FREE VIBRATION CHARACTERISTICS OF COMPOSITE PLATES WITH CIRCULAR HOLES AND SQUARE CUT-OUTS *

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Abstract

In this paper the influence of geometrical discontinuities on the vibration characteristics of simply supported composite plates is investigated. Lagranges' equations of motion have been employed for the analysis and in the case of free vibrations, these lead to an infinite system of frequency equations. Four groups of symmetric, antisymmetric and a combination of symmetric and antisymmetric modes are considered. The frequencies and the corresponding normalised eigen vectors are obtained using the root power method. Results have been obtained for square, simply supported plates with centrally located holes and square cut-outs for different fibre orientations. Ratios of principal Young's moduli of 1, 3, 10 and 40 have been used in the computations. The variation of the frequency with various parameters such as the ratio of the hole or cut-out size to plate length, modulus ratio, fibre-orientation angle, is considered. The fundamental frequency decreases for medium sized cut-outs for 0° and 15° fibre orientation angles; such a decrease is not observed for 30° and 45° orientations. The theoretical results are compared with the limited results available in the literature and also with the experimental results obtained in this study for isotropic and unidirectionally reinforced glass-epoxy plates with central circular holes.

I. Introduction

There is a need to study the vibration characteristics of plates and shells with geometrical discontinuities. These discontinuities of various sizes and shapes are required for lightening the structure, altering the resonant frequency, etc., in aerospace and marine structures which are made of composite materials. Several methods such as Rayleigh-Ritz [1], finite-element [2] and finite-difference [3] have been previously used to determine the natural frequencies of plates with discontinuities. The authors[4] have used a method which is similar to but more general than that used by Basdekas and Chi[5] to determine the natural frequencies of simply-supported and clamped-clamped composite plates with square, central cut-outs.

In the present study of composite plates with circular holes, Lagrange's equations of motion have been employed and for free vibrations these lead to an infinite system of frequency equations. A sixteen point Gauss-quadrature formula has been used to determine the integrals (Ref. 4) and there is a need for a numerical integration scheme. Four groups of symmetric, antisymmetric and a combination of symmetric and antisymmetric modes are considered. A suitable size of the frequency equation is chosen depending upon the accuracy and the computation time involved. Root power method has been used in the computation of frequencies and corresponding eigenvectors. Experiments have also been conducted on unidirectionally reinforced E-glass-epoxy plates with and without holes.

Results have been obtained for square, simply supported plates with centrally located circular holes for different fibre orientations. Theoretical results for unidirectionally reinforced E-glass-epoxy plates are verified with experimental results and are also compared with those for a plate with central square cut-outs. The theoretical mode shapes have been obtained for a square cut-out as well as a circular hole. A comparison of the mode shapes for the central circular hole and the central square cut-out is made. The effect of hole size on the normalized eigenvectors and the effect of anisotropy for a particular hole size on the natural frequency and the corresponding eigenvectors are investigated.

II. Theory

The Lagrange's equations of motion, in the presence of external forces, have the form

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_r}\right) - \frac{\partial T}{\partial q_r} + \frac{\partial V}{\partial q_r} = P_r \qquad (1)$$
$$r = 1, 2, \ldots$$

where T is the kinetic energy of the system, V is the potential energy, q_r are the time-dependent generalized displacement coordinates and P_r are the time-dependent generalized forces which include the effect of discontinuities. The expressions for the potential energy and kinetic energy of a plate have been given in Reference 4.

The displacement, w, can be expressed as

$$w(x,y,t) = \sum_{m=1}^{R} \sum_{n=1}^{S} q_{mn}(t)\phi_{mn}(x,y) \qquad (2)$$

where $q_{mn}(t)$ is the generalized displacement coordinate in the mn mode. $\phi_{mn}(x,y)$ is the admissible function in the mn mode and can be given as a product of beam characteristic functions in x and y directions.

For a simply supported plate,

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$$\phi_{mn}(x,y) = \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \qquad (3)$$

The function $\phi_{mn}(x,y)$ in Equation (3) satisfies the geometric boundary conditions but not the natural boundary conditions, except in the case of an isotropic plate.

Substituting the expressions for potential and kinetic energies in Equation (1) and taking into account the presence of holes or cut-outs, the following equation is obtained for free vibration:

$$\sum_{m=1}^{R}\sum_{n=1}^{S}\left(M_{mn}^{ij} - \beta_{mn}^{ij}\right)\ddot{q}_{mn} + \sum_{m=1}^{R}\sum_{n=1}^{S}\left(C_{mn}^{ij} - \alpha_{mn}^{ij}\right)q_{mn} = P_{Fij} = 0 \qquad (4)$$

$$i = 1,2,\ldots R$$

$$j = 1,2,\ldots S$$

The expressions for the expressions for M_{mn}^{ij}, C_{mn}^{ij}, β_{mn}^{ij} and α_{mn}^{ij} are given elsewhere (Ref.4).

The expressions for β_{mn}^{ij} and α_{mn}^{ij} get simplified for a cut-out, however, for a circular hole there is no such simplification and therefore, a numerical integration scheme has been employed. Equation (4) defines an eigenvalue problem and its solution gives natural frequencies and corresponding eigenvectors.

III. Characterization of the Composite Material

In order to study the dynamic response of unidirectionally reinforced composite structural elements, it is necessary to characterize the dynamic behaviour of these materials. Bert and Clary[6] have evaluated different experimental methods for determining dynamic modulus and damping of composite materials. Based on this evaluation, forced vibration response technique which is simple and where modulus can be determined from the resonant frequency and damping from, the half-power point frequencies, was used. The specimens tested were glass-fibre reinforced epoxy beams (cut from unidirectionally reinforced sheets) with fibre orientations of 0°, 15°, 30°, 45° and 90° with respect to the beam axis. The beams were of the dimensions of 1.0 in. width, 0.09 in. thick and 9.1 in. long.

The beams were mounted on an electrodynamic shaker which was excited by an oscillator through a power amplifier. Electrical resistance foil strain gauges were mounted on the top and bottom surfaces of the beams to record bending strains. The clamped end of the beams was given a sinusoidal harmonic motion and the output of the strain gauges was seen on an oscilloscope. The resonant frequency was determined by seeing the amplitude of the strain gauge output and then adjusting the exciting frequency to get the maximum strain gauge output. The beams were excited over a frequency range of 10-600 cps. The resonant frequency was measured on an electronic counter. The dynamic modulus was then determined. The effect of shear correction was also considered. It should be noted that as the modulus was a function of frequency, an average value has been used in the computations for plates with cut-outs.

The static modulus was determined by testing tensile specimens of 0°, 45° and 90° fibre orientations with strain gauges on an Instron machine.

The dynamic and static properties of unidirectionally reinforced E-glass epoxy composite material used in this study are given in Table 1.

Table 1. Static and Dynamic Elastic Constants

Constant	Static($\times 10^6$ psi)	Dynamic($\times 10^6$ psi)
E_{11}	2.02	2.08
E_{22}	0.53	0.74
G_{12}	0.18	0.27
ν_{12}	0.35	0.35

IV. Experimental Study

The test plates used in the experimental investigation other than aluminium plates were cut from a unidirectionally reinforced E-glass epoxy sheet. Square plates with 0°, 15°, 30° and 45° fibre orientations were tested. In order to study the vibration characteristics of plates, the following steps were taken:
(a) vibration characteristics for the unidirectionally reinforced plates without holes were found,
(b) small holes were drilled in the plates, the vibration characteristics were found,
(c) the holes were enlarged to size on a high speed routing machine using aluminium templates, the vibration characteristics were found,
(d) the previous step was repeated for all hole sizes required. The hole sizes used in the test were 0.45 in., 1.8 in. and 3.6 in. diameter.

A test fixture was designed and fabricated to study the vibrations of plates. It consisted of four vertical members bolted to the foundation and four horizontal members supported by the vertical members. On the horizontal members a thick plate with an internal square opening was kept. The thick plate could be secured on the horizontal members. Provision was made on the thick plate to place rollers for conducting experiments on simply-supported plates. Simply supported conditions were achieved by placing the test plate between two sets of rollers. The upper set of rollers was held in position on another plate with an internal square opening. The test plate was held between rollers by bolts. Tests were carried out first on a solid aluminium plate and the torque on the bolts to hold the thick plates with rollers was appropriately chosen to give a good agreement between the theoretical and experimental results. The same testing conditions were maintained throughout the course of experiment.

The test plates were excited acoustically by means of a loud speaker. The signal was fed to the loud speaker from an oscillator through an amplifier and the amplitude of the signal could be controlled both from the oscillator and the amplifier. The signal frequency was measured by an electronic counter. The mode shapes were visualized by sprinkling aluminium oxide granules on the plates.

Because of the loud speaker power limitation, only the first three symmetric modes could be excited. The experimental mode shapes have not been reported here, but in general they were very similar to the computed mode shapes.

V. Results and Discussion

Natural frequencies and mode shapes have been determined for square, simply supported plates with central circular holes for different fibre orientations. For a comparison of experimental and theoretical frequencies, the computations for unidirectionally reinforced E-glass-epoxy plates were made taking the dynamic properties from Table 1. Results are shown in non-dimensionalized form.

The variation of frequency with hole-size parameter (d/a), defined as the ratio of the diameter of the hole to the length of the plate, is shown in Figures 1 to 4 for fibre orientations 0°, 15°, 30° and 45°. The theoretical results are compared with those for a plate with central square cut-outs and also with experimental results. The frequencies have been subscripted (11, 12, etc.) on the basis of largest amplitude coefficient in the corresponding eigenvectors.

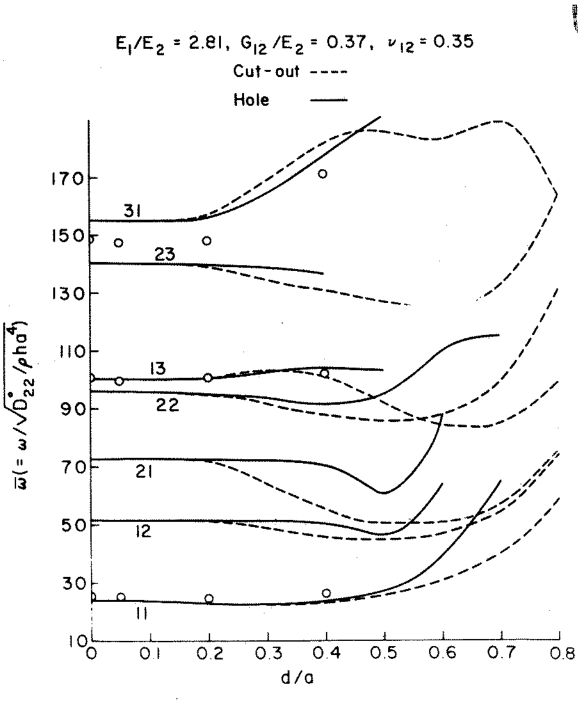

Figure 1. Variation of Frequency with Hole Size for θ = 0°

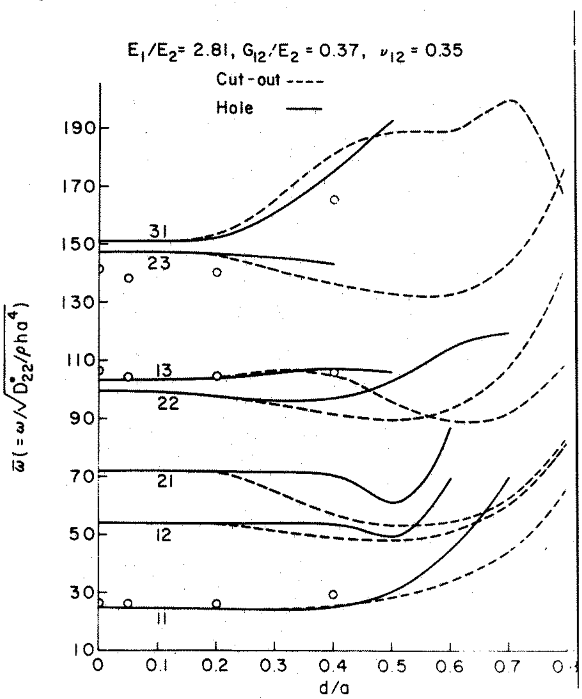

Figure 2. Variation of Frequency with Hole Size for θ = 15°

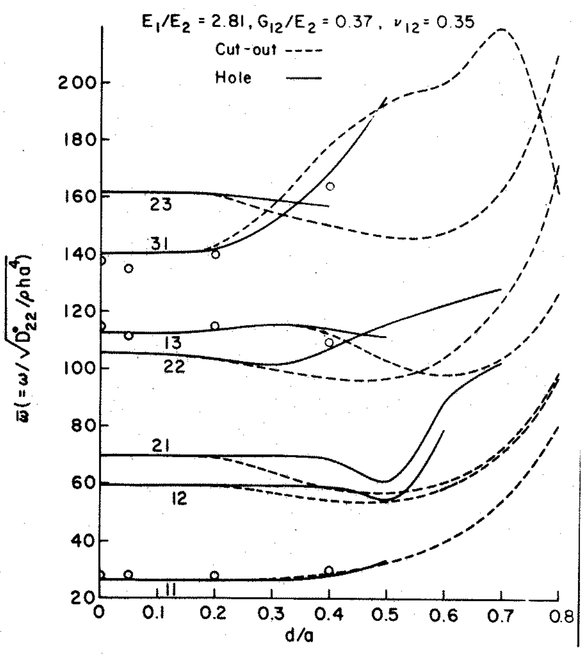

Figure 3. Variation of Frequency with Hole Size for θ = 30°

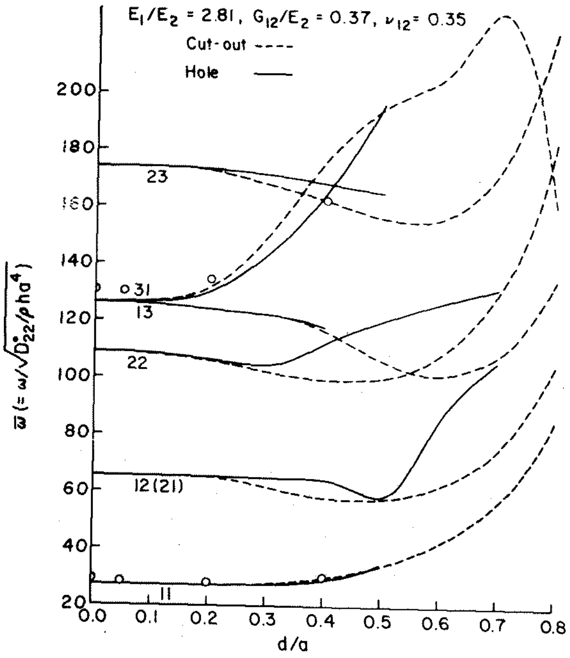

Figure 4. Variation of Frequency with Hole Size for $\theta = 45°$

It is seen that for all fibre orientations, ω_{11} increases with increase in d/a ratio. This is not true for frequencies ω_{12}, ω_{21} and ω_{23}. These are constant for smaller values of d/a and decrease as d/a is increased. However, further increase in the hole size increases the frequencies. The frequency ω_{22} behaves in a different manner. It decreases with increase in the hole size but is found to increase with larger d/a ratios. For smaller values of d/a there is no appreciable change in ω_{31} but it increases as the hole size is increased. For all the angles, modes corresponding to ω_{22} and ω_{13} are found to interchange. It is interesting to note that frequency curves for ω_{12} and ω_{21} come closer and those for ω_{13} and ω_{22} separate from each other as the fibre orientation angle is increased.

Experimental results for the first three symmetric modes are compared with theoretical results and the agreement is found to be good. In general it is seen from Figures 1 to 4 that the variation of different frequencies with d/a for a hole and for a cut-out is similar. At smaller d/a ratios (0.15 to 0.2), the frequency values for a cut-out as well as for a hole are practically the same.

The theoretical mode shapes for a unidirectional E-glass-epoxy plate with a hole are shown in Figures 5 and 6. The mode shapes and the corresponding frequencies for $\theta = 0°$ are shown in Fig.5 and those for $\theta = 45°$ are shown in Fig.6. The symmetric modes are found to distort due to the presence of the hole.

A comparison of the mode shapes and frequencies is made between a cut-out and a hole in a plate. The mode shapes are found to be similar for the same mode.

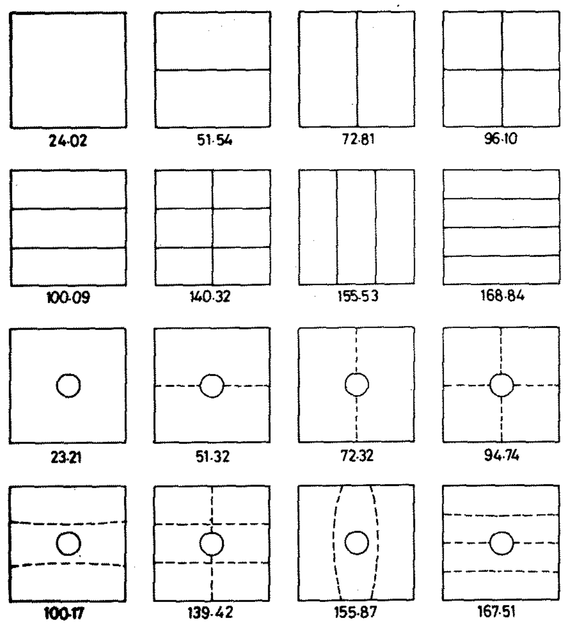

Figure 5. Frequencies and Mode Shapes for $\theta = 0°$

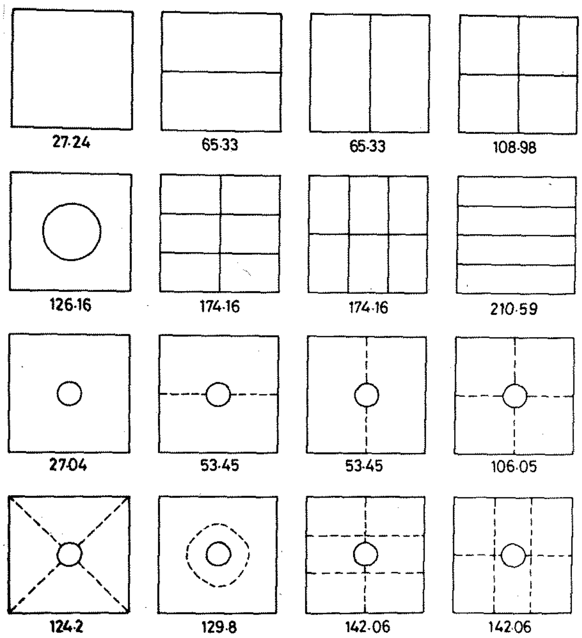

Figure 6. Frequencies and Mode Shapes for $\theta = 45°$

A comparison of the normalized eigenvectors for different d/a ratios in the case of $\theta = 0°$ and frequency ω_{13} is shown in Table 2. It is observed that as the d/a ratio is increased, the influence of other modes is more pronounced.

A comparison of the frequency values and normalized eigenvectors for a square, simply supported plated with different modulus ratios for a particular mode is given in Table 3. The frequency increases with increasing modulus ratio, for composite plates. The eigenvectors for unidirectionally reinforced plates are similar. This suggests that through the frequency value is changed, there is practically no change in the mode shape.

Table 2. Comparison of Normalized Eigenvectors for $\theta = 0°$, ω_{13}

Amplitude coefficients	d/a = 0.0	d/a = 0.2	d/a = 0.5
A_{11}	0.0	-0.1222	-0.4976
A_{13}	1.0	1.0728	1.1274
A_{15}	0.0	-0.0318	-0.0599
A_{17}	0.0	0.0150	-0.0621
A_{31}	0.0	-0.0114	-0.4327
A_{33}	0.0	-0.0222	0.0640
A_{35}	0.0	0.0196	-0.0038
A_{37}	0.0	-0.0110	-0.0420
A_{51}	0.0	-0.0083	-0.0527
A_{53}	0.0	0.0096	-0.0183
A_{55}	0.0	-0.0088	0.0036
A_{57}	0.0	0.0060	-0.0095
A_{71}	0.0	-0.0042	0.0360
A_{73}	0.0	-0.0042	-0.0271
A_{75}	0.0	0.0037	0.0116
A_{77}	0.0	-0.0027	-0.0016

Table 3. Comparison of Frequencies and Amplitude Coefficients for $\theta = 0°$, ω_{13}

	$\frac{E_1}{E_2}=1$ (isotropic)	$\frac{E_1}{E_2} = 3$	$\frac{E_1}{E} = 10$	$\frac{E_1}{E_2} = 40$
Amplitude Coefficients	ω_{13}=147.79	ω_{13}=108.33	ω_{13}=158.12	ω_{13}=169.46
A_{11}	-0.9919	-0.5289	-1.2741	1.0026
A_{13}	0.8510	1.1176	1.0456	1.1300
A_{15}	0.0140	-0.0190	1.1371	0.5063
A_{17}	0.0069	-0.0729	-0.8099	-0.2311
A_{31}	0.8510	-0.4047	0.0983	-0.0019
A_{33}	0.5767	0.0710	0.3197	0.0997
A_{35}	-0.0101	-0.0342	-0.7797	-0.1956
A_{37}	-0.0071	0.0650	0.7890	0.1920
A_{51}	0.0140	0.0256	-0.4053	-0.0411
A_{53}	-0.0101	0.0003	0.2603	0.0196
A_{55}	-0.0040	0.0040	0.0049	0.0240
A_{57}	0.0106	-0.0216	-0.2632	-0.0353
A_{71}	0.0069	0.0467	0.3256	0.0527
A_{73}	-0.0071	-0.0395	-0.2844	-0.0473
A_{75}	0.0106	0.0191	0.1672	0.0310
A_{77}	-0.0084	-0.0010	-0.0117	-0.0063

VI. Conclusions

1. The fundamental frequency increases with increase in hole size for all fibre orientations, as in isotropic plates.
2. The fundamental frequency for plates without holes decreases with increase in fibre orientation.
3. In general the behaviour of different frequencies for a hole and for a cut-out is similar. The effect of shape is seen only at larger d/a ratios.
4. The symmetric mode shapes distort in the presence of geometrical discontinuities (hole and square cut-out).
5. A good agreement between the experimental and theoretical results has been obtained. This indicates that the present method of analysis which is simple can be used to solve more complicated problems.

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