FLIGHT TRAJECTORY OPTIMIZATION FOR AN EFFICIENT AIR TRANSPORTATION SYSTEM

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Abstract

An efficient, reliable and futuristic Air Transportation System (ATS) is the key to handle the anticipated rapid changes in the aviation industry over the next few decades. Trajectory Based Operations (TBO) could be considered as one of the most significant solutions to cater for these problems. This paper discusses the development of a 4D optimal flight trajectory based on aircraft performance, weather forecasts, Air Traffic Control (ATC) database and aircraft operational data. Dynamic Programming is utilized in the optimization process for its qualities of avoiding iterative calculations, predicting the required amount of calculations and feasibility of applying inequality constraint conditions. Optimal flight trajectories were generated for a single aircraft for its climb, cruise and descent phases in a provided 4D grid platform based on a busy domestic flight route, between Tokyo (Haneda) and Fukuoka. Flight data were recorded on a commercial GPS data logger inside an airliner cabin and used to compare with optimum solutions obtained from the model. Consumed fuel consumption and flight time were evaluated to understand the benefits gained by the optimization.

1 Introduction

The air traffic is expected to increase rapidly in a significant rate over the next few decades [1]. Also, the modern aviation industry has been greatly affected by sky rocketing fuel prices and economic instability in the global business market. Research & Development (R&D) projects such as NextGen of United States, SESAR of Europe and CARATS of Japan are dedicated to modernize the present Air Transportation System (ATS) and cater for these demands in foreseeable future [2], [3], [4].

At present busy airports such as Haneda and Narita International Airports in Japan mainly concentrate on increasing their ground capacities to be able to accommodate more aircraft and passengers, handling the terminal airspace effectively to avoid departure and arrival delays and to prevent noise pollution among residential areas. On the contrary, the present ATC system consists of many restrictions that are obligated to be followed by pilots during their flight missions.

Trajectory Based Operations (TBO) could be considered as one of the most significant solutions for achieving a greater overall efficiency and treat these conventional rules and restrictions. Conflict free 4D-flight trajectories optimized for wind conditions and aircraft performance such as specific range would greatly reduce unnecessary fuel consumption and flight time. Particularly, jet stream winds (westerly winds) over the subtropical East Asia and the Western Pacific is an important atmospheric circulation system and has a considerable influence over the airspace of Japan [5].

4D-trajectory optimization is formulated as an optimal control problem and had been extensively studied over the last few decades [6]. Various mathematical programming approaches have been proved to be effective in dealing with the optimization process [7], [8] and Dynamic Programming (DP) is considered as one of the most effective tools [9] for its unique qualities which are described later in chapter 3.

This paper discusses the development of a 4D-optimal flight trajectory by applying it to the flight route between Haneda and Fukuoka. The fuel consumption is evaluated as the cost index for its climb, cruise and descent flight phases by reviewing the aircraft performance, weather forecasts and aircraft operational data. DP method is used to achieve the optimum solution for a twin engine, wide-body, long-range jet passenger aircraft model.

Furthermore, analysis on GPS data is discussed which were used to calculate the reference flight data for evaluation. Optimal flight trajectories are achieved with the proposed model, and gained benefits are quantitatively evaluated and compared with flight data obtained by the GPS data logger.

2 Aircraft Equations of Motion

2.1 Definition

The aircraft's equations of motion are described by a point mass approximation, that is, the state variables which represent the aircraft's attitude are omitted. State variables to derive its 3D translational motion are depicted on Fig. 1. The heading angle ψ_a, flight path angle γ_a and engine thrust T are used as control variables and the equations of motion are solved with respect to the state variables of latitude ϕ, longitude θ, altitude H and aircraft's true airspeed V_{TAS}.

$$\begin{aligned}
\dot{\phi} &= \frac{V_{TAS}\cos\gamma_a\cos\psi_a + W_v(\phi,\theta,H)}{R+H} \\
\dot{\theta} &= \frac{V_{TAS}\cos\gamma_a\sin\psi_a + W_u(\phi,\theta,H)}{(R+H)\cos\phi} \\
\dot{H} &= V_{TAS}\sin\gamma_a \\
m\cdot\dot{V}_{TAS} &= T - D - mg\sin\gamma_a
\end{aligned} \tag{1}$$

where D the aerodynamic drag, g the gravity acceleration, m the aircraft mass and R is the Earth's radius. W_u and W_v are defined here as zonal and meridional wind components respectively, considering the eastward and northward flows are positive. In the trajectory optimization analysis, which will be discussed in later chapters, differential equations are discretized for a transition of two neighboring grid points depicted by Fig. 1.

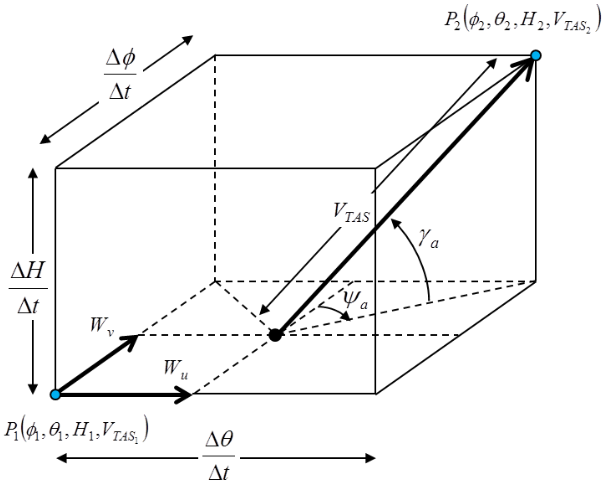


Fig. 1. Definition of point mass equations

Transition of $(\phi_1,\theta_1,H_1,V_{TAS_1}) \rightarrow (\phi_2,\theta_2,H_2,V_{TAS_2})$ is considered to calculate the control variables γ_a, ψ_a and Δt, the time cost between the two reviewed points, where the state variable V_{TAS} is represented by a mean value as Eq. (2).

$$V_{TAS} = \frac{V_{TAS_1} + V_{TAS_2}}{2} \tag{2}$$

It means that the change rate of true airspeed is assumed to be constant at each transition. By obtaining these values and calculating the aerodynamic drag by using the relationship between lift and drag coefficients provided by aircraft's aerodynamic model, we could gain the engine thrust by Eq. (3) (the discrete force equilibrium equation in Eq. (1)), where constant flight path angle in the transition is assumed, i.e. $L = mg\cos\gamma_a$.

$$\frac{\Delta V_{TAS}}{\Delta t} = \frac{T}{m} - \frac{D}{m} - g\sin\gamma_a \tag{3}$$

2.2 Geometrical formulation

The general concept of generating a flight route in this study is shown on Fig. 2, where the Earth is an assumed sphere with radius R and the aircraft's position is located by latitude ϕ, longitude θ and altitude H in a polar coordinate frame XYZ. The Great Circle Route (GCR) defined as the shortest distance between two locations, is reviewed here as the standard flight route. Let $\mathbf{r}_0$ and $\mathbf{r}_\mathrm{f}$ be the geocentric unit vectors to departure point P_0 and arrival point P_f respectively. The down range angle η_f between these two points is obtained by using the scalar product of above unit vectors and the distance of GCR could be given as $R\eta_\mathrm{f}$.

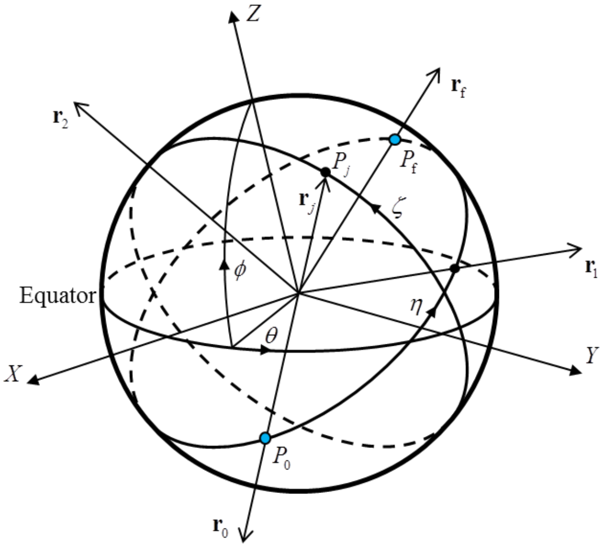


Fig. 2. Definition of polar coordinate axes and flight route

We define a unit vector $\mathbf{r}_2$ orthogonal to the GCR and its normal unit vector $\mathbf{r}_1$ and establish a set of points in order to generate the flight route between the two desired locations.

$$\mathbf{r}_0 = \begin{bmatrix} \cos\phi_0 \cos\theta_0 \\ \cos\phi_0 \sin\theta_0 \\ \sin\phi_0 \end{bmatrix}, \mathbf{r}_\mathrm{f} = \begin{bmatrix} \cos\phi_\mathrm{f} \cos\theta_\mathrm{f} \\ \cos\phi_\mathrm{f} \sin\theta_\mathrm{f} \\ \sin\phi_\mathrm{f} \end{bmatrix}$$
$$\eta_\mathrm{f} = \cos^{-1}\left(\mathbf{r}_0^T \cdot \mathbf{r}_\mathrm{f}\right) \quad (4)$$
$$\mathbf{r}_2 = \frac{\mathbf{r}_0 \times \mathbf{r}_\mathrm{f}}{\left|\mathbf{r}_0 \times \mathbf{r}_\mathrm{f}\right|} \ , \ \mathbf{r}_1 = \frac{\mathbf{r}_2 \times \mathbf{r}_0}{\left|\mathbf{r}_2 \times \mathbf{r}_0\right|}$$

These two unit vectors are given in Eq. (4). The geocentric unit vector of j th point of N equidistant points on GCR is given as,

$$\mathbf{r}_{j_0} = \cos\left(\frac{\eta_0 j}{N+1}\right)\mathbf{r}_0 + \sin\left(\frac{\eta_0 j}{N+1}\right)\mathbf{r}_1 \quad (5)$$

Here we consider that the aircraft optimizes its route by considering the influence of wind and accordingly deviates from GCR. This is the cross range angle ζ and the geocentric unit vector of this position P_j (normally deviated from GCR) is defined. Longitude and latitude of aircraft position can be calculated from the geocentric unit vector as shown in Eq. (6).

$$\mathbf{r}_j = \cos\zeta_j \left\{ \cos\left(\frac{\eta_0 j}{N+1}\right)\mathbf{r}_0 + \sin\left(\frac{\eta_0 j}{N+1}\right)\mathbf{r}_1 \right\} + \sin\zeta_j \mathbf{r}_2 \quad (6)$$

Assumptions are made in this study such that the vertical component of wind vector is negligible, magnitude of W_u and W_v are only position dependent and remain constant throughout the flight time. We also assume the heading angle variation and flight path angle variation at each grid point do not give significant effect on the performance. Also the set of equations at Eq. (1) are proposed as a result of tradeoff between obtaining a precise optimal solution for a real aircraft flight model and carrying out numerical computation within a limited time period. These models including the meteorological data, atmosphere model, aerodynamic model, fuel consumption model and engine thrust model are discussed later in this paper.

3 Dynamic Programming Approach

Generally, a steady flight is assumed to evaluate the performance optimization of an aircraft for its climb, cruise and descent phases. In a case we assume that prognostic wind data is in possession and the influence of wind is considered, it creates a necessity of a dynamic optimization. Various analytical methods are available to obtain a numerical solution in optimal control problems. For this study DP method is the most suitable to solve a discrete deterministic process to achieve optimal flight trajectories in presence of winds.

Richard Bellman's principle of optimality, by definition, is that regardless of initial values and decisions, the rest of the decisions have to be able to constitute an optimal policy with regard to the state resulting from the initial decision [10]. The unique qualities of applying DP in an optimal control problem could be listed as follows;

The advantages are,

- Capability of gaining a global optimum solution without any local optimum concerns.
- Predictability of complexity as iterative computation to achieve a solution converged to the optimal value function is not required.
- Simplicity of applying inequality constraint conditions of state and control variables.

However, there are several practical issues to worry about.

- Necessity of increasing the amount of calculations to reduce the discretization error and enhance the accuracy of the solution gained.
- Extreme increase in required amount of computations with the increase in dimension of state space and with the number of discrete points for each state, which is known as the 'Curse of Dimensionality'.
- Necessity of specific techniques to reduce the discretization error.

3.1 Performance Index

This paper mainly concentrates on minimizing the fuel consumed by the subjected aircraft. Therefore the performance index J_{opt} is established as following.

$$J_{opt} = \int_{t_0}^{t_f} FF \, dt \tag{7}$$

where $FF[\mathrm{kg/s}]$ is the fuel flow, t_0 the departure time and t_f the arrival time.

The state space created by state variables in Eq. (1) is defined by a grid scheme to utilize the DP method. The distribution of wind over the reviewed airspace is assumed constant within the flight time. Hence, the equations of motion are independent functions of time which enable us to utilize a monotonically varied state variable η as the independent variable and reduce the dimension of state space by 1. Trajectory transition between each two grid points in the scheme (refers to as a 'section' on the grid) is considered to perform the algorithm. As mentioned in chapter 2, we assume that the rate of change between two grid points is constant. This is achieved by treating the utilized control variables as constants within a section.

The optimization problem is to steer the system, the set of equations in Eq. (1) from an initial state $(\eta_0, \zeta_0, H_0, V_0)$ at t_0 to a final state $(\eta_f, \zeta_f, H_f, V_f)$ at t_f so that the fuel consumed is minimized. We assume that an arbitrary state $(\eta_i, \zeta(\eta_i), H(\eta_i), V(\eta_i))$ is reviewed at down range angle η_i on the grid where we have already achieved an optimal trajectory with a given initial state. We could define this minimized performance index as a function of state variables as $J_{opt}(\eta_i, \zeta(\eta_i), H(\eta_i), V(\eta_i))$. The optimal solution or the optimal trajectory from current state to the next state $(\eta_{i+1}, \zeta(\eta_{i+1}), H(\eta_{i+1}), V(\eta_{i+1}))$ possesses the relationship expressed in Eq. (8).

$$\begin{aligned} & J_{opt}\left(\eta_{i+1}, \zeta_{j_{i+1}}(\eta_{i+1}), H_{k_{i+1}}(\eta_{i+1}), V_{l_{i+1}}(\eta_{i+1})\right) = \\ & \min\left\{J_{opt}\left(\eta_i, \zeta_{j_i}(\eta_i), H_{k_i}(\eta_i), V_{l_i}(\eta_i)\right) + \Delta J\right\} \end{aligned} \tag{8}$$

The 'min' term declares the selection of optimized performance index among the combinations of all transitions upto η_{i+1} th section of the grid as depicted on Fig. 3. ΔJ in Eq. (9) is the increase of performance index for each transition.

$$\begin{aligned} & \Delta J = \left(\eta_i, \zeta_{j_i}(\eta_i), H_{k_i}(\eta_i), V_{l_i}(\eta_i)\right) \\ & \rightarrow \left(\eta_{i+1}, \zeta_{j_{i+1}}(\eta_{i+1}), H_{k_{i+1}}(\eta_{i+1}), V_{l_{i+1}}(\eta_{i+1})\right) \end{aligned} \tag{9}$$

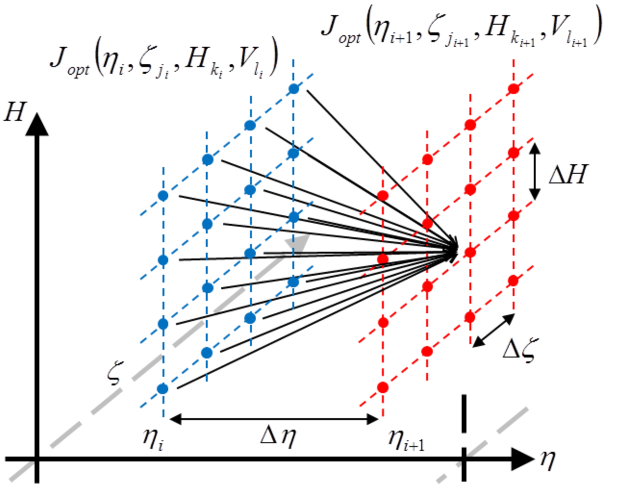


Fig. 3. Optimization process with dynamic programming (Dimension of velocity is omitted to simplify the image)

4 Models Used in Optimization Process

4.1 Meteorological Data

The Japan Meteorological Agency provides a variety of weather data, Grid Point Value (GPV), depending on the type of Numerical Weather Prediction (NWP) model. The NWP model, Meso Scale Model (MSM) is used in this paper to obtain wind data and its basic characteristics are given on table 1. where the obtained physical quantities on each barometric pressure surface include geopotential altitude, ambient temperature, upward flow of wind, relative humidity and wind speed (zonal and meridional components).

Table 1. Weather prediction model overview [11]

Model	MSM (Meso Scale) (GPV) non hydrostatic model
Area	Japan and its adjacent waters Lat. $22.4-47.6\,[\mathrm{deg}]$ Lon. $120-150\,[\mathrm{deg}]$
Resolution	Lat. $0.1\,[\mathrm{deg}]$, Lon. $0.125\,[\mathrm{deg}]$
Barometric surface altitudes	1000, 975, 950, 925, 900, 820, 800, 700, 600, 500, 400, 300, 250, 200, 150, 100 [hPa]

Bilinear and linear interpolations were used to obtain the required wind components at the reviewed grid points for optimization.

4.2 Atmosphere Model

In this study we use the International Standard Atmosphere for the case of no wind ideal atmosphere [12].

4.3 Aircraft Performance Model

To discuss an aircraft's performance especially to conjecture the fuel consumption in respect to a specific flight condition, we require a model capable of providing sufficient and precise data. BADA (Base of Aircraft Data) 3.9 developed and maintained by EUROCONTROL is used here as the performance model [13].
Flight envelope is the region of velocity-altitude plane where an airplane can safely fly during its operation [14]. The specific range in the flight envelope provides a fundamental understanding on static fuel consumption per distance of an aircraft. The specific range computed for above mentioned aircraft BADA model is shown in Fig. 4. The flight operational data on table 2. was used as limitations to calculate the maximum Specific Range $SR\,[\mathrm{m/kg}]$ for 3 standard aircraft masses provided by BADA model. Cruising performance of an aircraft is represented by SR defined in Eq. (10) as the flight distance per unit quantity of fuel [15].

$$SR = \frac{V_{TAS}}{cT} \tag{10}$$

where $c\,[\mathrm{kg/kgf/hr}]$ is the specific fuel consumption. The optimized results are shown in table 3. Results show that the aircraft could gain a maximum SR by flying close to the service ceiling except for the maximum mass case.

When a complete flight including climb, cruise and descent phases is considered, we need to review the variations in thrust, fuel consumption, airspeed and other parameters to perform a dynamic optimization. To calculate these variations we use the models defined in BADA. The aerodynamic model provides the useful equations to compute the lift and drag. The fuel consumption model provides information to gain the fuel flow at climb, cruise and descent phases of flight. The nominal fuel

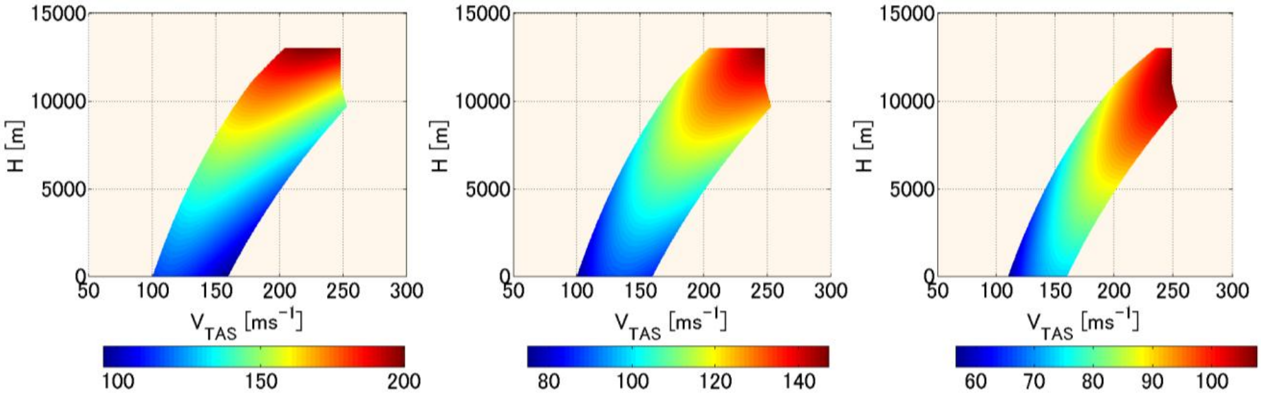


Fig. 4. Specific Range [m/kg]
(a) Minimum mass: 138100[kg] (b) Reference mass: 208700[kg] (c) Maximum mass: 287000[kg]

Table 2. Limitations for static performance optimization

Max. operating speed (CAS)	310[kt]
Min. operating speed (CAS)	149×1.3 [kt]
Max. operating Mach No.	0.84
Max. operating altitude	41000[ft]

Table 3. Results on static performance optimization

Mass	Optimal altitude [m]	Optimal velocity V_{TAS} [ms^{-1}]	Max. *SR* [m/kg]
minimum	13000	227.78	200.13
reference	13000	247.85	147.17
maximum	11600	247.85	107.48

flow is a function of true airspeed while the minimum fuel flow is a function of altitude which applies for idle thrust descent conditions. Fuel flow for cruise phase is corrected by a cruise fuel flow factor. The thrust model provides the maximum thrust available in each flight phase.

5 GPS Data Analysis

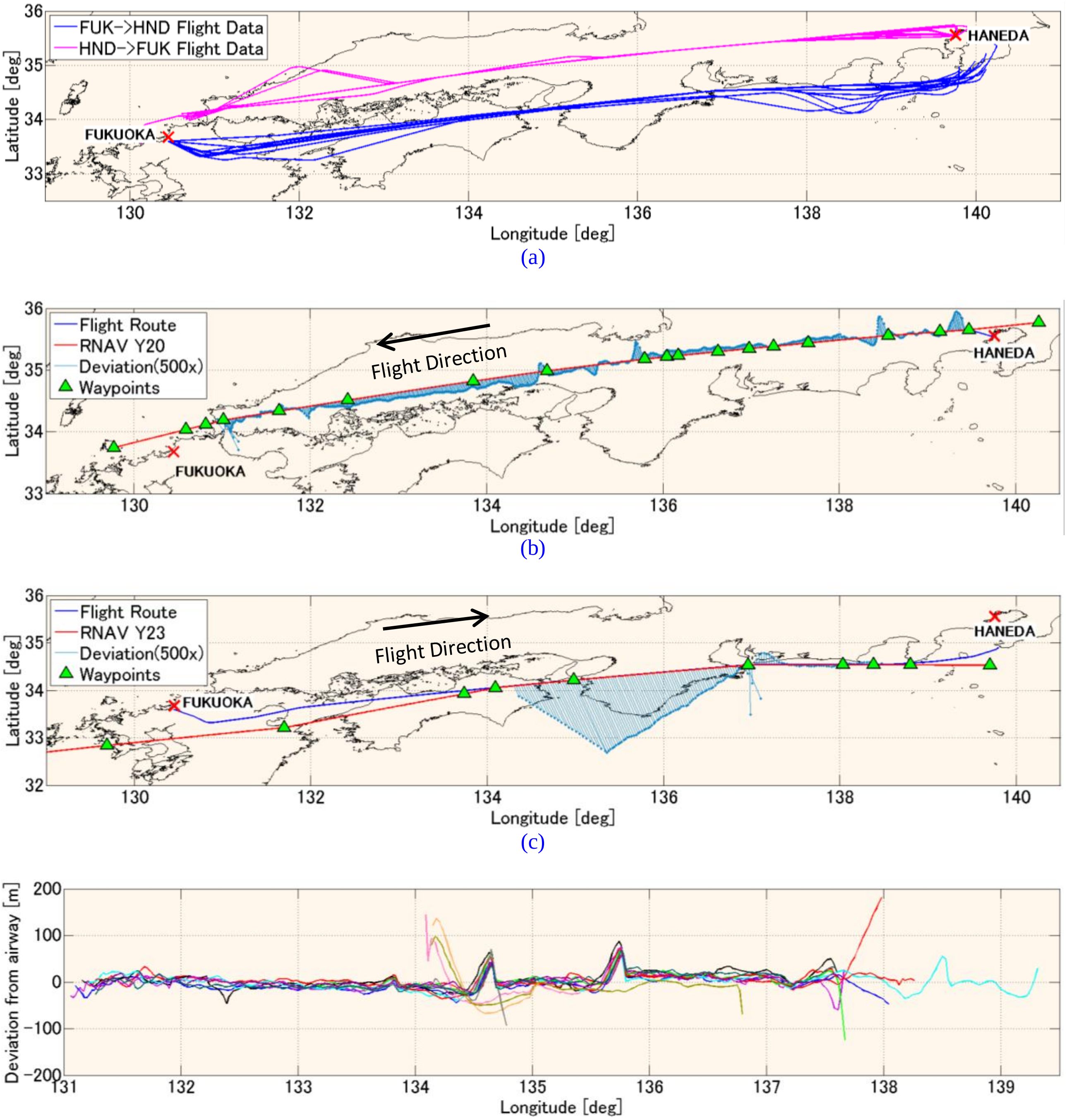
To understand the structure of modern ATC system and to discuss the benefits of proposed optimal trajectory model we need to pursue a standard for data discussion [16]. We use a commercial GPS data logger (Globalsat DG-100 GPS Logger) inside the cabin of an airborne aircraft (after departure warnings are turned off and before landing warnings are turned on) to record the aircraft's position (latitude, longitude, and geometric height) and the ground speed.

A series of flight data were recorded over a domestic flight route between Haneda and Fukuoka as shown in Fig. 5(a). Respective wind data were obtained from the meteorological data to compute the pressure altitude and true airspeed. The data compared are from flights with the same type of aircraft. Information provided by [17] was used to plot the airway data RNAV Y20 (Haneda→Fukuoka on Fig. 5(b)) and RNAV Y23 (Fukuoka→Haneda on Fig. 5(c)). The lateral deviation of flight routes from its predetermined airway were calculated as shown in Fig. 5(d). The maximum deviation was recorded only at a magnitude of several dozen meters [16]. This proves that modern aircraft are,

- capable of cruising with high precision with provided onboard flight data on their Flight Management Systems (FMS), not only for lateral deviation from the airway but also pressure altitude from the assigned flight level.
- usually bound to follow vector control procedures before entering the terminal airspace of busy airports in order to adjust the arrival sequence.

6 Results

In this section, 4D-optimal flight trajectories with the proposed model are evaluated. They are compared with the analyzed flight data and the gained benefits are discussed. "Flight data" here are not recorded by onboard equipment, but they are analyzed from GPS data and meteorological



(a)

(b)

(c)

(d)

Fig. 5. (a) Recorded flight data over Haneda ⇔ Fukuoka flight route (b) Flight route based on airway RNAV Y20 (c) Flight route based on airway RNAV Y23 (d) Lateral Deviation from airway for recorded flight data

data as explained in chapter 5. The initial and final altitude and calibrated airspeed V_{CAS}, are fixed according to the respective flight data. In the proposed model, the grid space for state variables is given as following.

Altitude: $3000\,[\mathrm{m}] \sim 13000\,[\mathrm{m}]$

Calibrated airspeed: $100\,[\mathrm{ms}^{-1}] \sim 160\,[\mathrm{ms}^{-1}]$

Cross range angle: $0.5\,[\mathrm{deg}]$ on each side from Great Circle Route $(56\,[\mathrm{km}])$

To decide a meaningful grid space and also to prevent the model by surpassing limits of the reviewed aircraft's service ceiling we use the following operational limitations.

Maximum operating velocity (CAS): $310\,[\mathrm{kt}]$

Maximum operating Mach number: 0.84

6.1 Fukuoka→Haneda Flight Route

Table 4. Fukuoka →Haneda flight route

Date	Initial Altitude [m]	Final Altitude [m]	Initial Velocity V_{CAS} [ms^{-1}]	Final Velocity V_{CAS} [ms^{-1}]	Initial time for NWP data (JST)
2011/07/23	3611.8	3750.2	158	156	15:00
2011/09/02	3010	3016.3	141	122	09:00
2011/10/08	3338.2	3030.6	151	132	15:00
2011/10/15	4129.5	3012.1	126	112	15:00

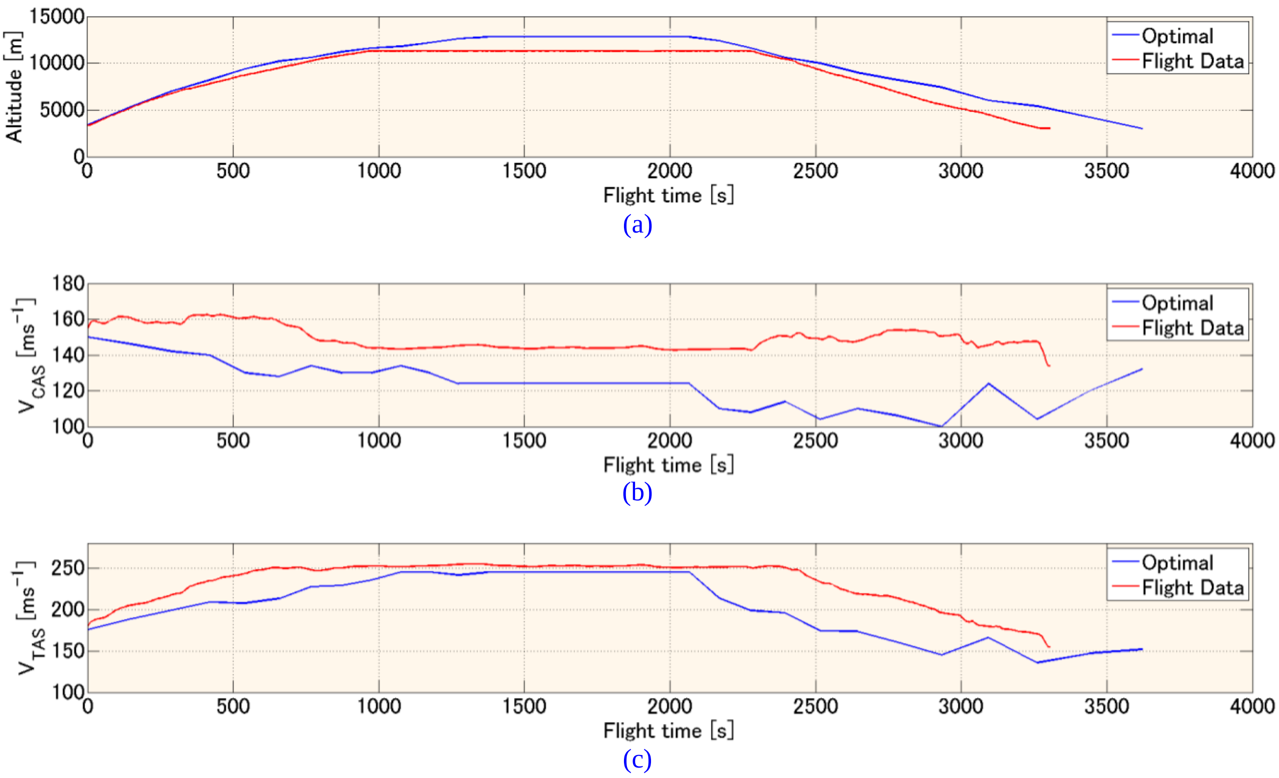


Fig. 6. 4D-optimal flight trajectory compared with flight route on 08 October 2011
(a) Altitude comparison (b) Calibrated airspeed comparison (c) True airspeed comparison

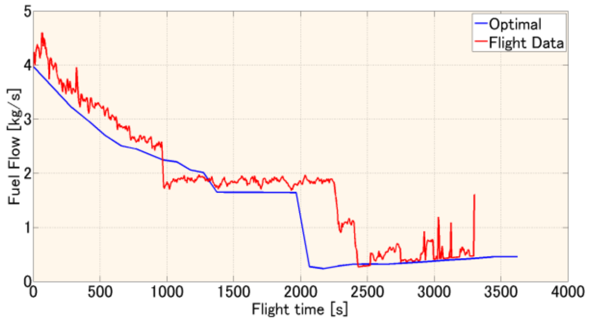


Fig. 7. Fuel flow comparison (2011/10/08)

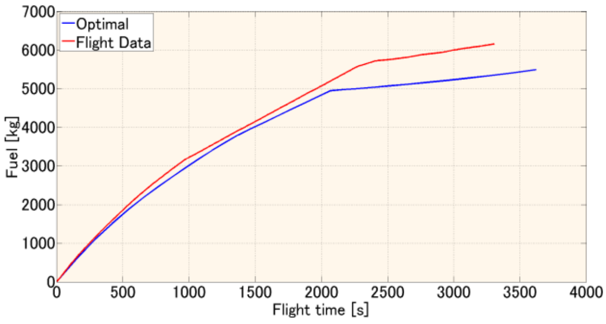


Fig. 8. Fuel consumption comparison (2011/10/08)

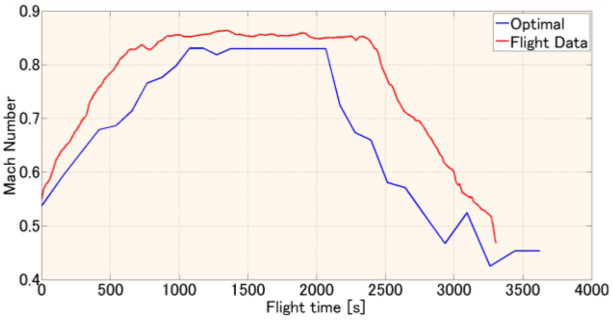


Fig. 9. Mach number comparison (2011/10/08)

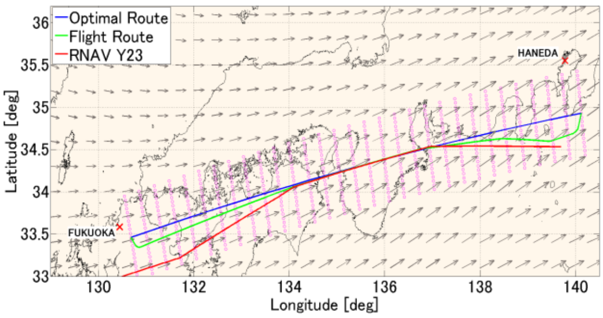


Fig. 10. Flight route comparison (2011/10/08)

Table 4. shows the data evaluated for Fukuoka→Haneda flight route. The comparison of flight time and fuel consumption is shown in table 5.

Table 5. Comparison of flight time and fuel consumption

Flight data		Trajectory Model		Fuel efficiency
Flight time [s]	Consumed fuel [kg]	Flight time [s]	Consumed fuel [kg]	
3490	6633.00	3710	5957.76	10.18%
3930	7011.43	4067	6290.62	10.28%
3305	6164.03	3621	5493.14	10.88%
4220	6893.51	3760	5613.89	18.56%

From these results we could understand that optimal trajectory obtains a lower airspeed while maintaining a higher altitude to gain a large specific range. On the contrary the original flight route is at a lower altitude with a higher airspeed resulting a larger Mach number. This has resulted a larger value of fuel flow compared to the fuel flow of trajectory model. Although fuel flow calculation needs aircraft mass information, which is difficult to obtain, reference mass is used in both cases of the GPS flight data analysis and the optimal trajectory. The rate of descent for optimal trajectory is smaller compared to the steeper descent of original flight route. Hence the flight time for optimal trajectory is larger than the compared flight time of original data. As the aircraft experiences tailwinds in most cases, the optimal flight route is same as the GCR. The fuel efficiency is calculated as the ratio of fuel consumption difference to the fuel consumption of original flight route.

The average fuel efficiency is around 10% except for the flight case on October 15.

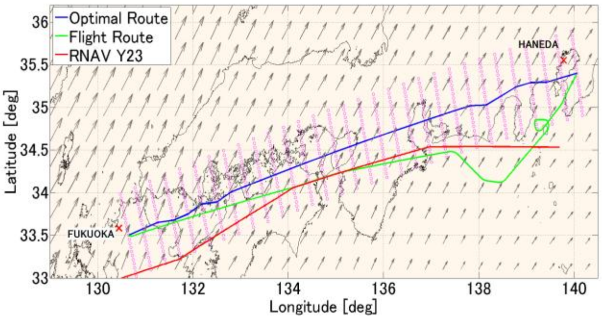


Fig. 11. Flight route comparison (2011/10/15)

The flight case on 2011/10/15 is depicted on Fig. 11. The original flight route has probably followed vector control procedures as it circles around the terminal airspace of Haneda airport to negotiate with the approach sequence. This has resulted more in flight time and fuel consumption compared to the trajectory model which has led to a fuel efficiency of 18.56%.

6.2 Haneda→Fukuoka Flight Route

In this sub chapter, the study discusses on 3 cases on trajectory comparison and gained benefits over the Haneda→Fukuoka flight route.

In this flight optimal flight trajectory is greatly affected with strong headwinds. The original flight route follows the airway RNAV Y20 throughout most of its flight time. It has a change in cruising altitude and has increased the airspeed to keep the Mach number close to the operational limit. As before the original flight route implements a steep descent and has used a considerable amount of fuel compared to its optimized route. On the contrary, the optimal trajectory deviates laterally to avoid strong headwinds while flying at a higher altitude to achieve a large specific range. Then it continues its descent with a small rate and as a result, it consumes more flight time compared to the original flight route. Table 6. shows the data evaluated and Table 7. shows the results on comparison over the Haneda→Fukuoka flight route. The results for case on October 10 are figured and the rest are quantitatively evaluated.

7 Conclusions

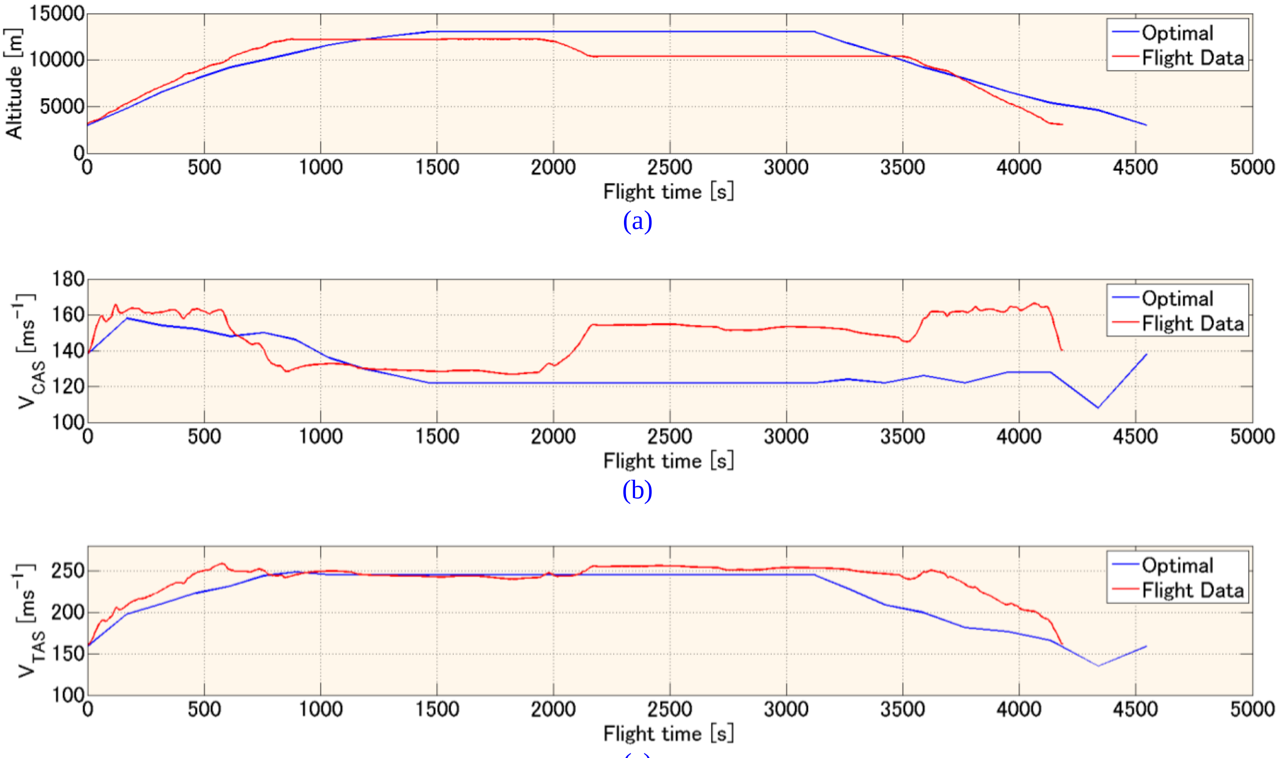
We could conclude this study with an overview of this research and with a discussion on gained results and remained numerous challenges to be overcome.

Optimization of a jet passenger aircraft for climb, cruise and descent phases has been formulated as an optimal control problem by considering its inherent performance characteristics assuming no ATC constraints are engaged. The aircraft's 3D-position and airspeed were considered as state variables to implement the optimization in a provided grid space with heading angle, flight path angle and engine thrust as control variables. One advantage is that this study could be easily extended for numerous constraints such as fixed waypoints and arrival times.

DP method could be considered as an ideal optimization tool for its capability to handle constraints of state and control variables as well as the flexibility to adjust according to the variation of model. The 'Curse of Dimensionality' with the increase of state variables is a drawback but DP is a potential tool for 4D-optimal flight trajectory applications.

Table 6. Haneda →Fukuoka flight route

Date	Initial Altitude [m]	Final Altitude [m]	Initial Velocity V_{CAS} [ms^{-1}]	Final Velocity V_{CAS} [ms^{-1}]	Initial time for NWP data (JST)
2011/07/31	4884.8	3750.2	152	129	21:00
2011/08/28	3352.0	3362.3	140	157	21:00
2011/10/10	3098.8	3062.9	138	138	15:00



(a)

(b)

(c)

Fig. 12. 4D-optimal flight trajectory compared with flight route on 10 October 2011
(a) Altitude comparison (b) Calibrated airspeed comparison (c) True airspeed comparison

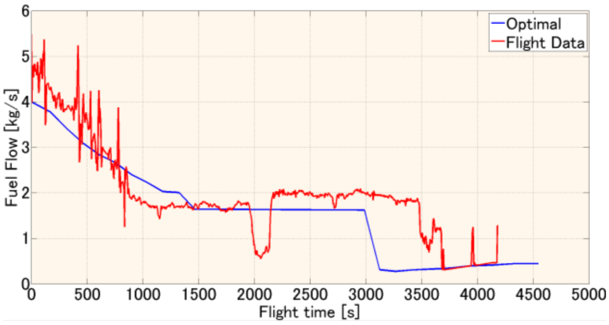


Fig. 13. Fuel flow comparison (2011/10/10)

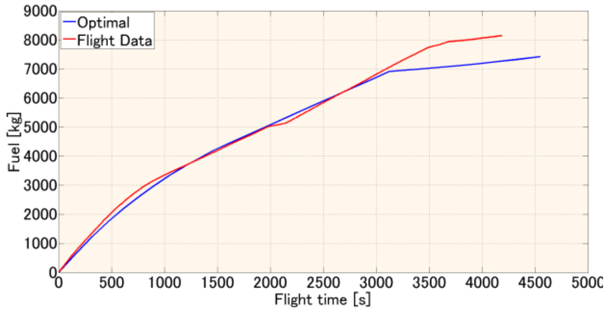


Fig. 14. Fuel consumption comparison (2011/10/10)

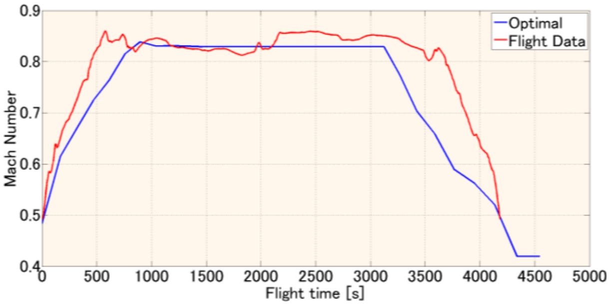


Fig. 15. Mach number comparison (2011/10/10)

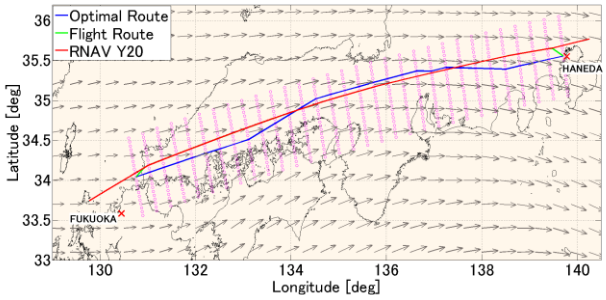


Fig. 16. Flight route comparison (2011/10/10)

Table 7. Results for Haneda→Fukuoka Flight route

Flight data		Trajectory Model		Fuel efficiency
Flight time [s]	Consumed fuel [kg]	Flight time [s]	Consumed fuel [kg]	
3550	6168.49	3808	5814.33	5.74%
3700	6995.25	3989	6494.96	7.15%
4185	8157	4546	7424.4	8.98%

The aircraft performance model, BADA model provides valuable data on numerous aircraft and proves to be a very useful performance model. The model's constraints and data definition according to different flight conditions helped us to achieve logical results to compare with the recorded GPS flight data. The accuracy of wind data provided by the NWP models remains as a future study which is vital in the case of generating onboard optimal flight trajectories in a futuristic ATS environment. Extension of our approach to achieve a complete 4D-optimal flight trajectory model by including initial climb and final approach would be possible and there accuracy of wind data is necessary [18].

Using a simple GPS data logger to record a very few number of flight data parameters in an airliner cabin was sufficient to implement a substantial research and understand the structure of modern ATC system. A possibility of obtaining more precise flight data would greatly increase the accuracy of our proposed model as well as obtaining more realistic benefits by data evaluation.

The proposed model and recorded GPS flight data have been quantitatively compared to understand the gained benefits. The fuel consumption optimized aircraft tends to fly at high altitudes with low airspeeds to maintain a larger specific range and close to the service ceiling. This leads to consume more flight time compared to modern flight procedures which tends to fly at lower altitudes with considerably high airspeeds by consuming more fuel. It could be explicitly stated that a tradeoff between fuel consumption and flight time is important in an overall optimization [19].

The constrained workspace of this study has to be emphasized, namely providing free airspace for one passenger aircraft to perform a 4D-optimal trajectory and not considering any ATC restrictions and constraints. However the main objective in this research is to generate a fundamental, yet reliable optimal flight trajectory model to lay the foundation for future studies in the field of ATM. Considering multiple aircraft makes the optimal control problem more complex and confusing. Hence Conflict Detection and Resolution (CD&R) methods also have to be considered to achieve an overall optimization of the system. Yet, using modern computational hardware with high performance capabilities and above discussed *easy to apply* methods would eventually provide sufficient background to expand this form of research to the foreseeable future.

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Copyright Statement

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