STABILIZATION OF LONGITUDINAL AIRCRAFT MOTION USING MODEL PREDICTIVE CONTROL AND EXACT LINEARIZATION

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Abstract

This work deals with constrained control of nonlinear model of an aircraft. A methodology applying for this control is based on an exact linearization (EL) in connection with linear model predictive control (MPC) techniques.

1 Introduction

The purpose of this control methodology is the flight recovery problem. Primary task of an automatic recovery system is to regain control over an aircraft in case of unintended maneuvers or uncontrolled situations (e.g. pilot loses orientation). In such situation nonlinear movement is assumed and therefore classical control methods cannot be used. Those require model with small deviations and control around the equilibrium. Whole procedure has several limiting factors. Above all there are stall angle of attack and sideslip angle (separation of the flow – aerodynamic limitation), pitch, roll and yaw rate (mechanical limitations of the plane), aerodynamic loads (physiological pilot limitations) and a control limit (amplitude and rate of control surfaces deflection and thrust of engine). Based on these facts combination of input constrained MPC with EL were chosen. An important advantage is that EL model can be controlled with a linear MPC with fixed model. On the other hand it is not easy to define constraints of linearized model.

As a controlled platform for a plane behavior a model of flying laboratory Vilík (Fig. 2) was created. In this case it was modeled as a nonlinear form of longitudinal behavior with two inputs.

2 Problem formulation

Pure mathematical equation of motion for aircraft model is in non-linear form:

$$\dot{x} = f(x,u) \tag{1}$$

where is state vector, is input vector and f is differentiable field on . Now we can define projection of our non-linear space to new linear space and change inputs:

$$x = \Phi(\xi), \quad u = \alpha(\xi, v) \tag{2}$$

where ξ is state of new linear model and v is new virtual input. Most of nonlinear dynamic models (aircraft model too) can be described as control affine system:

$$\dot{x} = f(x) + g(x)u \tag{3}$$

By the help of Lie derivative it is possible to create static feedback that transforms nonlinear to linear model.

$$\dot{x} = f(x) + g(x)u \quad \Rightarrow \quad \begin{aligned} \dot{\xi} &= \mathbf{A}\xi + \mathbf{B}v \\ v &= \mathbf{D}(x)u + \boldsymbol{\alpha}(x) \end{aligned} \tag{4}$$

Matrices **A** and **B** are depended on transformation form. They are typically composed from series of integrators. Matrices **D** and **α** are determined by Lie derivatives and will be described below.

MPC is a type of control which uses optimal state-feedback and predictive strategy for optimal design sequence of control action with reference to future states and outputs of the system. MPC is based on a linear, discrete time state-space model of the model. Quadratic programming (QP) was typically used for solution of MPC. A main advantage of QP is a simple implementation of inputs/outputs constraints. Quadratic objective function for linearized system is:

$$J(v) = q\sum_{k=1}^{Tp}(\xi_k - w_k)^2 + r\sum_{k=1}^{Tp} v_k{}^2 \tag{5}$$

where w is reference vector and q and r are state respectively input weights.

3 Mathematical model of an aircraft

A formulation of aircraft flight dynamics is derived from Newton's second law of motion. This can be investigated from the viewpoint of stability or flying qualities and can be used for modeling of aircraft motion [1, 2]. There are following simplifying assumptions: the aircraft is rigid body with fixed mass, gravitational acceleration is constant and the aircraft is without structural deformation. Following equation and variables are relative to the body axes system (Fig, 1.).

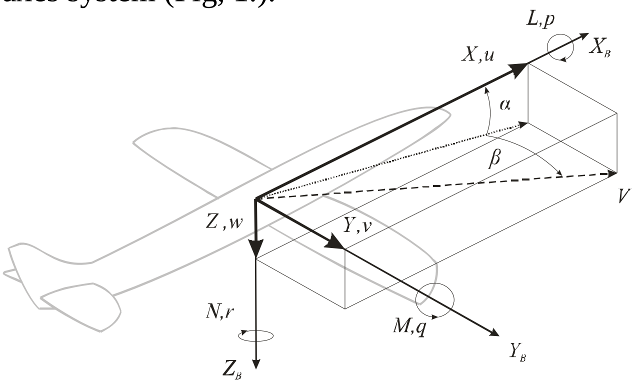

Fig. 1 Body axis components; definition of moments *L, M, N*; forces *X, Y, Z;* velocities *u, v, w*; angular rates *p, q, r*; angle of attack α and sideslip angle β

3.1 Equation of motion

The general forces and moments equations can be described by Newton's second law of motion in translational and rotational form [3]. We consider rotational axis system, where the derivate operator applied to vectors has two parts: one for the rate of change of the vector, and one for axis system rotation.

$$\mathbf{F} = m\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{\Omega}\times\mathbf{v}\right) = \mathbf{F_w} + \mathbf{F_a} + \mathbf{F_t} \tag{6}$$

$$\mathbf{M} = \frac{\partial(\mathbf{I}\cdot\mathbf{\Omega})}{\partial t} + \mathbf{\Omega}\times\mathbf{I}\cdot\mathbf{\Omega} = \mathbf{M_a} \tag{7}$$

Where $\mathbf{F}$ represents the sum of all externally applied forces, m is the mass of aircraft, $\mathbf{v}$ is the velocity vector, $\mathbf{\Omega}$ is the total angular velocity of the aircraft, $\mathbf{M}$ represents the sum of all applied torques and $\mathbf{I}$ is moment of inertia matrix. Thereby we get six component equations of motion, where on the left side there are time derivatives of angular rates and velocities and on the right side there are description of motion and external influences. That is where the external forces depend on the weight vector $\mathbf{F_w}$, the aerodynamic force vector $\mathbf{F_a}$ and the thrust vector $\mathbf{F_t}$ respectively. We assume the thrust produced by the engine acts only parallel to the aircraft's longitudinal axis. External moments are those due to aerodynamics.

3.2 Aerodynamic model

The aircraft aerodynamic model mainly depends on the following factors: the airspeed V (Mach number or Reynolds number respectively) and density of the airflow ρ; the geometry of the aircraft (wing area S, wing span b and mean aerodynamic chord c_{MAC}); the orientation of the aircraft relative to the airflow [4] (angle of attack α and side slip angle β); the control surface deflections δ and the angular rates p, q, r. There are other variables such as the time derivatives of the aerodynamic angles that also play a role, but these effects are less prominent. The volume of the aerodynamic forces and moments is determined by the amount of air variables in different directions. In the standard way of modeling the aerodynamic forces and moments are given by the following equations:

$$\mathbf{F_a} = \bar{q}\cdot S\cdot c_i(\alpha,\beta,\delta,p,q,r,\ldots) \quad i = L,D,Y \tag{8}$$

$$\begin{aligned}\mathbf{M_a} &= \bar{q}\cdot S\cdot c_i(\alpha,\beta,\delta,p,q,r,\ldots)\cdot x_i \\ i &= l,m,n \qquad x_i = b, c_{MAC}, b\end{aligned} \tag{9}$$

where $\bar{q} = 1/2\,\rho V^2$ is dynamic pressure. Each dimensionless aerodynamic coefficient from (8) and (9) can be described as a function of other flight parameters:

$$c_L = c_L(\alpha,\beta,\delta,p,q,r,M,\mathrm{Re},\ldots) \tag{10}$$

This description is too detailed for mathematical modeling and too difficult for measurement/computation. Therefore it is described in a simplified way where the effect of each flight parameter is separated. For example the aerodynamic lift coefficient is then given by:

$$c_L = c_{L0} + c_{L\alpha} \cdot \alpha + c_{L\bar{q}} \frac{q \cdot c_{MAC}}{2 \cdot V} + c_{L\delta_e} \cdot \delta_e \qquad (11)$$

The aerodynamic coefficient equals summation of initial value (value on lift line at zero angle of attack), product of lift line derivatives and angle of attack, product of non-dimensional pitch rate derivatives and non-dimensional pitch rate and product of control derivatives and control deviation (elevator). The aerodynamic coefficients (more precisely aerodynamic derivatives) are usually obtained from wind tunnel or by computational methods [5]. In this case data from flight tests [6, 7] were used (Fig.2), where the aerodynamic derivatives were estimated by maximum likelihood method [8].

Fig. 2 model Vilík during flight test

3.3 Longitudinal part of motion

The aircraft equations of motion in general form are a set of coupled nonlinear differential equations. However, in our case these equations can be simplified. Assuming zero lateral states (side slip angle, roll and yaw rates, roll angle), a separated longitudinal motion in nonlinear form can be obtained:

$$\begin{aligned}
\dot{x}_1 &= c_1 x_1^2 + c_2 x_1^2 x_2 + c_3 x_1^2 u_1 + c_4 u_2 \cos(x_2) \\
&\quad + c_5 \sin(x_2 - x_4) \\
\dot{x}_2 &= c_6 x_1 + c_7 x_1 x_2 + c_8 x_3 + c_9 x_1 u_1 + x_3 \\
&\quad - c_4 x_1^{-1} u_2 \sin(x_2) + c_5 x_1^{-1} \cos(x_2 - x_4) \\
\dot{x}_3 &= c_{10} x_1^2 + c_{11} x_1^2 x_2 + c_{12} x_1 x_3 + c_{13} u_1 x_1^2 \\
\dot{x}_4 &= x_3
\end{aligned} \qquad (12)$$

The longitudinal motion of aircraft is a dynamic system with four states/outputs $y = x = [V, \alpha, q, \theta]$ (consecutive airspeed, angle of attack, pitch rate and pitch angle) and two inputs $u = [\delta_e, F_t]$ (elevator deflection and thrust of engine). All parts which are independent on state variables are substituted by constants (see Appendix).

3.4 Steady flight

Computation of equilibrium is a classical way of how to determine initial values of states and inputs of nonlinear model. In this case we have four equations and six unknowns (four states and two inputs). However, we can use additional equations for vertical speed (i.e. change of altitude) that we assume zero:

$$\dot{h} = x_1 \sin(x_4 - x_2) = 0 \quad \rightarrow \quad x_4 = x_2 \qquad (13)$$

Further more typically an attitude of airspeed and pitch rate (x_3) is assumed zero in equilibrium. Thereby we get three equations with three unknowns:

$$\begin{aligned}
0 &= c_1 \tilde{x}_1^2 + c_2 \tilde{x}_1^2 x_2 + c_3 \tilde{x}_1^2 u_1 + c_4 u_2 \cos(x_2) \\
0 &= c_6 \tilde{x}_1 + c_7 \tilde{x}_1 x_2 + c_9 \tilde{x}_1 u_1 - c_4 \tilde{x}_1^{-1} u_2 \sin(x_2) + c_5 x_1^{-1} \\
0 &= c_{10} \tilde{x}_1^2 + c_{11} \tilde{x}_1^2 x_2 + c_{13} u_1 \tilde{x}_1^2
\end{aligned} \qquad (14)$$

where $\tilde{x}_1$ is chosen airspeed. Analytically solution is due to goniometric functions relatively complicated. Therefore Nelder-Mead simplex method is used for solving of this static optimization problem.

4 Exact linearization

An assumption for this transformation is an accomplishment of an invertible matrix D [9]. In our case it is necessary to use dynamic feedback for accomplishment of this assumption. It means, that the elevator deflection (input) is added in integrator thereby elevator rate is the new input and elevator deflection is changed into state. We define the elevator input as a new state:

$$u_1 = x_5 \tag{15}$$

Now we determine nonlinear transformation $\Phi: X \to \xi$. The system in new coordinates can be expressed as:

$$\dot{\xi}_1 = \xi_2 \tag{16a}$$
$$\dot{\xi}_2 = v_1 \tag{16b}$$
$$\dot{\xi}_3 = \xi_4 \tag{16c}$$
$$\dot{\xi}_4 = \xi_5 \tag{16d}$$
$$\dot{\xi}_5 = v_2 \tag{16e}$$

where the first new state is chosen as:

$$\begin{aligned} \xi_1 &= x_1 \sin(x_2) \\ \dot{\xi}_1 &= \dot{x}_1 \sin(x_2) + x_1 \cos(x_2)\dot{x}_2 \end{aligned} \tag{17}$$

by substituting derivatives of state from (12) to (17) and proper arrangement we get:

$$\begin{aligned} &\xi_2 = \dot{\xi}_1 = x_1^2 \sin(x_2)(c_1 + c_2 x_2 + c_3 x_5) + \ldots \\ &x_1^2 \cos(x_2)(c_6 + c_7 x_2 + c_9 x_5) + c_5 \cos(x_4) + \ldots \\ &x_1 x_3 \cos(x_2)(1 + c_8) \end{aligned} \tag{18}$$

The result of the last term is too complicated to be solved by analytical method. Therefore Matlab symbolic toolbox for solution (16b) was used. Second part of decoupled system is determined by last three equations (16c), (16d), (16e):

$$\begin{aligned} \xi_3 &= x_4 \\ \xi_4 &= x_3 \\ \xi_5 &= c_{10} x_1^2 + c_{11} x_1^2 x_2 + c_{12} x_1 x_3 + c_{13} u_1 x_1^2 \end{aligned} \tag{19}$$

Last three state derivatives are also solved by symbolic toolbox. From (16) it is easy to form state and input matrices (4):

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \tag{20}$$

Last part of completion of the exact linearization is to provide second equation from (4). Using affine system assumption we can easily determine matrices **D** and $\boldsymbol{\alpha}$ as a part which is dependent on input u (matrix **D**) and the rest (matrix $\boldsymbol{\alpha}$). The feedback linearizing control law is given by:

$$u = \mathbf{D}(x)^{-1}(v - \boldsymbol{\alpha}(x)) \tag{21}$$

5 Model Predictive control

Conventionally MPC is formulated in discrete time[10]. Assuming controlled aircraft model (4) the description by the linear discrete time difference equation is:

$$\xi_{k+1} = \mathbf{A}\xi_k + \mathbf{B}v_k \tag{22}$$

Following states on prediction horizon T_p can be described as follows:

$$\begin{aligned} &\xi_{k+2} = \mathbf{A}\xi_{k+1} + \mathbf{B}v_{k+1} = \mathbf{A}^2\xi_k + \mathbf{AB}v_k + \mathbf{B}v_{k+1} \\ &\xi_{k+3} = \mathbf{A}\xi_{k+2} + \mathbf{B}v_{k+2} = \mathbf{A}^3\xi_k + \mathbf{A}^2\mathbf{B}v_k + \mathbf{AB}v_{k+1} + \mathbf{B}v_{k+2} \\ &\vdots \\ &\xi_{k+Tp-1} = \mathbf{A}^{Tp}\xi_k + \mathbf{A}^{Tp-1}\mathbf{B}v_k + \mathbf{A}^{Tp-2}\mathbf{B}v_{k+1} + \ldots + \mathbf{B}v_{k+Tp} \end{aligned} \tag{23}$$

Last equation can be rewritten to matrix form:

$$\bar{\xi} = \bar{\mathbf{A}}\xi_k + \bar{\mathbf{B}}\bar{v} \tag{24}$$

where each variable with bar is described as follows:

$$\begin{aligned} &\bar{\xi} = \begin{bmatrix} \xi_{k+1} \\ \xi_{k+2} \\ \vdots \\ \xi_{k+Tp-1} \end{bmatrix} \quad \bar{\mathbf{A}} = \begin{bmatrix} \mathbf{A} \\ \mathbf{A}^2 \\ \vdots \\ \mathbf{A}^{Tp} \end{bmatrix} \\ &\bar{\mathbf{B}} = \begin{bmatrix} \mathbf{B} & & & \\ \mathbf{AB} & \mathbf{B} & & \\ \vdots & & \ddots & \\ \mathbf{A}^{Tp-1}\mathbf{B} & \mathbf{A}^{Tp-2}\mathbf{B} & \cdots & \mathbf{B} \end{bmatrix} \quad \bar{v} = \begin{bmatrix} v_k \\ v_{k+1} \\ \vdots \\ v_{k+Tp} \end{bmatrix} \end{aligned} \tag{25}$$

Now we can define final form of objective function for MPC with exact linearized system:

$$\underset{\min \bar{v}}{J}(\bar{v}) = (\bar{\mathbf{A}}\xi_k + \bar{\mathbf{B}}\bar{v} - w_k)^T \mathbf{Q}(\bar{\mathbf{A}}\xi_k + \bar{\mathbf{B}}\bar{v} - w_k) + \bar{v}^T\mathbf{R}\bar{v} \tag{26}$$

Subject to:

$$u_{\min} \le u \le u_{\max} \tag{27a}$$
$$\mathbf{D}(x)u_{\min} + \boldsymbol{\alpha}(x) \le \bar{v} \le \mathbf{D}(x)u_{\max} + \boldsymbol{\alpha}(x) \tag{27b}$$

where **Q** and **R** are weight matrices of states and inputs respectively. The box constraints (27a) of original model were transformed into state dependent constraints (27b). Figure 3 illustrates the feasible area of u_1 and u_2. It has four corners *a,b,c,d*. The coordinates of corners are (u_{1min}, u_{2max}), (u_{1max}, u_{2max}), (u_{1max}, u_{2min}), (u_{1min}, u_{2min}), respectively.

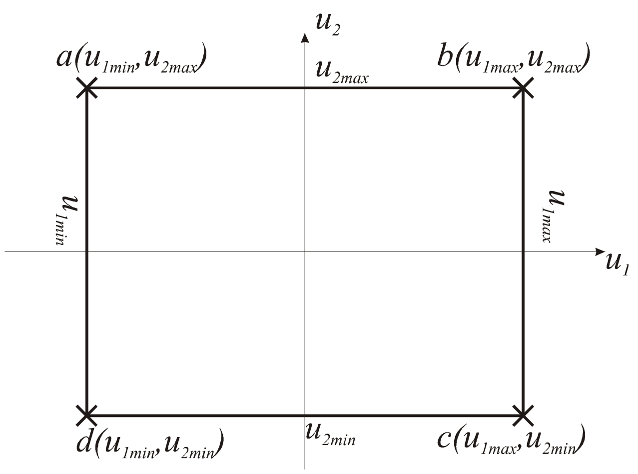

Fig. 3 Feasible area of the control inputs original coordinates

Corners of original model were step by step transformed in to new coordinates (27b). Figure 4 illustrates the feasible area of virtual inputs v_1 and v_2 after each point in the feasible area of u_1 and u_2 is multiplied by matrix $\mathbf{D}$ and translated by $\boldsymbol{\alpha}$.

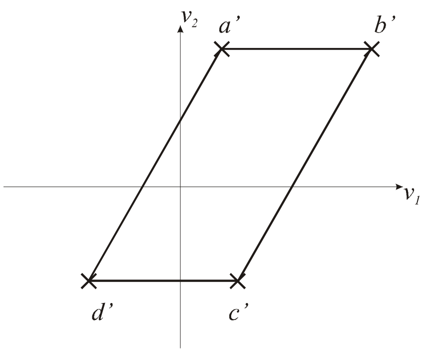

Fig. 4 Feasible area of the virtual control inputs

The new coordinates have the following relationship with the old ones:

$$\begin{aligned} a' &= \mathbf{D}(x)a + \boldsymbol{\alpha}(x) \\ b' &= \mathbf{D}(x)b + \boldsymbol{\alpha}(x) \\ c' &= \mathbf{D}(x)c + \boldsymbol{\alpha}(x) \\ d' &= \mathbf{D}(x)d + \boldsymbol{\alpha}(x) \end{aligned} \tag{28}$$

The feasible area in Figure 4 of the transformed inputs is not suitable for use with the predictive control algorithm. However, the box constraints of original input can be rewritten by following procedure. Let's have two points from the previous problem.

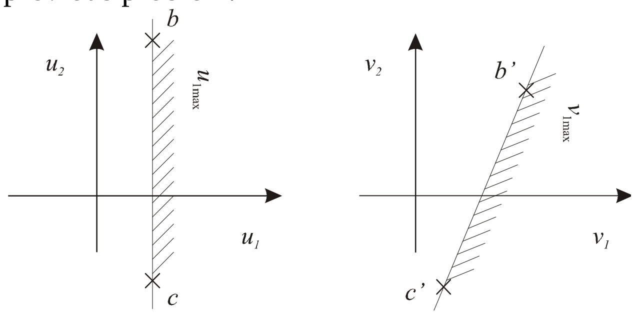

Fig. 5 constrained transformation

Constrained point *c*, *b* are transformed into *c'* and *b'*.

$$\begin{aligned} c' &= \mathbf{D}(x)c + \boldsymbol{\alpha}(x) \\ b' &= \mathbf{D}(x)b + \boldsymbol{\alpha}(x) \end{aligned} \tag{29}$$

Line between new points can be described:

$$\begin{aligned} v_2(c') &= k + l v_1(c') \\ v_2(b') &= k + l v_1(b') \end{aligned} \tag{30}$$

where line parameters *k* and *l* are solution of two equations with two unknowns:

$$\begin{aligned} l &= \frac{v_2(c') - v_2(b')}{v_1(c') - v_1(b')} \\ k &= \frac{v_1(b')v_2(c') - v_2(b')v_1(b')}{v_1(c') - v_1(b')} \end{aligned} \tag{31}$$

The constrained line is now defined as:

$$v_2 = k + l v_1 \tag{32}$$

Final relationship between original and virtual constraints is formulated as follows:

$$\begin{aligned} u_1 &\le u_{1\max} \\ v_2 &\le k + l v_1 \end{aligned} \tag{33}$$

Each point pairs (*c'*, *b'*), (*b'*, *a'*), (*a'*, *d'*), (*d'*, *c'*) are consequently used for computation of virtual input constraints (33) which create complete trapeze constraints for virtual inputs. The computational process is shown on figure 6. This procedure creates enclosed area for virtual constraints.

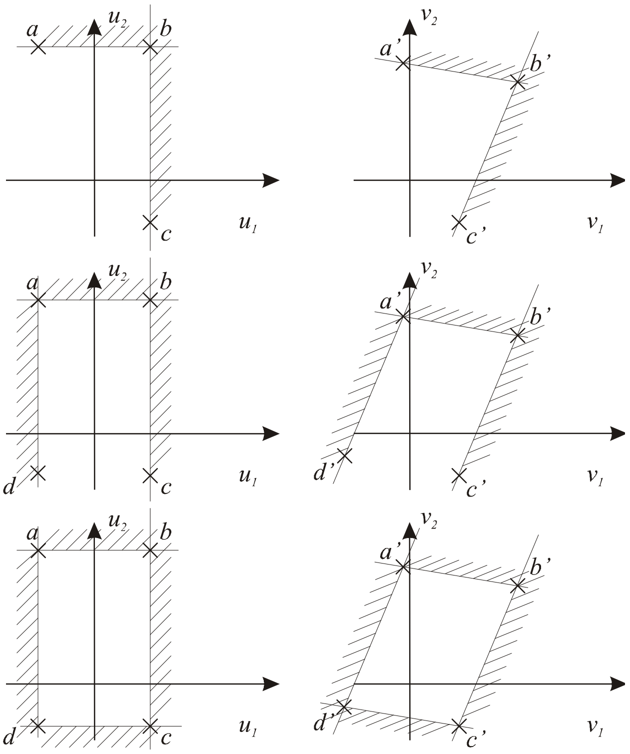

Fig. 6 constraints computational process

Algorithm

Initialization

1 Model preparation (35)
2 Steady state flight condition (14)

Simulation

3 Solve open loop –aircraft equation (3) with EL(21) and virtual inputs v

MPC

4 Open loop simulation on prediction horizon Tp (12)(21)
5 Constraints computation (33)
v = MPC solution (26)(27b)
while constrained condition
6 Open loop computation on prediction horizon Tp (12)(21)
7 Constraints computation (33)
8 v = MPC solution (26)(27b)
end while

end MPC

return to Simulation 3 until $t_{sim} < T_{final}$

end Simulation

5 Case study

In this section, we present algorithm and simulation results using MATLAB and SIMULINK.

We suppose iterative process that is described below. At initialization step we start from steady flight condition (14) that satisfies input constraints and constant (35) for nonlinear simulation (12) is computed.

MPC process starts with nonlinear state computation of the prediction horizon Tp. This is open loop solution of aircraft equation (12) and feedback linearizing control (21) with actual state as initial condition and virtual inputs v on the horizon prediction. Obtained output is used to construct the constraints of the virtual input v at each step of the prediction horizon.

Next we use the calculated input constrained sequence to generate new virtual inputs (26) subject to (27b). Constrained conditions are controlled and if they are not satisfactory the MPC process is repeated. After decision of satisfied condition is next simulation step computed.

Following algorithm summarizes the steps of the proposed methodology.

The performance of this scheme has been verified via simulation. The aircraft is controlled from steady regime and consequently its flight is disturbed by a wind gust (addition to angle of attack). Character of wind disturbance is shown in Fig. 7 (dash line).

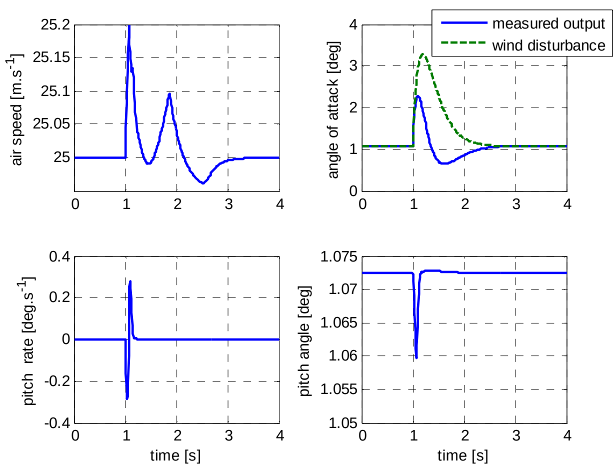

Fig. 7 States of aircraft longitudinal motion

Wind gust amplitude is two degree and we can see from Fig. 7 that real angle of attack error is smaller than one degree. The error means difference between actual state and reference which is obtained from steady flight solution.

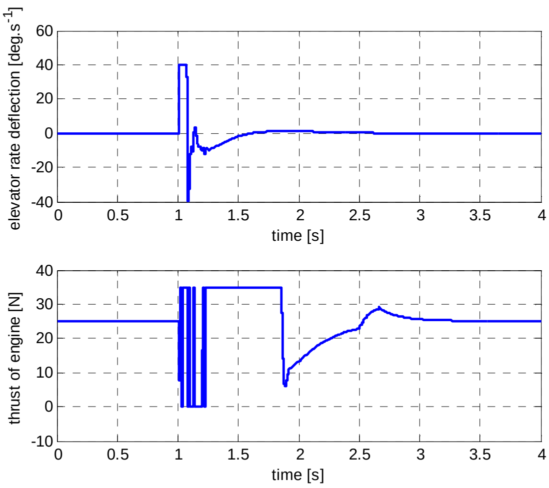

Fig. 8 inputs during wind disturbances

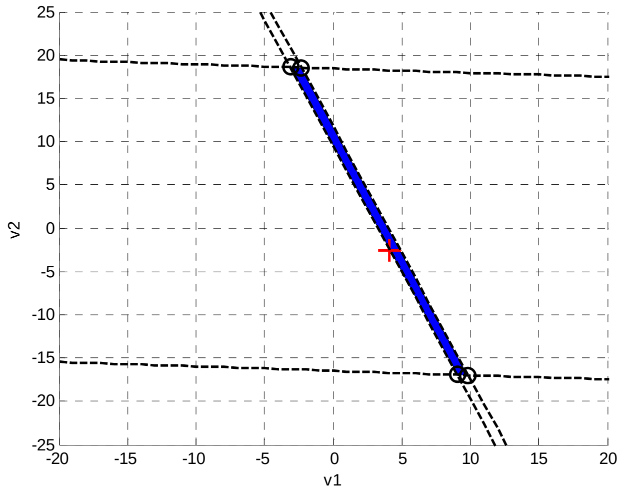

Fig. 9 constrained virtual inputs

Previous figure shows constrained inputs. Due to dynamic exact linearization is new input defined (15). Therefore elevator rate deflection as original input used instead of elevator deflection. Both inputs satisfy amplitude limits (elevator rate deflection -40÷40 [°/s], thrust of engine 0÷35 [N]). Virtual inputs used for feedback linearizing control are shown in Fig. 9. There are four lines which correspond to constrained transformation (33) and define feasible control area (filled surface) . The cross represents value of MPC solution. Figure 10 shows the same problem with the difference that the feasible areas of linearized model on the prediction horizon are shown. On the feasible area of each prediction step the virtual input value is marked by a circle. All values are inside the feasible region thereby is graphically demonstrated fulfillment of constrained condition.

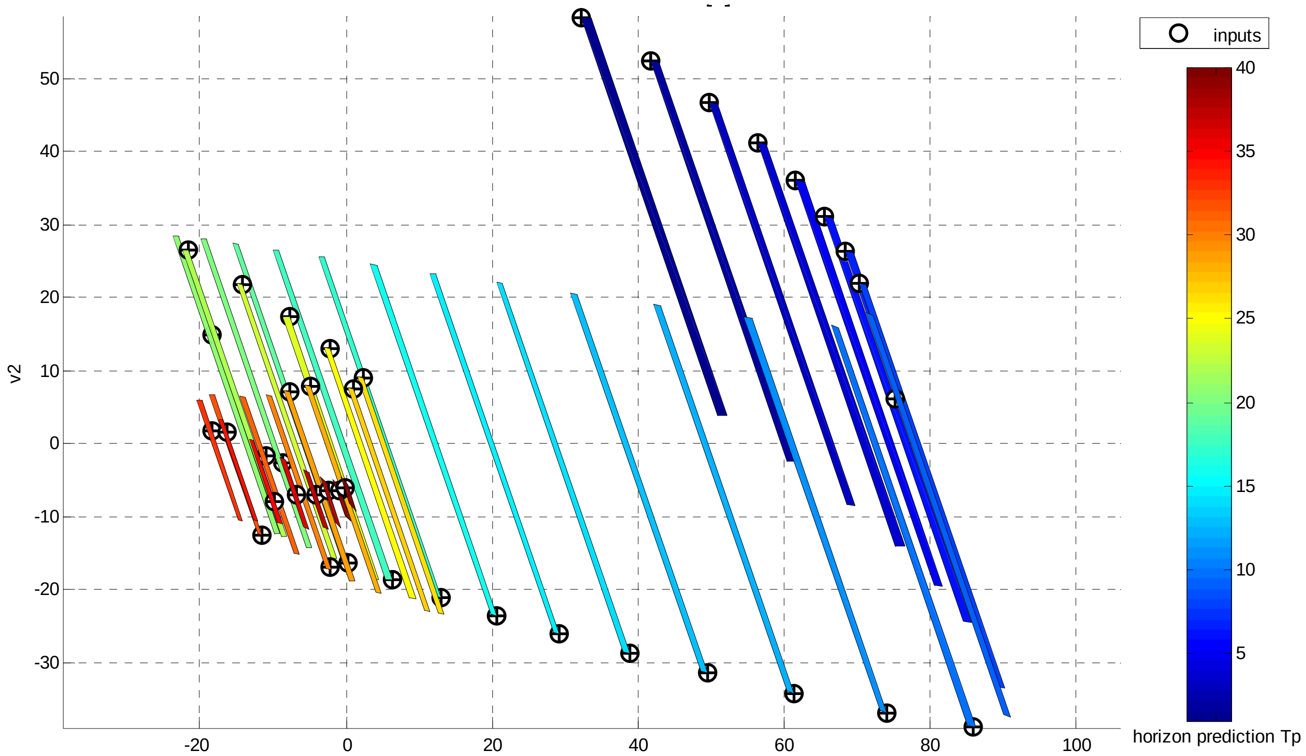

Fig. 10 feasible areas of linearized model on the prediction horizon

Appendix

The longitudinal part of equations of motion derived from (6), (7) are described in aerodynamic axis system:

$$
\begin{aligned}
\dot{V} &= -\bar{q}\frac{S}{m}\left(c_{D0} + c_{D\alpha}\alpha + c_{D\delta e}\delta e\right) + \frac{F_T}{m}\cos\alpha \\
&\quad + g\sin(\alpha - \theta) \\
\dot{\alpha} &= -\bar{q}\frac{S}{Vm}\left(c_{L0} + c_{L\alpha}\cdot\alpha + c_{L\bar{q}}\frac{q\cdot c_{MAC}}{2\cdot V} + c_{L\delta_e}\cdot\delta_e\right) \\
&\quad + q - \frac{F_t}{mV}\sin\alpha + \frac{g}{V}\cos(\alpha - \theta) \\
\dot{q} &= \bar{q}\frac{Sc_{MAC}}{I_y}\left(c_{m0} + c_{m\alpha}\cdot\alpha + c_{m\bar{q}}\frac{q\cdot c_{MAC}}{2\cdot V} + c_{m\delta_e}\cdot\delta_e\right) \\
\dot{\theta} &= q
\end{aligned}
\tag{34}
$$

Each constants of nonlinear equation (12) are defined as follows:

$$
\begin{aligned}
c_1 &= -\frac{1}{2}\frac{\rho S}{m}c_{D0} & c_2 &= -\frac{1}{2}\frac{\rho S}{m}c_{D\alpha} \\
c_3 &= -\frac{1}{2}\frac{\rho S}{m}c_{D\delta e} & c_4 &= \frac{1}{m} \\
c_5 &= g & c_6 &= -\frac{1}{2}\frac{\rho S}{m}c_{L0} \\
c_7 &= -\frac{1}{2}\frac{\rho S}{m}c_{L\alpha} & c_8 &= -\frac{1}{4}\frac{\rho S}{m}c_{MAC}c_{L\bar{q}} \\
c_9 &= -\frac{1}{2}\frac{\rho S}{m}c_{L\delta e} & c_{10} &= \frac{1}{2}\frac{\rho S}{I_y}c_{MAC}c_{m0} \\
c_{11} &= \frac{1}{2}\frac{\rho S}{I_y}c_{MAC}c_{m\alpha} & c_{12} &= \frac{1}{4}\frac{\rho S}{I_y}c_{MAC}^2 c_{m\bar{q}} \\
c_{13} &= \frac{1}{2}\frac{\rho S}{I_y}c_{MAC}c_{m\delta e}
\end{aligned}
\tag{35}
$$

Conclusion

In this work, a new method was provided in order to apply standard MPC techniques for exact linearizable system with inputs constraints. An iterative process was designed and at every step the optimization problem involved only linear dynamics and constrints. Future work will investigate the extension of restrictions on the status and speed inputs.

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