Finding Tuning Frequency for Symmetric and Anti-Symmetric Lamb Waves Using FEM

F. Mollaei*, M. R. Moosavi*, A. Abedian,I. Golzad***
***MSc Student,Aerospace Eng. Dept., Sharif University of Technology**
**** Associate Prof.,Aerospace Eng. Dept., Sharif University of Technology**
abedian@sharif.edu

Keywords: *piezoelectric, finite element method, tuning, lamb wave*

Abstract

In the area of life assessment, none-destructive methods of inspection and evaluation (NDI/NDE) play important role to ensure if a structure is still healthy enough to get used of. Using ultrasonic stress waves is one of the easiest and cheapest methods of NDI. Accordingly, different modes of lamb wave are mostly of interest..There are various methods for generating lamb waves. One of these methods is using piezoelectric wafer actuator/sensors (PWAS).

Accordingly, one of the most important parts of this method is to generate right mode via an appropriate choose of actuating frequency. In this text using a new method for finding tuning frequencies of different lamb wave modes proposed. For this reason finite element Method (FEM) used to predict the behavior of propagated lamb waves in a simple thin structure. The proposed method mainly considers the stress fields of each mode of lamb wave to decompose the generated wave in any actuating frequency. Good coincidence of the produced results with the previous results depicts the new methods accuracy.

1Introduction

Health monitoring of aging structures (SHM) is a major concern of the engineering community. This need is even more intense in the case of aging aerospace structures [1].

Embedded nondestructive evaluation (NDE) is an emerging technology that will allow the transition of conventional ultrasonic methods to embedded structural health monitoring (SHM) systems such as those envisioned for vehicle health management (VHM). Although NDE techniques such as ultrasonic testing can provide significant details about the nature of damage, these techniques generally require direct access to the structure. PZT patches, because they are small, light, cheap, and useful as *built-in* sensor systems offer a special opportunity to use in. Lamb wave method is considered to this study as strategy for PZT based damage detection. The Lamb wave-based method identifies the changes in the transmission velocity or energy of elastic wave associated with damage [2].

Previously a number of advanced NDE and conventional methodologies were used to monitor the bridge behavior during its service life. F Lanza et al did an acoustic emission test for failure monitoring of a carbon-reinforced-polymer cable for potential use in the bridge. The test allowed detection of early failure at 52%, 65%, 82% and 87% of the cable's ultimate load. An array of acoustic emission sensors were employed for early warning of possible failure of the structure in service[4].

De Lima and Hamilton and subsequently Deng analyzed the problem of nonlinear guided waves in isotropic plates by using normal mode decomposition and forced response as suggested by Auld. F Lanza et al start with the same formulation adopted by de Lima and Hamilton to study the behavior of guided Lamb waves at higher harmonics. The formulation reduces a nonlinear problem to a forced linear problem which concludes that anti-symmetric Rayleigh–Lamb waves are only allowed at odd harmonics, whereas symmetric Rayleigh–Lamb waves are allowed at all (odd or even) harmonics [5].

Advanced signal processing and pattern recognition techniques such as continuous wavelet transform (CWT) and support vector machine (SVM) are used in the current system. Firstly, PZT patches were used in conjunction with the impedance and Lamb waves to detect the presence and growth of artificial cracks on a1/8 scale model for a vertical truss member of Seongsu Bridge, Seoul, Korea, which collapsed in 1994.[2]

In this context, a FEM modelfor finding tuning frequencies of different lamb wave modes *i.e.* symmetric and anti-symmetric in simple thin structural solid is proposed. Different frequency ranges were studied and during every discrete step of analysis, a single frequency was analyzed. The main idea for decomposing the two modes in this work was based on subtracting the mean amplitude of observed through the thickness stress field from the total stress distribution and describe it as the anti-symmetric part and consider the average of stress as the symmetric part of the propagated wave. Then the results of present study with results of analytical solutions from ref [1] were compared.

2 Governing Equations

As brought in ref [1], a harmonic shear-stress boundary excitation as in Eq. 1 is applied to the upper surface of the plate, i.e.

$$\tau_a(x,d)=\begin{cases}\tau_0\sinh(\Gamma x), & |x|<a\\ 0, & O.W\end{cases} \qquad (1)$$

Since the excitation is harmonic, the solution is also to be assumed harmonic and the potential functions $\emptyset$ and ψ satisfy the wave equations.

For solving the wave equations we also apply the Fourier transform. Following notation is introduced as in ref. [1]

$$p^2=\frac{\omega^2}{c_p^2}-\xi^2 \qquad (2)$$

$$q^2=\frac{\omega^2}{c_s^2}-\xi^2 \qquad (3)$$

In this equation C_p and C_s are longitudinal (pressure) and transverse (shear) wave speeds, respectively. By using potential functions we reach the displacement equations and solve the equations in two forms (anti-symmetric and symmetric). Assuming anti-symmetric displacement and stresses about the mid-plane and noting that positive shear stresses have opposite directions on the upper and lower surfaces; strain wave solution at the plate's upper surface can be obtained by:

$$D_A=(\xi^2-q^2)^2\sin(ph)\times\cos(qh) + 4\xi^2 pq\cos ph\,\sin qh \qquad (5)$$

$$N_A=\xi q(\xi^2+q^2)\sin(ph)\times\sin(qh) \qquad (6)$$

$$\tilde{\varepsilon}_x=-i\frac{\tilde{\tau}}{2\mu}\frac{N_A}{D_A} \qquad (7)$$

Following a procedure similar to that applied to the anti-symmetric case one can reach the symmetric solution. Then the total solution for strain that is the sum of the symmetric and anti-symmetric solutions is obtained, i.e.

$$\tilde{\varepsilon}_x=-i\frac{\tilde{\tau}}{2\mu}\left(\frac{N_S}{D_S}+\frac{N_A}{D_A}\right) \qquad (8)$$

Applying the inverse Fourier transform yields the strain wave solution

$$\varepsilon_x(x,t) = -\frac{a\tau_0}{\mu}\sum_{\xi^S}(Sin\xi^S a)\frac{N_S(\xi^S)}{D_S'(\xi^S)}e^{i(\xi^S x-\omega t)} - \frac{a\tau_0}{\mu}\sum_{\xi^A}(sin\xi^A a)\frac{N_A(\xi^A)}{D_A'(\xi^A)}e^{i(\xi^A x-\omega t)} \qquad (9)$$

Where D_S' and D_A' represent the derivatives of D_A and D_S with respect to ξ.

$$u_x(x,t) = -\frac{a^2\tau_0}{\mu}\sum_{\xi^S}\frac{Sin\xi^S a}{\xi^S}\frac{N_S(\xi^S)}{D_S'(\xi^S)}e^{i(\xi^S x-\omega t)} - \frac{a^2\tau_0}{\mu}\sum_{\xi^A}\frac{sin\xi^A a}{\xi^A}\frac{N_A(\xi^A)}{D_A'(\xi^A)}e^{i(\xi^A x-\omega t)} \qquad (10)$$

At low frequencies, only two Lamb-wave modes exist, S0 and A0, and the general solution has only two terms for low frequency, i.e.,

$$\varepsilon_x(x,t) = \frac{1}{2\mu}\frac{\tilde{\tau}(\xi_0^S)N_S(\xi_0^S)}{D_S'(\xi_0^S)}e^{i(\xi_0^S x-\omega t)} + \frac{1}{2\mu}\frac{\tilde{\tau}(\xi_0^A)N_A(\xi_0^A)}{D_A'(\xi_0^A)}e^{i(\xi_0^A x-\omega t)} \quad (11)$$

The corresponding expression for displacement is:

$$u_x(x,t) = \frac{1}{2\mu}\frac{1}{i\xi_0^S}\frac{\tilde{\tau}(\xi_0^S)N_S(\xi_0^S)}{D_S'(\xi_0^S)}e^{i(\xi_0^S x-\omega t)} + \frac{1}{2\mu}\frac{1}{\xi_0^A}\frac{\tilde{\tau}(\xi_0^A)N_A(\xi_0^A)}{D_A'(\xi_0^A)}e^{i(\xi_0^A x-\omega t)} \quad (12)$$

3 Wave Decomposition and Results

3.1 Mode shapes

According to the previous works, different mode shapes can be recognized as strain field through the thickness. By this explanation, the acquired strain field from a propagated wave on plate-like structures can be decomposed to two different modes by assuming strain distribution.

In the current work, the first mode of each wave type is considered and the simplicity of these two modes will help to decompose the strain field. This is due to the fact that the symmetric wave is a type of strain field in which the strain is uniform through the thickness and the first mode of anti-symmetric wave is denoted by a strain field which is completely anti-symmetric relative to the mid-plane of the structure.

3.2 FEM Model

Due to previous efforts for modeling lamb-waves in three-dimensional, very fine elements in the same order of the wavelength should be used. Therefore, a complete 3-D modeling is not desirable for the tuning purposes. However, the work done by Giurgiutiu [1] in which good results were found for the properties of APC-850 using 2-D geometry with the conditions of both plane strain and axisymmetric coincide well with 3-D modeling of wave propagation.

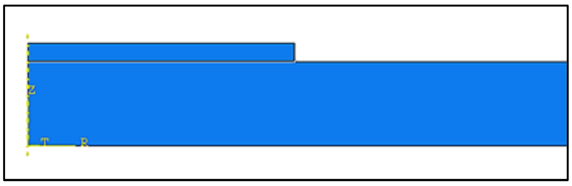

Figure 1- Schematic of Modeling

As this method is mainly developed to find tuning frequency of waves that generated by both square and circular shaped PWAS, axisymmetric condition has been taken into account.

A schematic of final model is presented in Fig.1, which shows the section of a big rectangular thin structure with a PWAS attached on the top and the center line is the axis of symmetry of the model.The dimensions of the structure are 1.07 mm thickness and 20cm length and the PWAS is a cylinder with 0.224mm thickness and diameter of 6.4 mm.

3.3 Results

For different time frames, the decomposition process was accomplished and the ratio of the maximum strain of each mode to the absolute maximum of strain distribution was found.

The main point was to choose the correct time frame in which the wave is on the peak i.e. maximum strain energy in a selected section during a time span. According to this procedure, comprehend series of FEM solution was made for a model in which the strain field is easy to follow.

The section in which the strain field is investigated is located 40 mm away from the centerline. This spacing was chosen to be about two times of the wavelength.

The results of each frame were normalized to check the coincidence with analytical results of equation (12) and experimental results of ref. [1].

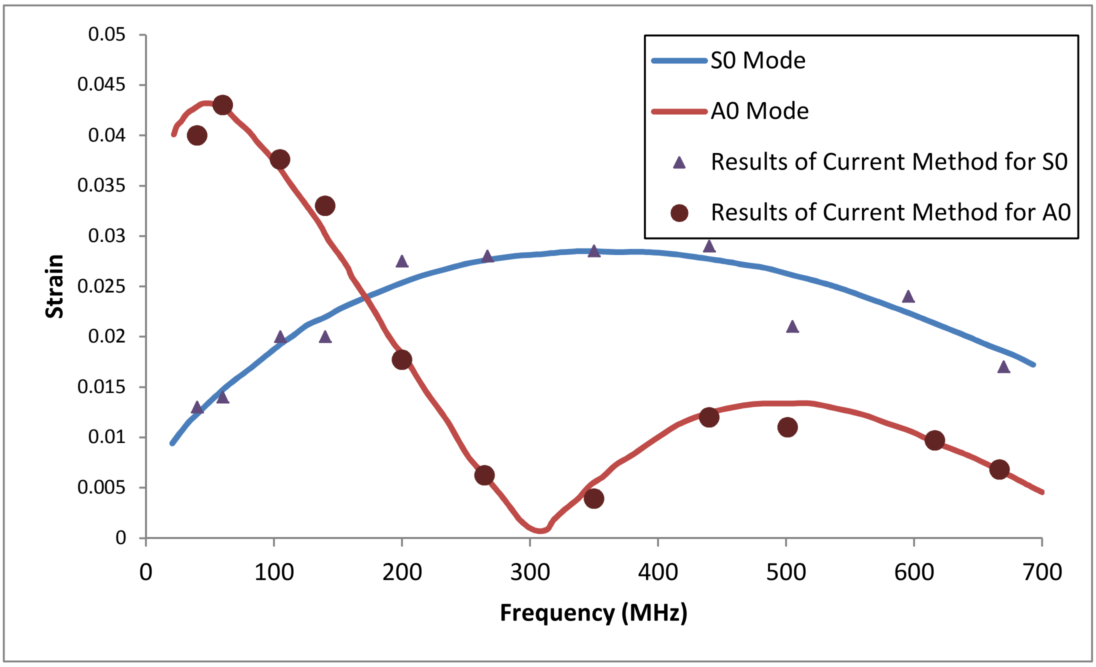

Figure 2-Comparison with Theoretical Results of Aluminum Plate of 1.007mm Thickness

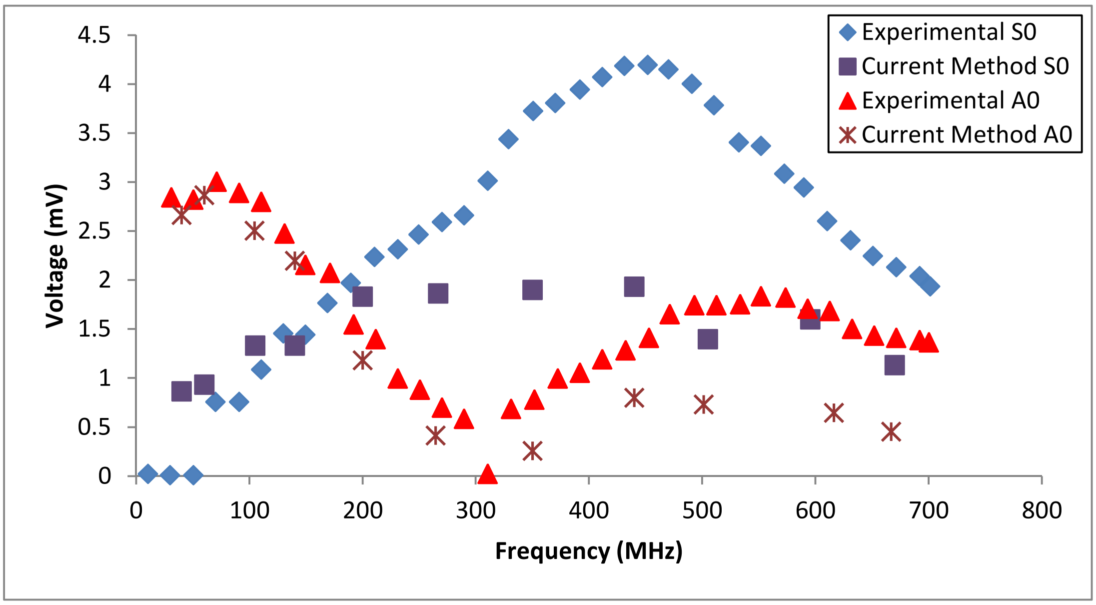

Figure 3- Comparison with Experimental Results of Ref. [1]

Fig.2 shows a plot that includes results of analytical equation and current method for 1mm thin Aluminum plate.

Fig. 3 also includes the results of 1mm thin Aluminum plate and the experimental results in scale of voltage gathered from sensor surface. To make a good coincidence with these data series, the maximum strain which used for normalization was multiplied by an appropriate factor to get the scale of voltage.

4 Conclusions

As brought above, a correct trend in portion of each mode shows the validity of the proposed method.

Also it is good to mention that the numerical studies need much lower efforts due to axisymmetric condition and 2-D simplification. This will cause an easy way to find tuning frequency for different metals and different cross sectional geometries.

References

[1]-Victor Giurgiutiu and Lingyu Yu; the Embedded Ultrasonic Structural Radar with Piezoelectric Wafer Active Sensors for Damage Detection in Cylindrical Shell Structures; 45th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics & Materials Conference and 12th AIAA/ASME/AHS Adaptive Structures Forum, Palm Springs, CA, 19-22 Apr. 2004, paper # AIAA-2004-1983

[2] Seunghee Park, Chung-Bang Yun, YongraeRoh, Jong-Jae Lee; PZT-based active damage detection Techniques for steel bridge components; Smart Mater. Struct. 15 (2006) 957–966

[3] Victor Giurgiutiu, Lingyu Yu; Embedded Ultrasonic Structural Radar with Piezoelectric Wafer Active Sensors for Damage Detection in Cylindrical Shell Structures; 45th, AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics & Materials Conference and 12th, AIAA/ASME/AHS Adaptive Structures Forum, Palm Springs, CA, 19-22 Apr. 2004, paper # AIAA-2004-1983

[4] Francesco Lanza di Scalea, Piervincenzo Rizzo, VistaspKarbhari and FriederSeible,;Health monitoring of UCSD's I-S/Gilman Advanced Technology Bridge ; University ofCalifornia, San Diego, USA

[5] AnkitSrivastava, Francesco Lanza di Scalea;On the existence of antisymmetric or symmetric Lamb waves at nonlinear higher harmonics; Journal of Sound and Vibration 323 (2009) 932–943