A PEG-BASED ASCENDING GUIDANCE ALGORITHM FOR RAMJET-POWERED VEHICLES

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Keywords: *Missile Guidance, PEG, Supersonic engine vehicle*

Abstract

This paper addresses guidance of ramjet vehicles to minimize propellant consumption along the ascending trajectory. An extension of the powered explicit guidance (PEG) algorithm is proposed to (a) incorporate parameter estimation of velocity to be gained and the burn time under unknown thrust profile, and (b) take into account aerodynamic forces on the vehicle flying at supersonic speed, while taking advantage of fuel efficiency of linear tangent-based approaches. Numerical simulations demonstrate feasibility of the proposed approach.

1 Introduction

Ramjet propulsion systems have been studied since 1910s [1]. Advantages of ramjet propulsion over other air-breathing propulsion systems include fuel efficiency, light vehicle weight, and simple structure; these advantages may enable the use of a ramjet-powered vehicle as a high-speed precision guided weapon [2]. Operation of a vehicle with a supersonic engine is often limited due to interaction between the vehicle motion and the combustion process. Therefore, a specialized guidance scheme is needed for ramjet missiles in order to consider maneuver limitations and thus to fully exploit the potential benefits of ramjet vehicles such as fuel efficiency.

Unfortunately, however, perhaps for security reasons, there has been little open research on guidance algorithms for ramjet missiles. Instead, there has been research on a fuel-efficient guidance framework in the context of launch vehicle guidance [3, 4, 5, 6], termed powered explicit guidance (PEG). The PEG algorithm does not require the intermediate information (e.g., position, velocity) to create the vehicle's pitch command, and is known to be used for vehicles with large variation in the thrust-weight ratio. Also, in principle the PEG, which is based on the linear tangent law, provides a minimum-time trajectory once the final orbit requirements are satisfied.

Note that the ascending phase of a ramjet missile has similarities with the first phase of launch vehicles: (a) final constraints of the phase must be satisfied, and (b) the thrust-weight ratio of the vehicle varies significantly. In addition, for a given propulsion strategy, a minimum-fuel ascent trajectory is often very close to the minimum-time trajectory with a linear tangent scheme. Thus, a PEG approach can be a good solution for ramjet vehicle guidance for which propellent saving is crucial; therefore, this work proposes a PEG-based method for ascending guidance of a ramjet missile.

However, since PEG was originally developed for vehicles operating in exoatmosphere, it cannot directly be applied to guidance of ramjet missiles flying in the air. The main contribution of this work is to present required modifications to the PEG algorithm to incorporate additional aerodynamic terms such as lift and drag. Numerical examples on two-dimensional ascending guidance demonstrate feasibility of the proposed method for supersonic ramjet missiles.

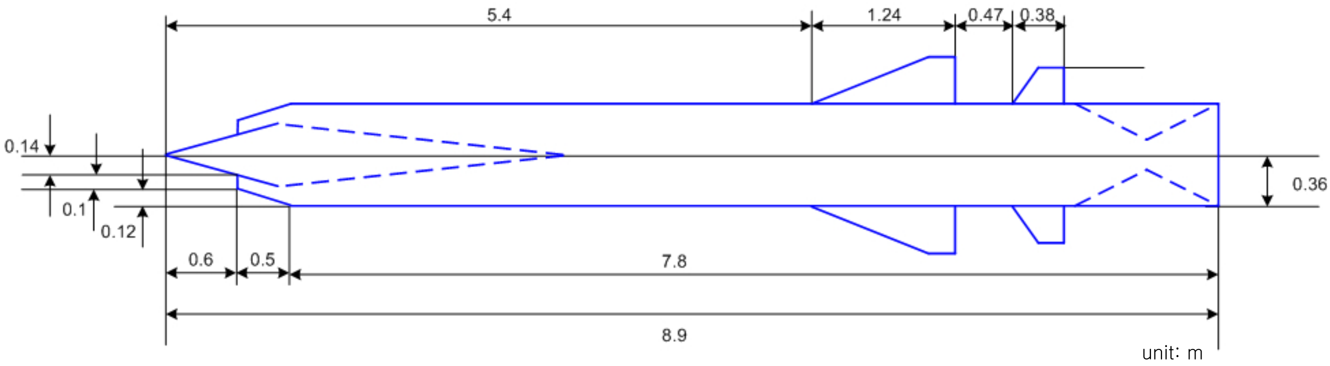

Fig. 1 Ramjet Missile Configuration

2 Preliminaries

2.1 Missile Model

There are several missiles which are operated by ramjet engine. The missile model used in this research is based on Yakhont [7] which is a medium-range ship to ship missile.

In the simulation, the ramjet missile configuration is needed to obtain aerodynamic data. A reference configuration which is used in this paper is shown in Fig. 1. This work considers a missile configuration to roughly model the Yakhont[8] whose image is available on the Internet. Aerodynamic coefficients can be obtained using the shape of the missile and an aerodynamic coefficient generation program . The lift coefficients and drag coefficients covers Mach 1.5 to 3.3 with angle of attack between $-6°$ and $6°$.

Thrust modeling was conducted. Thrust can be defined as a function of the Mach number, altitude, angle of attack, nozzle throttle area, and rate of fuel injection. The ramjet engine does not have its own compressor, so it needs the combustion process of compressing air with its own shape and fuel ignition. Base of this combustion process, the suction rate of air, combustion efficiency, and thrust properties are changed according to the flight conditions. Also, as the rate of fuel injection and nozzle throttle area change, the temperature and pressure of the combustor change. Generally, the nozzle throttle area is used to maintain stability of the combustor environment, and thrust is not sensitively changed by the nozzle throttle area. In this paper, the effect of the nozzle throttle area is assumed to be fixed during thrust evaluation.

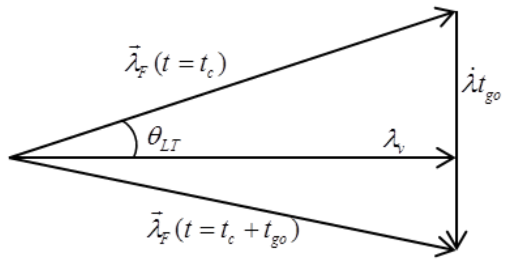

Fig. 2 Linear Tangent Steering Law[4]

2.2 PEG Algorithm[4]

This section explains PEG algorithm which is already-developed and studied by previous researchers, as a preliminary of the paper.

A general PEG algorithm has the advantage of not requiring the intermediate position and velocity to acquire the vehicle's pitch command to accomplish minimum time guidance and reach the final orbit requirements[9]. PEG is an algorithm based on linear tangent law. It has been developed to handle the space shuttle. Linear tangent law is irrelevant to the magnitude of the thrust vector, but relates to the direction of the vector.

The vehicle equation of motion can be expressed as follows

$$\frac{F}{m}\vec{u}_F + \vec{g}_F = \ddot{\vec{r}} \tag{1}$$

Fig. 2 represents the basic concept of the Linear Tangent Steering Law [4]. Based on Fig.2, the thrust vector can be written as (2)

$$\vec{u}_F = \frac{\vec{\lambda}_v + \dot{\vec{\lambda}}(t - t_\lambda)}{\sqrt{1 + \dot{\vec{\lambda}}^2 (t - t_\lambda)^2}} \tag{2}$$

λ_F is a vector defining the commanded thrust direction, λ_v is a unit vector in the direction of the velocity to be gained, $\dot{\lambda}$ is a vector which is normal to λ_v. t_λ means the time chosen that the total velocity changes due to thrust along λ_v, and t_c is current time.

The PEG algorithm consists of three parts. First, the steering parameters are calculated. The steering parameters λ_v and t_λ can be obtained from (4), (4):

$$\vec{\lambda}_v = \frac{\vec{v}_{go}}{L} \tag{3}$$

$$\vec{t}_\lambda = \frac{J}{L} \tag{4}$$

where

$$\begin{array}{ll} L = \int_0^{t_{go}} (F/m)dt & S = \int_0^{t_{go}} \int_0^t (F/m)\,dsdt \\ J = \int_0^{t_{go}} (F/m)\,dt & Q = \int_0^{t_{go}} \int_0^t (F/m)\,tdsdt \\ H = \int_0^{t_{go}} (F/m)\,t^2 dt & P = \int_0^{t_{go}} \int_0^t (F/m)\,t^2 dsdt \end{array} \tag{5}$$

Finally $\dot{\vec{\lambda}}$ can be represented as (6).

$$\dot{\vec{\lambda}} = \frac{\vec{r}_{go} - S\vec{\lambda}_v}{Q - St_\lambda} \tag{6}$$

From (6), the control input can be calculated as (2).

The second step is the prediction of the cutoff state. In this step, final states are predicted by using parameters which are obtained in the first step.

$$\vec{v}_{thrust} = \vec{\lambda}_v \left[L - \frac{1}{2}\dot{\lambda}^2 (H - t_\lambda J) \right] \tag{7}$$

$$\vec{r}_{thrust} = \vec{\lambda}_v \left[S - \frac{1}{2}\dot{\lambda}^2 (P - 2t_\lambda Q + t^2 J) \right] + \dot{\vec{\lambda}}(Q - St_\lambda) \tag{8}$$

$$\vec{v}_p = \vec{v} + \vec{v}_{thrust} + \vec{v}_{grav} \tag{9}$$

$$\vec{r}_p = \vec{r} + \vec{v}t_{go} + \vec{r}_{thrust} + \vec{r}_{grav} \tag{10}$$

Finally, correction in the $\vec{v}_{go}$ is needed to achieve null terminal errors. $\vec{v}_{miss}$ which is the difference between the predicted final velocity and desired final velocity is calculated as (11) After that, $\vec{v}_{go}$ is updated by using the following term.

$$\vec{v}_{miss} = \vec{v}_p - \vec{v}_d \tag{11}$$

$$\vec{v}_{go}(new) = \vec{v}_{go}(old) - \vec{v}_{miss} \tag{12}$$

A general PEG algorithm mentioned in this section has the advantage of not requiring intermediate position and velocity and generates vehicle's pitch command to accomplish minimum time guidance to reach final orbit requirements. Because there are many similarities between launch vehicle and ramjet missile, PEG can be applied as a ramjet missile guidance with fixed propulsion strategy.

3 Modification of PEG Algorithm

The objective of PEG algorithm usually used in launch vehicle is minimizing time of fuel consumption during the launch phase. If thrust is fixed, the result of minimizing time of fuel consumption problem is almost same as the result of minimizing fuel consumption problem. So PEG algorithm can be used as a guidance algorithm for ramjet supersonic missiles which have an objective of minimizing propellent consumption.

However PEG algorithm does not consider about aerodynamic forces. Also thrust profile needs to be known to apply PEG algorithm to vehicles. To integrate the algorithm into ramjet missile which flies at supersonic speed, additional modification to consider aerodynamic force and unknown thrust profile is needed.

The thrust of a supersonic missile varies with the flight environment and flight conditions of the vehicle such as the Mach number, altitude, angle of attack, and equivalent ratio, and it cannot be estimated beforehand. It has the problem that it is impossible to calculate the initial $\vec{v}_{go}$ and t_{go} directly. So it is impossible to apply the general PEG algorithm directly to the supersonic engine missile guidance. To apply PEG to missile guid-

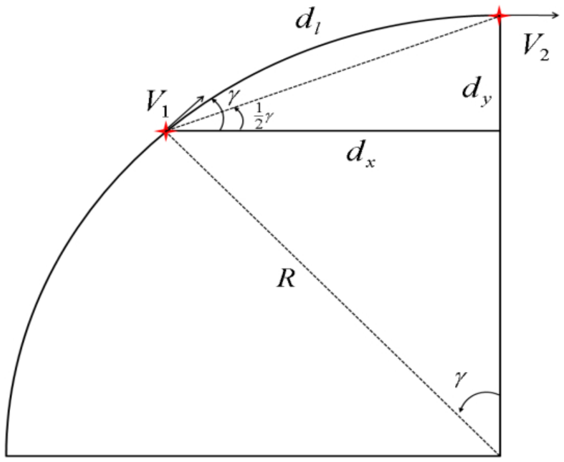

Fig. 3 Geometry for calculation time-to-go

ance, it is necessary to make modifications to the PEG algorithm.

V_{go} Calculation: Initially the thrust profile is unknown, so $\vec{v}_{go}$ which is an important parameter in PEG algorithm cannot be calculated. As a solution for this problem, the equation for the $\vec{v}_{go}$ is replaced as the difference between the current velocity and the desired cutoff velocity. The equation is shown as below. In this modification, the assumption that the vehicle can always approach the desired position using its thrust is made.

$$\vec{v}_{go} = (\vec{v}_d - \vec{v}) - \vec{v}_{grav} \tag{13}$$

t_{go} Estimation: The absence of information regarding the thrust profile also makes it impossible to directly calculate the t_{go} which indicates the remaining flight time. Because this parameter is needed in the integration calculation process, a simple method is suggested in this paper. For the estimation, the assumption that the vehicle follows a circular arc trajectory as shown in Fig. (3) is made. In Fig. (3), the radius R can be calculated through (14).

$$R\sin\gamma = d_y/\tan\frac{\gamma}{2} \tag{14}$$

d_l is the distance between the current position and the final destination, and it can be calculated by (15)

$$d_l = \gamma(rad)R \tag{15}$$

If the vehicle travels at a velocity equal to the average of the vehicles current velocity and final velocity, t_{go} can be represented as follows.

$$t_{go} = \frac{d_l}{V_1 + V_2} = \frac{\gamma d_y}{(V_1 + V_2)\sin\gamma\tan\frac{\gamma}{2}} \tag{16}$$

Aerodynamic Force Consideration: Generally, PEG algorithm is used for the space shuttle which is operated in the exoatmosphere. So there is no consideration of lift and drag terms in the PEG algorithm. On the other hand, since missiles are sensitive to lift and drag, the lift and drag terms need to be additionally considered. For lift and drag consideration, a small angle of attack of the vehicle is assumed.

In (17), $\vec{v}_L$ïĄšrepresents the velocity variation depending on lift. This paper assumes that direction of the lift and direction of the thrust is always perpendicular, and from this assumption $\vec{v}_L$ can be estimated.

$$\vec{v}_{go} = (\vec{v}_d - \vec{v}) - \vec{v}_{grav} - \vec{v}_L \tag{17}$$

In (18), $\vec{v}_{thrust}$ can be estimated as, $\vec{v}_{thrust} = \vec{v}_d - \vec{v}$, and β is a tuning variable.

$$\vec{v}_{thrust} \cdot \vec{v}_L = 0 \tag{18}$$

$$|\vec{v}_{thrust}| = \beta|\vec{v}_L| \tag{19}$$

$\vec{r}_L$ represents the position obtained from acceleration which is generated from lift. To consider the lift force, $\vec{r}_L$ is added to calculation of $\vec{r}_{go}$.

$$\vec{r}_{go} = \vec{r}_d - \vec{r} - \vec{v}t_{go} - \vec{r}_L \tag{20}$$

$$\vec{r}_L = \int_0^{t_{go}}\int_0^t \vec{a}_L ds dt = \frac{1}{2}\vec{v}_L t_{go} \tag{21}$$

From the assumption, drag vector has the same direction with thrust. So, for the consideration of drag, the additional magnitude of drag is added to the thrust.

4 Simulation Results

4.1 Trajectory Optimization

The objective of trajectory optimization is to obtain the optimal ascending trajectory of a supersonic engine missile. Because there was no previous research of specific guidance algorithm for ramjet missile, it is difficult to measure the utility of the result obtained from this paper. Hence, the result of the optimal trajectory will be used as a criterion of judgment to evaluate the suitability of the suggested algorithms which are introduced in the following chapters. The major consideration of ramjet engine missile is a fuel consumption. The cost function for trajectory optimization is defined as (22) which minimizes total fuel consumption.

$$\min J = \int_{t_0}^{t_f} \dot{m}_f dt \tag{22}$$

The results of the simulation show ascending trajectories, fuel consumption rate and velocity profile of the ascending phase. The command obtained from result of trajectory optimization is applied to 6DOF simulation.

4.2 Modified PEG

6DOF simulation is conducted with MPEG. Missile model for 6DOF simulation is obtained from a previous section. In this paper, equivalence ratio for generating thrust is fixed as constant value. Simulation is conducted several times with different initial condition and final constraints.

Table. 1 represent several cases with different conditions for simulations. For each of the cases, 6DOF simulation results of trajectory optimization and MPEG which show the total fuel consumption and total ascending time are denoted in Table. 2. As you can see in the table total fuel consumption of MPEG is 0.9% - 17% larger than trajectory optimization results. About time variables, some results of MPEG shows shorter ascending time compare with results of trajectory optimization. Considering that result from trajectory optimization is the optimal, result of the MPEG shows promise of application. Figs. 4 through 7 show exemplary simulation results of modified PEG algorithm for case 3. Simulation results show that they satisfy final constraints, and the angle of attack of a vehicle varies between -0.5° and 4.5 °.

Table 1 Simulation Condition

	Case	Initial Condition	Final Condition
1.	Altitude(km)	2.1	16
	Mach	2.1	2.5
	Path angle (deg)	18	0
2.	Altitude(km)	2.1	16
	Mach	2.1	2.5
	Path angle (deg)	36	0
3.	Altitude(km)	2.1	16
	Mach	2.1	2.9
	Path angle (deg)	18	0
4.	Altitude(km)	2.1	16
	Mach	2.1	2.9
	Path angle (deg)	36	0

Table 2 Simulation Result

	units: Time - sec Fuel used - kg	Trajectory Optimization	MPEG
1.	Time	89.1	92.1
	Fuel used	291	342
2.	Time	101.0	95.9
	Fuel used	335	338
3.	Time	100.0	123.8
	Fuel used	343	375
4.	Time	104.5	137.1
	Fuel used	365	377

5 Conclusion

This paper has presented a PEG-based guidance algorithm for supersonic engine missiles. A modified PEG algorithm that allows for considering additional aerodynamic effects and real time guidance is suggested. Numerical simulations have verified reasonable performance of the proposed method, compared to the trajectory optimization result.

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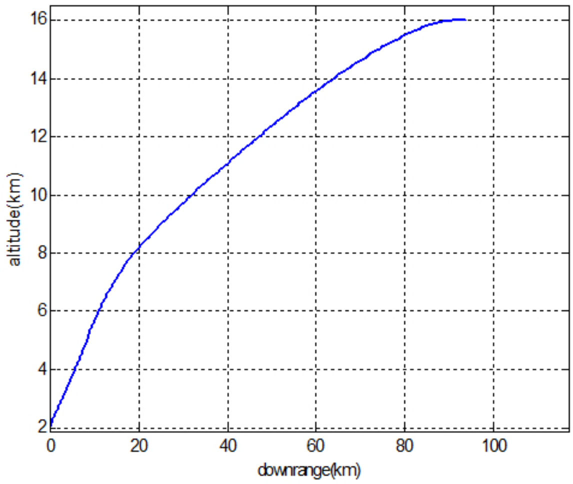

Fig. 4 Trajectory

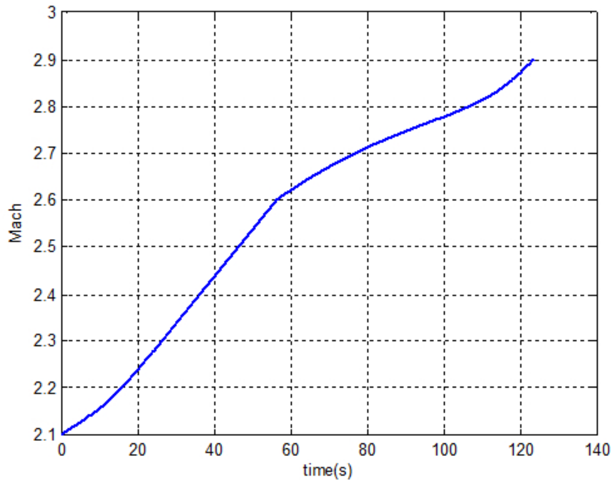

Fig. 5 Mach

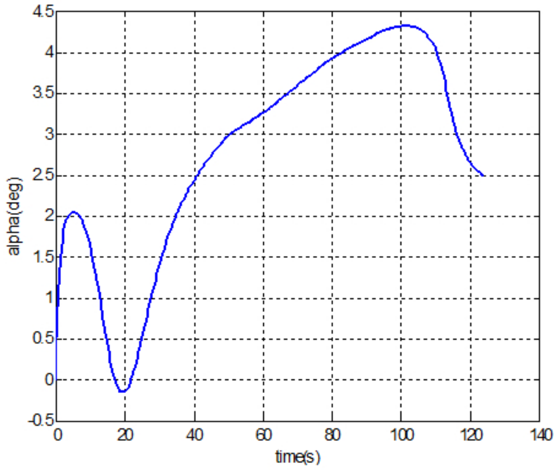

Fig. 6 Angle of attack

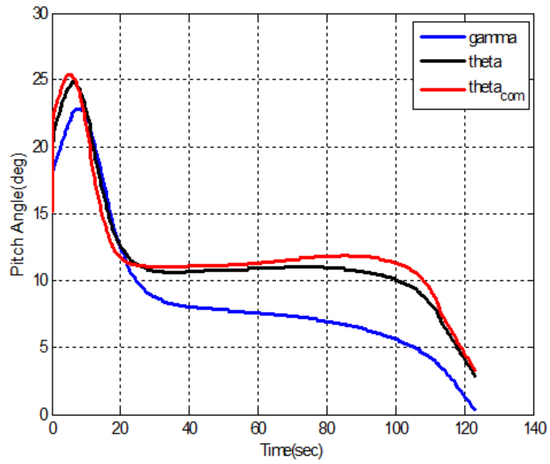

Fig. 7 Pitch command, Flight path angle

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