 7 seconds until the thrust has reached its maximal value if initial conditions were set to 25% (apparent high order dynamical behavior of the engines due to internal flow restrictions). On the other hand, if a step-input from 60% to 70% is introduced in throttle position, then the development of thrust is similar to a first order lag characteristic.

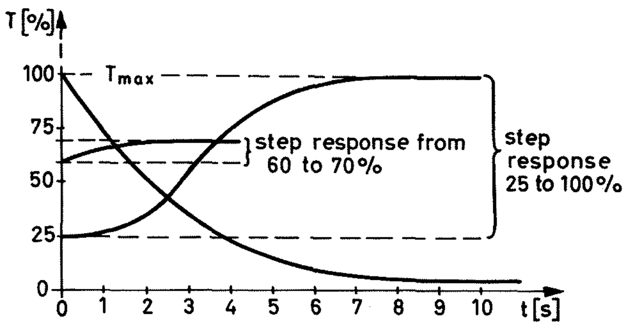

Fig. 6 Dynamical behavior of a CF6-50C-engine

For the dynamic simulation of this rather complicated behavior a method described in [23] was used. This method is based on the representation of the thrust evolution in the bounded phase plane as shown in fig. 7. In principle the time varying development of thrust is computed numerically by an iteration routine

$$T(t) = T_0 + \int_0^t \dot{T}(T,\Delta T)\, d\tau \qquad (11),$$

where T is the actual thrust and ΔT beeing the difference between desired and actual trust.

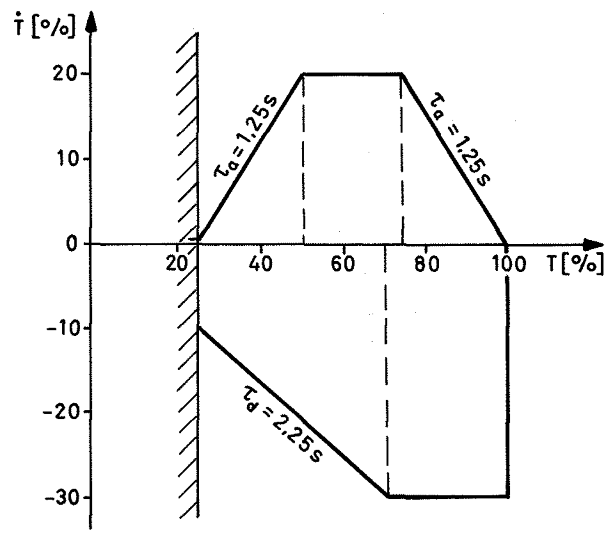

Fig. 7 Thrust behavior in the bounded phase plane

The time dependent derivative $\dot{T}$ to be integrated may be computed with regard to the fig. 6 and 7 in the following manner,

$$\dot{T}(t) = [T_{desired}(t) - T_{actual}(t)]/\tau_c = \Delta T(t)/\tau_c \qquad (12),$$

where the time constant τ_c can be determined from fig.6 (accelerating time constant τ_a = 1.25s, decelerating time constant τ_d = 2.25s). If the computed value of $\dot{T}$ is located inside the bounded phase plane of fig.7, then this value is subject of integration. If on the contrary this value is found to be outside these boundaries, then the admissible value $\dot{T}$ on the limit corresponding to the actual thrust T has to be taken.

By this method presented in [23], with the introduction of maximal permitted thrust variations to consider the internal flow restrictions in the engine, it is possible to simulate the highly nonlinear and time varying dynamical behavior of high order by a simple time constant of first order.

In this way only little efforts in programming and in computer time are required and the accuracy of the simulation is pretty good.

V. Structures of the control inputs

As a connecting link between search algorithm and mathematical model a structure for the time dependent control inputs has to be defined (fig.1). Two different types of structures - Tschebyscheff-Polynomials and Spline function systems - have been used in the study for representing the evolutions of the flap deflections δ_F and the pitch angle θ (the thrust T is given by the step response of the engines). In principle the open-loop controls may be expressed in function of time or depending upon the distance to the runway. For illustration purposes these controls have been expressed here in function of distance to the aim point at the beginning of the runway (see fig. 8).

a) Approximation with Tschebyscheff-polynomials

Because of their convergence properties Tschebyscheff-polynomials were applied:

$$\left.\begin{aligned} \delta_F(x) &= c_{10}\cdot T_0(x) + c_{11}\cdot T_1(x) + \dots \\ \theta(x) &= c_{20}\cdot T_0(x) + c_{21}\cdot T_1(x) + \dots \end{aligned}\right\} \quad (13)$$

with $T_0 = 1\ ;\ T_1 = 2(x - x_a)/(x_b - x_a) - 1$

$$T_n = 2\cdot T_1\cdot T_{n-1} - T_{n-2}\ ,$$

the distance $x_b - x_a$ being the interval over which the control functions are defined.

In the frame of the presented study Tschebyscheff-polynomials up to the 6th degree were used demanding 7 coefficients for each input variable. One coefficient however can be safed , respectively, because the initial value of each function at the beginning of the go-around maneuver is given as a known command setting for the stationary landing approach flight.

b) Approximation with Spline function systems

Beside the above mentioned Tschebyscheff-polynomials also Spline function systems have been applied for the representation of the control inputs. These functions are characterized by the fact that between given points of support, which need not to be located aequidistantly, a polynomial of 3rd degree is fitted. This is done in a way so that neighboring polynomials have at the common point of support the same function value, the same derivative and the same curvature.

For the go-around maneuvers considered between 7 and 9 points of support have been selected. The amplitudes (function values) at these points were delivered by the static optimization algorithm, with exception of the initial values known a-priori by the adjustment of the commands before the go-around. In total the search algorithm had to manipulate up to 16 coefficients to find the optimal evolutions of the two interesting input variables δ_F and θ.

VI. The search algorithm used [24]

There are numerous sophisticated algorithms for the determination of a local extremum of a multi-variable function. Their usefulness however is somewhat reduced by the fact that they are often difficult to learn and to apply. The program "EXTREM" employed for this study on the contrary has been developed particularly with regard to a simple application, also when constraints have to be considered. The program is easy to use and has a small size (less than 100 FORTRAN-statements, list of the program in [24]), it requires little storage (working space depending upon the number K of variables to be optimized is 7 * K), the evaluation of the function derivatives is not necessary and it is possible to consider a variety of boundaries.

In principle the algorithm performs the following tasks during an optimization stage comprising successive approximate searches for an extremum along K directions: choosing a search direction, determining the extremum along a line, defining the step sizes and considering the involved constraints.

Choice of search directions: The algorithm discerns two types of search directions, namely one main search direction approximately along the gradient of the function whose extremum has to be found and K-1 secondary search directions which are orthogonal to the main search direction and to themselves. A main search direction is given by the line going through the best point of the last stage and the best point of the preceding stage.

Determination of the optimum along a line: Starting from the best point attained in the meantime the algorithm performs along the considered direction a search step forward and backward and determines the optimum along this line by fitting a parabola through the 3 evaluated function values and by computing the extremum of this hypothetic parabola.

Definition of step sizes: If during any parabolic interpolation the distance between the new point and the old is smaller than one quarter of the current step size, then this step size is devided by 4. If an extrapolated point lies more than 20 times the current step size away from the old point, then the step size will be multiplied by 2.

Consideration of boundaries: The program given by the user to evaluate the performance index checkes if the set of coefficients delivered by the search algorithm violates any possibly given boundary. If this is the case the signal flow is returned to the search algorithm which delivers another set of coefficients computed in a particular way that satisfies the constraints.

The optimization procedure stops if during the last stage the variations of the function value or of the argument vector are smaller than chosen values or if the number of stages equals a predefined number.

VII. Computation of optimal go-around maneuvers under different conditions

After the programs for the definition of the quality criterion, for the simulation of the go-around maneuvers and for representing the input variables by a structure of functions with unknown coefficients have been explained the results of the optimization investigations will be discussed.

a) Assumptions considered for the maneuvers

Before initiating the simulated go-around it is assumed that the aircraft moves in an undisturbed atmosphere or in a head wind situation above the wind shear zone in a normal landing configuration (undercarriage and flaps extended) in an altitude of h = 100 m and at a distance of x_0 = -2 km before the aim point (see fig. 8).

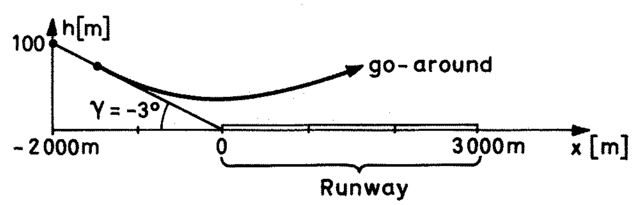

Fig. 8 Landing approach with go-around

The aircraft progresses with an airspeed of V_a = 135 kn ≈ 250 km/h ≈ 70 m/s and a flight-path angle γ = -3° to the aim point.

The decision to perform a go-around maneuver is taken at a distance of x_D = -1500m, that is approximately 2 seconds after reaching the possible wind shear zone. This response delay of 2 s for deciding a go-around after encountering a wind shear is very much too optimistic. With reference to [3] the reaction time of a pilot may vary between 3 and 10 s. For simplification purposes it is further assumed that the undercarriage is retracted 3 seconds after the decision for a go-around.

For the computation of the area between flight-path and ground (quality criterions (2) and (3) in section 3) the domain between x_0 = -2 km and x_E= +3 km (end of the runway) has been considered (see fig. 8). Sometimes this distance of s = 5 km was prologed to s = 10 km and s = 30 km, that is up to x_E = 8 km and x_E = 28 km.

b) Go-around maneuver in undisturbed atmosphere with full power

It is assumed that the pilot or the automatic guidance device is allowed to manipulate the inputs only in the range of u = 1.5 km between x_D = -1.5km and x=0 (approximate beginning of the runway, see fig.9). Over this range Spline function systems with 7 aequidistant points of support have been defined to represent the input variables of the flap deflection $\delta_F(x)$ and the pitch angle $\theta(x)$. The search algorithm computes iteratively the K = 2 * 6 = 12 amplitudes of the input variables at the free support points (number 1 to 6 on fig. 9) so that the area between flight-path and ground (criterion (2) in section 3) in the range s = 5 km between x_0 = - 2 km and x_E = 3 km (end of the runway) is maximized.

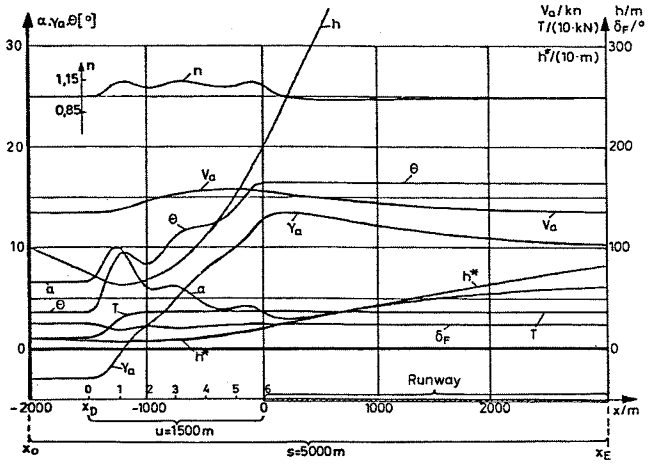

Fig. 9 Go-around in undisturbed atmosphere with full power above s = 5 km

The resulting flight-path is exhibited in fig. 9 with the trajectory of the aircraft given by the altitude h and h^* on two different scales. In view to maximize the mentioned area the following actions have to be undertaken on the command inputs: as soon as a go-around is decided at a distance of x_D = -1.5 km before the runway the throttle has to be pushed to its maximum position, the resulting step response of the thrust T is shown also on fig. 9. The input command of the pitch angle θ is increased continously in a way not to violate the maximum allowed load factor of n = n = 1.15, as indicated on the figure. Surprisingly the flap deflection originally in a position of δ_F = 25° does not diminish noteworthy during the go-around maneuver. It seems that in view to a low C_D/C_A-ratio (see fig. 5) the angle of attack α should be increased and the flap deflection δ_F decreased.

But the fig. 10 exhibits that if a very high lift coefficient C_L is required, e.g. C_L = 2.0, then a higher δ_F and a lower α is favorable for a desired low drag coefficient C_D. In this and the following figures also the evolutions of the aerodynamic airspeed V_a and the aerodynamic flight-path angle γ_a are presented.

The next fig. 11 shows what are the input controls and the resulting state variables for a go-around maneuver over the range of s = 30 km, with a domain of u = 7.5 km where the input variables may be manipulated. Again Spline function systems were used to represent the evolutions of the controls, but the number of the now non aequidistant points of support was increased (numbers 0 to 8 on the abszissa of the figure) so that K = 2 * 8 = 16 coefficients had to be optimized by the search algorithm.

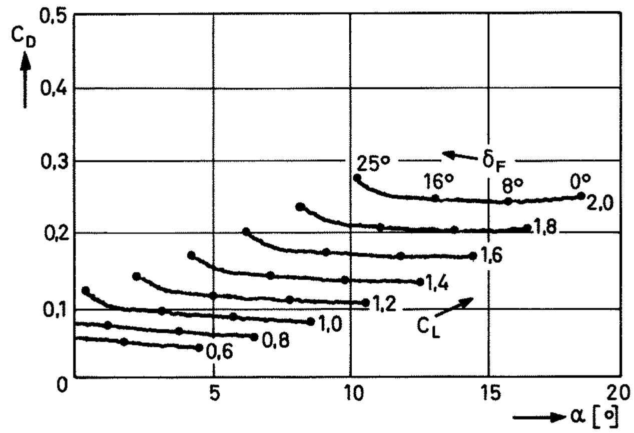

Fig. 10 Lift and drag coefficients

Now the range over which the area has to be computed for the performance index (2) is very much larger and it is worth to increase the airspeed V_a up to the velocity of best climbing.

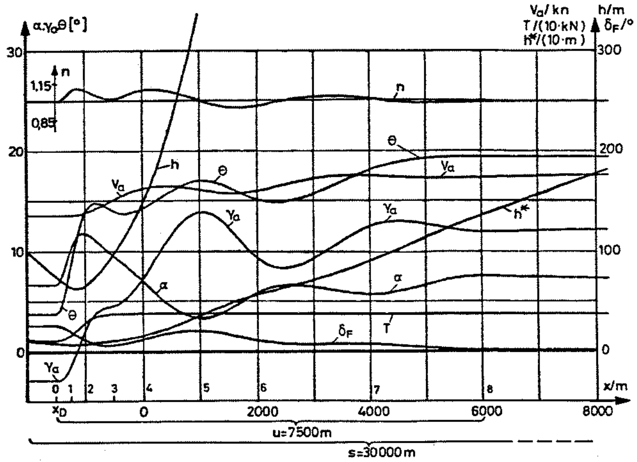

Fig. 11 Go-around in undisturbed atmosphere with full power above s = 30 km (trajectory represented only up to x = 8 km)

Because the required lift coefficient diminishes with the airspeed the flaps are retracted accordingly to fig. 10, as to be expected with regard to a low C_D/C_L-ratio.

c) Go-around maneuver in undisturbed atmosphere with half power

For this circumstances the quality criterion (3), simultaneous maximization of the minimal distance to ground and maximization of the area between flight path and ground, had to be applied. The input variables represented by Spline-functions may be manipulated again over a range of u = 7.5 km. In a first flight part the aircraft is pitched up in a way to meet the constraints of $\dot{\theta} \leq 8°/s$ and α_{max} = 12°. Then the aircraft moves on during a second flight part in a constant altitude until it reaches nearly the best climb airspeed. During the third and last part of the flight the climb is introduced in view to maximise the area under the flight-path. This is done by transforming partially the kinetic energy of the aircraft into potential energy. Obviously the first part of the flight is performed in a way to maximize the minimal distance to ground and the second and third parts of the trajectory are due to the maximization of the area between flight-path and ground.

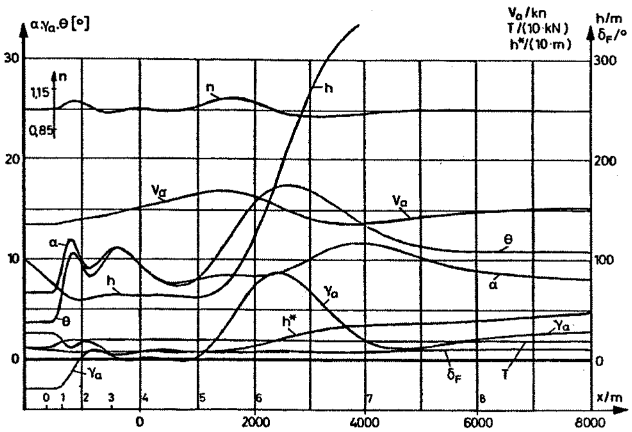

Fig. 12 Go-around in undisturbed atmosphere with half power above s = 10 km

d) Go-around maneuver under wind shear with full-power

The following fig. 13 and 14 shall demonstrate that two different structures of input functions deliver similar results if the other remaining conditions are the same.

The trajectories of the fig. 13 were obtained using a Spline-Structure for the control inputs with K = 2 * 6 = 12 coefficients to be optimized. It is visible that over the range of u = 1.5 km, where the commands of the aircraft may be manipulated, the load factor is nearly at its maximum possible value of n = 1.15. This is mainly achieved by increasing the pitch angle θ more or less continuously until the maximum admissible value of θ_{max} = 20°.

In fig. 14 the results are shown for the same conditions but with a Tschebyscheff-structure with again K = 2 * 6 = 12 coefficients for the input-variables. The results are similar with exception of the mentioned load factor. This difference is due to the fact that the Tschebyscheff-polynomial system may exhibit a jump in the evolution of its derivative between the stationary flight conditions and the range where the controls can be manipulated, that is at the points x_D = -1.5 km and x_0 = 0. (In the Spline function system the derivatives of θ and δ_F are forced to be zero at the indicated points what improves the comfort for the passengers.) So with the Tschebyscheff polynomials more "mobility" is given to the search algorithm. Advantage is taken of it, what yields a more abrupt pulling out of the aircraft when the wind shear is reached (the maximum load factor is attained sooner in fig. 14 than in fig. 13). As to be expected, the quality criterion (2) used here, namely the maximization of the area between flight-path and ground, yields a slightly better value with the Tschebyscheff-structure than with the Spline-functions (see table at the end of this contribution).

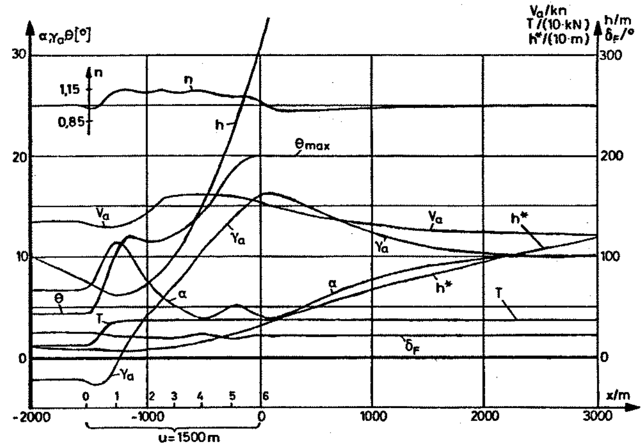

Fig. 13 Go-around under wind shear with full power over s = 5 km (Spline-structure)

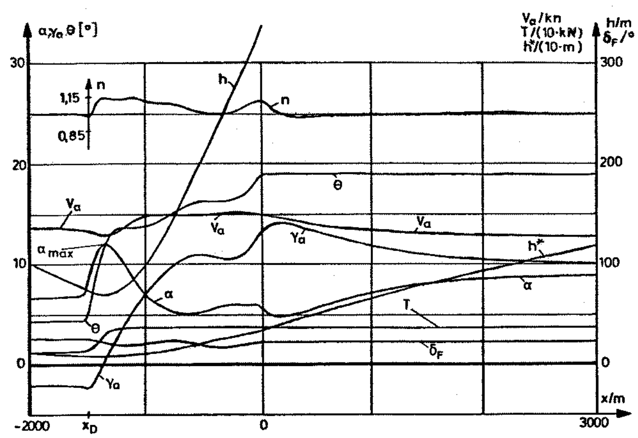

Fig. 14 Go-around under wind shear with full power over s = 5 km (Tschebyscheff-structure)

The effect of the wind shear is clearly visible on both fig. 13 and 14 with the evolutions of the variables γ_a, V_a and n, especially near the point x_D = 1.5 km where the go-around is decided, this in comparision to the fig. 9 where no wind shear was assumed to occur.

e) Go-around maneuver under wind shear with half power

In this particular dangerous and difficult situation for the pilot the maximum admissible value for the angle of attack was allowed to increase up to the rate of α_{max} = 15°, that is 3° more than usually admitted but still approximately 5° away from the angle of attack with maximum lift coefficient.

With the fig. 15 an optimal go-around maneuver under heavy tail wind shear and only half power available shall be explained. The input controls were allowed to be manipulated by a Spline-structure along a distance of u = 7.5 km (position of the points of support see fig. 12) and the range over which the area had to be computed for the quality criterion (2) was chosen to be s = 10 km.

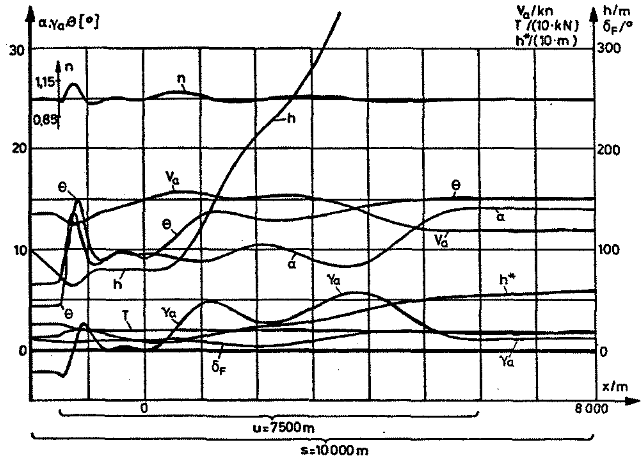

Fig. 15 Go-around under wind shear with half power over s = 10 km

Again the whole trajectory can be characterized by three different flight sections:

- Although merely the simple quality criterion (2) has been foreseen for this go-around (only maximizing of the area between flight-path and ground, no maximizing of the minimal distance to ground), the search algorithm has found out that during a first very short flight part the aircraft must be pulled out as quickly as possible from the wind shear zone. While during the go-around of fig. 8 (only half power available but no wind shear) the aircraft was simply leveled off in a way to reach a horizontal motion, here, under the effect of the considered tail wind shear, the search algorithm drives the inputs to values so that the aircraft leaves with the maximum possible load factor of $n = n_{max}$ = 1.15 the adverse disturbance.
- As soon as the altitude of h = 80 m is reached (beginning of the wind shear zone, see fig. 2), this altitude is hold and the aircraft is accelerated during a longer second flight period in a horizontal motion to gain kinetic energy in view to a better climb speed.
- When a more favorable velocity for climb has been reached, the climbout trajectory is initiated by increasing the pitch angle. During the progress of this third and last flight section for a go-around maneuver the airspeed diminishes again with regard to a growth of altitude (decrease of kinetic energy to the advantage of potential energy).

Note that the flap deflection angle δ_F shows during the whole go-around maneuver rather high values. At the last part of the considered flight e.g., where the input variables remain constant, a lift coefficient C_L slightly above 2.0 is required (value delivered by the printouts of the optimization runs). Considering fig. 10 it turns out that the search algorithm has found, in view to a low C_D/C_L-ratio, the best combination of flap setting and angle of attack with δ_{FE} = 15.6° and α_E = 14° (resulting by a pitch angle of $\theta = \alpha + \gamma_a$ = 15.1°).

The last fig. 16 shows a similar go-around maneuver under wind shear, but with an extended range s = 30 km over which the area is computed for the performance index. Again the aircraft is pulled out of the wind shear zone during a short first flight part with a maximum admissible load factor of n_{max} = 1.15 until it reaches the approximate altitude of h = 80 m where the wind shear begins. Since the total flight is now very much longer it is worth to increase in a second flight section with horizontal motion (here much longer than in fig. 15) the airspeed nearly up to the velocity of best climb. Towards the end of the trajectory (not visible on th figure) the required lift coefficient is only about $C_L \approx$ 1.0 becouse of the higher airspeed of the aircraft. The resulting flap setting δ_{FE} = 0.1 and the angle of attack α_E = 9° at the end of the flight at x_E = 28 km (see table) are closely related to the minimum drag value of fig. 10 under the considered condition.

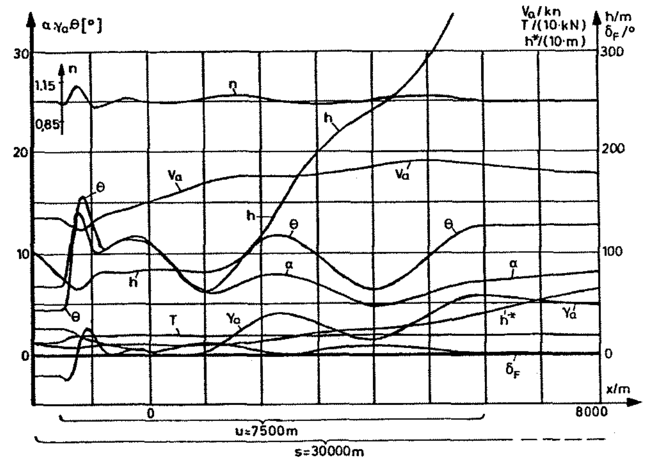

Fig. 16 Go-around under wind shear with half power over s = 30 km (trajectory represented only up to x = 8 km)

Observation

For some go-around maneuvers the additional constraint of $\ddot{h}_E > 0$ was involved, meaning that the aircraft had to fly with a positive acceleration in vertical direction during the flight position following the flight section where the controls were allowed to be manipulated. As an example in fig. 15 this constraint is included and it is observable that the aerodynamic flight-path angle γ_a increases slightly from x = 6 km (end of the range where the inputs may be changed) to x_E = 8 km, end of the considered flight. On the contrary at the go-around maneuver of fig. 16 this constraint is not met and accordingly the state variable γ_a diminishes from x = 6 km until the end of the flight at x_E = 28 km, not visible on this figure.

In the table 1 the most interesting results of the considered go-around maneuvers are exhibited:

Fig. no.	Sp/Ts	s	u	QC	K	$\ddot{h}_E$	α_{max}	A	a_{min}	x_E	δ_{FE}	θ_E	V_{aE}	γ_{aE}
without wind, full power:														
9	Sp	5	1.5	2	12	<	12	1.80	--	3	24.1	16.4	136	10.3
11	"	30	7,5	"	16	>	"	87,25	62.0	28	1,1	19.5	180	12.5
without wind, half power:														
12	Sp	10	7.5	3	16	>	12	2.37	58.7	8	9.5	10.8	153	2.8
with wind shear, full power:														
13	Sp	5	1.5	2	12	<	12	2.60	--	3	21.6	20.0	122	9.8
14	Ts	"	"	"	"	"	"	2.64	--	"	21.9	18.9	127	10.1
with wind shear, half power:														
15	Sp	10	7.5	2	16	>	15	3.10	63.1	8	15.6	15.1	118	1.1
16	"	30	"	3	"	<	"	32.23	63.1	28	0.1	12.7	168	3.7

Table 1 Main results of the considered go-around maneuvers

Comments related to the table 1:

- Fig. no. – number of the figure in this paper
- Sp/Ts – hint if Spline or Tschebyscheff-structures have been used for representing the control inputs
- s – range in km above which the area between flight-path and ground is computed for the performance index
- u – domain in km along which the input variables may be manipulated
- QC – number of the quality criterion considered (see section III of this paper)
- K – number of the coefficients to be optimized
- $\ddot{h}_E$ – vertical acceleration required after completion of the flight part with variable inputs ($\ddot{h}$ > 0, only positive vertical acceleration accepted; $\ddot{h}$ < 0, any vertical acceleration allowed)
- α_{max} – maximum angle of attack allowed in °
- A – area in km² between flight-path and ground over the range of s
- a_{min} – minimum distance to ground in m during the go-around maneuver
- x_E – end of the trajectory in km
- δ_{FE} – flap setting in ° at the end of the trajectory
- θ_E – pitch angle in ° at the end of the trajectory
- V_{aE} – airspeed in kn at the end of the trajectory
- γ_{aE} – aerodynamic flight-path angle in ° at the end of trajectory

IIX. Conclusions

The presented study on the determination of optimal go-around maneuvers under the influence of wind shear has shown that the described computer aided optimization procedure is an easy to use and flexibly applicable tool to calculate open-loop inputs for any dynamical system. The method reduces to static optimization by representing the inputs as a structure of functions. The coefficients of the structure are driven by a search algorithm to values which optimize a quality criterion adjoined to the system. Since the procedure is not concerned with the details of the dynamical system to be optimized it works also for complicated nonlinear systems of high order restricted by numerous constraints.

The results of the investigations on optimal go-around maneuvers may be summarized as follows:
The evolutions of the obtained trajectories show that after the decision for a go-around the aircraft should be pulled out during a first short period of time in a manner to satisfy the given safety- and comfort constraints until the adverse wind shear zone has been left or, under better wind conditions, until a horizontal motion as been reached. In a second flight period an acceleration of the aircraft

yields a better velocity for the following climb. Depending on the available thrust a simultaneous climb is possible or at least the attained altitude must be hold. As soon as the speed of optimal climb has been nearly reached the desired climb can be introduced in a third flight period.

The optimization studies have confirmed qualitatively the corresponding procedures of the flight hand books for go-arounds with restricted power, that is pull out, acceleration, and climb. Referring to the manner, how these trajectories should be achieved, the obtained optimization results show however noteworthy conclusions. So it turned out that contrary to the current recommendations a go-around with extended flaps is advantageous in certain circumstances. Especially in disturbed atmosphere this flap setting gives an additional safety margin against stall.

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