ESTIMATES OF THE STABILITY DERIVATIVES OF A HELICOPTER FROM FLIGHT MEASUREMENTS

D.G. Gould, W.S. Hindson
National Aeronautical Establishment
National Research Council of Canada
Ottawa, Canada

ABSTRACT

A least-squares quasilinearization procedure has been used to obtain estimates of the dominant lateral-directional and longitudinal stability derivatives from in-flight response tests of a single rotor, medium sized helicopter. The particular adaptation of the classical least-squares method had two features, believed to be unique, to reduce the influence on the resulting derivative estimates of peculiarities of the model and of the particular circumstances of the tests.

1. INTRODUCTION

There are many valid reasons for seeking improved methods of estimating aerodynamic stability derivatives from in-flight measurements of response to control inputs. There is a continuing need for full scale measurements in order to make better theoretical estimates of stability derivatives and to assess the significance of the aerodynamic forces on various elements of aircraft. The handling qualities of a particular aircraft are of general interest only if they are accompanied by good estimates of response parameters. A knowledge of the stability derivatives leading to response to turbulence is necessary to properly estimate structural design loads and in the design of systems to improve ride qualities. The design of autopilots and aided or automatic landing systems requires a good knowledge of the response to both control inputs and turbulence.

The work reported upon herein came about primarily because of a need for a knowledge of the values of the aerodynamic stability derivatives of the Bell 205 helicopter which has been adapted by the Flight Research Laboratory of the National Research Council of Canada to an in-flight V/STOL aircraft simulator (Ref. 1). The least-squares quasilinearization procedure was the basis of the method used.

The direct implementation of least-squares mathematical techniques to a particular response test leads to parameter estimates that minimize the square of the differences between measured responses and those computed from a model of the aircraft. The resulting parameter estimates reflect any peculiar circumstances or features of the mathematical model and the particular test analysed. The concern of the engineer, however, is to extract aerodynamic stability derivatives that are, to the maximum extent possible, independent of the peculiarities of the test and analysis so that the results may be used to predict the response in more general circumstances.

The adaptation of the least-squares quasilinearization theory used has two features, believed to be unique, to reduce the influence on the resulting stability derivative estimates of peculiarities of the aircraft mathematical model and the circumstances of the tests. The theoretical development of the procedures is given first followed by examples of the implementation of these procedures to in-flight tests with the Bell 205 helicopter.

2. STABILITY DERIVATIVE ESTIMATE METHOD

2.1 Least-Squares Quasilinearization

The literature is profuse, almost to the point of confusion, with parameter identification techniques referred to by different names which, if not exactly equivalent, are at least more closely a result of the same fundamental classical theory of least squares (Ref. 2). The essential elements of the least-squares, quasilinearization procedure are reviewed briefly below.

Consider the linear or non-linear system modelled by the equation set

$$F(X_i, U_j, \lambda_k) = 0 \qquad (2.1.1)$$

whose solution is $X_i(U_j, \lambda_k)$, where the only constraint imposed is that the column parameter vector λ_k be constant over the time period of interest. In this representation, U_j is a known system vector forcing function and X_i is the column state vector describing the response of the system.

To the first order, the change in system response, X_i, due to a small change in the parameter vector, $\Delta\lambda_k$, is

$$X_i(U_j, \lambda_k + \Delta\lambda_k) = X_i(U_j, \lambda_k) + \frac{\partial X_i(U_j, \lambda_k)}{\partial \lambda_k} \cdot \Delta\lambda_k \qquad (2.1.2)$$

The corresponding value of the cost function (the integral of the square of the difference between observed and modelled states) is

$$J = \int_0^t \left\{ \left[Y_i - X_i(U_j, \lambda_k) - \frac{\partial X_i(U_j, \lambda_k)}{\partial \lambda_k} \cdot \Delta\lambda_k \right]^T W_i \left[Y_i - X_i(U_j, \lambda_k) - \frac{\partial X_i(U_j, \lambda_k)}{\partial \lambda_k} \Delta\lambda_k \right] \right\} dt \qquad (2.1.3)$$

where Y_i is the column vector of observed states and W_i is a weighting matrix reflecting the relative accuracy in measurement of the state variables. It may also reflect the relative importance assigned to the observed state variables. With the assumption that an extremum in J has been reached, then $\frac{\partial J}{\partial \Delta \lambda_k} = 0$ yields a recursive relationship for successive changes in the parameter vector in order to minimize the cost function:

$$\Delta\lambda_k = \left[\int_0^t \left\{ \left[\frac{\partial X_i(U_j, \lambda_k)}{\partial \lambda_k} \right]^T W_i \left[\frac{\partial X_i(U_j, \lambda_k)}{\partial \lambda_k} \right] \right\} dt \right]^{-1}$$

$$\left[\int_0^t \left\{ \left[\frac{\partial X_i(U_j, \lambda_k)}{\partial \lambda_k} \right]^T W_i \left[Y_i - X_i(U_j, \lambda_k) \right] \right\} dt \right] \qquad (2.1.4)$$

It should be noted, although it is not the case of present interest, that if the parameter sensitivity functions,

$$\frac{\partial X_i}{\partial \lambda_k},$$

are independent of λ_k, Equation 2.1.2 is exact, Equation 2.1.4 is an explicit expression, and iterative solutions are not required. In the present analysis, since 2.1.1 is a set of first order differential equations (linear or non-linear) the sensitivity functions are dependent on λ_k and iterative solutions are necessary.

The procedure outlined above bears noticeable similarity to earlier gradient methods in which each parameter was perturbed by a prescribed amount and the J associated with each perturbation determined to give an approximate value of

$$\frac{\partial J}{\partial \lambda_k} = \frac{\Delta J}{\Delta \lambda_k}.$$

Difficulties were often encountered with judging the perturbation magnitudes and sometimes convergence was dependent on this choice. With the advent of modern computing techniques the sensitivity functions are calculated directly from differentiation of Equation 2.1.1:

$$\frac{\partial F(X_i, U_j, \lambda_k)}{\partial \lambda_k} = 0 \qquad (2.1.5)$$

and if the model contains derivatives with respect to some other independent variable such as time, it is assumed that orders of differentiation can be reversed, so that

$$\frac{\partial}{\partial \lambda_k}\left(\frac{dX_i}{dt}\right) = \frac{d}{dt}\left(\frac{\partial X_i}{\partial \lambda_k}\right) \qquad (2.1.6)$$

In the event that fairly reliable estimates of some parameters are known, then it is possible to control the amount of departure from these first estimates during the course of the iterations through an additional term in the cost function (Ref. 3).

$$[\lambda_k - \lambda_{k_o}]^T \, D_k \, [\lambda_k - \lambda_{k_o}] \qquad (2.1.7)$$

With this additional term in the cost function, the recursive relationship of 2.1.4 for $\Delta\lambda_k$ now becomes

$$\Delta\lambda_k = \left[\int_0^t \left\{ \left[\frac{\partial X_i}{\partial \lambda_k} \right]^T W_i \left[\frac{\partial X_i}{\partial \lambda_k} \right] \right\} dt + D_k \right]^{-1}$$

$$\left[\int_0^t \left\{ \left[\frac{\partial X_i}{\partial \lambda_k} \right]^T W_i \, [Y_i - X_i] \right\} dt - D_k(\lambda_k - \lambda_{k_o}) \right] \qquad (2.1.8)$$

The vector D_k in the above expressions is usually referred to as the *a priori* weight vector in that it enables the analyst to make intelligent use of information with respect to the parameters from additional sources. For example, in response tests with conventional aircraft it is usually possible to estimate from aerodynamic theory the value of the parameter Z_w to good accuracy and an appropriate *a priori* weight for that parameter will keep its value within the bounds prescribed.

It is frequently the case when the number of parameters to be estimated is large that the inverted matrix in Equation 2.1.4 is poorly conditioned because of approximate linear relationships among the state variables or their time derivatives and the parameter sensitivity functions. This problem can be alleviated but often not eliminated by a careful choice of control inputs (the system vector forcing function). It may be seen from expression 2.1.8 that the *a priori* weight vector makes the sensitivity matrix better conditioned in that terms are added to the diagonal elements. Thus, if reliable *a priori* information is available, the addition of the *a priori* weight term in the cost function can be of considerable benefit in preventing divergent solutions.

2.2 Algebraic Constraints Among the Stability Derivatives

The response in the longitudinal plane of a helicopter may usually be modelled to within a reasonable accuracy by the following set of first order linear differential equations

$$\dot{U} = X_{\dot{u}}\dot{U} + X_u U + X_{\dot{w}}\dot{W} + X_w W + X_{\dot{Q}}\dot{Q} + (X_Q - W_o)Q - g\theta$$

$$+ X_{\dot{\delta}_c}\dot{\delta}_c + X_{\delta_c}\delta_c + X_{\dot{\delta}_e}\dot{\delta}_e + X_{\delta_e}\delta_e$$

$$\dot{W} = Z_{\dot{u}}\dot{U} + Z_u U + Z_{\dot{w}}\dot{W} + Z_w W + Z_{\dot{Q}}\dot{Q} + (Z_Q + U_o)Q$$

$$+ Z_{\dot{\delta}_c}\dot{\delta}_c + Z_{\delta_c}\delta_c + Z_{\dot{\delta}_e}\dot{\delta}_e + Z_{\delta_e}\delta_e$$

$$\dot{Q} = M_{\dot{u}}\dot{U} + M_u U + M_{\dot{w}}\dot{W} + M_w W + M_{\dot{Q}}\dot{Q} + M_Q Q$$

$$+ M_{\dot{\delta}_c}\dot{\delta}_c + M_{\delta_c}\delta_c + M_{\dot{\delta}_e}\dot{\delta}_e + M_{\delta_e}\delta_e$$

$$\dot{\theta} = Q \qquad (2.2.1)$$

If the U, W, Q and θ response variables are taken as the state vector $X_i(i = 1, 2, 3, 4)$, the $\dot{\delta}_c$, δ_c, $\dot{\delta}_e$ and δ_c control input variables as the vector forcing function $U_j(j = 1, 2, 3, 4)$ and the stability derivatives $X_{\dot{u}}$, X_u, ---, M_{δ_e} as the parameter vector $\lambda_k(k = 1, 2, ---, 30)$, the least squares quasilinearization procedure outlined in the foregoing paragraphs can be used to obtain estimates of the 30 stability derivatives. In fact, it would be very unlikely that convergent solutions could be obtained because of the poorly conditioned sensitivity function matrix and, even in the event solutions were obtained, many of the stability derivative estimates would be physically unrealistic and hence of little value to the engineer.

The linear model assumed for the formulation of the Equation Set 2.2.1 is approximate and while the response associated with neglected non-linear terms may be small, the change in derivative estimates that comes about in an attempt to make the linear model give the same response as the exact model, may be large. This effect is related to the magnitude of the response allied with neglected terms for the particular test analysed and is likely to be quite different depending upon the characteristics of the control input. Furthermore, during the time of a particular test, there may be small atmospheric inputs present which are unknown and not allowed for in the model. These also, even if the response associated with them is small, have a significant influence on the parameter estimates.

The use of an *a priori* weight vector in the manner described in Section 2.1 is a useful and powerful aid for ensuring that resulting parameter estimates are not strongly influenced by peculiarities associated with a particular set of test data. In order to use the method, however, it is necessary to know the confidence associated with each of the initial parameter estimates. Certain parameters such as Z_w can be predicted with confidence for conventional fixed-wing aircraft but if the aircraft is a helicopter or other V/STOL aircraft the complexity resulting from interfering aircraft components is usually such that theoretical estimates are uncertain and adequate wind tunnel tests are not normally available.

A procedure adopted for the results given in this paper, and in more detail in References 4 and 5, expresses the stability derivatives in the Equation Set 2.2.1 in terms of the aerodynamic forces acting on the major components of the helicopter, moment arms and inertial parameters. For example, the total derivatives X_w and Z_w are broken into elements associated with the major helicopter components as follows:

$$X_w = X_{WMR} + X_{WTR} + X_{WFT} + X_{WFUS}$$

$$Z_w = (1 - \epsilon_{TR} - \epsilon_{FT})Z_{WMR} + Z_{WTR} + Z_{WFT} + Z_{WFUS} \tag{2.2.2}$$

where the subscripts MR, TR, FT and FUS refer to the main rotor, tail rotor, fixed tail and fuselage respectively. The terms ϵ_{TR} and ϵ_{FT} represent down-wash factors at the tail rotor and fixed tail resulting when the Z force on the main rotor is changed. The corresponding pitching moment derivative, M_w is

$$M_w = \frac{m}{I_{yy}}\Big[-h_{MR}X_{WMR} + (\epsilon_{TR}\ell_{TR} + \epsilon_{FT}\ell_{FT})Z_{WMR} - \ell_{TR}Z_{WTR} - \ell_{FT}Z_{WFT} + \ell_{FUS}Z_{WFUS}\Big] \tag{2.2.3}$$

where m is the helicopter mass, I_{yy} is the pitching moment of inertia, and $h_{MR}, \ell_{TR}, \ell_{FT}, \ell_{FUS}$ represent the moment arms from the reference axes to the effective aerodynamic centres of the main rotor, tail rotor, fixed tail and fuselage, respectively.

Similar expressions to those of 2.2.2 and 2.2.3 were developed for each of the stability derivatives of the Equation Set 2.2.1 in terms of new parameters such as those in the above expressions. Thirty-two new parameters, designated P_ℓ, were used in the expressions for the thirty stability derivatives, designated R_m. The algebraic constraints provided by these expressions make it such that a change in one parameter P_ℓ, say X_{WMR}, not only changes X_w but also changes M_w. Furthermore, most of these new parameters ($h_{MR}, \ell_{TR}, \ell_{FT}$ --- etc.) could be estimated *a priori* and the confidence in the estimates established for use in setting the elements of an *a priori* weight vector.

The sensitivity functions with respect to the original stability derivatives,

$$\frac{\partial X_i}{\partial R_m}$$

were calculated from the sets of sensitivity equations obtained by taking derivatives of the Equation Set 2.2.1 with respect to the parameters R_m appearing in these equations. The partial derivatives

$$\frac{\partial R_m}{\partial P_\ell},$$

expressing the sensitivity of each of the stability derivatives to the parameters P_ℓ, were obtained from the expressions such as 2.2.2 and 2.2.3 and the sensitivity functions with respect to the new set of parameters P_ℓ calculated from

$$\frac{\partial X_i}{\partial P_\ell} = \sum_{m=1}^{30} \frac{\partial R_m}{\partial P_\ell} \cdot \frac{\partial X_i}{\partial R_m} \tag{2.2.4}$$

Algebraic complexity is introduced using this procedure but it has the very significant advantage of providing constraints between variations in the different parameters. Because the problem is formulated in terms of parameters more fundamental than are the stability derivatives, it is easier to establish realistic levels of confidence in the initial estimates and assign an *a priori* weight vector.

2.3 Conglomerate Analysis of Several Similar Tests

If small inputs from atmospheric unsteadiness are present during a particular test the Equation Set 2.1.1 used to model the system properly includes an additional input vector, Ω_n, and an additional parameter vector, β_s. If the model equations are a set of linear differential equations, as in the present case, the total state vector describing the system response, X_{T_i}, can be written as the superposition of the responses resulting from the control input vector, U_j, and the atmospheric input vector, Ω_n; namely;

$$X_{T_i}(U_j, \lambda_k, \Omega_n, \beta_s) = X_i(U_j, \lambda_k) + X_{A_i}(\Omega_n, \beta_s) \tag{2.3.1}$$

The recursive relationship for successive changes in the parameter vector now becomes

$$\Delta\lambda'_k = \Delta\lambda_k - \left[\int_0^t \left\{\left[\frac{\partial X_i}{\partial \lambda_k}\right]^T W_i \left[\frac{\partial X_i}{\partial \lambda_k}\right]\right\} dt\right]^{-1} \left[\int_0^t \left\{\left[\frac{\partial X_i}{\partial \lambda_k}\right]^T W_i\, X_{A_i}\right\} dt\right] \tag{2.3.2}$$

where the $\Delta\lambda_k$ is the change in the parameter vector in the absence of atmospheric inputs given by Equation 2.1.4 (Equation 2.1.8 if *a priori* weight is used). The other term in Equation 2.3.2 therefore represents a bias that may result in the parameter vector caused by atmospheric inputs.

If the dimension of the cost function is extended to include N independent tests, where it is expected that the parameter vector should be the same in each of the included tests, then

$$J_N = J^{(1)} + J^{(2)} + \dots J^{(N)} \qquad (2.3.3)$$

and the recursive relationship becomes

$$\Delta\lambda'_{k_N} = [A]^{-1}\left[\sum_{n=1}^{N}\int_0^{tn}\left\{\left[\frac{\partial X_i^{(n)}}{\partial\lambda_k}\right]^T W_i^{(n)}\left[Y_i^{(n)} - X_i^{(n)}\right]\right\}dt\right]$$

$$- [A]^{-1}\left[\sum_{n=1}^{N}\int_0^{tn}\left\{\left[\frac{\partial X_i^{(n)}}{\partial\lambda_k}\right]^T W_i^{(n)} X_{A_i}^{(n)}\right\}dt\right]$$

$$= \Delta\lambda_{k_N} - [A]^{-1}\left[\sum_{n=1}^{N}\int_0^{tn}\left\{\left[\frac{\partial X_i^{(n)}}{\partial\lambda_k}\right]^T W_i^{(n)} X_{A_i}^{(n)}\right\}dt\right] \qquad (2.3.4)$$

where

$$[A] = \left[\sum_{n=1}^{N}\int_0^{tn}\left\{\left[\frac{\partial X_i^{(n)}}{\partial\lambda_k}\right]^T W_i^{(n)}\left[\frac{\partial X_i^{(n)}}{\partial\lambda_k}\right]\right\}dt\right] \qquad (2.3.5)$$

The first term $\Delta\lambda_{k_N}$ is the parameter vector estimate in the absence of atmospheric unsteadiness while the second term represents the bias in the estimates resulting from these unknown small inputs.

The probability of atmospheric inputs being the same, or more precisely of the correlation between the sensitivity function vector and the response function vector to atmospheric disturbances being the same, among a number of tests conducted from the same reference condition, is very small. Consequently the magnitude of the bias term in Equation 2.3.4 is likely to be reduced relative to that when only one test is included as in Equation 2.3.2. If N is large the resulting bias will have a high probability of being negligible but from the point of view of the practical engineer, the confidence in the estimates will be considerably enhanced if N is 3 or 4.

It should be noted that this procedure is not likely to remove bias in the estimates that result from inadequacies of the model in predicting response to the known control inputs since the correlation of the unmodelled response (corresponding to the term X_{A_i} in 2.3.4) with $\frac{\partial X_i}{\partial\lambda_k}$ may be the same in successive tests, particularly if the known inputs are similar in each of the tests. The modelling procedure outlined in Section 2.2, which is based on an appreciation of the aerodynamic peculiarities of the unknown system, hopefully reduces concern in this area.

This procedure of treating a conglomerate of independent runs in the cost function with a common parameter vector is similar in concept to a procedure sometimes known as the Bayesian maximum likelihood estimator (Ref. 6). The formal derivation of the maximum likelihood estimator comes from a statistical approach to the problem and strictly speaking requires that the unknown inputs, regarded as noise, are random with a Gaussian distribution. The more usual approach is to determine derivative estimates from special tests conducted in approximately turbulence free conditions. The unknown atmospheric inputs that may be present, and certainly those that cause the greatest problem, are small amplitude low frequency disturbances having a period of the same order as the record length and it is not possible to describe these in a meaningful statistical manner.

3. EXAMPLES OF APPLICATION TO FLIGHT TESTS WITH THE BELL 205 HELICOPTER

The procedures of Section 2 have been used to obtain estimates of both the lateral-directional and longitudinal stability derivatives of the Bell 205 helicopter (Fig. 1). Details of the tests and analyses along with complete results are given in References 4 and 5. Twenty-one lateral-directional derivatives were obtained using nineteen fundamental parameters of the nature of those designated by P_ℓ in Section 2.2, while thirty longitudinal derivatives were found from thirty-two parameters. The observed responses used in the cost function were roll rate, yaw rate and lateral velocity for the lateral-directional response tests and longitudinal velocity, normal velocity, pitch rate and normal acceleration for the longitudinal tests.

The estimates obtained for the 10 dominant lateral-directional derivatives are given in Table 1 for three separate tests. The values found when each was treated separately all appear reasonable and the computed or modelled responses fit the measured responses quite well as shown for one of the tests in Figure 3a). One is able to conclude from these results that the procedure outlined in Section 2.2, which set up algebraic constraints among the elements of the parameter vector and which enabled the use of an *a priori* weight vector, was successful in that convergent solutions were obtained and the estimates appear to be physically realistic. On the other hand, each of the three tests were performed from the same reference condition, so that hopefully the derivative values would have been closely the same from the separate tests. There were, in fact, considerable differences particularly in the dominant roll damping derivative estimates.

Included in Table 1 are the single set of derivative estimates obtained by treating these three separate tests as a conglomerate in the cost function after the fashion of Section 2.3. The root mean square error between the modelled and measured responses for each run using the derivative values from each test analysed separately are compared with those obtained using the derivative estimates from the conglomerate analysis in Figure 2. The response time histories for one of the runs are also included in Figure 3b) using the derivatives from the conglomerate analysis for comparison with the responses using the parameter estimates from that test alone. These two Figures show that the common set of derivatives from the conglomerate analysis produced model responses for each of the three different tests nearly as well as those produced from the separate results obtained for each test.

FIGURE 1 BELL 205 HELICOPTER

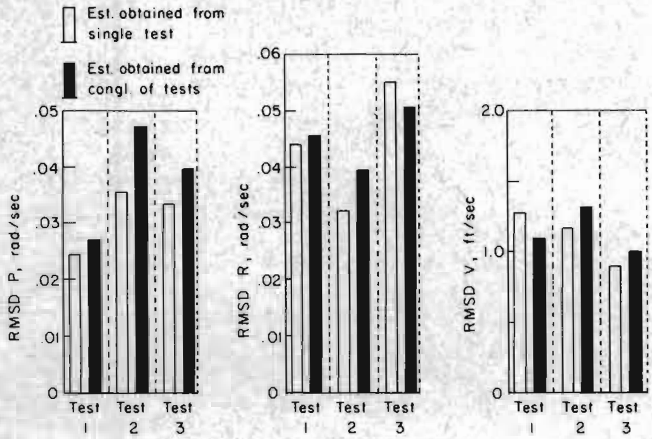

FIGURE 2 COMPARISON OF ROOT MEAN SQUARE DIFFERENCES, LATERAL-DIRECTIONAL RESPONSE, 70 KNTS

TABLE 1 ESTIMATED VALUES OF LATERAL-DIRECTIONAL ROTARY DERIVATIVES, 70 KNTS

	Values Obtained from Analysis of Separate Tests			Values Obtained from Congl. of Tests
	1	2	3	
L_p	−0.820	−0.523	−0.922	−0.806
N_p	−0.048	−0.022	−0.069	−0.037
L_R	0.122	0.105	0.200	0.174
N_R	−1.393	−1.029	−1.254	−1.303
L_V	−0.0164	−0.0137	−0.0147	−0.0147
N_V	0.0189	0.0192	0.0146	0.0177
L_{δ_a}	0.183	0.178	0.191	0.184
N_{δ_a}	0.024	0.024	0.030	0.025
L_{δ_r}	−0.254	−0.218	−0.259	−0.260
N_{δ_r}	0.561	0.467	0.431	0.493

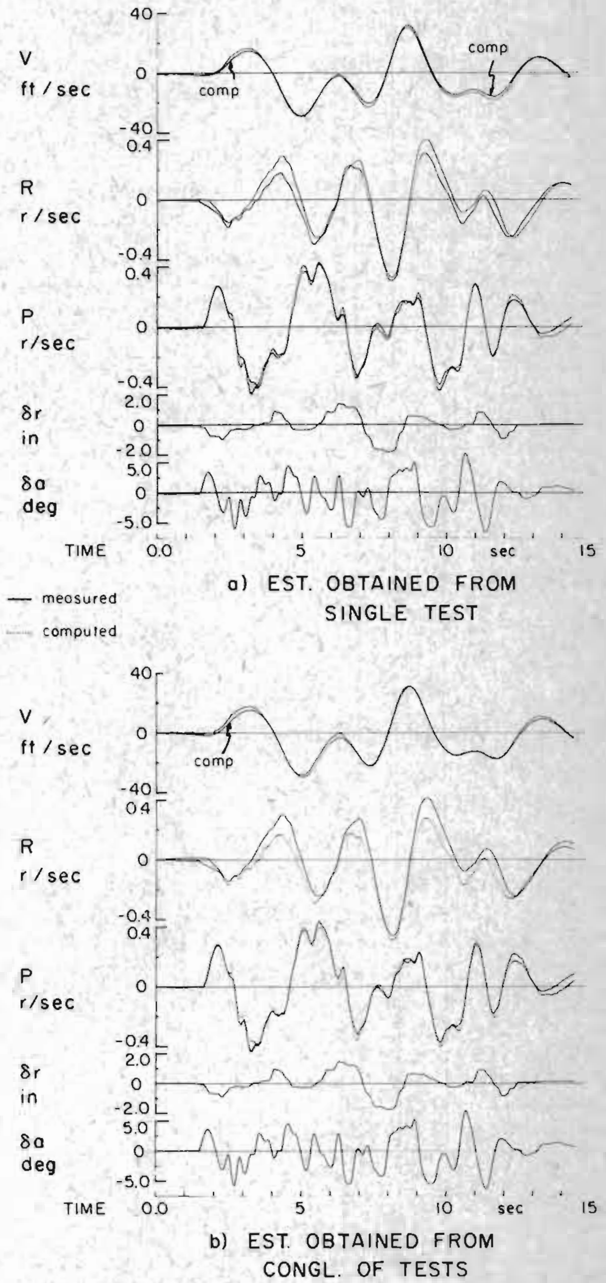

FIGURE 3 COMPARISON OF MEASURED AND MODELLED RESPONSE, TEST 1, 70 KNTS

Estimates for fifteen of the most significant longitudinal response derivatives are given in Table 2 for four different tests. As in the case of the lateral-directional response experiments there are considerable differences among the estimates for a particular derivative for the different tests in spite of the fact that the reference flight condition was the same for each.

Also included in Table 2 are the estimates resulting from including tests 4 and 5 together, tests 6 and 7 together, tests 4, 5 and 6 together and finally tests 4, 5, 6 and 7 together as conglomerates. The root mean square error between the four modelled and measured response variables (Q, a_z, U and W) are compared in Figure 4 for each of the cases of Table 2. The differences between estimates for runs 4 and 5 analyzed as a conglomerate and runs 6 and 7 analysed as a conglomerate are considerably less than was the variability when each run was taken separately, with the exception of the Z_{δ_c} derivative. The estimated values obtained by taking tests 4, 5 and 6 together differ somewhat from those with only two tests taken together but the difference is not large. The estimated derivative values changed a small amount from those obtained with three tests in a conglomerate when all four were analysed together.

The comparison of the root mean square errors between the modelled and fitted responses for each of the cases of Table 2 (Fig. 4) show that although the errors generally increase somewhat for each test when the common stability derivatives from the conglomerate analysis are used, the increase is not large. In fact for test 4, the errors between the modelled and measured U and W responses were smaller when the common derivatives were used than when the derivatives obtained for this test separately were used. The weighted error, which is not shown, did of course increase.

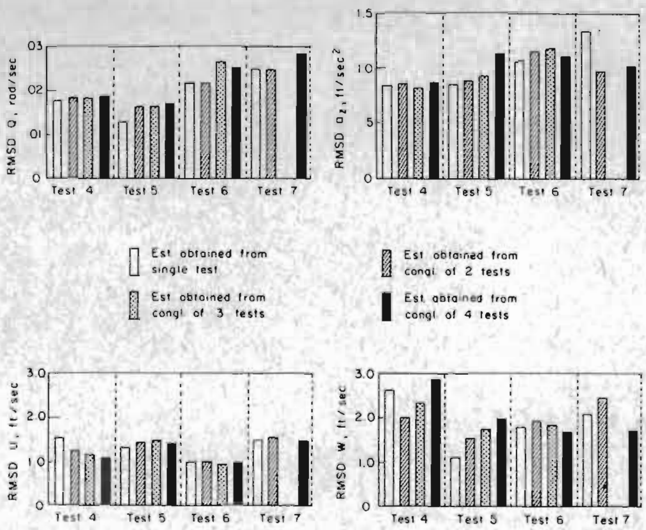

FIGURE 4 COMPARISON OF ROOT MEAN SQUARE DIFFERENCES, LONGITUDINAL RESPONSE, 70 KNTS

In Figure 5, the errors are given for tests 4 and 5 for the responses of test 4 modelled with the derivatives from test 5 and vice versa. It may be seen that the errors in this case are greater than those obtained using the common derivatives with the exception of the W and Q responses of test 4.

TABLE 2 ESTIMATED VALUES OF LONGITUDINAL DERIVATIVES — 70 KNTS

	Values Obtained From Analysis of Separate Tests				Values Obtained From Analysis of Conglomerate of Tests				Algebraic Average of Tests 4, 5, 6, 7
	4	5	6	7	4 & 5	6 & 7	4, 5 & 6	4,5,6&7	
X_u	−0.0220	−0.0305	−0.0235	−0.0584	−0.0317	−0.0235	−0.0265	−0.0334	−0.0336
Z_u	−0.0534	−0.0427	−0.0458	−0.0600	−0.0518	−0.0240	−0.0479	−0.0288	−0.0505
M_u	0.0009	0.0020	0.0011	0.0062	0.0021	0.0006	0.0015	0.0023	0.0025
X_w	−0.0351	−0.0495	−0.0342	−0.0640	−0.0065	−0.0408	−0.0207	−0.0432	−0.0457
Z_w	−0.4597	−0.5559	−0.7675	−0.4854	−0.5809	−0.5196	−0.6321	−0.5713	−0.5671
M_w	−0.0043	−0.0085	−0.0041	−0.0215	−0.0082	−0.0134	−0.0089	−0.0091	−0.0096
X_Q	1.417	1.478	1.499	1.436	1.547	1.500	1.571	1.461	1.457
Z_Q	−40.24	−40.58	−40.32	−41.08	−40.11	−40.65	−40.21	−40.43	−40.56
M_Q	−0.8329	−0.3566	−0.4264	−0.1398	−0.6900	−0.4204	−0.6852	−0.5993	−0.4389
X_{δ_c}	2.94	2.55	0.56	0.53	1.95	0.46	1.63	1.11	1.64
Z_{δ_c}	−19.55	−26.20	−23.25	−16.03	−23.87	−14.89	−23.09	−19.72	−21.28
M_{δ_c}	−0.150	−0.125	0.153	−0.011	−0.174	−0.013	−0.154	−0.103	−0.033
X_{δ_e}	−5.02	−5.02	−5.04	−4.99	−5.05	−5.06	−5.08	−5.06	−5.02
Z_{δ_e}	−19.56	−19.88	−19.83	−19.68	−20.05	−19.81	−20.17	−20.08	−19.73
M_{δ_e}	0.835	0.772	0.832	0.670	0.731	0.713	0.732	0.706	0.706

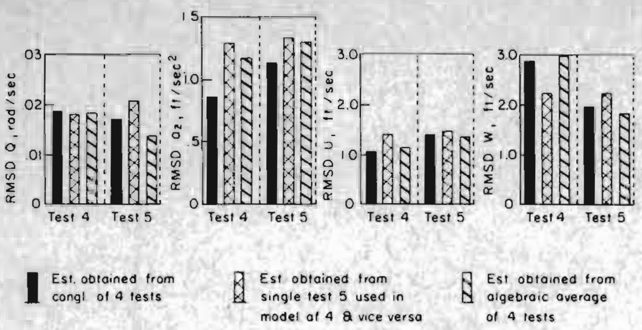

FIGURE 5 COMPARISON OF ROOT MEAN SQUARE DIFFERENCES, LONGITUDINAL RESPONSE, 70 KNTS

Also included in Figure 5 are the errors found for tests 4 and 5 using derivative values that result from taking the simple algebraic average of the estimates from the four tests taken individually. These errors are larger than those obtained using the common derivatives from the conglomerate analysis except for the Q and W responses of test 5. The conglomerate analysis is a more formally correct procedure to obtain the best common estimates for a number of tests from the same reference condition than is the simple algebraic average. These results show that the root mean square errors are slightly reduced when the more formally correct procedure is used.

It is difficult to make an assessment of the accuracy of the final stability derivative estimates. There was considerable variation among the stability derivative values obtained from individual tests. The most likely causes of these differences were the presence of unknown, probably low frequency atmospheric inputs, and an indeterminateness between two or more derivatives. To the extent that the conglomerate analysis was successful in removing these problems, the greatest uncertainty in the estimated values at a given reference flight speed is removed. Certainly the confidence in being able to use these common derivatives for more general responses is considerably enhanced.

As is indeed intended, the *a priori* weight vector used has considerahle influence on the final derivative estimates. The onus is placed on the engineer to make reasonable estimates of the allowable variation in each parameter from the initial values to arrive at the relative magnitude of the elements of the *a priori* weight vector. The confidence that may be given to these estimates of the allowable variation in each parameter is considerably improved when the model is formulated in the manner of Section 2.2.

The resolution of the least-squares method for each set of parameter estimates may be determined from the results of the last two or three steps in the iteration process. The parameter estimates oscillate in a coupled manner between successive iteration steps with only very small oscillations in the value of J. These variations are indicative of the resolution but not the absolute accuracy of the method. Estimates of the resolution of the method are given in References 4 and 5 and in all cases they are small compared to the variations between derivative values obtained from separate tests at the same reference flight condition.

4. CONCLUSIONS

A least-squares quasilinearization procedure was used to obtain estimates of twenty-one lateral-directional and thirty longitudinal stability derivatives from in-flight response tests of the Bell 205 helicopter. The particular adaptation of the classical least-squares method had two features, believed to be unique, to reduce the influence on the resulting derivative estimates of peculiarities of the mathematical model and the circumstances of the tests.

The first of these features consisted of the formulation of the model in terms of the aerodynamic forces acting on the major components of the helicopter, the moment arms from the reference axes to the effective aerodynamic centres of these forces, and inertia parameters. This formulation provided algebraic constraints among the stability derivatives in terms of these parameters, and made it possible to determine allowable variations in the parameters and establish an *a priori* weight vector for inclusion in the cost function. This procedure can increase significantly the confidence that may be given to the final derivative values, but the onus is placed on the engineering analyst to make good estimates of allowable variations in the parameters so that the final results are not unduly biased.

The second feature involved an increase in the dimension of the cost function so that a number of independent tests from the same reference flight condition could be taken as a conglomerate with a common parameter vector. This procedure is capable of considerably reducing the influence on the derivative estimates of small unknown atmospheric inputs that may be present during a particular series of tests. It also provides a formally correct procedure for determining the best average parameter estimates from independent tests at the same reference flight condition.

5. REFERENCES

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