REVIEW OF NUMERICAL METHODS FOR THE PROBLEM OF THE SUPERSONIC FLOW AROUND BODIES AT ANGLE OF ATTACK

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Abstract

Numerical methods for three-dimensional steady supersonic inviscid flow around bodies modelling vehicles and their separate parts are reviewed. Three main groups of methods are cosidered: finite-difference net methods,method of characteristics, method of integral relations (including method of lines). A number of different examples with numerical results concerning flow structure and aerodynamic properties under various flight conditions are discussed. In some cases nonequilibrium processes in the air are taken into account.

I. Introduction

One of the most important problems in supersonic aerodynamics is the determination of flow-fields, forces and moments acting upon a flying vehicle, as well as the determination of heat flows. About two decades ago the physical experiment in wind tunnels played the main role in this research. Today,however, the computational experiment has become dominate. One of the reasons of it is an extremely high price of experiments in wind tunnels, besides there is a tendency for both price and time of such experiments to grow exponentially.

Once computers were invented they have been used widely in aerodynamics ever since. Now a new branch of that science - computational aerohydrodynamics has been developed. The computational experiment has definite advantages: high precision, possibility of investigation of separate influence of different factors. It can also be used when physical simulation is impossible in principle (for example, at hypersonic speeds). The success in the development of numerical methods and increase of the computer capacity make it possible to solve more and more complex problems, passing from the treatment of separate aerodynamical elements to a whole vehicle, taking into account high temperature effects (physical-chemical processes, radiation). Much success has been achieved in study of inviscous gas flows, which furnishes enough information in a number of interesting cases. Further consideration of viscosity may be carried out in the frame work of the boundary layer theory.

A review of the development and the present state of numerical methods for three-dimensional steady supersonic inviscid flows around bodies with shock waves is presented in this paper. The methods use models of perfect gas with constant ratio of specific heats γ and also of equilibrium and nonequilibrium reacting gases. The radiation effects are not discussed because numerical methods of radiation gasdynamics are an individual topic.

Three main groups of the methods are reviewed: the finite-difference net methods, the method of characteristics, the method of integral relations with method of lines. The finite-element method is not touched upon because it is not yet utilized for computing three-dimensional nonisentropic supersonic flows. In the survey the main attention is paid to a series of practically important cases of supersonic three-dimensional flows. Some numerical results of flow researches both for the main parts of vehicles and for vehicles as a whole are presented. Flow structures and aerodynamic properties of bodies under various flight conditions, in particular,the influence of nonequilibrium processes are discussed. The numerical results,belonging mainly to different Russian scientists,are cited because, they have made a great contribution to the development of computational aerodynamics and these results are not well known abroad.

The more detailed survey by the same authors (1) dealing with the numerical methods for both two-dimensional and three-dimensional supersonic flows and containing over four hundreds references has preceded this review.

II. Finite-difference net methods

Finite-difference net methods for calculating supersonic flows have received wide applications. In these methods original differential equations have been approximated by finite-difference expressions using a net that is not connected with characteristic directions. In spite of a large number of the methods they can be classified according to some features.

There are explicit and implicit, one-step and multi-step, one-reference plane and multi-reference plane finite-difference schemes. Calculations in all schemes are carried out step by step from certain reference plane to the next one. In any explicit scheme the solution in the subsequent reference plane depends on the previous plane only, but in any implicit scheme it depends on preceding and subsequent reference planes. Explicit schemes are submitted to a stability condition and therefore to a step size restriction. Implicit schemes have no such restriction but they require iterations. In two-step schemes there are two stages of solution: at first intermediate values are obtained, and then final ones are calculated. These schemes are classified as splitting schemes. There are analytical, geometrical and physical splittings. The order of accuracy of a scheme is determined by the error of the solution. Second-order accuracy schemes are more often used for flow problems.

There are two approaches in calculating shock waves - shock fitting and shock capturing. According to the first approach, all the shock waves are treated as strong discontinuities being boundaries of the corresponding subdomains of the solution. According to the second approach the existence of shock waves is not explicitly supposed. They appear in numerical solution as narrow zones with large gradients of gasdynamic functions.

When a subsonic region arises in steady supersonic flow, the time-dependent stationing principle often is applied. In this case the complicated steady elliptic-hyperbolical problem is replaced by the simpler unsteady hyperbolical problem owing to the addition of the time as the fourth independent variable. For conical flow the self-similar stationing principle is usually used.

All the above mentioned features may occur in finite-difference methods in different combinations which leads to a large variety of computional algorithms. Certain methods are effective for certain problems only. We shall discuss the finite-difference methods which were practically applied for computations of three-dimensional supersonic flows.

Explicit first-order method. This method (which is known as Godunov method) with many applications is reviewed in the book (2). At first it has been developed by Godunov, Zabrodin and Prokopov (3) for axisymmetric supersonic flow problem using the time-dependent stationing principle. Its extension to three-dimensional steady problems has been carried out later by Ivanov et al (4). The governing equations are written here in the integral form. The solution (for each pair of neighbouring cells) of the self-similar problem of interaction of two semi-infinite streams having the parameters from preceding reference plane is the basis of the algorithm. The method can be suitable for both smooth and discontinuous solutions. For smooth solutions it has the first order accuracy. The method is an effective tool for the investigation of very complicated flows with discontinuites of various kinds. However, for smooth flows it becomes noneconomical, because of the step size restriction. Besides it is difficult to adapt it to the case of reacting gas flows.

As an example of the application of this method the results by Ivanov and Nikitina (5) for the flow-field about the front part of the vehicle at $M_\infty=5$ and angle of attack $\alpha=5°$ are shown in Fig.1. Here solid and dashed lines represent the body, double lines represent the bow shock, and thin lines are the isobars (P/P_∞).

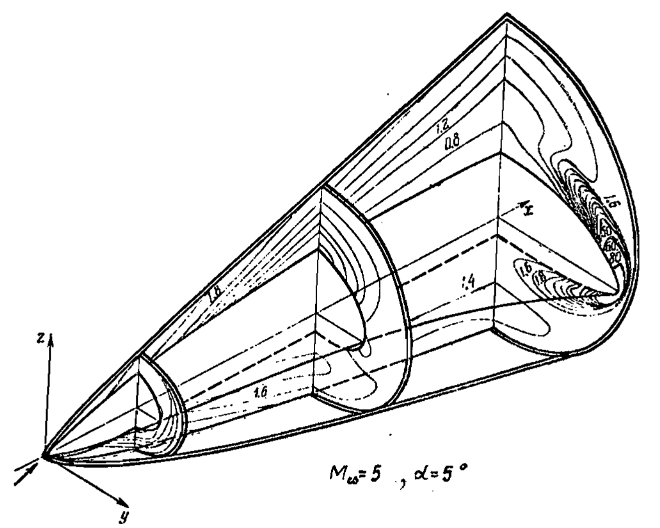

Fig.1 Flow about vehicle nose.

With some modification of this method and the application of shock capturing approach the supersonic flow about blunt cone at angle of attack has been computed by Kolgan (6), while the flows about two delta shaped vehicles with swept $\chi=70°$ and different convex lower side at $M_\infty=5$, $\alpha=5°$ has been calculated by Kosykh and Minailos (7). The cross-flow is given in Fig.2, where the bow shock attached to the body apex, the embedded shock, the characteristic, the cross-flow sonic line are drawn by solid, dashed and dash-dotted lines.

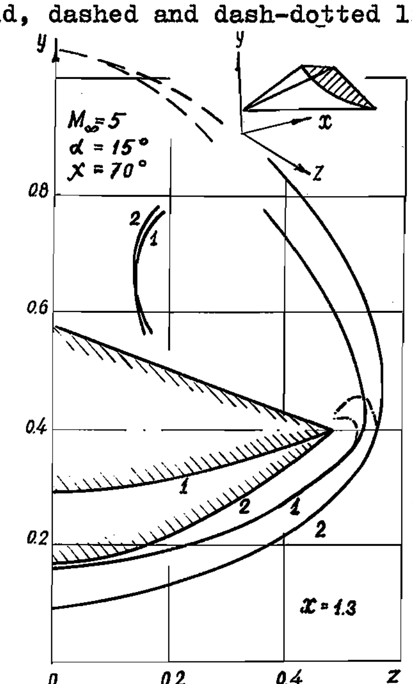

Fig.2 Cross-flows about two deltaplanes.

Implicit second-order method. It has been suggested by Babenko and Voskresensky (8). This method was the first method specially developed for three-dimensional supersonic flows and it is used widely in various modifications now. Computational algorithm advances the solution through the successive reference planes along the body. The solution of the local boundary-value problems on the rays, which lie in meridional planes and pass from body to bow shock wave, is the basic element of the algorithm. These local problems are solved from ray to ray by the sweep method, known in Russian literature as the "progonka" method. Because these problems are interconnected through the right parts of the finite-difference equations, the iteration procedure is necessary.

The detailed description of the method with stability and convergence analysis and some improvements are given in the book by Babenko et al (9). Afterwards Yu.N. D'yakonov, Yu.Ya.Mikhailov, Yu.B.Radvogin and others have improved the stability of the method in complicated cases.

A great number of numerical investigations for three-dimensional purely supersonic flows about pointed and blunted bodies were carried out. The numerical solution of the classical problem of the supersonic perfect gas flow about circular cone at angle of attack was obtained in (9). The results are represented as the tables of gasdynamical functions for the cones with semi-angles $\omega = 10° \div 45°$ at $M_\infty = 2 \div 7$ and angles of attack $\alpha = 0 \div 20°$. Some examples of equilibrium air flows about pointed bodies are also given here.

The flow about a circular cone at an angle of attack that exceeds the cone semi-angle is investigated by Bachmanova et al (10), while the case of elliptical cones is considered by Vetlutskii and Ganimedov (11). Three-dimensional flow about strongly flattened elliptical cones and non-conical body are studied by Voskresensky et al (12). Fig.3 shows some results for the non-conical flattened body of deltaplane type. The plot reveals a sharp pressure maximum on the side of the vehicle.

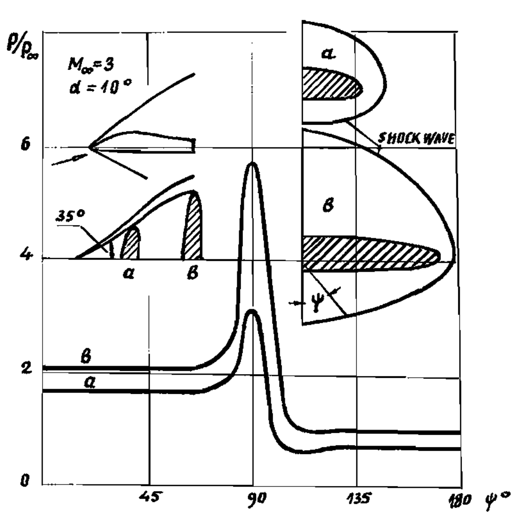

Fig.3 Flow about deltaplane.

The method has been extended by Voskresensky (13) to the case of profilied delta wings with shock wave attached to the leading edges. The table (14) of the flow-field about flat delta wings with swept $\chi = 45° \div 75°$ at $M_\infty = 1.7 \div 10$ and $\alpha \le 15°$ were calculated. The flow-fields about delta, swept and rhombic-shaped sharp tip wings were determined (15). These calculations reveal that transition from flat surface on the lower side of delta wing to convex surface radically changes the character of the loading distribution on the wing. It ascertains the great importance of the lower delta wing form surface.

The supersonic domain of streams about blunted bodies at angles of attack was calculated by some authors. The perfect and equilibrium gas flow about spherically blunted direct and inverse cones were investigated by D'yakonov et al (16). Spherically blunted body with elliptical cross-section was considered by Mikhailov et al (17). Some results calculated by Radvogin for the blunted complicated body at $M_\infty = 15$, $\alpha = 10°$ are shown in Fig.4. The body cross-section, the shock wave and the isobars are presented here.

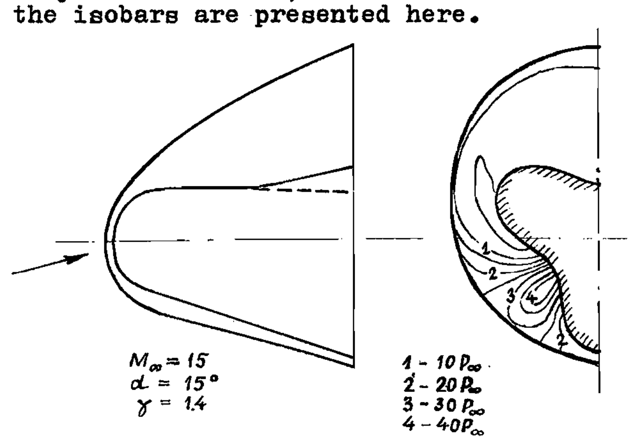

Fig.4 Flow around blunted body.

In order to calculate mixed subsonic-supersonic flows around blunt nose part of bodies Babenko and Rusanov (18) have modified the method. They have taken governing unsteady equations and used time-dependent stationing principle. At first, axisymmetric case was considered and later Rusanov (19) and Babenko (20) have extended the method to a three-dimensional case. Rusanov and Lyubimov (21) have considered the front parts of spherical and elliptical paraboloids at $M_\infty = 4 \div 10$ and $\alpha \le 15°$ and Babenko et al (22) - ellipsoids of revolution with large axis which lie across free stream at $M_\infty = 6 \div 20$. The method also was modified and used by Voskresensky (23) for the mixed flow about the front part of an arbitrary wing with blunted airfoil. In Fig.5 one example of the computations is presented where the shock wave in front of the delta wing at $M_\infty = 3.5$ and $\alpha = 5°$ is shown.

The method in its original form was intended for the calculation of smooth flows. Embedded shock waves of weak intensity only could be computed introducing some dissipative terms in the scheme.

Weiland (24) has modified the method taking the conservation form of the governing equations and computed the flow about a blunt nosed cylinder at $M_\infty=1.4$, $\alpha=10^\circ$ capturing an embedded shock wave.

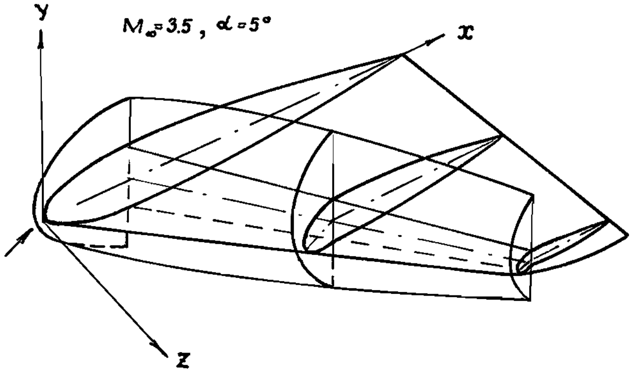

Fig.5 Shock wave in flow about delta wing.

Three-dimensional supersonic nonequilibrium flows are very complicated for numerical analysis. An unpublished example of such calculation for the ellipsoid of revolution at $M_\infty=15$, $\alpha=24^\circ$, $H=60$ km. was kindly presented by Yu.B.Radvogin. The shock wave, the sonic lines and the temperature distributions in the symmetry plane are shown in Fig.6.

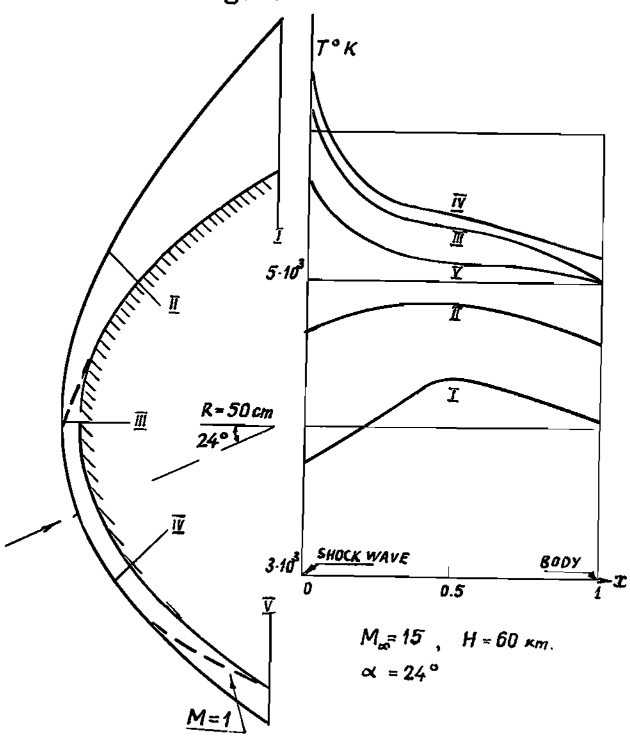

Fig.6 Nonequilibrium flow about ellipsoid nose.

To reduce the number of nodal points the method was modified in some works by introduction of trigonometrical approximation in respect of meridional angle. In this manner Mikhailov and Savinov(25) have calculated a perfect air flow about the front part of ellipsoids at angles of attack and yaw, while Savinov and Shkadova in (26) studied nonequilibrium carbondioxide flows about the front parts of blunted cone and segment-conical body. The relaxational equations in(26) are integrated in the symmetry plane only, while in other meridional planes nonequilibrium effects were taken into account totally by trigonometrical interpolation of a frozen adiabatic coefficient and an energy influence function. The same approach was utilized by Burdelny and Minostev (27) for purely supersonic domain of nonequilibrium three-dimensional flow about segment-conical body.

Splitting methods. The numerical solution of three-dimensional flow problem using the time-dependent stationing principle requires the computers with high capacity. Bohachevsky and Mates (28) calculated with the help of simple one-step explicit Lax scheme the flow-field about the nose part of reentry vehicle "Apollo" at angle of attack.

However, explicit schemes with splitting are more efficient for multi-dimensional problems. The splitting can also improve both the stability and the convergence of the scheme. According to Yanenko, three kinds of splitting can be distinguished:

1) analytical splitting on which two-step and multi-step schemes are based;
2) geometrical splitting on which the fraction-step method or alternating direction method is based;
3) physical splitting on which methods of particles are based.

Two-step schemes. The first successful two-step scheme was Lax-Wendroff second-order scheme. MacCormack suggested another two-step second-order scheme, the so-called "predictor-corrector" scheme. In this scheme at the stage "predictor" the stability is provided and at the stage "corrector" the conservation laws are fulfiled and the accuracy is increased.

Let us discuss some applications of these schemes for three-dimensional supersonic flow. At first, we shall consider several cases of simple aerodynamical bodies and then of more complicated configurations. In all these cases(with the exception of three below mentioned cases) MacCormack scheme has been utilized. Using the time-dependent principle and shock-capturing approach, Kutler and Lomax (29) calculated some flows about circular cone at large angles of attack, flat delta wing, delta wing with half-cone. In the same manner the flat delta wing with subsonic leading edges was considered by Bazzhin and Chelysheva (30). Fitting the bow shock wave, Pandolfi (31) studied flow about elliptical cones at angles of attack and yaw. Using the same approach the flat delta wing with half-cone was calculated by Lobanovskii(32). An interesting three-dimensional internal corner flow problem is solved by different authors. A system of shock waves found by Marconi (33) is sketched in Fig.7.

Using Lax-Wendroff scheme, Li computed (34) nonequilibrium oxygen and nitrogen flow about the front part of ellipsoid at angle of attack. Among works where the mixed flow about blunt nose together with subsequent supersonic domain are calculated let us note the work by Walkden et al (35). Here the flat delta wing with blunt leading edges at angle of attack is considered.

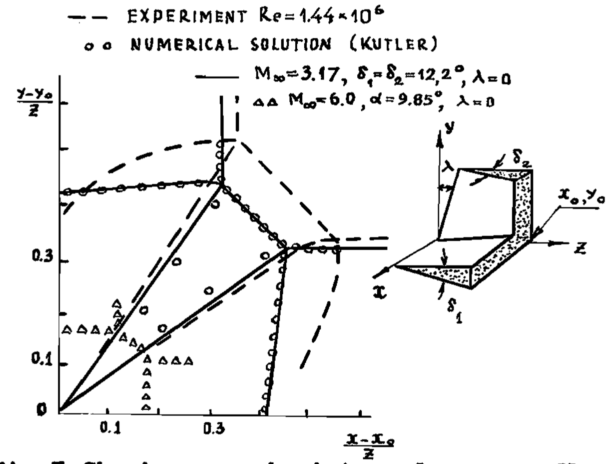

Fig.7 Shock waves in internal corner flow.

Moretti (36) has recently suggested two-step " λ - scheme" and determined the supersonic flow about strongly flattened elliptical cone at angle of attack. In this scheme fixed computational net is used. However, the governing equations have characteristic form. The net points are chosen so that the computational and physical regions of influence should be close.

The increase of computer capacity up to tens of millions operations per second makes it possible to calculate supersonic flows about complicated configurations resembling real vehicles. In such calculations splitting schemes and bow shock fitting approach are used as a rule. Embedded shocks are either fitted or captured. Marconi and Salas (37) computed the flow about the nose of an aircraft. The embedded shocks, appearing in calculation, were fitted as boundaries of subdomains of the solution. The numerical and experimental values of the pressure coefficient C_p on the upper side of the fuselage is plotted in Fig.8 at $M_\infty = 2.2$ and $\alpha = 5°, 10°$. The pressure peak is due to the embedded shock before the cockpit.

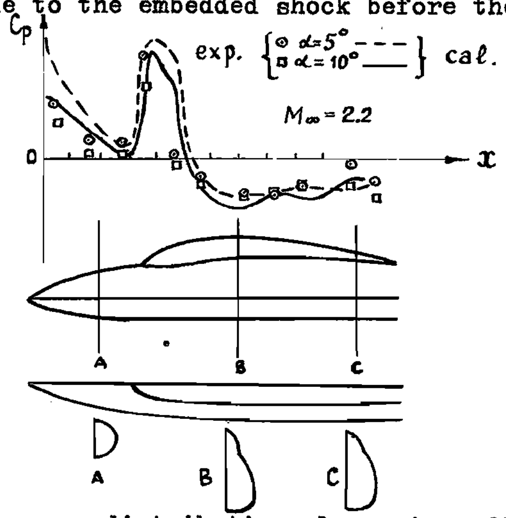

Fig.8 Pressure distribution along aircraft nose.

Other authors using the conservation form of governing equations spread embedded shocks. Kutler and Lomax (38) determined the flow-fields about the front part of a spacecraft in the perfect and equilibrium air. The local subsonic domains are ignored in these calculations. D'Attore et al (39) computed the flow about the model of the aircraft B-1 at $M_\infty = 1.6, 2.2$ and $\alpha = 2°, 3°$. The authors used Lax scheme with dissipative terms. The flow-field was divided into several computational domains.

<u>Fraction-step method and method of particles.</u> In the fraction-step method (or in the alternating direction method) the multi-dimensional finite-difference operators are split into one-dimensional. Thus the problem is reduced to the solution of sequence of one-dimensional problems. At first the splitting scheme was suggested by D.M.Peaceman and H.H.Rachford and also by J.Douglas for heat conduction problems. Later, the scheme of geometrical splitting was worked out by other authors, in particular, Yanenko (40) made a considerable contribution in the development of the fraction-step method.

Rizzi and Inouye (41) used splitting in their method of "finite volumes". The governing equations in their work were written in the integral form, and the average values of functions in cells were determined by means of volume integral. They investigated the case of a blunted cone. Rizzi and Bailey calculated the nonequilibrium flow about the front part of a spacecraft vehicle. In Fig.9 the cross-sections of the bow shock wave for this vehicle at $M_\infty = 21.7$, $\alpha = 41°$ and $H \approx 65$ km. are shown (42).

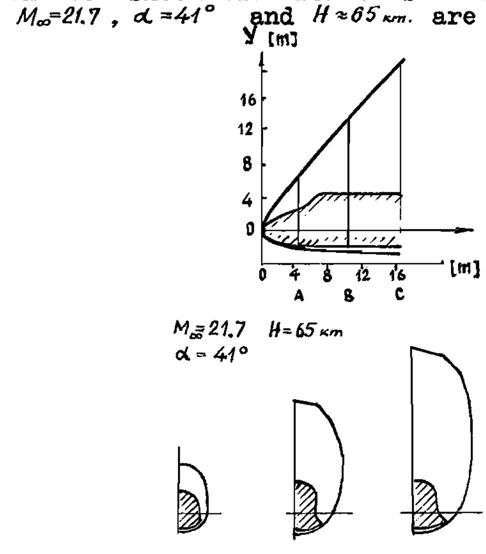

Fig.9 Shock wave in nonequilibrium flow about spacecraft vehicle.

The splitting by physical processes is involved in methods of particles: PIC by F.H.Harlow, FLIC by R.A.Gentry, R.E.Martin, B.J.Daly, in the method of large particles by O.M.Belotserkovskii and Yu.M.Davydov. Thus Euler and Lagrange considerations of the medium movement can be used in turn and therefore their best features can be combined. These methods for the supersonic flow problems are applied with the time-dependent stationing principle and the shock-capturing approach. In three-dimensional case these methods are practically not used now, because for sufficient accuracy they require computers of very high capacity. Here only two examples concerning parallelepiped in supersonic stream are given by Pyzik and Tarnogrodzky (43) and Davydov (44).

III. Method of characteristics

The method of characteristics is used for purely supersonic flows of an inviscid gas, when the gasdynamical equations belong to hyperbolic type. The method is based on treatment of charecteristic surfaces, where the original system of governing equations is replaced by the equivalent system of characteristic compatibility relations depending on only two variables in a three-dimensional case. These relations are represented in a finite-difference form on a characteristic net which is not rectangular, beforehand unknown, but is created in the course of the solution. There are characteristic nets of direct and inverse types. In the first case characteristic surfaces are issued downstream and their intersection gives a new net point. In the second case, the solution is advanced using successive reference (initial data) surfaces (where one independent variable is constant) with fixed net points in respect of two other variables. Here characteristic surfaces are issued upstream to the preceding reference surface where some interpolations are employed.

The method of characteristics has the following advantages: better treatment of region of influence of solution, simpler control of stability, convenient computional algorithms for net points on the body and on the shock wave with simultaneous determination of streamlines (which are required to calculate nonequilibrium flows, since the relaxation equations are described along streamlines). However, the method is not universal, being applied only to hyperbolic domains. Use of the direct net gives irregular position of net points, demands the calculation of their locations, results in low accuracy for small intersection angles of characteristic surfaces. It is reasonable to apply the method of characteristics computing flows with few shock waves, which are treated as discontinities, and solving variational problems when rigorous treatment of region of influence is necessary. It should be noted that some three-dimensional finite-difference net methods includes the characteristic algorithms for boundary points on a body and a shock wave.

It is possible to construct various three-dimensional characteristic computational schemes having different elementary cells, order of accuracy and being of simplicial or nonsimplicial types. The review and the analysis of such schemes are given in (45). They may be divided into the schemes with bicharacteristics and the schemes with characteristic lines on coordinate planes.

Schemes with bicharacteristics. The direct tetrahedral scheme of Rusanov has been practically realized by Podladchikov (46). Here the region of influence is determined exactly, but the irregularity of net resulting in growing calculational errors restricts the application of the scheme to small angles of attack and small body lengths.

The inverse tetrahedral scheme with three bicharacteristics having the first order of accuracy has been proposed by Minostsev (47) for smooth bodies. There all the functions on each reference plane between a body and a bow shock wave are represented by continuous interpolations. Perfect gas flows about inverted cones with segment-shaped bluntness have been calculated by means of the scheme. Two projections of the flow pattern about the body with the semi-angle $\omega=-30°$ at $M_\infty=\infty$ $\alpha=30°$, $\gamma=1.4$ are drawn in Fig.10. Solid lines show the streamlines on the body, dashed lines show the isobars on the body, which are continued over the symmetry plane, dash-dotted line shows the sonic line. The pressure on the leeward side of the flow is seen to be close to zero.

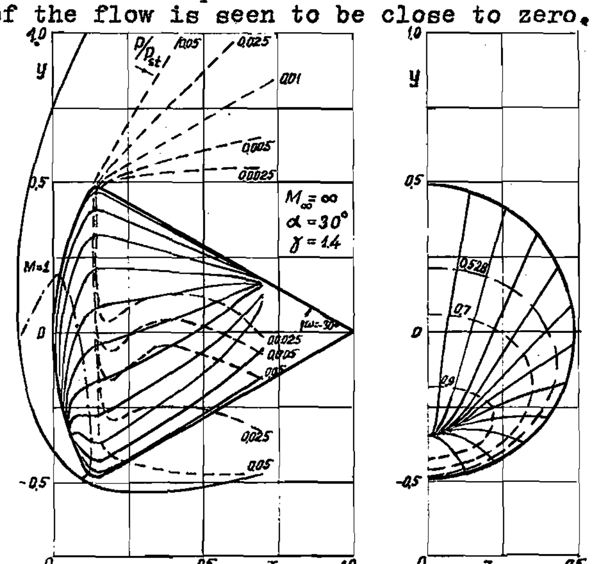

Fig.10 Flow about segment-conical body.

The inverse pentahedral scheme with four bicharacteristics has been developed by Magomedov (49). He has proceeded from Butler's idea to introduce the fourth additional bicharacteristic in order to eliminate the differentials along the nonbicharacteristic directions involved in the compatibility relations. Computations of perfect and equilibrium air flows about spherically blunted cones at an angle of attack have been carried out with this scheme. Chu (50) has suggested a scheme in which the number of bicharacteristics used is more than necessary and the overdeterminate system of equations obtained in the calculational procedures is solved by minimizing residual function.

Schemes with characteristic lines on coordinate planes. It turns out to be a fruitful idea to treat two-dimensional traces of characteristic surfaces on coordinate (meridional) planes and to connect the values of functions on them by some interpolations. The approach, in which such interpolations are carried out still in the original system of equations to eliminate the third independent variable, is particulary simple. Such a scheme is utilized by Moretti (51), however it has a number of demerits (linear interpolations, direct net, the first order of accuracy, instability in Neumann's sence).

An effective scheme using two-dimensional characteristic compatibility relations has been worked out by Katskova and Chushkin (52). They have applied continuous trigonometric interpolations, inverse net, normalized independent variables, an iterative procedure which assures the second order of accuracy. Supersonic perfect gas flows about blunted cones (52) and about ducted bodies (53) at angles of attack were computed with the help of this scheme. Later on it was extended to the nonequilibrium case (Chushkin (45)). Various three-dimensional supersonic flows with exact treatment of nonequilibrium physical-chemical processes (dissociation, combustion) have been analysed for the first time by this scheme (54). We present in Fig.11 some results for direct $(\omega > 0)$ and inverse $(\omega < 0)$ elliptical cones with ellipsoidal bluntness ($b/a = 1.5$, $b = 1\,m$) in the stream of nonequilibrium dissociating oxygen at $M_\infty = 10$, $\alpha = 10°$, $p_\infty = 0.001\,atm$, $T_\infty = 300°K$. The shock wave and the temperature distribution along windward and leeward body generators are drawn by solid and dashed lines, respectively.

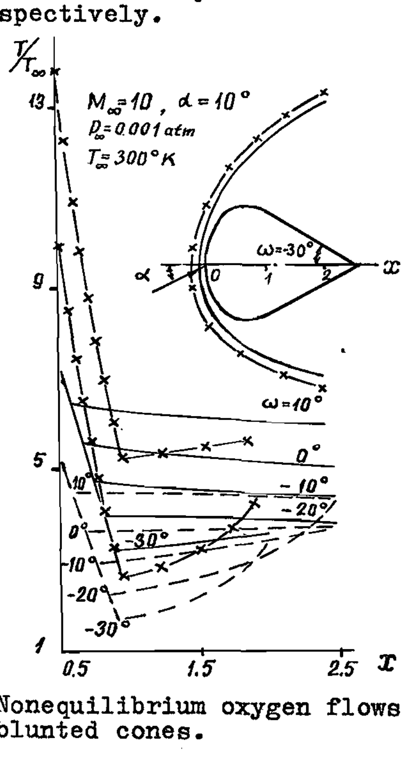

Fig.11 Nonequilibrium oxygen flows about blunted cones.

The corresponding data for frozen flow illustrating the dissociation effect are depicted by crosses for the cone $\omega = -30°$.

This scheme is very efficient for nonequilibrium flows. Rakich et al (55) have employed it with some modification (choosing as the reference surfaces the planes normal to the body surface, rather than to the body axis) to calculate three-dimensional nonequilibrium air flow about the front part of an aerospace vehicle. A scheme of similar type, but with direct net has been constructed by Grigor'ev and Magomedov (56).

An explicit net-characteristic scheme has been worked out by Magomedov and Kholodov (57). They have developed the idea of Courant-Isaacson-Rees. Here traces of characteristic surfaces on two coordinate planes are considered, the solution is advanced following to successive reference planes with a fixed net. Finite-difference approximation of characteristic compatibility relations and linear (or quadratic) interpolations with respect to the net points are carried out for each elementary cell. Flow-fields about various blunted bodies in perfect and equilibrium air have been investigated with the aid of this scheme. The results obtained by Belotserkovskii and Kholodov (58) for the cone having the spherical nose and the segmental aft are presented in Fig.12. Here the shock waves and the body pressure distributions in the flow symmetry plane are shown for $M_\infty = 2$, $\gamma = 1.4$ and various angles of attack $\alpha = 0° \div 180°$.

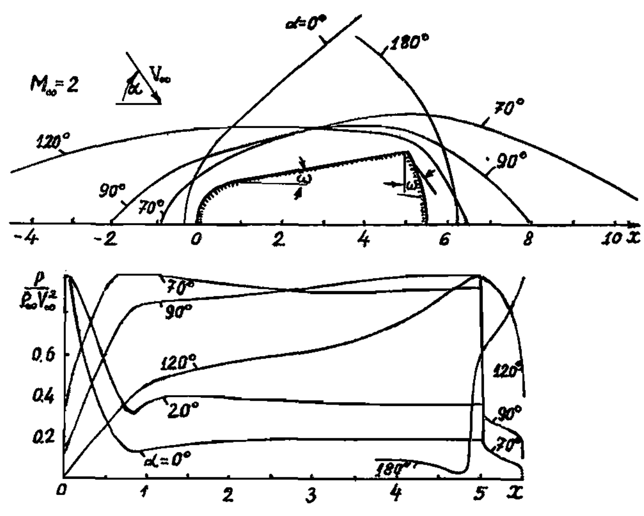

Fig.12 Flow about blunted body.

IV. Method of integral relations and method of lines

Method of integral relations. It was the method by which the first numerical solution of aerodynamic problems using computers were obtained. Dorodnicyn (59) proposed this method in 1951 developing the method of lines. Here the governing system of equations is taken in the divergent form and the integration region is subdivided into N nonintersecting strips. This system is integrated across each strip. The application of certain interpolations with nodes on the strip boundaries reduces the obtained integral relations to the approximating system of ordinary differenial equations. Dorodnicyn (60) has also given the generalized form of the method of integral relations where the approximating system, which better approximates the solution, is obtained owing to the introduction of certain weight functions.

The advantage of the method of integral relations is the actual elimination of one independent variable with the exact integration in respect of another variable that results in extreme simplicity of computational algorithm and little computer storage required. Boundary conditions are very easily realized in the calculational procedure, in particular in cases of singularities on boundaries (discontinity of slope or curvature of body surface, singularity on the axis in the cylindrical coordinates). Sufficient accuracy of solution is reached in many cases for a small number of strips (N=1+3). However, if a boundary-value problem of high order arises, the efficiency of the method diminishes.

The method of integral relations has been extensively applied to compute mixed flow about nose part of blunted bodies at supersonic free stream. Belotserkovskii (61) has obtained the first solution of the blunt-body problem for the direct formulation. The mixed flow behind the detached shock wave may be calculated by two different schemes in which the strips are chosen along or across the shock layer respectively. The method of integral relations has been used by some authors to compute supersonic flows about cones (see, for example,(54)) and delta flat wings at angles of attack.

When a three-dimensional flow about a body at angle of attack is considered, at first the governing three-dimensional system of equations is reduced to the two-dimensional one by means of trigonometric polinomials in respect of the meridional angle, then the two-dimensional method of integral relations is applied to the system obtained. Minailos (62) has utilized this approach in the blunt-body problem, considering the whole shock layer as a single strip.

Chushkin has solved the three-dimensional problem of flow about a cone of finite length, having small bluntness and the semi-angle greater than the limiting one. To remove certain calculation difficulties due to large gradients of gasdynamic functions near the body apex, this author as in (63) has introduced special variables. The shock waves and body pressure distributions calculated by him are plotted in Fig.13 for the cone with semi-angle $\omega = 75°$ at $M_\infty = 4$, $\gamma = 1.4$ and a number of α. When the angle of attack increases, large pressure gradients arise in the vicinity of small bluntness, and the local supersonic zone appears there for greater values of α. The decrease of bluntness radius and increase of free-stream Mach number have similar influence.

To compute three-dimensional mixed flows of perfect, equilibrium and nonequilibrium gases about bluntnesses, Golomazov (64) has developed the scheme of the method of integral relations with strips across shock layer.

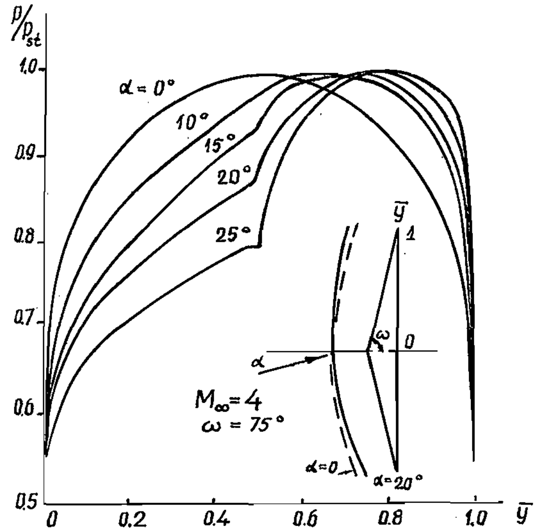

Fig.13 Flow about blunted large-angle cone.

To numerically analyse three-dimensional purely supersonic flows Chushkin (65) has worked out an algorithm of the method of integral relations, using approximations across N strips between the body and the bow shock and carrying out trigonometrical approximations with K meridional planes between the windward ($\psi = 0°$) and leeward ($\psi = 180°$) planes of symmetry. As an application some external flows about ducted bodies at angles of attack with attached shock wave have been investigated. The pressure distributions along five generators $\psi = const$ are presented in Fig.14 for the ducted body, having shape of truncated cone with semi-angle $\omega = 15°$ at $M_\infty = 5$, $\alpha = 10°$, $\gamma = 1.4$. The convergence of numerical solution for various K is also illustrated and the solution is compared with the calculation by the method of characteristics (53)(black points in the graph). Downstream the pressure tends to the corresponding value for a sharp cone (triangles in the graph,(9)).

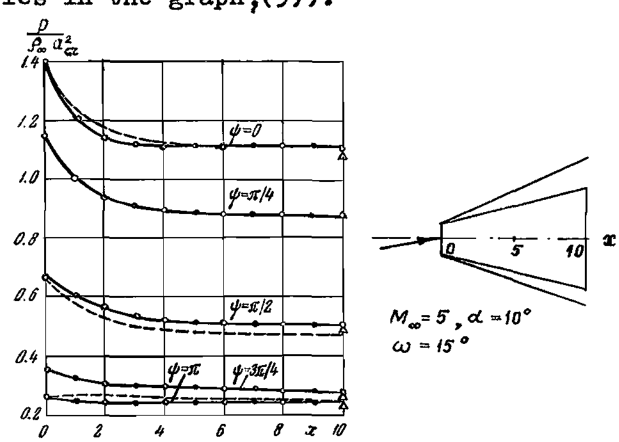

Fig.14 Flow about ducted body.

Method of lines. This method has been

used by Telenin and Tinyakov (66) for three-dimensional mixed flows about smooth bluntnesses. The approximating system of ordinary differential equations integrated along a series of rays from the detached shock wave to the body nose, is obtained as in the second scheme of the method of integral relations. However in distinction from the latter method, now the governing equations are not taken in the divergent form and are not solved in the minimal region of influence.

The application of the method of lines has been already presented above in Fig.10 for a segment-conical body in the perfect gas stream (48). Studying this body at angle of attack in nonequilibrium air stream Shkadova (67) has extended the method of lines to this case, having worked out an effective two-point implicit scheme to integrate the relaxation equations near the equilibrium, where they become stiff. Some results for the segment-conical body with radius of $2\ cm$ are given in Fig.15 for two angles of attack α and free-stream parameters $M_\infty = 20$, $p_\infty = 10^{-7}\ atm$, $T_\infty = 250°K$ when nonequilibrium effects are important. Here the distributions of pressure $\bar{p} = p/\rho_\infty V_{max}^2$ and of atomic nitrogen concentration C along the body in the flow symmetry plane are shown. The corresponding pressure curve for the perfect air is also depicted by dashed line. As it is seen, nonequilibrium effects increase the asymmetry of the flow.

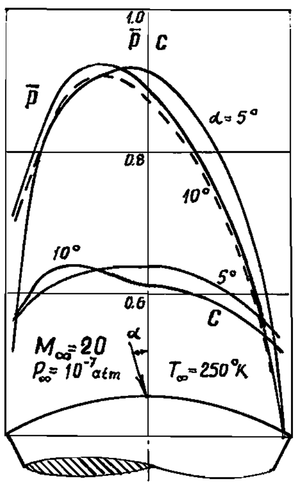

Fig.15 Nonequilibrium flows about segment-conical bluntness.

A series of authors have solved by the method of lines the supersonic conical flow problem for the two-dimensional formulation. The work by Fletcher (68) should be noted, who has calculated the perfect air flows about a circular cone. He has explicitly taken into account the embedded shock and the irrotational singular point detached from the body surface.

A modified scheme of the method of lines has been developed by Golomazov and Zyuzin (69), utilizing piecewise continuous approximations and a special calculational procedure near body surface. An example of their computations (70) is presented in Fig.16 for the blunted cone with a large semi-angle ($\omega = 60°$) at $M_\infty = 10$, $\gamma = 1.4$. In a small interval of angles of attack, a local supersonic zone on the leeward side near the detached shock wave appears which has a bend there. The corresponding behaviour of sonic lines is seen; they have at $\alpha = 5°$ and $5.5°$ two branches, which join together at $\alpha = 6°$. There is also the single sonic line at $\alpha = 0°$. This behaviour of the leeward flow-field for body angle close to the limiting one is due to the transition of flow from the type determined by the spherical bluntness to the type determined by the cone generator.

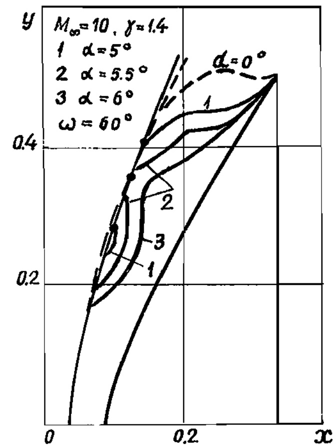

Fig.16 Flows around blunted cone.

V. Conclusion

For solving the problem of three-dimensional supersonic steady inviscid flow a number of authors have worked out three following types of numerical methods: the finite-difference net method, the method of characteristics and the method of integral relations (including the method of lines). The first type of these methods has widespread applications.

Now this problem may be considered to be mainly solved for classical aerodynamic body shapes in supersonic streams of perfect and real high-temperature gases. The tables of flow-fields have been calculated with sufficient accuracy for a series of cases (sharp cones, blunted cones and wedges, delta wings).

The present-day question is the numerical computation of complete configuration of vehicles in three-dimensional supersonic streams. The successful solution of this problem is possible by means of existing numerical methods, but it demands the use of very powerful computers.

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