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Abstract

In this paper a method is presented for the design of multi-element airfoils in incompressible flow such that a priori specified aerodynamic and geometric requirements are fulfilled approximately. An object function is formed by summing the squared weighted deviations from the prescribed pressure distribution, zero normal velocity and geometric conditions. Minimizing the object function results in the simultaneous determination of the shape of the airfoil and the real pressure distribution.

1. Introduction

One of the most important problems of aerodynamic design is that of the design of wings that satisfy certain a priori specified aerodynamic requirements. Procedures for wing design often involve the use of so-called inverse calculation methods which are intended to determine the shape of a wing that produces a given pressure distribution.

One of the best known difficulties that may arise at the application of such inverse methods is the fact that an arbitrary prescribed pressure distribution can lead to physically impossible shapes with e.g. locally negative thickness. However, engineering arguments available to the designer will often allow to prescribe the pressure distribution in a qualitatively or quantitatively approximated sense only. A plausible use of this fact can be made by exchanging the implied amount of freedom for control over the geometry to be determined.

Most methods that are found in the literature (e.g. Refs. 2 and 3) leave it to the designer to modify the prescribed pressure distribution such that realistic shapes will be determined. In reference 4 a method is described which contains as a special feature a procedure for the a posteriori modification of the pressure distribution and hence of the 2D airfoil which generates it, in order to satisfy additional geometric or aerodynamic requirements. Another approach for implementing aerodynamic or geometric constraints is described in reference 5. It is based on minimization of some specified parameter (e.g. the drag) by variation of the coefficients of a polynomial which describes the airfoil contour. For each guess of the airfoil contour the flow is determined by means of a finite difference method. At NLR some exploratory investigations have been carried out based on the idea (Ref. 1) of developing a least squares technique for the design problem such that an a priori specified balance is obtained between the prescribed pressure distribution and the geometric requirements. As a try out the two-dimensional incompressible case has been considered. For this case a singularity method for the simulation of the flow is the obvious choice. The present paper gives a brief description of the method as well as some illustrative numerical results.

† This investigation was performed under contract with the Netherlands Agency for Aerospace Programs (NIVR).

2. The flow problem

The two-dimensional incompressible inviscid flow around a multi-element airfoil is considered. The problem to be solved is to determine the shape of the airfoil such that it will produce a specified pressure distribution. It is assumed that the pressure distribution on all airfoil elements will be given as a function of the x-coordinate of an axis system together with a guess of the true shape of the airfoil. On each element one point will be fixed.

Physically realistic shapes will be ensured by adding the constraint that the deviation from the original guess will be small. It will be shown in section 4 that this requirement can be relaxed considerably if the prescribed pressure distribution is realistic itself.

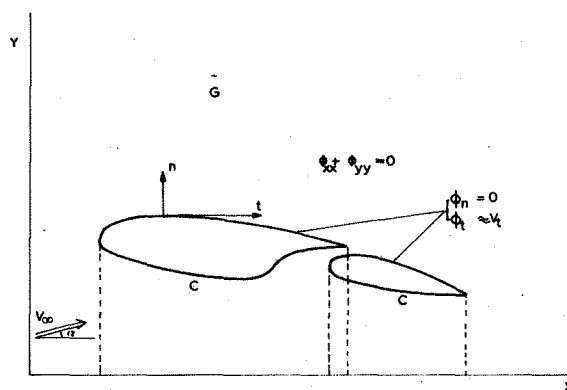


Figure 1. The flow problem

Formulated in terms of a velocity potential the flow problem is described by the equations (see Fig. 1)

$$\phi_{xx} + \phi_{yy} = 0 \quad \text{in the flow domain } G \quad (2.1)$$

$$\phi_n = 0 \quad (2.2)$$

$$\phi_t = V_t = \pm \sqrt{1 - C_p} \quad \text{at the unknown airfoil contour } C \quad (2.3)$$

where  $C_p$  is the prescribed pressure coefficient;  $V_t$  is taken positive going clock-wise from the stagnation point and negative going anti clock-wise.

The geometrical constraint is formulated as:

$$y(x) \approx \tilde{y}(x) \quad (2.4)$$

where  $y(x)$  represents the equation of the airfoil contour to be determined and  $\tilde{y}(x)$  represents the guess of the airfoil shape.

### 3. Solution method

#### 3.1 Basic idea

An obvious choice for simulating the flow is the application of a vorticity or doublet distribution along the airfoil contour because in that way it is ensured that there exists a velocity potential that satisfies eq. (2.1). In the present method a vorticity distribution has been chosen because of the direct relationship between the vorticity and the tangential velocity along the airfoil contour:

$$\frac{\partial \phi^+}{\partial t} - \frac{\partial \phi^-}{\partial t} = 2\pi \gamma(t) \quad (3.1)$$

where the + and - sign refer to the outer and inner side of the airfoil respectively and where  $\gamma$  denotes the local vorticity (see fig. 2).

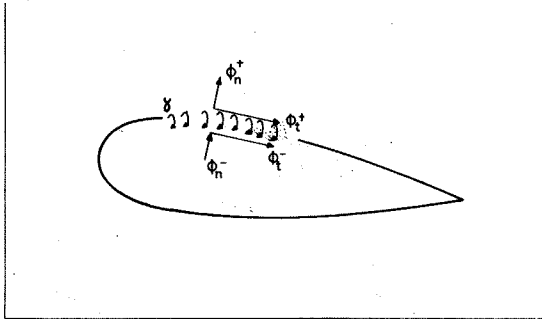


Figure 2. The vorticity distribution

If in all contour points of the airfoil  $\frac{\partial \phi^-}{\partial t} = 0$ , the velocity potential  $\phi$  itself is constant along the inner side. Solving the Dirichlet problem for the interior with  $\phi = \text{constant}$  along the contour leads to  $\phi = \text{constant}$  in the interior. Then also  $\frac{\partial \phi^-}{\partial n} = 0$  and thus according to the fact that for a vorticity distribution

$$\frac{\partial \phi^+}{\partial n} - \frac{\partial \phi^-}{\partial n} = 0 \quad (3.2)$$

also  $\frac{\partial \phi^+}{\partial n} = 0$ . From these considerations it follows that the flow problem formulated in section 2 can be reformulated as: Determine an airfoil contour (C) together with a vorticity distribution ( $\gamma$ ) such that

$$\left. \begin{aligned} \phi_t^+ &\approx V_t \\ \phi_t^- &= 0 \end{aligned} \right\} \text{ on } C \quad (3.3)$$

$$\phi_t^- = 0 \quad (3.4)$$

The tangential velocity on the contour of the airfoil can be expressed in terms of the vorticity distribution:

$$\phi_t^+ = V_\infty \cos \alpha \cdot n_y - V_\infty \sin \alpha \cdot n_x + \pi \gamma(t) - \int_0^s \gamma(s) \frac{\partial}{\partial t} \left\{ \tan^{-1} \left( \frac{y-\eta}{x-\xi} \right) \right\} ds \quad (3.5)$$

$$\phi_t^- = V_\infty \cos \alpha \cdot n_y - V_\infty \sin \alpha \cdot n_x - \pi \gamma(t) - \int_0^s \gamma(s) \frac{\partial}{\partial t} \left\{ \tan^{-1} \left( \frac{y-\eta}{x-\xi} \right) \right\} ds \quad (3.6)$$

Forming now the least squares functional

$$F = \int_0^S (F_t^2 + F_n^2 + F_c^2) dt \quad (3.7)$$

where

$$F_t = W_t (\phi_t^+ - V_t) \quad (3.8)$$

$$F_n = W_n \phi_t^- \quad (3.9)$$

$$F_c = W_c (y - \tilde{y}) \quad (3.10)$$

then through the minimization of this functional the functions  $y$  and  $\gamma$  and the angle of attack  $\alpha$  will be determined such that the equations (3.3) and (3.4) will be fulfilled in a least squares sense.

The fact that eq. (3.4) is satisfied in a least squares sense instead of by collocation simplifies the optimization procedure and introduces some extra flexibility.

The weighting functions  $W$  may be used to counter balance the different requirements expressed by the equations (2.2) through (2.4). It will be clear that for  $W_n = 0$  the problem reduces to a Dirichlet problem, while for  $W_t = 0$  a Neumann problem results.

#### 3.2 Numerical approximations

As the minimization of the functional  $F$  will have to be performed numerically a function representation must be chosen which should involve a relatively small number of unknown parameters and at the same time should offer sufficient flexibility in contour representation as well as the possibility to describe the velocity distribution with sufficient accuracy.

Within the present method a cubic Hermite polynomial representation has been chosen:

$$f(\tau) = \sum_j \left\{ f_j h_{1j}(\tau) + f'_j h_{2j}(\tau) + f_{j+1} h_{3j}(\tau) + f'_{j+1} h_{4j}(\tau) \right\} \quad (3.11)$$

Here  $f_j$  and  $f'_{j+1}$  are the values of  $f$  and  $f'_j$  and  $f_{j+1}$  are the derivatives of  $f$  with respect to  $\tau$  at the nodal points  $j$  and  $j+1$  respectively.

The cubic functions  $h$  are defined by:

$$h_{1j} = (1+2u)(1-u)^2 \quad h_{3j} = (3-2u)u^2 \quad (3.12)$$

$$h_{2j} = (\tau_{j+1} - \tau_j)u(1-u)^2 \quad h_{4j} = (\tau_j - \tau_{j+1})(1-u)u^2$$

$$\text{with } u = \frac{\tau - \tau_j}{\tau_{j+1} - \tau_j} \quad (3.13)$$

Thus the airfoil co-ordinates and the vorticity are considered as functions of a parameter  $\tau$ . An obvious good choice for this parameter would be the arclength measured along the airfoil contour. However, as the contour is unknown a priori and updating the parameter choice during the minimization process would be too costly,  $\tau$  is chosen to be the arclength measured along the initial guess

of the airfoil contour.

As a result of the Hermite polynomial representation the airfoil has a smooth closed contour with continuous first derivatives (except at the trailing edge) so that  $\phi_t$  can be evaluated at any point of the contour. By the further requirement of zero net vorticity at the trailing edge no other solutions will be admitted than those satisfying the Kutta condition of smooth flow at the trailing edge.

It may be remarked that a quadratic approximation of  $\gamma$  would have been sufficient to ensure a consistent set of function approximations (see e.g. Ref. 9). However, from programming point of view it seemed attractive to use one kind of approximation for all functions involved.

The integrals of eqs (3.5) through (3.7) are evaluated by means of Lobatto quadrature formulae (see e.g. Ref. 10).

Because any airfoil contour can be derived from an initial guess of the airfoil contour by changing the function  $y(z)$  only, the values of  $y$  and  $y'$ ,  $\gamma$  and  $\gamma'$  at the nodal points and the angle of attack  $\alpha$  are the parameters with respect to which the functional  $F$  will have to be minimized.

### 3.3 Solution to the minimization problem

The minimization problem is solved by means of a method that has been applied to several different problems at NLR and that has proven to be very efficient (see Ref. 6). It is based on a method proposed by Fletcher (Ref. 7). The generalized inverse of the matrix of derivatives of  $F$  to the minimization parameters is determined following Lawson and Hanson (Ref. 8).

The function  $F$  of equation (3.7) has essentially the form:

$$F = \sum_{k=1}^N \{f_k(\bar{z})\}^2$$

where  $\bar{z}$  is the vector of the parameters  $y_j, \gamma_j, y'_j, \gamma'_j$  and  $\alpha$  and where the  $f_k$  form together a vector  $\bar{f}$ .

This function is minimized by applying the following computational algorithm:

- 1) given  $\bar{z} = \bar{z}^{(i)}$  compute the vector of residuals  $\bar{f}^{(i)}$  and the matrix of derivatives  $J^{(i)}$ ; determine a search direction by computing  $\bar{s}^{(i)} = -J^{(i)+}\bar{f}^{(i)}$  where  $J^{(i)+}$  is the generalized inverse of  $J^{(i)}$
- 2) set  $\bar{z}^{(i+1)} = \bar{z}^{(i)} + \lambda \bar{s}^{(i)}$  and determine  $\lambda > 0$  such that  $F(\bar{z}^{(i+1)}) < F(\bar{z}^{(i)})$
- 3) if preset conditions with respect to the variation of the parameters  $z$  or with respect to the residuals  $f_k$  are fulfilled, convergence is considered to be attained; then terminate the iteration process, otherwise repeat from 1).

If the  $f_k$  are linear functions of  $\bar{z}$  the solution to the minimization problem is attained in just one iteration step. This situation is simulated if the Neumann problem is considered by setting  $W_t = 0$ ,  $W_n = 1$  and making  $W_c$  relatively large and if the Dirichlet problem is considered by setting  $W_t = 1$ ,  $W_n = 0$  and  $W_c$  relatively large.

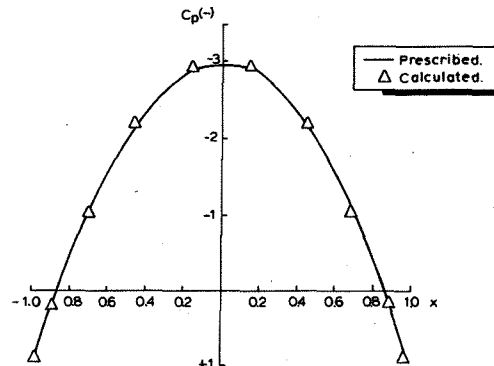
## 4. Numerical results

According to the method outlined above a computer programme has been developed for the design of airfoil shapes. On the CDC CYBER 72/14 the central processor computing time for a typical geometry of 25 panels is approximately 5 minutes. The solutions to six example cases are presented here.

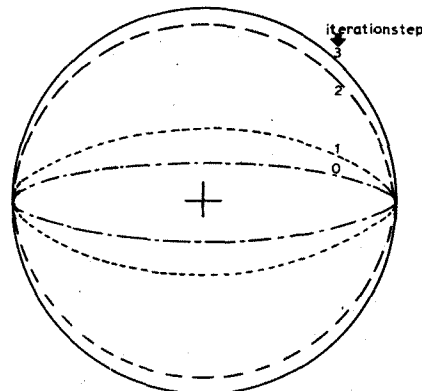
### Example 1

The objective of the first example is to show that the method is capable of reconstructing a known geometric shape from the corresponding pressure distribution.

Starting with an ellipse as initial guess of the geometry to be determined and with the exact pressure distribution of a circular cylinder in uniform flow as target, the circular shape is reconstructed straightforwardly by the computer programme, as shown in figure 3. In this case the weighting functions were chosen as  $W_t = W_n = 1$  and  $W_c = 0$ , showing that there is no need to apply the geometric constraint of equation (2.4) if the prescribed pressure distribution is known to be realistic, in the sense mentioned before.



a. Pressure distribution



b. Contour

Figure 3. Reconstruction of circular cylinder

### Example 2

In figure 4 the velocity distribution for a Von Karman-Trefftz airfoil obtained by solving the Neumann problem in the way described in section 3.3 ( $W_t = 0$ ,  $W_n = 1$ ,  $W_c = 100$ ), is compared with the "exact" velocity distribution.

The latter velocity distribution is the Hermite polynomial approximation of the exact solution obtained by means of conformal transformation. A reasonable prediction accuracy is obtained when a representation based on 13 nodal points (figure 4b) is used. The solution obtained by means of 23 nodal points is very satisfactory (fig. 4c).

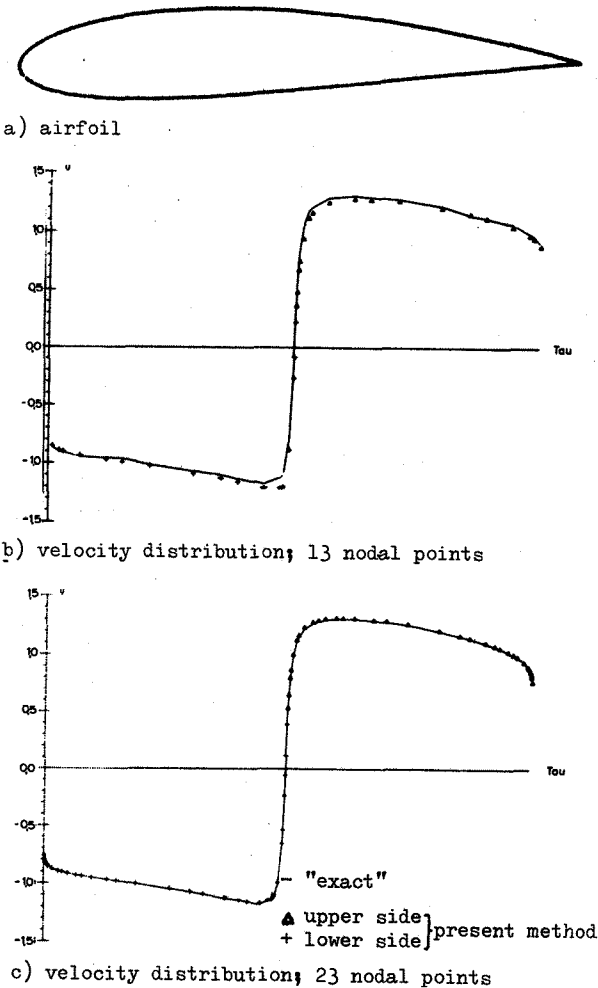


Figure 4. Velocity distribution on a Von Karman - Treftz airfoil

Example 3

The third example demonstrates compatibility between the inverse method described above and the direct method of applying eq. (3.4) in a sufficient number of contour points and solving the resulting linear equation system for the unknown vorticity distribution. By means of this direct method the velocity distribution on an airfoil-flap configuration has been calculated. Prescribing the latter distribution as "target" velocity distribution and starting with a combination of two symmetric profiles as initial guess of the airfoil shape, the application of the inverse method results in the reconstruction of the airfoil-flap configuration and the velocity distribution in three iteration steps, as is shown in figure 5. During the iteration process the mutual position of the airfoil elements is determined by fixing the trailing edges, i.e. the

trailing edges of the initial airfoil have the same co-ordinates as those of the airfoil resulting from the iteration process.

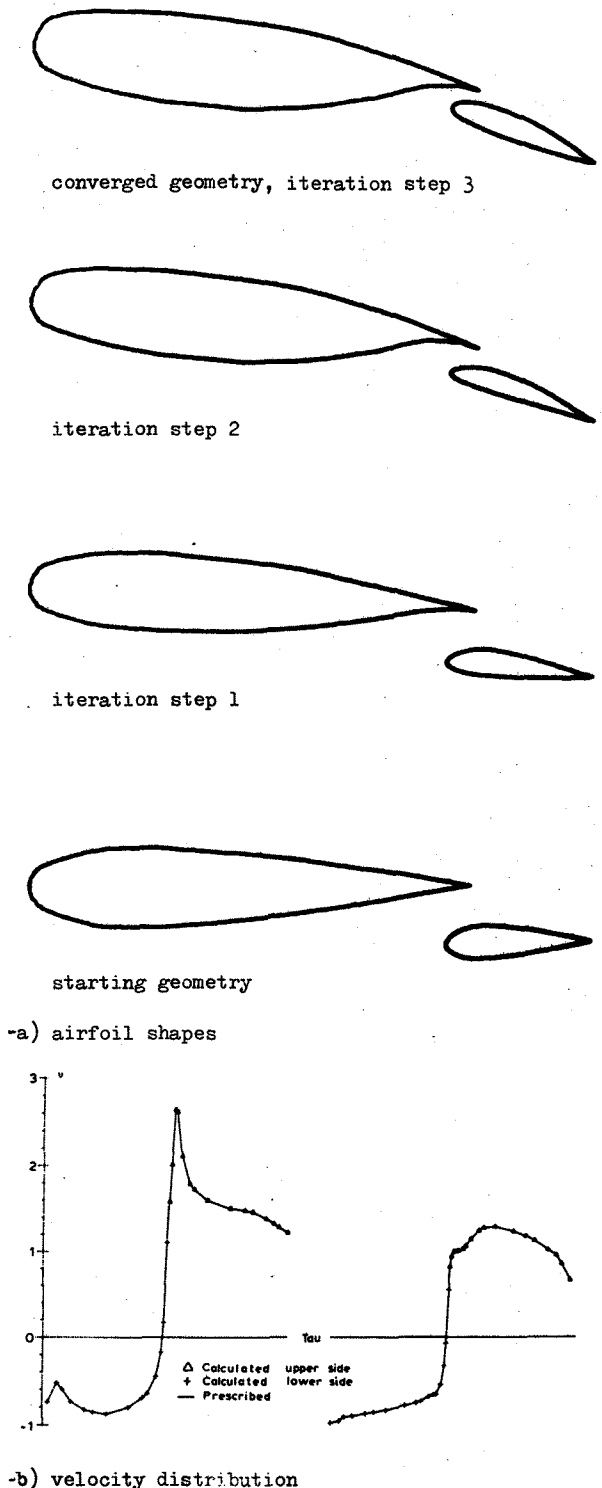
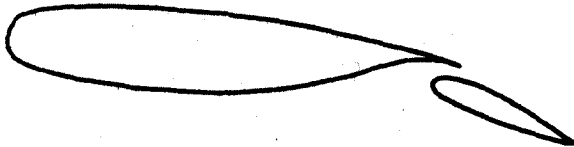


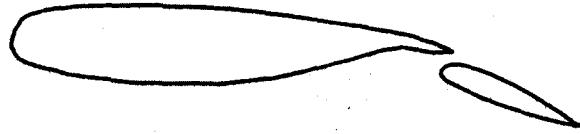
Figure 5. Reconstruction of an airfoil-flap configuration

Example 4

The next example shows how the present method could be used during a design process. It is supposed that a designer wants to alter the velocity distribution of figure 5 into one with a higher dumping velocity on the main profile and a rooftop like distribution on the flap. Starting with the airfoil of figure 5 as initial guess and fixing the trailing edge of the main airfoil and one point on the flap in the neighbourhood of the main airfoil, the iteration process results in a rather slight change of the airfoil (fig. 6a) which produces apparently not completely the desired velocity distribution (fig. 6b).



converged geometry



converged geometry



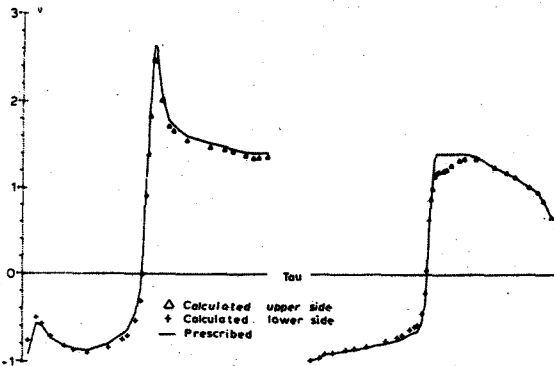
starting geometry



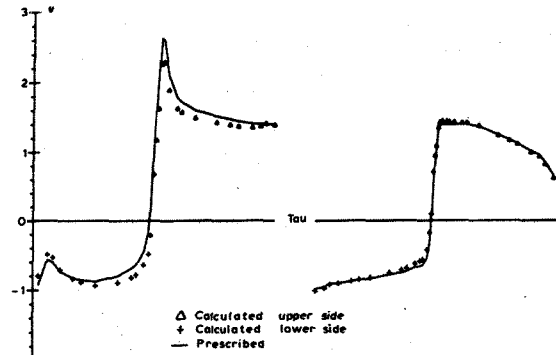
starting geometry

a) airfoil shapes

a) airfoil shapes



b) velocity distribution



b) velocity distribution

Figure 6. Inverse problem solution with fixed mutual position of main profile and flap

Figure 7. Inverse problem solution with a changed mutual position of main profile and flap

Accepting this as the closest possible approximation of the target with the given mutual position of the airfoil elements, a next step could be to alter the position of the flap with respect to the

airfoil and to start the iteration process again. Figure 7 shows the result which is obtained when the flap is moved downstream over 10 % of the flap chord.

Example 5

The fifth example shows a possible application of the geometrical constraint of eq. (2.4). The velocity distribution of example 4 is chosen as target again, but now it is tried to maintain the shape of the main profile and the mutual position of the elements and to reshape the flap only. Figure 8 shows the results of two such trials in comparison with the result of the former example. It can be seen that the increase of the weight factor  $W_0$  for the main profile leads eventually to maintaining the shape of the forward airfoil element.

However, this involves increasing deviations between real and target velocity distribution.

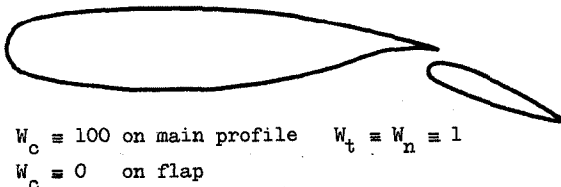
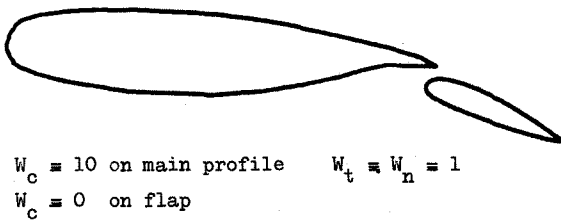
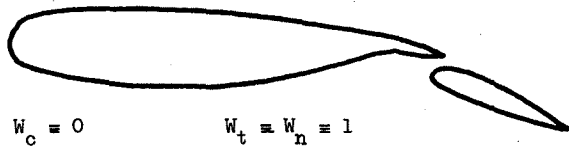


Figure 8a. Inverse problem solution applying a geometrical constraint to the main profile; airfoil shapes

Example 6

In the last example the geometry of fig. 5 is chosen as a start and the velocity distribution of example 4 is chosen as target again and the shape of the main airfoil is fixed by means of the weightfactor  $W_c$ . But now the position of the flap is fixed in x-direction only, so that it may rotate and translate in y-direction. The velocity distribution that can be realised in this way has a strong resemblance to the velocity distribution that results in the former example (with  $W_c=100$ ) except in the region of the rooftop target distribution. However, the resulting shape and position of the flap is very different from that of the former example. (see figure 9)

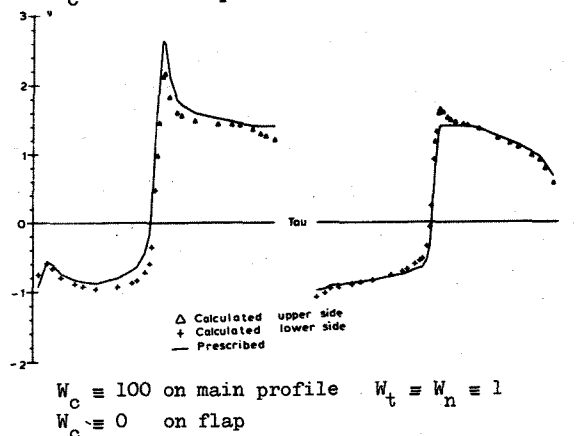
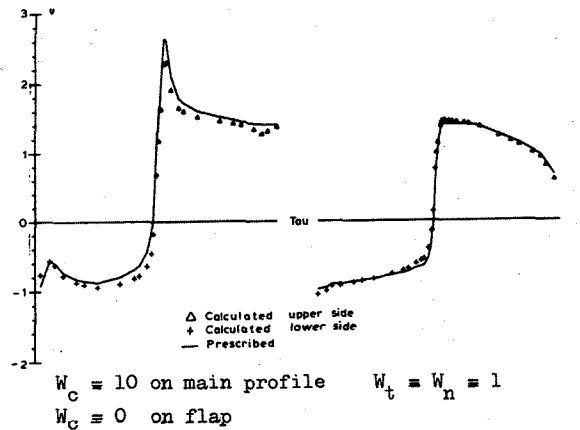
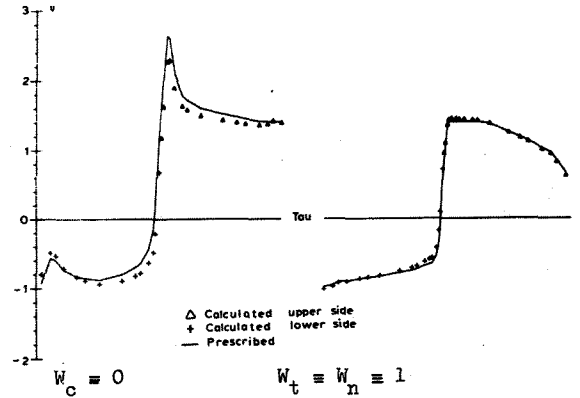


Figure 8b. Inverse problem solution applying a geometrical constraint to the main profile; velocity distributions

## 5. Conclusions

A method has been presented for the design of multi-element airfoils in incompressible flow which, subject to geometrical constraints, generate approximately a given distribution of the pressure coefficient. The method is based on parametric optimization of a least squares error function. A vorticity distribution is applied along the airfoil contour for the simulation of the flow.

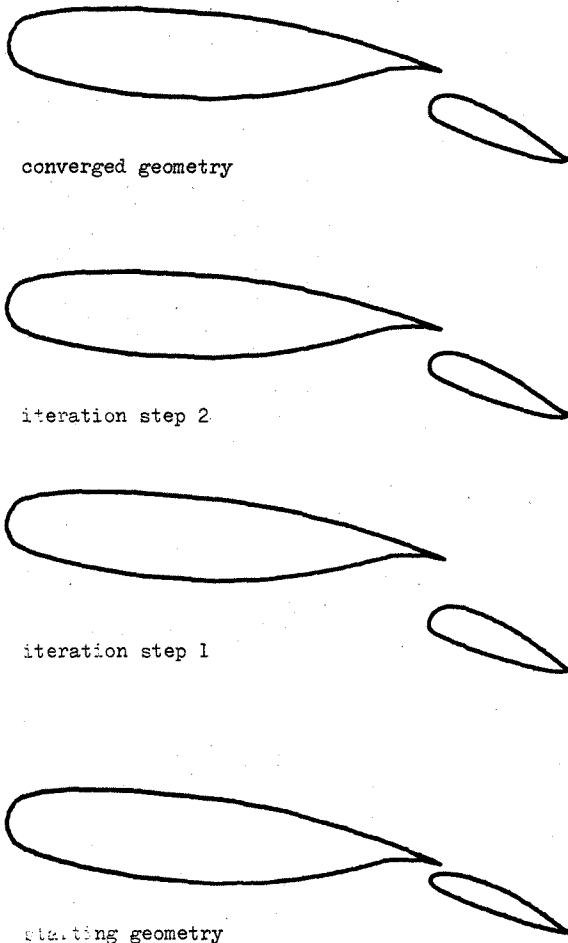
It can be concluded from the results that the approach described here holds very promising aspects for the process of wing design. The results indicate that there is probably no need to apply a geometrical constraint if realistic pressure distributions are prescribed.

In the event of uncertainty about the possibility to generate a realistic shape the geometrical constraint as introduced here can be applied. Some results of application of this constraint for maintaining part of the airfoil shape have been shown.

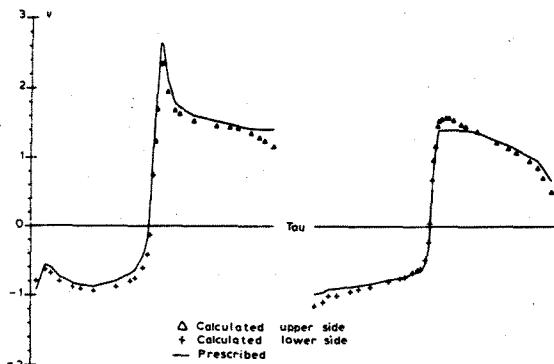
In the present method the optimization parameters have been limited to comprise the ordinates of the airfoil and the angle of attack. In principle, however, any other parameters such as flap angle or the mutual position of the elements could be included. The geometrical constraint as applied in the present method is rather simple, but the basic idea of the method allows certainly the application of more complicated constraints such as positive thickness and range of angle of attack.

## 6. References

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a) airfoil shapes



b) velocity distribution

Figure 9. Inverse problem solution with fixed shape of the main profile and free shape and vertical position of the flap.

List of symbols

$c_p = \frac{p - p_\infty}{\frac{1}{2} \rho V_\infty^2}$	pressure coefficient
$n$	normal to airfoil contour, taken positive when pointing outwards
$n_x, n_y$	direction cosines of the normal to the airfoil contour
$p$	pressure
$s, t$	arclength along airfoil contour taken positive in the direction from trailing edge to leading edge at the lower side and in the direction from leading edge to trailing edge at the upper side
$x, y$	orthogonal co-ordinate system
$C$	airfoil contour
$F, F_n, F_t, F_c$	least squares error functions
$G$	flow domain
$S$	circumference of the airfoil contour
$V$	velocity
$V_t$	tangential velocity
$W_c, W_n, W_t$	weighting functions (see eqs. (3.8) to (3.10))
$\alpha$	angle of attack
$\gamma$	vorticity
$\xi, \eta$	xy co-ordinates of a point on a line segment
$\rho$	density
$\phi$	velocity potential
$\phi_n, \phi_t$	normal and tangential velocity components respectively
$\tau$	independent variable

Subscripts

$j$	refers to a point from the set representing the airfoil contour
$\infty$	refers to the undisturbed flow
$-$	refers to the inner side of the airfoil
$+$	refers to the outer side of the airfoil