DISCRETE PROBABILITY DISTRIBUTION METHODS FOR FATIGUE LIFE PREDICTION

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Abstract

Mathematical difficulties in confidence limits calculation for three-parameter Weibull distributions may be evaded by the use of discrete error distribution formulae. Error distributions are generated from all possible combinations of 4 or more specimens of the test run, or - upward from 8 piece runs - by a special Monte Carlo process.

Ranking accuracy and low-cycle safe life investigations are being done in three ways: ranking formulae have been compared, reciprocal binomial fatigue life distributions and cross-plotting between different stress--levels for the complete Wőhler curve have been tried.

Notation:

a	constant	
b	constant	
i	rank order of test piece	
j	number of test pieces being evaluated	
k	number of possible combinations	
n	total number of test pieces	
p	failure probability density	
$\wp$	discrete scatter probability density	
x	$\frac{1}{a}\ln(N_i - N_{oI})$ from Eq. (15)	
y	$\ln(N_i - N_{oI})$	
N	number of load cycles to failure	
N_o	minimum fatigue life	
N_{100}	maximum fatigue life	
P	failure probability	
$\mathcal{P}$	scatter probability	
α	Weibull shape parameter	
β	scatter parameter, in (10) scale parameter	
ε	minimum life ratio	
ϑ	skewness parameter	
Δ	average inaccuracy	
σ	normal stress	N/mm^2
σ_B	ultimate tensile stress	N/mm^2

Subscripts:

i	i-th in order
j	weighed for a number of specimens j
w	as calculated with Weibull distribution
P	at failure probability P

1. INTRODUCTION

Cycles to failure values for a fatigue test series are always received - by the very nature of the work - in the form of a discrete sequence. Against this, basic theoretical and practical considerations require working life - failure probability relationships to be of the continuos type. There is a fair assortment of mathematically well-proven procedures for treating each of the respective distribution categories separately (see e.g.[1]) but transitions of the type required here are to be made by means of the more or less voluntary process of ranking.

Interesting as these problems are, researches reported on in the present paper have been motivated not so much by theoretical but by primary practical considerations. Improving on current confidence limits calculation procedures has been one of them. In addition, there are the correlative problems of test series irreguarities, ranking accuracy and analytical smoothing of the Wőhler curve. We have tried to get one step shead in them by investigating what would happen if transition from the discrete probability treatment to the contiuos distributions would be made not at the first step but somewhat later in the calculations.

2. CONFIDENCE LIMITS CALCULATIONS FOR WEIBULL DISTRIBUTIONS

2.1 Weibull Statistics

From among the probability distributions presently in use for smoothing of fatigue test results the author is preferring the so-called three-parameter Weibull type. Basic properties of Weibull distributions are well known (see e.g. Johnson[2] and Amstadter[3]) but full uniformity in practical details of calculation procedures and in formulae has not been reached as yet[4]. Theoretical as well as practical reasons are strongly recommanding concentration on determining the minimum fatigue life N_o as exactly as possible. Its physical reality has been re-emphasized a few years ago by Freudenthal[5] and our own practical experiences, too, are speaking for it. Following his nomenclature the equation for the three-parameter Weibull distribution reads[6]:

$$P(N) = 1 - \exp\left\{-\left(\frac{N-N_o}{\beta-N_o}\right)^{\alpha}\right\} \qquad (1a)$$

Personally we are prefering a slightly modified form:

$$P(N) = 1 - \exp\left\{-\left(\frac{N-N_o}{\beta}\right)^{\alpha}\right\} \qquad (1b)$$

and this will be used in the following.

In Eq. (1a) β is for the scale parameter, i.e. for the P=63.2 % failure probability fatigue life. We are marking the scatter parameter with β; a point of view justified in our opinion not so much by a simpler formula- but by conformity with the discrete type distribution functions to be spoken of later.

For ease of calculation Weibull distributions can be put in the form:

$$\frac{1}{1-P(N)} = \exp\left\{\left(\frac{N-N_o}{\beta}\right)^{\alpha}\right\} = \exp\left\{\left(\frac{N}{\beta}-\varepsilon\right)^{\alpha}\right\} \qquad (2)$$

where the minimum life ratio, in our notation

$$\varepsilon = \frac{N_o}{\beta} \qquad (3)$$

is a good first estimate of the statistical fatigue behavior. Eq. (2) is giving by double logarithmic transformation

$$\ln \ln \frac{1}{1-P(N)} = \alpha \ln(N-N_o) - \alpha \ln \beta \qquad (4)$$

being a straight-line relation for the correct value of the minimum fatigue life N_o.

Fatigue test series life sequences do not fit in correctly with this function necessitating smoothing e.g. by least squares procedures. Our computer programs are working on the principle of straight-line fit indicated by the maximum value of the correlation coefficient. This way a most likely combination of parameters α, β and N_o is worked out.

2.2 Analytical Confidence Band Calculations

Reliability of the best fit parameter combination may always be open to question and this is accounted for by appropriate confidecence limits. The customary analytical procedures for this purpose are setting out from two interconnected basic assumptions:

a/ if the number of test pieces could be increased to infinity then the experimental life distributions would hold to a perfect Weibull type one;

b/ irregularities in the finite experimental life distributions are therefore to be explained entirely by the random order of test pieces.

Assumptions a/ and b/ may be set forth and formed into analytical formulae using standard reliability resp. extremal statistics procedures. For a given or known set of parameters the method can give exact results[6] not so however in the much more frequent case of individual fatigue test series evaluation. Basic mathematical difficulties are restricting here the universal solution essentially to the two-parameter case. So Johnson[2] McCool[7] and others are working with zero minimum fatigue life, Márialigeti[8] is assuming the fest fit α to be the correct value, etc. Amstadter[3] is going another way: he is accepting the correctness of N_o and plotting the experimental points to the higher failure probabilities corresponding to the respective confidence level.

All these analytical procedures are giving confidence limits as function of the number of specimens tested and of Weibull parameters only, regardless of the good or bad fit of experimental points (see assumption a) !) . This view can be accepted if and only if convergence to the ideal distribution can be proven for each individual case. Sorry to say, a try done by the author has shown in some way inconclusive results even when working out averages for a couple of test series[9]. It has been therefore undertaken to develop a confidence bounds calculation procedure applicable without restrictions to the three-parameter case and to imperfect distributions, too.

2.3 Discrete Confidence Limits Procedure

Our provable individual knowledge of a given series of specimens is actually limited to the measured fatigue life values. In this abstract academic sense no further statements not even a maximum likelihood smoothing would be correct, so there is clearly a need to agree upon some auxiliary hypothese acceptable to common sense and compatible with practical experience.

In view of the difficulties concerning the proof of the Weibull convergence postulate we can fall back upon postulating the set of specimens tested to be a fair representative of the whole lot the qualification of which the testing has been intended for. This agreement authorizes the smoothing of results albeit for lack of further investigations with the possibility of incorrect extrapolations. Furthermore, we can generate an error probability distribution for whatever failure probability level by observing the errors we would made against the full series result if testing had not been made on the whole series of specimens.In other words, we can extrapolate to the higher number of pieces of the whole lot by a systematic observation of going back to part of the series tested.

Let us leave out a single piece from a series of n specimens. All of them are equivalent in .this respect, so this can be done in n different ways, i.e. for each of the specimens. If j is the numer of pieces left from a series of n specimens then the number k of possible combinations is:

$$k(n,j) = \binom{n}{j} \qquad (5)$$

At least 4 specimens are needed for a Weibull evaluation to be of any significance. Upwards from this all possible combinations should be worked out. Each of them gives after Weibull smoothing a fatigue life value for every failure probability level P we intend to investigate.

Debatting the uniqueness of the sample size j as a measure for the probable error does not mean discarding it wholly as such at the same time. It has to be the dominant parameter in every confidence band formula and in a case, as our's is, when an error distribution is formed from series of mixed sizes, allowance has to be taken for this, too.

Individual scatter points for three-parameter Weibull distributions are weighed by $\sqrt{j-3}$. The resulting composite scatter picture is incorporating irregularities of the sample lot, too. In this respect it is giving an unbiassed picture of the whole, at least as far as the sample series is representative of it.

Full combination with individual Weibull evaluation for each variant is a lengthy process, even for present-day computers. We have restricted its use therefore to runs of up to and including 7 pieces. For the larger runs we are using a special Monte Carlo process. In this procedure the total number of partial sequences is limited to say 100 while their relative number is held to that corresponding to Eq. (5).

An example of how this works out in practice for a moderately irregular 9 piece test series at the 10 % failure probability level is given on Fig. 1. Calculation of the

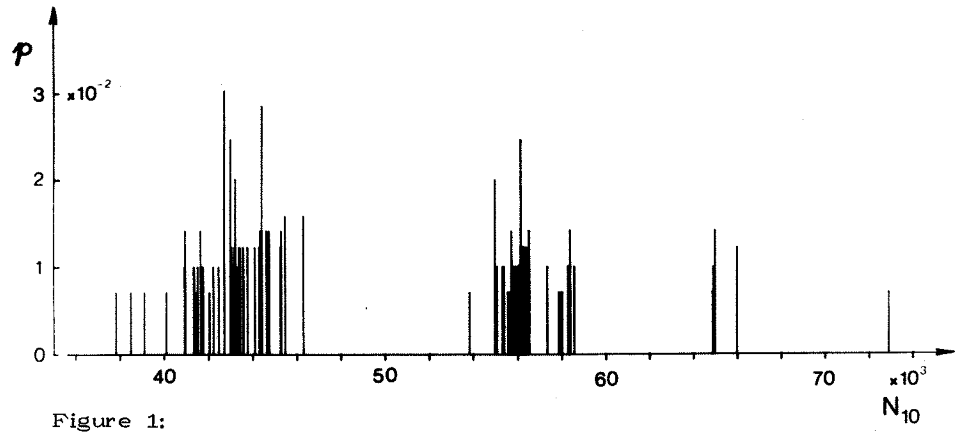

Figure 1:
Discrete Scatter Density at 10 % Failure Probability Level

scatter probability $\mathcal{P}$ from the discrete scatter density p is a straightforward process as is the working out of confidence limits.

The method is also applicable to fatigue life comparison problems as on Fig. 2 for lots A and B. The confidence number for superiority can be worked out by standard reliability methods.

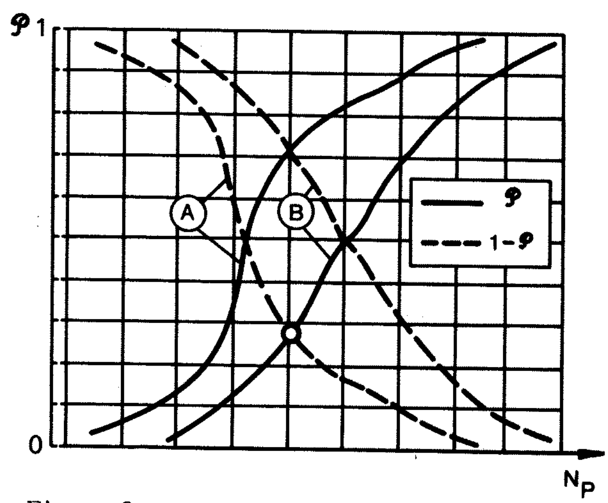

Figure 2:
Comparison of Two Lots for Fatigue Life

Our confidence limits calculation procedure is the result of several years of development. It has been used in its final form many times and practical experiences are most encouraging so far.

2.4 Goodness of Fit Control

Although strictly speaking a subject outside of confidence limits calculation, some words are to be said here about individual test series regularity classification, too. Confidence bands are good for giving scatter boundaries but for other purposes a single qualifying number concerning the regularity of the sequence comparative to the smoothing function is also needed.

Formerly we used the so-called weighed inaccuracy number derived from the Weibull correlation coefficient for this purpose[9]. Recently the problem of comparison between different smoothing functions has also arised. We had therefore to change over to the mean square type equation giving the average inaccuracy as

$$\Delta = \frac{1}{j}\sum_{i=1}^{j}\left(\frac{N_W - N_i}{N_W}\right)^2 \qquad (6a)$$

This error number too, can and has to be weighed for the size of the lot j:

$$\Delta_j = \frac{\sqrt{j-3}}{j}\sum_{i=1}^{j}\left(\frac{N_W - N_i}{N_W}\right)^2 \qquad (6b)$$

In the following this type of inaccuracy number will be used.

3. THE RANKING PROBLEM

3.1 The Need for Reliability Percentage Ranking

In every case when a discrete series of test data, e. g. of fatigue lives, is to be smoothed and eventually extrapolated by some continuos type distribution function it is necessary to assign discrete probability levels to each of the measured values. Several formulae are known for this purpose giving slightly different values to the ranks. Fortunately, for lots of at least medium sizes there is no danger of gross errors because of this. Nevertheless, possible errors may overshadow details of trends in convergence, in stress level influence, etc., making the correct treatment of some advanced topics difficult if not impossible.

By the very nature of the problem, there is no possibility of a direct proof by experiments for one of the respective formulae. We have therefore tried to get some numerical data on the magnitude and on trends of ranking errors by comparing different formulae.

3.2 Ranking Formulae and Relative Errors

Perhaps the most accepted equation for assigning the probability level P_i to the i-th item in a series of j specimens is:

$$P_i = \frac{i}{j+1} \qquad (7)$$

When working with normal distributions it is customary to use mean ranking as given by

$$P_i = \frac{i-0.5}{j} \qquad (8)$$

Schott et al.[10] are listing an empirical formula worth of consideration:

$$P_i = \frac{3i-1}{3j+1} \qquad (9)$$

Müller [12] is using:

$$P_i = \frac{i-0.3}{j+0.4} \qquad (10)$$

The author has developed the following empirical equation to start the iteration process for the median rank determination with:

$$P_i = \left[\left(1+\frac{40}{3j}\right)\frac{i}{j+1} + \left(2+\frac{40}{3j}\right)\frac{i-0.5}{j}\right] \Big/ \left(3+\frac{80}{3j}\right) \qquad (11)$$

Probability theory is giving for the median ranks the implicit relation:

$$\sum_{x=0}^{i-1} \binom{j}{x} P_i^x \left(1-P_i\right)^{j-x} = 0.5 \qquad (12)$$

According to reliability mathematics the correct process for statistical evaluation of discrete test data is by median ranks. If this view should turn out in the investigations to be exact in every respect then differences between Eqs. (7-11) and Eq. (12) would turn into absolute errors. While not committing ourselves to this as a final view we have nevertheless worked out magnitudes and trends as basis for comparison. Maximum differences for lots of j=3-50 pieces are as on Fig. 3.

In doing this a quite practical object, too, has been sought. Median rank tables are not very numerous in the available literature: the ASTM standard proposal[4] and Johnson[2] give a 4 digit table up to j=20. The reliability table for 50 percent confidence in the handbook of Amstadter[3] is applicable only to early failure problems and above j=20 to round lot sizes. We have worked out a 5 digit table up to j=100, it is to be mimeographed in limited numbers soon. Modern developments, as e.g. unified parametric evaluation[9] or unified Wöhler curve fitting, may require still higher number of specimens to be ranked collectively. In principle our computer programs are capable of handling it but computer time is gradually becoming prohibitive and storing them is occupying too much memory space. In such cases a simple approximate formula having acceptable error levels would give us badly needed help. Let us examine therefore trends in this respect.

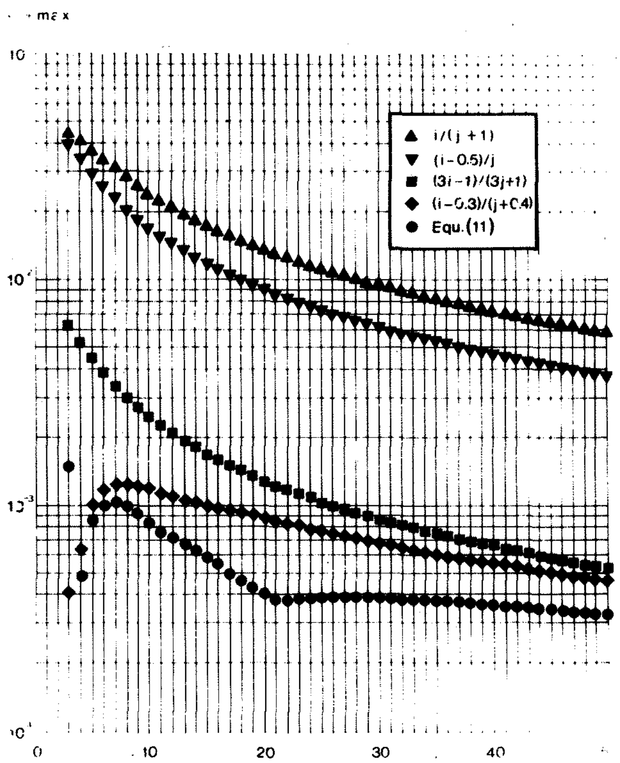

Figure 3:
Relative Errors of Ranking Formulae

From the first two formulae Eq. (8) is giving better results. In case of manual plotting without a pocket calculator this one is to be recommended. From among the two variants of medium complexity Eq. (10) is astonishingly good. Eq. (11), the starting *first approximation in our median rank computer program*, is going a step still further but at the price of some complexity. Its use is reasonable for older desk computers or for programmable pocket calculators. In the whole, ranking differences respective to the median ranks do not give concern because of gross inaccuracies.

4. FATIGUE TEST EVALUATION BY RECIPROCAL BINOMIAL DISTRIBUTIONS

4.1 Basic Concepts

Weibull smoothing of our test results has regularly shown an annoying trouble: for median life values below about 60.000 computer programs were unable to give any positive safe life value N_o[9] and in a representative low-cycle case a distinctly non-Weibullian behavior could be proven by progressive data accumulation simulation. The physical existence of a minimum fatigue life is obvious for all stress levels below ultimate but arbitrary fixing of some convenient value for it would break the otherwise well-founded principle of maximum correlation.

It has been decided on investigating the problem in detail and trying to eventually improve on it by switching over to a more flexible distribution type. Handbooks on applied mathematics (e.g. Ref.(1)), on reliability (see Ref. (3)), on fatigue (as Schott et al.[10]) and papers (e.g. Müller[12]) list several statistical functions to choose from. We are intending to make a multiple comparison between them for probable average errors but first of all, the possibility of using some sort of discerete type statistical distribution had to be examined. This decision has been motivated by the intention to get in addition complementary data on the ranking problem, too.

Prospective candidates have been selected by several aspects. They should:

- give a safe minimum fatigue life;
- have some phisical motivation;
- be unimodal;
- be as simple as possible.

Providing maximum life value, too, is regarded a welcome option but not absolutely essential.

Preference from among the variants tested will be given to the one

- giving an average inaccuracy generally not worse and for low-cycle work substantially better than the Weibull distribution;
- passing the progressive data accumulation simulation test.

The eventual final acceptance of a new type distribution function will depend on several years of favorable practical experience.

First of all, our attention has been drown to the binomial distribution. If after passing a safe life of N_o load cycles failure probability would follow the binomial model then differences in individual fatigue lives would form a reciprocal binomial sequence. In equation form:

$$N_i - N_{i-1} = a\left[\binom{j}{i-1} \vartheta^{i-1}(1-\vartheta)^{j-i+1}\right]^{-1} \qquad (13)$$

with the skewness parameter $0 < \vartheta < 1$. If a given sequence of test results has to be smoothed by linear regression we can refer to the full formula derived from Eq. (13):

$$N_i - N_0 = a\sum_{m=0}^{i-1}\left[\binom{j}{m} \vartheta^{m}(1-\vartheta)^{j-m}\right]^{-1} \qquad (14)$$

4.2 Variants of the Method

For $\vartheta \cong 0.5$ Eq. (14) can be regarded in some respect as a discrete counterpart to the normal distribution. This and the moderate success of our first trials using it led to the investigation of a second variant modelled to a certain extent as a skewed and discrete lognorm statistics with minimum fatigue life. The physical interpretation of this form would be that after passing beyond the safe minimum life the distribution of additional cycles to failure ratios is following the reciprocal binimial law. In the general case this leads to the somewhat unusual form:

$$\ln(N_i - N_{0I}) = a\sum_{m=0}^{i-1}\left[\binom{j}{m} \vartheta^{m}(1-\vartheta)^{j-m}\right]^{-1} + \ln N_{0II} \qquad (15)$$

and the resulting safe life is the sum:

$$N_0 = N_{0I} + N_{0II} \qquad (16)$$

This peculiarity may be explained by going through the smoothing process as shown on Fig. 4.

There is no such universal plotting aid as the Weibull paper for this distribution type. We have to proceed as follows. First of all, an approximate value for N_{OI} is to be choosen. More about this later. Then a first estimation for the skewness ϑ, too, is required. This

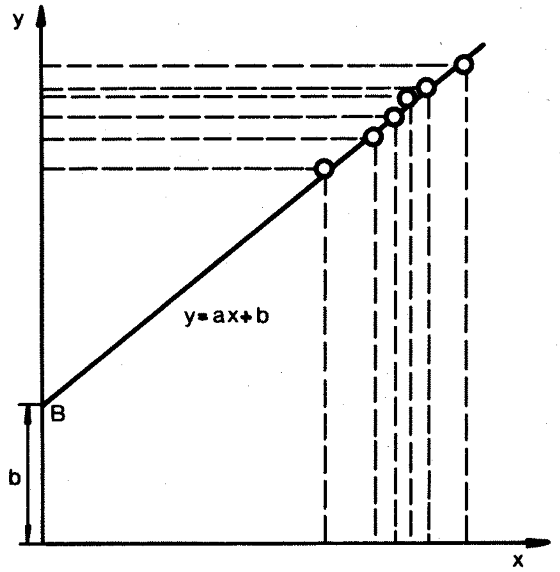

Figure 4:
Least Squares Fit for Reciprocal Binomial Distributions

alone enables us to calculate the plotting scale x, i.e. the sequence of sums in Eq.(15). The scala y representing the left-side of Eq.(15) is fixed by test results N_i and by the value of N_{oI} postulated. Linear regression through points defined by the two scala is giving point of intersection B and with it b, the second term in the right side of Eq.(15). Hence the dual form of N_o.

Because of the logarithmic scala it is not possible to reduce N_{oII} to zero. Two other possibilities remain: zero can be posited for N_{oI} in Eq.(15) or the duality has to be accepted.

4.3 Problems and Experiences

First trials have been made with evaluation by Eq.(14). The only calculation problem concerning it was that of finding the correct value for ϑ. First approximation can be made by a special method of moments and from this an iteration process is working out the value for maximum correlation coefficient or for minimum average error. As already mentioned, first results did not come up fully to expectations, perhaps because of lack of special experience.

Eq.(15) has been tried next. Basic problem with this type of evaluation is the correct choice for N_{oI}. Having it, optimal ϑ calculation is as above. At first we aimed at perfection by specifying minimum average inaccuracy number as condition for it. We have got some very promising results but for want of a correct and sufficient proof for tracing the idea back to laws of nature we are rather cautious about it.

We have experimented also with the variant $N_{oI}=0$. It is a quite easy procedure to work with but minimum fatigue life values frequently seemed us unreal by erring too much to the safe side and inaccuracy, too, did not show consistent improvements against the Weibull method.

Most test calculations, including progressive data accumulation simulations, have been done with N_{oI} as determined by our standard Weibull procedure. Much can be brought up in prise of it but the improvement in accuracy over the zero N_{oI} case might be sometimes more. Anyhow, we intend to make much more trial calculations before committing ourselves to use one of the variants for industry work or discarding the whole by declaring it to be good only for special theoretical investigations.

In this latter respect we can book already some success. The method is giving also maximum fatigue life values N_{100} by expanding Eq.(14) resp. Eq.(15) to i=j+1 summarizing thus to j and using the last term of the binimial sequence, too. This possibility might be usefull for a more exact investigation of the unlimited fatigue life problem.

It is usual to speak of crack initiation and crack propagation periods in the fatigue life to failure. Manson and his co-workers had even tried to base a new damage accumulation calculation method on this(13). Though some tests are indicating the need for further improvements(14, 15) it may be regarded to be a valuable development giving always more accurate results than customary linear procedures. The greatest problem in its further improvement and practical use is the investment in labor and cost necessary for the two-level test series. If the dual safe fatigue life proves to be correct then the possibility of substituting it for the two-level tests, too, might be investigated.

Progress has also been made relating to the low-cycle respective faulty technology safe life problem but this leads on to the Wöhler-curve smoothing procedures.

5. UNIFIED WÖHLER-CURVE PLOTTING

5.1 The Concept of Unified Ranking

The classical Wöhler form of handling fatigue data as function of the stress level σ is in its simplified form deficient in several ways. Various attempts have been made to improve on this situation and e.g.the "Wolfsburg mesh" of Müller (11, 12) is acceptable for lots of good quality and in conditions it has been made for. We would like however to get also the following serviees:

- analytical cross-plotting between stress levels for lower bounded life distributions;
- development of the cross-plotting into unified evaluation for considerable improvement in accuracy.

We have already succeeded in doing unified evaluation of parametric technological tests by the weighing function method(9) so we set out upon the whole. The weighing method does not require constant parameter test series but in its present form it is giving the same values for α and ε all over. Shape parameter and minimum life ratio are strongly dependent on the stress-level so unified Wöhler-curve fitting cannot be done this way. It is not impossible to improve on the weighing method but the existence of constant stress-level test series enables us to work out a better solution to the problem. Our program on trial works as follows.

Standard Weibull plotting and smoothing is done first for each of the respective stress--level separately giving approximate values for α, β and ε. Least squares fit by suitable analytical functions $\alpha(\sigma)$, $\beta(\sigma)$ and $\varepsilon(\sigma)$ is the next step. In the weighing function method unified plotting is done over the weighed fatigue life giving the most probable order of specimens for fatigue endurance. The same can be made using the three analytical parameter functions and experimental load cycles to failure giving an "analytical" failure probability. This is computed from Eq. (2). It is enough for this purpose to calculate the exponent on the right side of the equation, i.e. the term in brackets.

From the rank order determined this way a new median rank value can be assigned to each test specimen. Now a second Weibull curve fitting has to be made, because of the different parameter values separately for each of the respective stress-levels. This results in revised parameter values for the load levels and a second parameter function calculation follows, etc. The process is to be repeated until there is no change in individual ranks during the whole cycle.

5.2 Trends in Weibull Parameters

Accuracy of the unified Wöhler-evaluation depends very much on the correctness of the analytical parameter functions. Several years of practical experience will be required to arrive at the full and final solution. For the moment we can only report on the first observations and reflexions.

The character of the scatter parameter function $\beta(\sigma)$ is expected to be similar to the so called neutral model or to the Stüssi type Wöhler function for 50 % failure probability[11]. Strictily speaking, this function as well as those for α and ε should be based on the stress ratio σ/σ_B as independent variable. But the stress ratio would be true only if genuine and effective stresses could be compared. In this case fracture strength should be substituted for ultimate but what about the true applied stress level? Practical experience has already thought us that corrections e.g. by effective peak factors are out of question for higher stress-levels. Trends in the minimum life ratio ε against load level are of prime importance. Standard Weibull procedures have given consistently a sharp decrease in it when approaching stresses for N_{50} around 60.000 going even to zero. Should it be attributed to the inflexibility of the Weibull function, to ranking errors or taken to be a law of nature?

In order to investigate this problem, three low-cycle fatigue test series made by Havas[16] with constant strain amplitude in the $N=10^1-10^2$ range have been evaluated by our Weibull program. To our greatest satisfaction we got very good life ratios, $\varepsilon = 3.82$, 5.29 and 1.54 respectively. Weighed inaccuracy numbers, too, turned out very good indicating the trustworthiness of the results.

After an examination of the load-strain diagrams recorded during the test the following can be stated:

a/ Weibull evaluation of strain-controlled low--cycle fatigue tests is giving correct positive minimum life values.

b/ Median ranking is therefore not to blame for the low-cycle safe life ratio problem.

c/ Load-controlled low-cycle tests on good--quality specimens have to give also positive values for the minimum life in spite of greater and more irregular scatter. Numerical determination of the true safe life may be tried either by a discrete probability distribution or by a novel Weibull N_o optimalization procedure.

Taking all these into consideration we are experimenting with linear first order approximations for $\alpha(\sigma)$ and $\varepsilon(\sigma)$. Experience will prove or correct our view.

6. CONCLUSIONS

It is possible to calculate confidence limits for three-parameter Weibull fatigue life distributions by a discrete probability method. Generation of the sample scatter distributions for the respective failure probability levels is done by combination of all possible partial sequences from the sample lot containing at least 4 specimens. For lots of more than 7 pieces a special Monte Carlo process has been developed for the same purpose. Experience with the procedure is very favorable so far.

Accuracy of various probability ranking formulae relative to the median rank type has been controlled. In case of non-availability of the latter there is a good sortiment of substitute equations of different complexity to choose from. In general, probable errors in ranking do not give cause for concern.

Experiments in new smoothing and evaluation procedures are being made using reciprocal binomial type discrete probability distributions. Basic computing rules have been worked out and some good results obtained but there is as yet no proved and definitive optimum principle for safe life determination.

A Weibull type unified Wöhler-field plotting program is on trial. Based on the principle of unified ranking, it is using analytical approximation functions for fitting different stress-levels. Trends in the Weibull parameters are being observed in order to find the best function types.

ACKNOWLEDGEMENT

The studies reported on in this paper were part of the research work done on the Department for Mechanics of the Faculty for Transport Engineering under the auspieces of Professor P. Michelberger and the author wishes to express his gratitude for his furthering of the work.

Valuable original test results have been contributed by Mr. I. Havas. Acknowledgement is due to our Computing Center led by Mr. A. Ágoston and to our laboratory personal running the tests.

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