ICAS-82-2.5.1

CRACKS INTERACTING WITH CONTACT FORCES - A FINITE ELEMENT STUDY ON LOADED HOLES

H. Ansell and B. Fredriksson
SAAB-SCANIA AB
Linköping, Sweden

ABSTRACT

A computerized method for the simultaneous solution of the contact and stress intensity factor problem has been developed. The solution is based upon the finite element method using virtual crack extension method for stress intensity factor calculations. The method is generally applicable to both two- and threedimensional problems and both through- and partthrough cracks could be studied.

The method has been applied to a lug with pressfitted sleeve and shaft. Both single- and doublesided cracks are studied. Contact pressures and stress intensity factors during crack propagating are presented. The stress intensity factors are compared with experimental results. The importance of the simultaneous solution of the contact and crack problem is demonstrated.

I. INTRODUCTION

Both military and civil aircraft are today designed for damage tolerance. For operating aircraft damage tolerance assessment are important when lifetime extensions are studied. Among other things this means that it is necessary to demonstrate time for cracks to propagate to critical length under given loads. This means application of fracture mechanics and study of crack propagation. A central parameter in reliable predictions of the growth of cracks is the stress intensity factor. This means a requirement of more detailed information about both load distribution and local stresses in the structure.

The most critical areas in aircraft structures are often attachment lugs and bolted and riveted joints. Cracks usually starts at loaded holes. In this areas both geometry and loading are very complex. It includes both contact and friction problems. Globally the contact and friction properties are important for internal load distribution and locally they are important for crack propagation studies. It might also be possible that cracks are closing not only due to plasticity at the crack tip but also due to reversed loads influencing the stress intensity range.

When designing the structure it is important to have methods to demonstrate both analytically and experimentally the integrity of the structure. In the analytical tool it is then important to simultaneously solve the contact and crack propagation problem. It is necessary for subsequent accurate crack propagation studies to have accurate information about the stress intensity factor and this is influenced by the contact forces.

The fatigue behavior of attachment lugs is a two-stage process, consisting of crack initiation and crack propagation. It is important to be able to predict both the time to initiation and the crack propagation life. Fatigue and fracture of lugs has been studied by numerous authors. Larsson(1) has developed a method for fatigue life prediction of lugs. The method is supported by fatigue data. This concept was further studied by Buch and Berkovits (2). The crack propagation in lugs was previously studied by Bäcklund and Ansell (3) and also by Zatz et al(4). Both studies were concerned with twodimensional lugs and both indicated importance of contact stress distributions. Threedimensional stress intensity calculations were performed by for instance Bartelds and de Koning(5) and Eiden and Reibaldi(6). They studied an attachment lug from Spacelab and calculated the stress intensity factor on the crack front for cracks in the pin and lug and also at the thread of the bolt. Only reference (3) involved the simultaneous solution of the contact problem.

If cracks emanates from a loaded hole, i.e. a hole filled with a bolt or rivet which transfer load the crack problem will interact with the contact problem. The contact forces between bolt and hole are depending on the crack length and the stress intensity factor depends on both magnitude and distribution of these forces and will be particularly important at short cracks. Since the major part of the lifetime is spent at short cracks the stress intensity factor should be particularly accurate for these. Earlier studies by Bäcklund and Ansell(3) for single side cracks in a lug, indicates a great influence of the hole boundary load distribution on the stress intensity factor. In those studies, comparisions of stress intensity factors were made between contact load distributions caused by an elastic bolt to apriori assumed load distributions. The results show that the stress intensity factors calculated with the usually assumed sinusoidally varying load distribution, do not approximate stress intensity factors calculated with the correct contact load distribution in an acceptable way. A better approximation is made with a constant load intensity, since a contact load distribution is almost constant in the central part of the hole boundary. (See Fig 1).

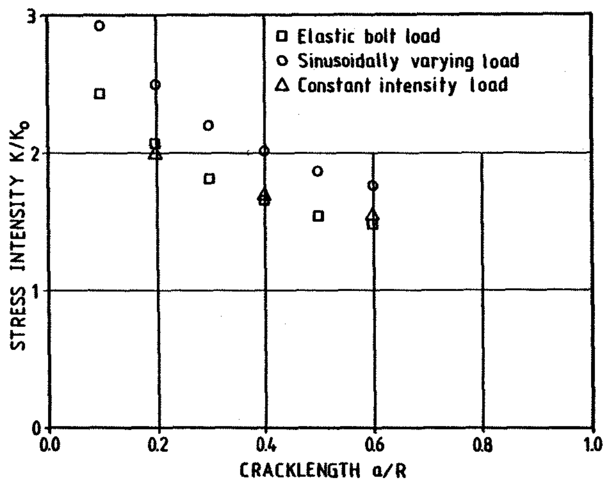

Fig 1. Stress intensity factors for three types of bearing pressure distributions. From ref.(3)

In the present paper we use a method that take the contact interaction effects into account in a correct way. This means that the contact problem is solved simultaneously to the stress intensity calculation. The solution of contact problems using the finite element method means minimization of the total potential energy with inequality constraints at the contacting nodes. Routines were developed and implemented into the general purpose FE-program ASKA by Torstenfelt(11). The routines developed are generally applicable to both twodimensional as well as three-dimensional contact problems. It is thus possible to study both through and part-through cracks. The method developed were applied to the lug with pressfitted sleeve and shaft. Both single and double sided cracks were studied. Contact pressures and stress intensity factors during crack propagation are presented. The stress intensity factors are compared with experimental results. The importance of the simultaneous solution of the contact and crack problem is demonstrated.

II. THEORY

The present method utilizes the finite element (FE-) method for the discretization and solution of the elasticity problem. When studying a conventional elasticity problem not involving contact problems the boundary conditions are prescribed loads p_i and prescribed displacements $\bar{u}_i$ on Ω_p and Ω_u respectively. The finite element displacement method means minimization of the total potential energy with equality constraints(7).

$$\text{Min}\ \{\pi = \int_V (\tfrac{1}{2}\sigma_{ij}\ \varepsilon_{ij} - X_i\ u_i)\ dV - \int_{\Omega_p} p_i\ u_i\ d\Omega \ \Big|\ \begin{matrix} u_i = \bar{u}_i \\ \text{on } \Omega_u \end{matrix}\ \} \qquad (1)$$

σ_{ij} = Stress tensor

ε_{ij} = Strain tensor

X_i = Body forces

u_i = Displacement vector

p_i = Surface loads

V = Volume occupied by the structure

Ω = $\Omega_p + \Omega_u$ = Surface of the structure

Introducing the FE-formulation we obtain in matrix notations.

$$\text{Min}\ \{\pi = \tfrac{1}{2}\underline{u}^t\ \underline{K}\ \underline{u} - \underline{P}^t\underline{u}\ \mid\ \underline{u} = \bar{\underline{u}} \text{ on } \Omega_u\} \qquad (2)$$

$\underline{u}$ = structure displacement vector

$\underline{K}$ = structure stiffness matrix

$\underline{P}$ = structure consistent load vector

The result is thus a minimum principle with so called equality constraints. The minimization will result in the classical linear system of equations to solve.

Considering two elastic structures A and B contacting each other we obtain contact boundary conditions which involves inequalities. The inequalities is introduced because points on either A or B approaching each other are not allowed to penetrate into the contacting material. At subsequent stages it is however possible that the contact is lost, if tensile stresses occur and there is no adherence capacity at the contact surfaces. If there for instance at the unloaded state are an initial gap h (x, y) or interference (negative gap) the differential normal displacement in A and B should satisfy

$$u_n^A(x,y) - u_n^B(x,y) \leq h(x,y) \text{ on } \Omega_c \qquad (3)$$

Ω_c = contact surface (unknown)

In addition the contact problem involves the equality constraints of prescribed displacements $\bar{u}_i$ and loads p_i. The solution of the elastic contact problem could be obtained from the principle of minimum potential energy π with inequality constraints.(7)

$$\text{Min}\ \{\pi = \int (\tfrac{1}{2}\sigma_{ij}\ \varepsilon_{ij} - X_i\ u_i)\ dV - \int_{\Omega_p} p_i\ u_i\ dV\ \Big|\ \begin{matrix} u_i = \bar{u}_i \text{ on } \Omega_n \\ u_n^A - u_n^B \leq h \text{ on } \Omega_c \end{matrix}\ \} \qquad (4)$$

$V = V_A + V_B$ = Volume occupied by structures A and B.
Introducing the FE-formulation we obtain in matrix notations.(7)

$$\text{Min } \{\pi = \tfrac{1}{2}\underline{u}^t \underline{K} \underline{u} - \underline{P}^t\underline{u} \mid \begin{array}{l} \underline{u} = \underline{u} \text{ on } \Omega_n \\ \underline{A}\,\underline{u} \leq \underline{h} \text{ on } \Omega_c \end{array} \} \quad (5)$$

This constitutes a quadratic programming problem with linear inequality constraints. The solution to this problem could be obtained from FE-programs by adding a module that automatically search for which contact nodepairs that are active in Eq (5). The conditions for checking wether the nodes are in contact or not are either the inequality condition in Eq (5) or if tensile normal stresses occur. Each changes of status means a transformation of the stiffness matrix and could be considered as a variable multipoint constraint transformation as described by Torstenfelt(11). The formulation of contact and also friction problems has been described in detail by Fredriksson(7)(9). The result of the FE-solution to the contact problem is in addition to stresses and displacements in the structures also generalized or consistent contact nodal forces, R_i. These forces should be transformed to normal contact pressure p_i. This could be done in a number of ways. A consistent way of doing it is to use an interpolation similar to the FE-method. Assume that the contact surface element (2D = line element, 3D = surface element) shape function is $\underline{\Phi}_e$. In terms of normal contact pressure p the consistent normal force $\underline{R}_e$ for element e could be written

$$\underline{R}_e = \int_{\Omega_e} \underline{\Phi}_e^t \; p \; d\,\Omega \quad (6)$$

Approximating p with $\underline{\Phi}_e$ we obtain

$$\underline{R}_e = (\int_{\Omega_e} \underline{\Phi}_e^t \, \underline{\Phi}_e \; d\,\Omega) \; \underline{p}_e = \underline{C}_e \, \underline{p}_e \quad (7)$$

The total normal contact force $\underline{R}$ are the sum over all contact surface elements of the element contact forces $\underline{R}_e$ and we obtain

$$\underline{R} = \underline{C}\,\underline{p}, \; \underline{C} = \Sigma \, \underline{C}_e \quad (8)$$

$\underline{C}$ is obtained by an assamblage process predicted by contact surface topology. It should be noted from Eq. (7) that $\underline{C}_e$ is equivalent to the consistent mass matrix for the surface element with density and thickness equals unity. The $\underline{C}_e$ matrix is thus available in general purpose programs. By using this "consistent mass" method of obtaining the contact pressure we have experienced that it gives accurate results away from contact boundaries and high gradients but is prone to oscillations close to such areas. The result is stabilizing if the "lumped mass" method is used instead. The lumped mass means that the diagonal mass matrix $\underline{C}_e$ is used in Eq. (7). The "lumped mass" method is used in this study. There are also other methods of obtaining contact stresses available.(11)

Stress intensity factor calculations by using the FE-method could be done in a number of ways.(10) A detailed study and evaluation of different methods has been performed by Bartelds and de Koning.(5) An attractive and also accurate method is to use the relationship between the stress intensity factor K_I and the energy release rate C at a virtual crack extension.

$$K_I^2 = \frac{E}{1 - n^2} \; C \quad (9)$$

$n = \nu$ (Poisson's ratio) for plane strain

$n = o$ for plane stress

The energy release rate C is then defined as the negative potential energy released per unit area of virtual crack extension

$$C = - \lim_{\Delta A \to o} \frac{\Delta \pi}{\Delta A} = -\pi' \quad (10)$$

ΔA = incremental virtual crack surface

Eq (9) is valid for mode I dominated cracks. At mixed modes it is not obvious how to separate the contributions from the different modes. The method is applicable also to threedimensional cracks. Which value of n to be used in Eq (9) is then depending on where on the crack front the K-value is to be calculated. If the crack is closing, terms are added in Eq (9) due to contact and friction.(8)(9) The calculation of C by using the FE-method is efficiently done by virtually shifting the nodes on the crack front.(5)(14) This shift then corresponds to a change ΔA in crack surface. The virtual shift of the FE-nodes means a change $\Delta\underline{K}$ in stiffness matrix. In matrix notations we then obtain from the FE-calculation

$$C = \tfrac{1}{2}\,\underline{u}^t \, (\frac{\Delta \underline{K}}{\Delta A}) \; \underline{u} \quad (11)$$

$\underline{u}$ is the displacement vector from the FE-solution prior to the virtual shift of the crack tip node. At the shift only $\Delta\underline{K}$ has to be computed.

III. FINITE ELEMENT MODEL

The previously described methods were used to simultaneously solve the contact and crack problem of a lug with pressfitted sleeve and shaft. The diametric interference between lug and sleeve was 0.084 mm and sleeve and shaft was net fitted. The analysis was performed with a shaft load F equal to 97 100 N producing a net stress equal to 68.8 MPa. The geometry of the lug is described in Fig 2. The FE-mesh is given in Fig 3. Quadratic 8-noded isoparametric elements were used. Crack tip elements were made singular by displacing the midnode to the 1/4-point.[13] At the transition from coarse to fine mesh for the crack tip substructure the multipoint constraint technique was used. The intermediate nodes were constrained according to the shape functions in the elements in order to maintain compatibility. The validity of the degree of refinement was checked and considered appropriate. The virtual shift of the crack tip when computing the stress intensity factor according to Eqs (9) and (11) was 10^{-4} times the crack tip element length. The complete analysis including contact problem solution, virtual crack tip shift and crack propagation is performed automatically.

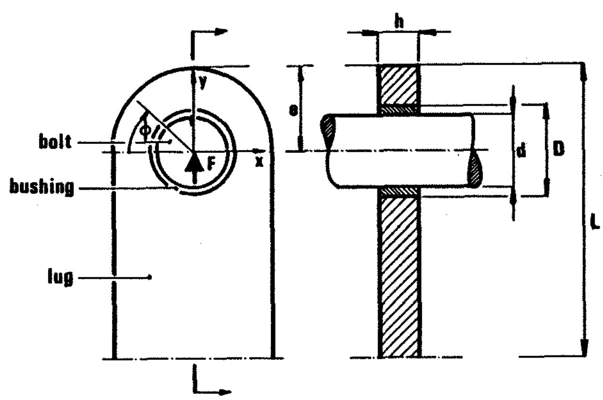

Fig 2. Analysed lug-sleeve-shaft structure.
L = 232.5 mm. e = 86.5 mm.
h = 14.0 mm. D = 72.0 mm.
d = 60.0 mm.

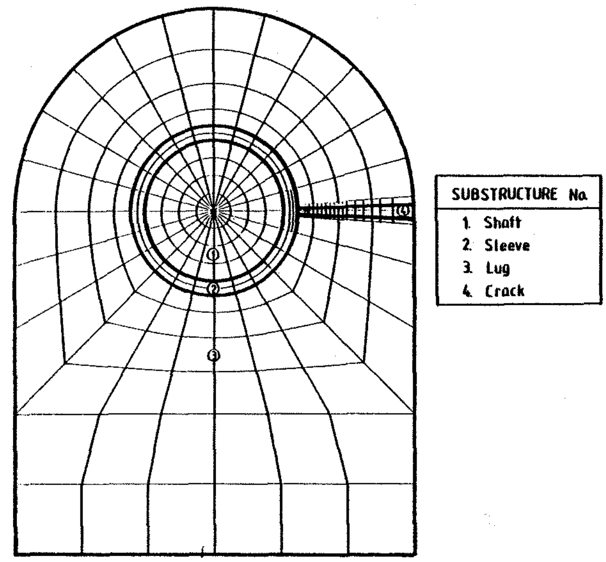

Fig 3. Lay-out of finite element mesh.

IV. EXPERIMENTAL PROCEDURE

One of the main purposes in this study is to correlate the theoretically obtained stress intensity factors to experimental data. One procedure to meet this request is to use results from crack growth tests since the crack growth rate can be presented as a function of stress intensity range. This means that the same crack growth rate is always measured irrespective of crack length, stress and component geometry for a specific stress intensity range. This method was found suitable since constant amplitude crack growth tests had been carried out on lugs with the same geometry, pressfit and loading as for the FE-analysed lug, Fig 1. The lug was manufactured from Zn-Mg-Cu-Ag aluminum alloy (Fuchs AZ74) with Young's moduli E = 69 800 MPa and Poisson's ratio ν = 0.33, the sleeve of Cr-Mo steel alloy with E = 206 000 MPa and ν = 0.32 and the shaft of Cr-Ni-Mo steel alloy with E = 196 000 MPa and ν = 0.32.

From the crack growth curve for the lug the crack growth rate is evaluated as a function of crack length.

$$(da/dN)_{lug} = fcn(a) \tag{12}$$

Before the crack growth rate of the lug can be used to obtain the experimental stress intensity factors, a reference test of crack growth rate with known stress intensity range has to be evaluated. The reference test has to agree to the lug in respect to material, plate thickness, stress ratio and environment, In this study compact tension (CT-WOL) test specimens was cut out from the individual lug specimens as shown i Fig 4. This procedure was found suitable in order to minimize the scatter in crack growth rate with respect to different material batches.

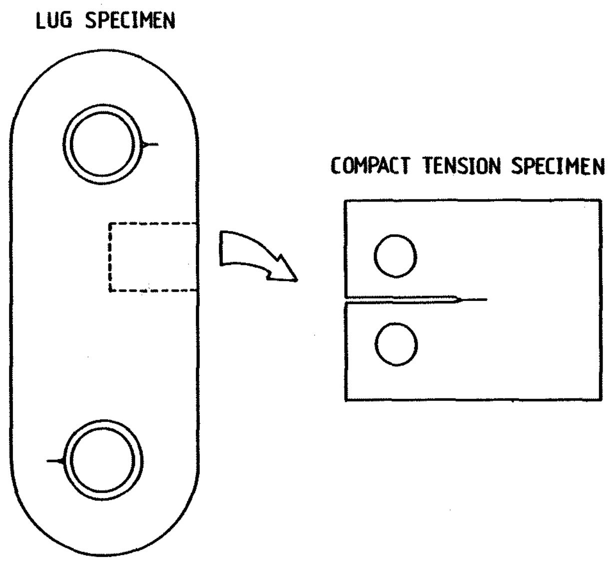

Fig 4. Individual reference test specimens cut from lug specimens.

From the crack growth curve for the reference specimen the crack growth rate is evaluated as a function of stress intensity range.

$$(da/dN)_{ref} = fcn(\Delta K_I) \tag{13}$$

This characteristics of the crack growth rate can then be transferred to the lug as a stated in the beginning. If a certain value of the crack growth rate of the lug is observed, it must have been produced by a specific stress intensity range found in the reference test at the same crack growth rate.

This procedure has earlier been used to obtain stress intensity factors for cracks in lugs by Schijve and Hoeymakers[15] and by Geijer[16]. The procedure is schematically shown in Fig 5.

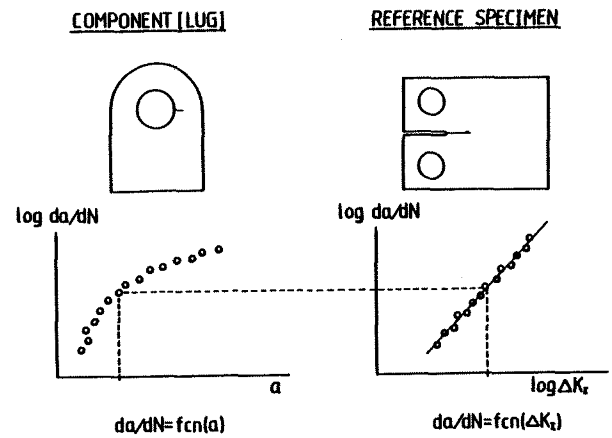

Fig 5. Procedure to obtain experimental stress intensity factors from crack growth tests.

V. RESULTS

The principal results from the theoretical FE-analysis of bearing pressure distributions and contact surface sizes between the sleeve and the lug are summarized and illustrated in Fig 6 and Fig 7. The illustrations show clearly the redistribution of the bearing pressures due to the propagating crack. These pressure distributions are calculated using the "lumped mass" method described in paragraph II. This method was selected since it showed an excellent capability to handle the pressure gradients. These gradients are caused by the discontinuity in the contact surface of the lug due to the crack. The illustrations show also that the contact surface size produced by the pressfit only, is changed due to the shaft load, F and contact is lost on part of the circumference.

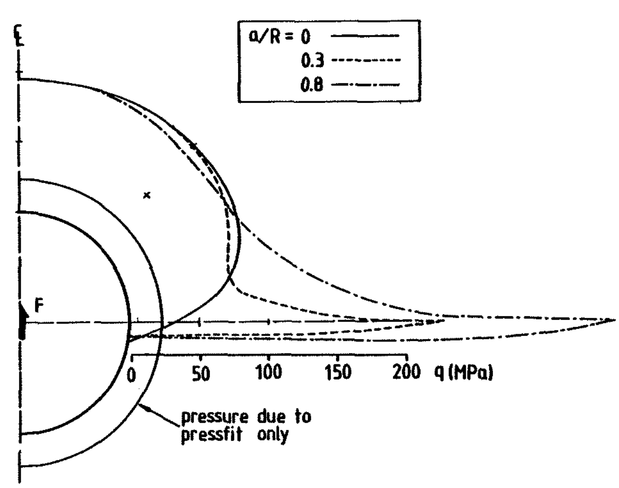

Fig 6. Bearing pressure distribution in lug-sleeve contact surface for a propagating double sided crack.

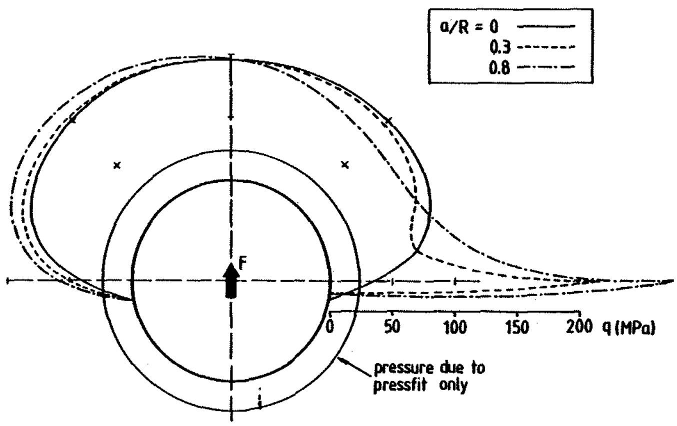

Fig 7. Bearing pressure distribution in lug-sleeve contact surface for a propagating single sided crack.

The results from the theoretical analysis of stress intensity factors are summarized in Table 1 and illustrated in Fig 8 and Fig 9. It is a common practice in the litterature to use a non-dimensional correction function depending on crack length only instead of the stress intensity factor. As a reference quantity it is useful to use a stress intensity which appear in a uniaxially loaded infinite plate with a central crack at the same stress and crack length.

$$K_0 = \sigma \sqrt{\pi a}$$

$$\sigma = \frac{F}{(2e-D)h} \quad \text{net stress}$$

a/R	K_0	Single sided cracks		Double sided cracks		% Difference
		K_I	K_I/K_0	K_I	K_I/K_0	
0,05	164	351	2,14	351	2,14	0,1
0,10	231	474	2,05	477	2,06	0,6
0,20	327	599	1,83	615	1,88	2,7
0,30	401	668	1,67	705	1,76	5,5
0,50	517	747	1,44	838	1,62	12,2
0,80	654	907	1,39	1118	1,71	23,3

Table 1. Theoretical stress intensity factors for single and double sided cracks. (MPa $\sqrt{\text{mm}}$).

Fig 8 and Fig 9 show a comparison between the experimental and the theoretically predicted stress intensity factors. Relativly close agreement between tests and calculations are shown. The experimental stress intensities indicates an anomaly for very short cracks, i.e. for crack lengths between 3.2 and 4.2 mm. Since the machined V-notch has a length of 3 mm, the corresponding fatigue cracks are 0.2-1.2 mm. In this initial phase is the experimantal crack growth rate significally lower than predicted. This may be caused by the initiation of cracks from a machined notch and by a low accuracy in determining the crack growth rate in the beginning of the crack growth curve. This observation was also made by Schijve and Hoeymakers.(16)

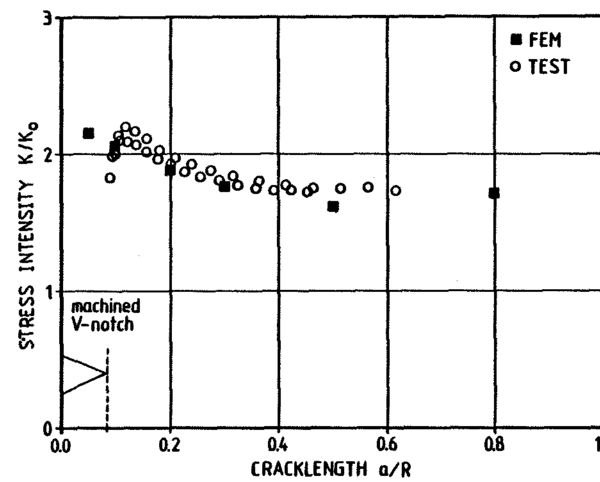

Fig 8. Comparision between experimental and theoretical stress intensity factors for double sided cracks.

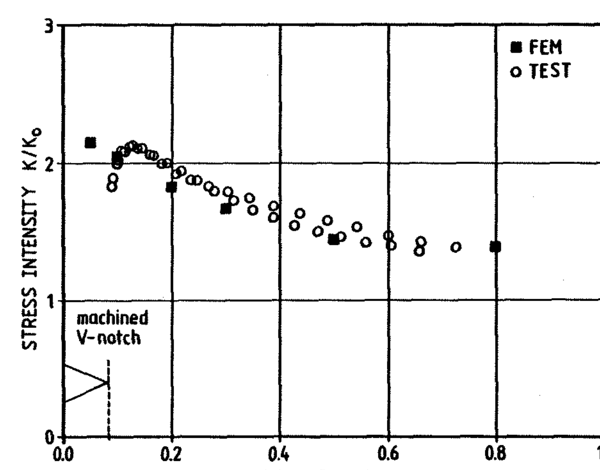

Fig 9. Comparision between experimental and theoretical stress intensity factors for double sided cracks.

VI. CONCLUSIONS

The method of simultaneously solving the contact and stress intensity problem of propagating cracks was shown to be useful and also necessary for accurate analysis. The results from previous analysis show that by apriori assuming a bearing pressure distribution the stress intensity could be in considerable error. The pressure is also redistributed during crack propagation as shown in this study. It is in a practical situation with varying lug geometries difficult to generally recommend what bearing pressure distribution that should be used for accurate stress intensity factor calculations. For two-dimensional lugs with very large 2e/D ratios there are analytical results available. Persson(17) has presented the solution to the plane stress or plane strain problem of an elastic shaft in an infint half plane. On the other hand, for lugs with small 2e/D ratios there are no analytical solution of bearing pressure distribution available and it is shown in this study that the bearing pressure distribution is significantly different from Perssons(17) solution which is close to sinusoidally. For geometrically more complex lugs such as wing rudder attachments where threedimensional effects might be of considerable importance it is almost impossible to apriori assume a pressure distribution. An analysis tool for the simultaneous solution of contact and stress intensity problem become a necessity.

The analytical tool applied in this work could also be used efficiently, as a complement to tests, for parameter studies. It could for instance be used to study influence of sleeve and lug interference on stress distribution and thus also on the stress intensity factor distribution. It could be used to study the stress intensity factor for cracks propagating in different directions. Both symmetrical and/or transverse loading could be analysed.

ACKNOWLEDGEMENT

The Swedish Air Materials Department are greatly acknowledged for their financial support of this work. The authors are also expressing their sincere gratitude to Mr Bo Torstenfelt at Linköping University, Institute of Technology for developing the crack propagation program and for placing his contact analysis program to our disposal.

REFERENCES

1. Larsson S-E.
The Development of a Calculation Method for the Fatigue Strength of Lugs and a study of Test Results for Lugs of Aluminum.
In Fatigue Design Procedures.
Oxford Pergamon Press 1962.

2. Buch A., Berkovits A.
Fatigue Life of Lugs under Service Loading - Test Results and Predictions.
Materialprüf 23 (1981) Nr 10 October.

3. Bäcklund J., Ansell H.
Cracks at Loaded Holes.
Proc. Euromech. Colloquium No 110 on Contact Problems and Load Transfer in Mechanical Assemblages.
Rimforsa, Sweden, Sept 27-29, 1978.

4. Zatz I.J., et al
An Application of the Energy Release Rate Concept to Crack Growth in Attachment Lugs.
In Structures, Structural Dynamics and Materials Conference, 22nd Atlanta Ga., April 6-8, 1981, Technical Papers. Part 1 (A81-29377 12-39)
New York. AIAA Inc 1981 p 402-15.

5. Bartelds G., de Koning, A.U.
Application of Finite Element Methods to the Analysis of cracks.
Reports NLR TR 78138U and NLR TR 79118U
National Aerospace Laboratory NLR, The Netherlands.

6. Eiden M., Reibaldi G.
Bruch/Rissmechanik Analyse eines Spacelab-Bolzeus Proc.
Finite Element Congress. Baden-Baden, 1981.

7. Fredriksson B., Rydholm G., Sjöblom P.
Variational Inequalities in Structural Mechanics with Emphasis on Contact Problems.
Proc Int Conf on Finite Elements in Nonlinear Solid and Structural Mechanics, Geilo Norway August/September 1977.

8. Fredriksson B., Rydholm G.
Mechanical and Temperature Contact in Fuel Rod and Cladding.
Nucl. Eng. and Design, Vol 48, No 1, June 1978, pp 69-79.

9. Fredriksson B.
Elastic Contact Problems in Fracture Mechanics.
Proc. Fourth Int. Conf. on Fracture, Vol 3, pp 427-35, Waterloo Canada, June 19-24, 1977.

10. Fredriksson B., Macherle J.
Stress Analysis Software for Fracture Mechanics
Linköping University. Institute of Technology Dept. of Mech. Eng.
Linköping, Sweden, 1978

11. Torstenfelt B.
Contact Problems with Friction in General Purpose Finite Element Computer Programs.
To be published in Computers & Structures.

12. Henshell R.D., Shaw K.G.
Crack Tip Elements are Unnecessary.
Int. J. Num. Meth. in Eng., Vol 9,
pp 495-507, 1975.

13. Barsoum R.S.
On the use of Isoparametric Finite
Elements in Linear Fracture Mechanics,
Int. J. Num. Meth. in Eng., Vol 10,
pp 25-37, 1976.

14. Hellen T.K.
On the Method of Virtual Crack Extensions.
Int. J. Num. Meth. in Eng. Vol 9,
1975, pp 187-207

15. Schijve J., Hoeymakers A.H.W.
Fatigue Crack Growth in Lugs and the
Stress Intensity Factor.
Report LR-273. Delft University of
Technology, Department of aerospace
engineering, Delft, Netherlands,
July 1978.

16. Geier W.
Strength Behaviour of Fatigue Cracked
Lugs.
Dissertation Technical University
Munich, 1980. Royal Aircraft
Establishment, Library Translation
2057, January 1981.

17. Persson B.G.A.
On the Stress Distribution of Cylindrical Elastic Bodies in Contact.
Chalmers University of Technology.
Dissertation. 1964.