REDUCED NONLINEAR FLIGHT DYNAMIC MODEL OF ELASTIC STRUCTURE AIRCRAFT

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Abstract

Flight dynamic model is given by equations of motion of unconstrained deformable body under aerodynamic forces, thurst load and weight. Dynamic model of deformable structure under aerodynamic forces is defined using finite element method. Reducing method is general for mechanical systems, which is based on the fact that very fast modes have a small energy and great damping. First step in procedure is to find nonlinear vector transformation of generalized coordinates which transform basic nonlinear model to a linear one. By using method of decoupling linear systems to subsystems of slow and fast modes it is possible to take out fast structures modes and find linear dependance between generalized transformed coordinates. Criterion for neglecting these modes is also presented. Second step is to find nonlinear dependance between basic generalized coordinates which leads to reduced nonlinear model of system. This procedure is applied for constructing reduced flight dynamic model with less dimensionality than basic one.

I. Introduction

Dynamic model of rigid structure aircraft, given in linear or nonlinear forms is usualy used in automatic flight control synthesis. In this paper is presented a synthesis of reduced flight dynamic model of elastic structure aircraft. Presented procedure is given in a general form for dynamical systems of "unsteady structures".

Flight dynamic model of elastic structure aircraft is synthesized in three parts. In first part is obtained dynamic model of unconstrained motion of deformable body, which is well known in literature. In this paper are given a final diferential equations of motion of deformable body under generalized forces. Dynamic model of elastic motion of structure is obtained in linear form, using variational principle. Structure is modeled with finite element method using discrete coordinates. Unsteady aerodynamic forces are presented by finite element method for solving equation of potential in the case of small perturbation velocites of flow. There are three spatial cases in synthesis flight dynamic model of elastic structure aircraft which depend on elastic characteristics of structure.

- dynamic systems of unsteady structures with decoupled external and structure dynamics. This dynamic model is equivalent to dynamic model of steady structure systems with steady aerodynamic flow;
- dynamic systems of unsteady structures with in undependent external dynamics whose influence on structure dynamics can not be neglected;
- dynamic systems with coupled external and structural dynamics.

In this paper all investigations are presented for the third case of aproximation. A procedure of reducing flight dynamics model of elastic structure aircraft is given in the third part of the paper. Formally, arbitrary nonlinear dynamic system has anequivalent linear model which is given by nonlinear transformation of generalized coordinates as a solution of the partial differential matrix equation. Approximate solution of this equation is obtained using Galerkin's method. We must note that there are no great diference in system of motion between real and linearised flight dynamic model of aircraft, and that is the reason for using of Galerkin's method. Convergence of this method is very fast. If equivalent linear dynamic system is with complex roots it is neccesary for this system to be stable because in any other case Galerkin's aproximation is inpossible or very complicated. In this case we must know all steady points of the system wich are defined by relation $\dot{x} = 0$, where x is a generalized coordinate of system motion.

Given equivalent linear system has a new analitycaly equivalent system, which is decoupled on subsystems of slow and fast modes. Linear transformation of generalized coordinates is a solution of the matrix Riccati diferential equation, (/8/ and /10/). Dynamic system with unsteady structure is always a system of small parameter with small energy of higher modes, which are negliable. If number od modes is less than number of generalized system coordinates a linear dependance between them must exsist. On this fact is based aproximate procedure of reducing. Now, we can define inverse procedure which gives nonlinear dynamic model with lower number of coordinates dimensionality than initial one. Nonlinear dependance between coordinates of initial system are also obtaines using Galerkin's method.

II. Notation

- u, v, w - velocity components in fixed inertial coordinate system
- p, q, r - components of angular velocity in coordinate system, fixed in arbitary point
- x_c, y_c, z_c - coordinates of center of gravity
- m - total mass of airplane
- $G=\{F_x, F_y, F_z, M_x, M_y, M_z\}$ vector of generalized forces
- $X=\{u,v,w, p,q,r\}$ - vector of generalized coordinates of motion
- $W=W(x,y,z,t)$ - vector of elastic displacement
- S - vector of generalised surface load
- $Q=\{q_i\}$ - vector of generalized coordinates of displacement
- J_x, J_y, J_z - inertial moments J_{xy}, J_{xz}, J_{yz}
- U - vector of control
- Y, Z - vector of transformed generalized coordinates of motion
- R, P - vector of transformed generalized coordinates of displacement
- H, J - vector function of transformed coordinates
- L, N - transformation matrix of generalized coordinates
- μ, ν - small parameter

III. Flight Dynamic Model of Elastic Structure Aircraft

Let us consider a motion of elastic unrestrained body in dynamic (fixed) coordinate system. Elastic deflections are defined in coordinate system which is fixed in arbitrary material point of body. Let $\vec{r}$ be a radius-vector of material point dynamical coordinate system. If the body is influenced by surface and inertial forces, denoted by vector $\vec{F}$ and $\vec{R}$ per unit of surface and volume, respectively, vector equations of equilibrium of forces and momenta are

$$\frac{d}{dt} \iiint_V \rho \frac{d\vec{r}}{dt} dV = \iiint_V \vec{R}\, dV + \iint \vec{F}\, dS$$

$$\frac{d}{dt} \iiint_V (\vec{r} \times \rho \frac{d\vec{r}}{dt})\, dV = \qquad (1)$$

$$= \iiint_V (\vec{r} \times \vec{R})\, dV + \iint_S (\vec{r} \times \vec{F})\, dS$$

where ρ is a mass per volume unit . Equations (1) in scalar form can be presented like:

$$F_x=m(\dot{u}+qw-rv)+\ddot{x}_c+2(q\dot{z}_c-r\dot{y}_c)-(q^2+r^2)x_c+$$
$$+(pq-\dot{r})y_c+(pr+\dot{q})z_c$$

$$F_y=m(\dot{v}+ru-pw)+\ddot{y}_c+2(r\dot{x}_c-p\dot{z}_c)-(p^2+r^2)y_c+$$
$$+(qr-\dot{p})z_c+(pq+\dot{r})x_c$$

$$F_z=m(\dot{w}+pv-qu)+\ddot{z}_c+2(p\dot{y}_c-q\dot{x}_c)-(p^2+q^2)z_c+$$
$$+(pr-\dot{q})x_c+(qr+\dot{p})y_c \qquad (2)$$

$$M_x=J_x\dot{p}+(J_z-J_y)qr+J_{xy}(pr-\dot{q})-J_{xz}(\dot{r}+pq)+$$
$$+J_{yz}(r^2-q^2)+\dot{J}_xp-\dot{J}_{xz}r-\dot{J}_{xy}q+$$
$$+m\left[(qv+rw)x_c+(\dot{w}-qu)y_c+(\dot{v}-ru)z_c+w\dot{y}_c-v\dot{z}_c\right]+$$
$$+q\int_m(x\dot{y}-\dot{x}y)dm+r\int_m(\dot{x}z-x\dot{z})dm+\int_m(y\ddot{z}-\ddot{y}z)dm$$

$$M_y=J_y\dot{q}+(J_x-J_z)pr+J_{yz}(pq-\dot{r})-J_{xy}(\dot{p}+qr)+$$
$$+J_{xz}(p^2-r^2)-\dot{J}_{xy}p+\dot{J}_yq-\dot{J}_{yz}r+$$
$$+m\left[(\dot{w}-pv)x_c+(pu+rw)y_c+(\dot{u}-rv)z_c-w\dot{x}_c+u\dot{z}_c\right]+$$
$$+p\int_m(\dot{x}y-x\dot{y})dm+r\int_m(y\dot{z}-\dot{y}z)dm+\int_m(\ddot{x}z-x\ddot{z})dm$$

$$M_z=J_z\dot{r}+(J_y-J_x)pq+J_{xz}(qr-\dot{p})J_{yz}(\dot{q}+pr)+$$
$$+J_{xy}(q^2-p^2)-\dot{J}_{xz}p-\dot{J}_{yz}q+\dot{J}_zr+$$
$$+m\left[(\dot{v}-pw)x_c+(\dot{u}-qw)y_c+(pu+qv)z_c+v\dot{x}_c-u\dot{y}_c\right]+$$
$$+p\int_m(\dot{x}z-x\dot{z})dm+q\int_m(\dot{v}z-y\dot{z})dm+\int_m(x\ddot{y}-\ddot{x}y)dm$$

Displacement can be presented by formal series

$$W(x,y,z,t) = V(x,y,z)\cdot Q(t)$$

where V is matrix of interpolation functions. Linear equations of motion for small disturbances of system can be formaly prezented by vector linear relation

$$G=\mathcal{L}_i(X,Q) \qquad (3)$$

where $\mathcal{L}_i$ is linear diferential operator. Using variational principle for small displacement of elastic structure we get a linear matrix diferential equation (Euler-Lagrange's equations) of elastic motion of structure

$$M\ddot{Q}+2P\dot{Q}+KQ=S \qquad (4)$$

where S is a vector of generalized forces and M,P,K respectively generalized matrices of mass, damping and stiffness. Motion of aerodynamic surface is given by approximate relation

$$h = y + W\cdot\vec{j}$$

Using a derivation of potencial in linear form and boundary conditions on the thin aerodynamic surface, we get a linear matrix relation for unsteady generalized aerodynamic forces

$$S=\mathcal{L}_a(X,Q,U) \qquad (5)$$

where $\mathcal{L}_a$ is linear diferential operator, and U is a control vector.
Steady aerodynamic generalized forces formaly are presented by relation

$$G=\mathcal{L}_A(X,Q,U) \qquad (6)$$

where $\mathcal{L}_A$ is linear diferential aerodynamic operator. Using equations (2), (4), (5) and (6) we get a nonlinear flight dynamic model of elastic structure aircraft in a vector form

$$\dot{X} = f(X,Q,U)$$
$$\dot{Q} = \mu A_{21}X + A_{22}Q + \nu B_2U \qquad (7)$$

or using equation (3), (4), (5) and (6) a linear flight dynamic model, given in a vector form

$$\dot{X} = A_{11}X + A_{12}Q + B_1U$$
$$\dot{Q} = \mu A_{21}X + A_{22}Q + \nu B_2U \qquad (8)$$

Given dynamic system presented by equations (7) and (8) are systems with small parameters μ and ν, which are a indicators of elasticity influence on aircraft motion. A procedure for reducing of nonlinear system (7) is given in the next part of the paper.

IV. Basic Equations

Let us consider a nonlinear dynamic system, given in a vector form by equation

$$\dot{X} = f(X,U,t) \quad (9)$$

where X is n-dimensional vector of generalized coordinates, U is 1-dimensional vector of control.

Linear system given by equation

$$\dot{Y} = A(t)\, Y + B(t)\, U \quad (10)$$

is formaly equivalent to system (9) if there is vector function $H(Y,U,t)$ presented by nonlinear transformation

$$X = Y + H(Y,U,t) \quad (11)$$

which is a solution of matrix partial differential equation

$$(\partial H/\partial Y)\cdot\left[A(t)\cdot Y + B(t)\cdot U\right] + A(t)\cdot Y + B(t)\cdot U + (\partial H/\partial U)\cdot\dot{U} - f\left[Y + H(Y,U),\ U,t\right] = 0 \quad (12)$$

with initial condition

$$H(Y_0,U_0) = H_0 = 0$$

Solution of equation (12) for the case of independent on time system is given in |3|. Approximate solution of equation (12) can be presented using Galerkin's approximation as follows:

$$H(Y,U,t) = T\cdot E(Y,U,t) \quad (13)$$

where matrix $T = [T_{ij}]$ is a solution of algebraic matrix equation

$$\alpha_j\cdot T_j + \gamma_j = \beta_j(T_j) \quad (14)$$

where are

$$\alpha_k = \int_{t_0}^{t} \{\partial E_k/\partial Y\left[A(t)\cdot Y + B(t)\cdot U\right] + (\partial E_k/\partial U)\,\dot{U}\}\cdot E_j(Y,U,t)\cdot dt$$

$$\beta_k = \int_{t_0}^{t_1} f\left[Y + H(Y,U)\ U,\ t\right]\cdot E_i(Y,U,t)\cdot dt \quad (15)$$

$$\gamma_k = \int_{t_0}^{t_1}\left[A(t)\cdot Y + B(t)\cdot U\right] E_i(Y,U,t)\ dt$$

Galerkin's approximation presented by relation (13)is given in a real time. If system (9) is indenpendent on time than aproximation (13) is defined by generalized coordinates which needs a possition of steady points of system motion. It is possible only if the system (10) is stable. If system (9) is steady, than matrices A and B are constant. In this case equations (14) has a different form, given by equations

$$\alpha_k = \iint_{YU}\{(\partial E_k/\partial Y)(A\cdot Y + B\cdot U) + (\partial E_k/\partial U)\cdot U\}\cdot E_i\ (Y,U)\cdot dY\cdot dU$$

$$\beta_k = \iint_{YU} f\left[(Y + H(Y,U),\ U\right]\cdot E_i(Y,U)\cdot dY\cdot dU \quad (16)$$

$$\gamma_k = \iint_{YU} (A\cdot Y + B\cdot U)\cdot E_i(Y,U)\cdot dY\cdot dT$$

Integration of equation (15) carried out in generalized coordinates, with condition that system (10) is stable. In this case vector of generalized coordinates cannot be known. The approximative equation (13) can be also presented like

$$H(Y,U,t) = T(t)\cdot E(Y,U)$$

Equations (14) can be integrated over generalized coordinates and control. Equations (14) are than transformed as follow:

$$\alpha_k = \iint_{YU}\{(\partial E_k/\partial Y)(A(t)\cdot Y + B(t)\cdot U) + (\partial E_k/\partial U)\,\dot{U}\}\cdot E_i\ (Y,U)\cdot dY\cdot dU$$

$$\beta_k = \iint_{YU} f\left[(Y + T(t)\cdot E(Y,U),\ U,t\right]\cdot E_i\ (Y,U)\cdot dY\cdot dU \quad (17)$$

$$\gamma_k = \iint_{YU} (A(t)\cdot Y + B(t)\cdot U)\cdot E_i(Y,U)\cdot dY\cdot dU$$

where matrix $T(t)$ is a solution of functional matrix equation

$$\alpha(t)\cdot\tau(t) + \gamma(t) = \beta\left[\tau(t)\right] \qquad (18)$$

This procedure does not need a knowledge of motion of system.

Let us consider a dynamic system with unsteady structure, given in subvector form by equation

$$\begin{aligned} \overset{\circ}{X} &= f_x(X,Q,U,t) \\ \overset{\circ}{Q} &= f_q(X,Q,U,t) \end{aligned} \qquad (19)$$

Linear unsteady dynamic system given by the next equation

$$\begin{vmatrix} \overset{\circ}{Y} \\ \overset{\circ}{R} \end{vmatrix} = \begin{vmatrix} A_{11}(t) & A_{12}(t) \\ A_{21}(t) & A_{22}(t) \end{vmatrix} \cdot \begin{vmatrix} Y \\ R \end{vmatrix} + \begin{vmatrix} B_1(t) \\ B_2(t) \end{vmatrix} U \qquad (20)$$

is formaly equivalent to system (19) if there is a solution of matrix partial differential equation (14). A linear transformation of generalized coordinates

$$\begin{vmatrix} Y \\ R \end{vmatrix} = \begin{vmatrix} I & -M \\ -L & (I+LM) \end{vmatrix} \cdot \begin{vmatrix} Z \\ P \end{vmatrix} \qquad (21)$$

gives as a decoupled dynamic system presented in a form

$$\begin{vmatrix} \overset{\circ}{Z} \\ \overset{\circ}{P} \end{vmatrix} = \begin{vmatrix} C_1(t) & 0 \\ 0 & C_2(t) \end{vmatrix} \cdot \begin{vmatrix} Z \\ P \end{vmatrix} + \begin{vmatrix} D_1(t) \\ D_2(t) \end{vmatrix} U \qquad (22)$$

Block-matrix of transformation (21) are solutions of Riccati matrix differential equations

$$\begin{aligned} \dot{L} &= L A_{12} L - L A_{11} + A_{22} L - A_{21} \\ \overset{\circ}{N} &= A_{11} N - A_{12} - A_{12} L N - \\ &\quad - N (A_{22} + L A_{12}) \end{aligned} \qquad (23)$$

For the case of steady dynamic system (19) equations (23) are algebraic Riccati matrix equations given in a form

$$\begin{aligned} & L A_{12} L - L A_{11} + A_{22} L - A_{21} = 0 \\ & A_{11} N - A_{12} - A_{12} L N - N(A_{22} + L A_{11}) = 0 \end{aligned} \qquad (24)$$

Matrix of system (22) are defined by relations

$$\begin{aligned} C_1(t) &= A_{11} - A_{12} L \qquad & D(t) &= B_1 + N D_2(t) \\ C(t) &= A_{22} + L A_{12} \qquad & D_2(t) &= B_2 + L B_1 \end{aligned} \qquad (25)$$

Using transformation (21) , transformation (11) has a final form given by next relation

$$\begin{vmatrix} X \\ Q \end{vmatrix} = \begin{vmatrix} I & N \\ L & I+L N \end{vmatrix} \begin{vmatrix} Z \\ P \end{vmatrix} + \begin{vmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{vmatrix} \begin{vmatrix} E_1(Z,P,U) \\ E_2(Z,P,U) \end{vmatrix} \qquad (26)$$

If there is a change of generalized coordinates (26) systems given by formulas (19) and (22) are formally equivalent.

If we write vector Q in a block form

$$Q = \{Q_1 \; Q_2\}$$

where and are l_1 and l_2 dimensional subvectors, respectively, system (19) can be writen in a mixed form

$$\begin{aligned} \overset{\circ}{X} &= f(X,Q_1,Q_2,U,t) \\ \overset{\circ}{Q}_1 &= A_{11}Q_1 + A_{12}Q_2 + C_1X + B_1U \\ \overset{\circ}{Q}_2 &= A_{21}Q_1 + A_{22}Q_2 + C_2X + B_2U \end{aligned} \qquad (27)$$

A next linear system, given by relation

$$\begin{aligned} \overset{\circ}{Y} &= A \;\; Y + D_1 \; R_1 + D_2R_2 + B \; U \\ \overset{\circ}{R}_1 &= A_{11}R_1 + A_{12}R_2 + C_1Y + B_1U \\ \overset{\circ}{R}_2 &= A_{21}R_1 + A_{22}R_2 + C_2Y + B_2U \end{aligned} \qquad (28)$$

is equivalent to system (27) if there is transformation (11) presented by change of generalized coordinates in form

$$\begin{vmatrix} X \\ Q_1 \\ Q_2 \end{vmatrix} = \begin{vmatrix} Y \\ R_1 \\ R_2 \end{vmatrix} + \begin{vmatrix} H\,(Y,R_1,R_2) \\ J_1(Y,R_1,R_2) \\ J_2(Y,R_1,R_2) \end{vmatrix} \tag{29}$$

which is a solution of matrix partial differential equation, given in form of equation (12). Transformation (21), writen in a form

$$\begin{vmatrix} Z \\ V_1 \\ \hdashline V_2 \end{vmatrix} = \begin{vmatrix} -L & I \\ \hdashline I-NL & N \end{vmatrix} \begin{vmatrix} Y \\ R_1 \\ \hdashline R_2 \end{vmatrix} \tag{30}$$

transformes system (28) into new decoupled one, given in a form

$$\begin{vmatrix} \dot{Z} \\ \dot{V}_1 \\ \dot{V}_2 \end{vmatrix} = \begin{vmatrix} F_{11} & F_{12} & 0 \\ F_{21} & F_{22} & 0 \\ 0 & 0 & F_{33} \end{vmatrix} \begin{vmatrix} Z \\ V_1 \\ V_2 \end{vmatrix} + \begin{vmatrix} K_1 \\ K_2 \\ K_3 \end{vmatrix} \tag{31}$$

where block-matrix are given by the formulas

$$\begin{vmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{vmatrix} = \begin{vmatrix} A & D_1 \\ C_1 & A_{11} \end{vmatrix} + \begin{vmatrix} D_2 \\ A_{12} \end{vmatrix} L$$

$$F_{33} = A_{22} + L \cdot \begin{vmatrix} D_2 \\ A_{12} \end{vmatrix} \tag{32}$$

$$\begin{vmatrix} K_1 \\ K_2 \end{vmatrix} = N \left\{ B_2 + L \cdot \begin{vmatrix} B \\ B_1 \end{vmatrix} \right\} + \begin{vmatrix} B \\ B_1 \end{vmatrix}$$

$$K_3 = B_2 + L \cdot \begin{vmatrix} B \\ B_1 \end{vmatrix}$$

Matrices   and   are solutions of Riccati matrix differential equations, given by relations

$$\dot{L} = L \begin{vmatrix} D_2 \\ A_{12} \end{vmatrix} L - L \begin{vmatrix} A & D_1 \\ C_1 & A_{11} \end{vmatrix} + A_{22} L - \begin{vmatrix} C_2 & A_2 \end{vmatrix}$$

$$\dot{N} \doteq \begin{vmatrix} A & D_1 \\ C_1 & A_{11} \end{vmatrix} N - \begin{vmatrix} D_2 \\ A_{12} \end{vmatrix} (\,I + L\,N\,) - \tag{33}$$

$$- N \left(A_{22} + L \begin{vmatrix} D_2 \\ A_{12} \end{vmatrix} \right)$$

If we assume that vector V_2 of the system (31) has a fast modes with small energy and great damping we can neglect these modes by relation

$$V_2 = 0 \tag{34}$$

which leads to linear dependance between coordinates, given by relation

$$R_2 = L \cdot \begin{vmatrix} Y \\ R_1 \end{vmatrix} \tag{35}$$

Matrix L can be writen as

$$L = \begin{vmatrix} L_1 & L_2 \end{vmatrix} \tag{36}$$

Using relation (35) we can obtain reduced linear dynamic model of system (27), given by vector equation

$$\begin{vmatrix} Y \\ R_1 \end{vmatrix} = \begin{vmatrix} A - D_2 L_1 & D_1 - D_2 L_2 \\ \hdashline A_{11} - A_{12} L_2 & C_1 - A_{12} L_1 \end{vmatrix} \cdot \begin{vmatrix} Y \\ R_1 \end{vmatrix} + \begin{vmatrix} B \\ B_1 \end{vmatrix} U \tag{37}$$

If relation (35) transforms into matrix partial diferential equation (12) it gives modificate partial matrix diferential equation in form (12). It is very easy to prove that there are no reduced system whose structural dynamics is described by linear submodel, as in equation (27).

Using relation (35) between transformed generalized coordinates, transformation (29) gives new form

$$\begin{vmatrix} X \\ Q_1 \\ Q_2 \end{vmatrix} = \begin{vmatrix} Y \\ R_1 \\ R_2 \end{vmatrix} + \begin{vmatrix} H\,(Y, R_1, -L_1Y-L_2R_1) \\ J_1(Y, R_1, -L_1Y-L_2R_1) \\ J_2(Y, R_1, -L_1Y-L_2R_1) \end{vmatrix} \quad (38)$$

Relation (38) is a parametric dependance between initial generalized coordinates X, Q_1 and Q_2 . Formally it can be writen in explicite form by equation

$$Q_2 = Q_2(\, Q_1,\, X\,) \quad (39)$$

which leads to a final reduced nonlinear dynamic model, given by vector differential equation

$$\begin{aligned} \mathring{X} &= f\left[X, Q_1,\, Q_2(Q_1, X), U, t\right] \\ \mathring{Q}_1 &= A_{11}Q_1 + A_{12}Q_2(Q_1, X) + C_1X + B_1U \end{aligned} \quad (40)$$

Let us consider now vector function as a vector series given in a form

$$Q_2(Q_1, X) = F \cdot Q(Q_1, X) \quad (41)$$

we can find approximate solution of equation (39) as a solution agebraic linear equations given by Galerkin´s method.

On the figure (1) it is shown influence of elasticity of aircraft structures on its dynamics. Criterion for neglecting higher modes of structures cannot be presented explicitely. In some cases on the Bode´s plots there are a new parts of curves with high frequency and great damping, which are not of interest. We shall define criterion for neglecting in a dynamic model of a rigid structure aircraft.

The criterion is defined by frequency with zero amplitude ratio on Bode´s plot given for dynamic model of rigid structure aircraft. All higher modes with great frequency than certain one can be neglected. This criterion is based on a fact that the Bode´s curves for elastic structure aircraft are in domain of less amplitude ratio which means that frequency with zero aplitude ratio is less than certain one.

Example

Influence of elastic structure on motion of the system is presented in this example using Bode´s plots. Numerical example is based on the example shown in 2 for the aircraft F-89, which is presented on figures with dashes. Full line presents plots for the case of short period approximation of elastic structure aircraft with two degrees of freedom of motion and one degree of elastic torsion of the wing, which stiffnes is hypothetical.

Conclusion

A numerical method for solving of system motion and for synthesis of automatic control using flight dynamic model of elastic structure aircr aft is very complicated because time constant of the system is very small for higher order modes. The influence of these modes on motion and control is negligable so we can consider the system without their influence. Number of modes of reduced dynamic model is lower than initial one, what gives a linear dependance between generalized coordinates. Using this method we can obtain dynamic model which includes only slow modes with lower number than initial one. In engineering first ten modes are of interest because their damping can be very small or , in the case of flutter there is negative damping, i.e. gaining. For solving vector function of transformation of generalized coordinates is used Galerkin´s approximation.

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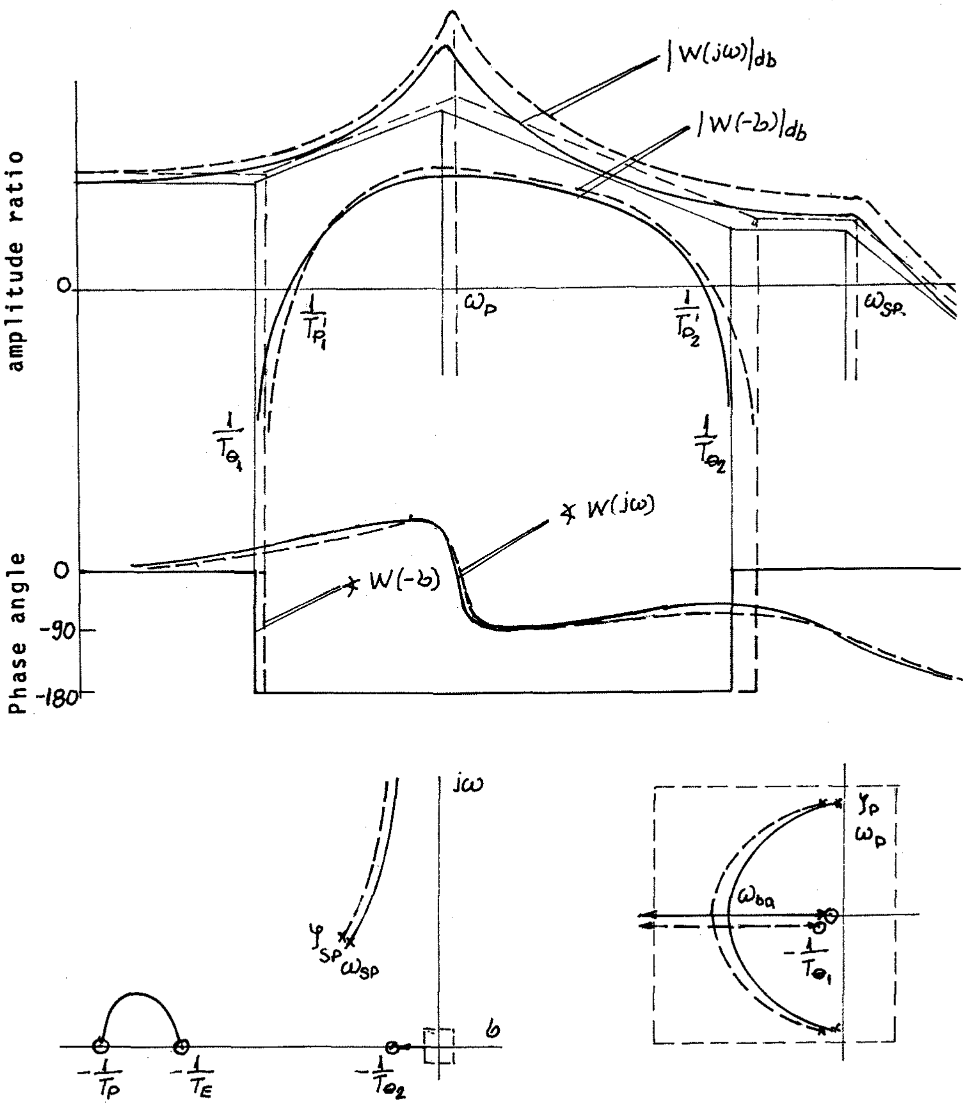
|W(jω)|db
|W(-b)|db
amplitude ratio
0
ωp
ωSP
Phase angle
0
-90
-180
W(jω)
W(-b)
jω
b
ωSP
ωp
ωba

Fig. 1.

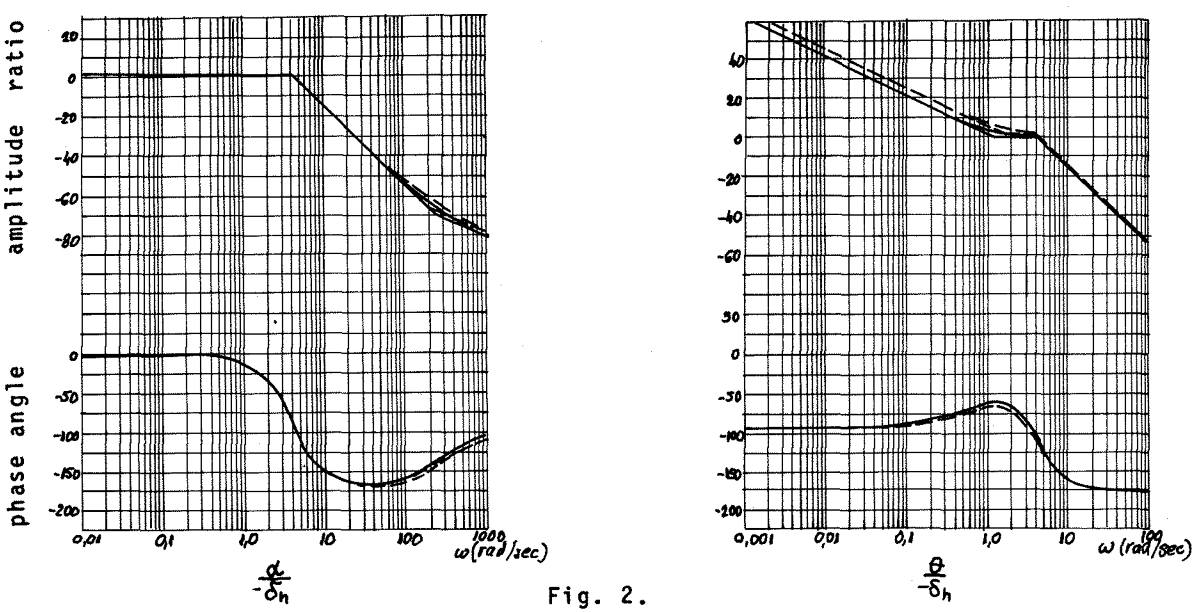
amplitude ratio
phase angle
20
0
-20
-40
-60
-80
0
-50
-100
-150
-200
0,01
0,1
1,0
10
100
1000
ω (rad/sec)
40
20
0
-20
-40
-60
30
0
-30
-100
-150
-200
0,001
0,01
0,1
1,0
10
100
ω (rad/sec)

Fig. 2.

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