AUTOPILOT DESIGN FOR AN AGILE MISSILE USING L1 ADAPTIVE BACKSTEPPING CONTROL

Chang-Hun Lee*, Min-Jea Tahk, and Byung-Eul Jun*****
***KAIST, **KAIST, ***Agency for Defense Development**
chlee@fdcl.kaist.ac.kr;mjtahk@fdcl.kaist.ac.kr;mountrees@gmail.com

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Abstract

This paper deals with an agile missile pitch autopilot design using L1 adaptive backstepping control methodology. The flight phases of the agile missile systems can be classified into three phases; launch, agile turn, and end-game. This paper is focused on the agile turn phase, which is a fast 180 degree turn to engage a rear-hemisphere located target after launch phase, under the presence of the aerodynamic uncertainties. To attain a fast response, the angle-of-attack is chosen to be the control variable. Since L1 adaptive control can guarantee robustness against uncertainty and a fast response during the transient phase, it is suitable for a control methodology of the agile turn. The performances of the proposed controller are investigated and demonstrated through numerical simulations.

1 Introduction

During the course of years, the control systems of the agile missiles have been extensively studied by many researchers [1-6]. In general, the flight phases of the agile missile systems consist of three different phases; launch, agile turn, and end-game [2]. Among three different phases, the controller design of the agile turn phase introduces many challenging problems. The agile turn is defined to be a fast 180 degree turn maneuver to engage a rear-hemisphere located target after launch phase. The reason, why handling of the agile turn is difficult, is the presence of a large variation of the aerodynamic uncertainties induced by a high angle-of-attack maneuver. Therefore, a robust control approach is needed in order to compensate such the aerodynamic uncertainties.

One of solution is that the controller predicts the model uncertainties and adapts the model parameters, which is called the adaptive control methodology [5-6]. Although this approach can have robustness under the presences of the model uncertainties, the transient performance is poor due to the adaptation time nullifying the model error. Hence, the conventional adaptive control methodology may not be suitable for a control method of the agile missile systems that require a fast transient performance.

As a remedy, in this paper, we propose an agile missile controller based on backstepping control methodology with L1 adaptive scheme. From previous works [8-9], it has been shown that L1 adaptive scheme can provide the robustness and a fast response during the transient phase. In addition, since the normal acceleration control of the missile systems faces with the nonminimum phase phenomena, we choose the angle-of-attack for the control variable. Additionally, according to references [8-9], the commanding of angle-of-attack is more desirable than that of normal acceleration for achieving a fast response. Finally, the performance of the proposed controller is demonstrated by numerical simulations.

This paper consists of five sections. In section 2, a nonlinear missile model is explained. The proposed controller is provided in section 3. The simulation results are shown in section 4. In section 5, we conclude this study.

2 Nonlinear Missile Model

In this study, a nonlinear model of the longitudinal missile motion is considered as shown in Fig. 1.

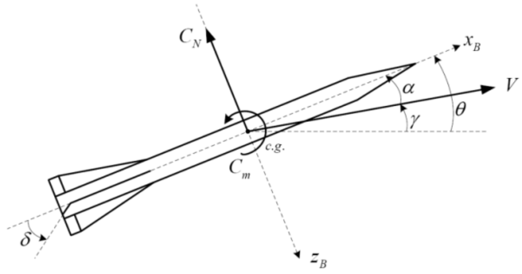

Fig. 1 The missile geometry

Under the assumptions that the missile body is rigid and the gravity force is compensated, the equations of motion are determined as follows:

$$\dot{\alpha} = -\frac{QS}{mV}\left(C_{N_0}(\alpha, M) + C_{N_\delta}(\alpha, M)\delta\right) + q \quad (1)$$

$$\dot{q} = \frac{QSl}{I_{yy}}\left(C_{m_0}(\alpha, M) + C_{m_q}(M)\frac{lq}{2V} + C_{m_\delta}(\alpha, M)\delta\right) \quad (2)$$

where α, q, and δ denote the angle-of-attack, the pitch rate, and the fin deflection angle, respectively. The notations of m, I_{yy}, S, and l indicate the mass, the pitching moment of inertia, the reference area, and the reference length, respectively. The velocity, the dynamic pressure, and Mach number are denoted by V, Q, and M, respectively. The aerodynamic coefficients in Eqs. (1) and (2) are given by the function of the angle-of-attack and Mach number. There coefficients are computed form the predetermined data tables, which are obtained by the wind tunnel test.

In the high angle-of-attack regime, the missile undergoes the aerodynamic uncertainties. From Eq. (1), we have the following equations under the presence of the aerodynamic uncertainties.

$$\dot{x}_1 = f_1 + \Delta f_1 + x_2 + (g_1 + \Delta g_1)u \quad (3)$$

$$\dot{x}_2 = f_2 + \Delta f_2 + (g_2 + \Delta g_2)u \quad (4)$$

where,

$$x_1 = \alpha, \qquad x_2 = q, \qquad u = \delta \quad (5)$$

$$f_1 = K_\alpha C_{N_0}, \quad f_2 = K_q\left(C_{m_0} + C_{m_q}(ql/2V)\right) \quad (6)$$

$$g_1 = K_\alpha C_{N_\delta}, \qquad g_2 = K_q C_{m_\delta} \quad (7)$$

$$K_\alpha = -(QS/mV), \qquad K_q = (QSl/I_{yy}) \quad (8)$$

and the notations of Δ represents the model error caused by the aerodynamic uncertainties in the high angle-of-attack regime.

3 L1 Adaptive Backstepping Controller

3.1 Model Derivation

In order to apply the backstepping control methodology, a strict feedback form of system equation is required. In the missile systems, the magnitude of control force $(g_1 + \Delta g_1)u$ in the right hand side of Eq. (3) can be generally negligible due to $|(g_2 + \Delta g_2)u| \gg |(g_1 + \Delta g_1)u|$. Therefore, Eqs. (3) and (4) can be rewritten in a strict feedback form as follows:

$$\dot{x}_1 = f_1 + x_2 + \Delta_1 \quad (9)$$

$$\dot{x}_2 = f_2 + g_2\omega u + \Delta_2 \quad (10)$$

where

$$\Delta_1 = \Delta f_1 + (g_1 + \Delta g_1)u, \quad \Delta_2 = \Delta f_2 \quad (11)$$

$$\omega = 1 + \Delta g_2 / g_2 \quad (12)$$

In Eqs. (9) and (10), Δ_1 and Δ_2 can be regarded as the total model errors due to the aerodynamic uncertainties and the neglecting term of $(g_1 + \Delta g_1)u$. By introducing the linear parameterization [9], the model uncertainties can be parameterized as follows:

$$\Delta_1 = \theta_1|x_1| + \sigma_1 \quad (13)$$

$$\Delta_2 = \theta_2\|x\|_\infty + \sigma_2 \quad (14)$$

Substituting Eqs. (13) and (14) into Eqs. (9) and (10) yields the following equation.

$$\dot{x}_1 = f_1 + x_2 + \theta_1|x_1| + \sigma_1 \quad (15)$$

$$\dot{x}_2 = f_2 + g_2\omega u + \theta_2\|x\|_\infty + \sigma_2 \quad (16)$$

where θ_1, θ_2, σ_1, σ_2, and ω are unknown parameters. The adaptation schemes of these variables will be discussed in section 3.3.

3.2 State Predictor Design

In this section, we discuss the state predictor design. First, let us define the desired error dynamics of state predictor as follows:

$$\dot{\tilde{x}}_1 = -K_1\tilde{x}_1, \qquad \dot{\tilde{x}}_2 = -K_2\tilde{x}_2 \quad (17)$$

where $\tilde{x} \triangleq \hat{x} - x$ represents the prediction error. $\hat{x}$ and x are the predicted state and the true state, respectively. The gains of K_1 and K_2 decide the convergence speed of the prediction error. Substituting Eqs. (15) and (16) into Eq. (17) provides the following state predictor.

$$\dot{\hat{x}}_1 = -K_1\tilde{x}_1 + f_1 + x_2 + \theta_1|x_1| + \sigma_1 \quad (18)$$

$$\dot{\hat{x}}_2 = -K_2\tilde{x}_2 + f_2 + g_2\omega u + \theta_2\|x\|_\infty + \sigma_2 \quad (19)$$

Since θ_1, θ_2, σ_1, σ_2, and ω are unknown parameters in Eqs. (18) and (19), they should be estimated as follows:

$$\dot{\hat{x}}_1 = -K_1\tilde{x}_1 + f_1 + x_2 + \hat{\theta}_1|x_1| + \hat{\sigma}_1 \quad (20)$$

$$\dot{\hat{x}}_2 = -K_2\tilde{x}_2 + f_2 + g_2\omega u + \hat{\theta}_2\|x\|_\infty + \hat{\sigma}_2 \quad (21)$$

3.3 Adaptive Law Design

This section derives the adaptive law based on Lyapunov function. By using Eqs. (15), (16), (20), and (21), the prediction error dynamics in Eq. (17) can be rewritten as follows:

$$\dot{\tilde{x}}_1 = \dot{\hat{x}} - \dot{x}_1 = -K_1\tilde{x}_1 + \tilde{\theta}_1|x_1| + \tilde{\sigma}_1 \quad (22)$$

$$\dot{\tilde{x}}_2 = \dot{\hat{x}}_2 - \dot{x}_2 = -K_2\tilde{x}_2 + g_2\tilde{\omega}u + \tilde{\theta}_2\|x\|_\infty + \tilde{\sigma}_2 \quad (23)$$

where $\tilde{\theta}_1 = \hat{\theta}_1 - \theta_1$, $\tilde{\theta}_2 = \hat{\theta}_2 - \theta_2$, $\tilde{\sigma}_1 = \hat{\sigma}_1 - \sigma_1$ $\tilde{\sigma}_2 = \hat{\sigma}_2 - \sigma_2$, and $\tilde{\omega} = \hat{\omega} - \omega$.

Let us consider the following Lyapunov function.

$$V_1 = \frac{1}{2}\tilde{x}_1^2 + \frac{1}{2}\tilde{x}_2^2 + \frac{1}{2\Gamma}\left(\tilde{\omega}^2 + \tilde{\theta}_1^2 + \tilde{\theta}_2^2 + \tilde{\sigma}_1^2 + \tilde{\sigma}_2^2\right) \quad (24)$$

Taking the time-derivative of V_1 yields:

$$\dot{V}_1 = -K_1\tilde{x}_1^2 - K_2\tilde{x}_2^2 + C_1 + C_2 + C_3 + C_4 + C_5 \quad (25)$$

where

$$C_1 = \tilde{\theta}_1\tilde{x}_1|x_1| + (1/\Gamma)\tilde{\theta}_1\dot{\tilde{\theta}}_1 \quad (26)$$

$$C_2 = \tilde{\theta}_2\tilde{x}_2\|x_2\|_\infty + (1/\Gamma)\tilde{\theta}_2\dot{\tilde{\theta}}_2 \quad (27)$$

$$C_3 = \tilde{\sigma}_1\tilde{x}_1 + (1/\Gamma)\tilde{\sigma}_1\dot{\tilde{\sigma}}_1 \quad (28)$$

$$C_4 = \tilde{\sigma}_2\tilde{x}_2 + (1/\Gamma)\tilde{\sigma}_2\dot{\tilde{\sigma}}_2 \quad (29)$$

$$C_5 = \tilde{\omega}\tilde{x}_2 g_2 u + (1/\Gamma)\tilde{\omega}\dot{\tilde{\omega}} \quad (30)$$

In order to guarantee the asymptotic stability of the prediction error, we enforce C_1 through C_5 to be zero. Then, the time-derivative of $\dot{V}_1$ is always negative definite as follows:

$$\dot{V}_1 = -K_1\tilde{x}_1^2 - K_2\tilde{x}_2^2 < 0 \quad (31)$$

From the condition of nullifying C_1 to C_5, the following adaptive law can be obtained.

$$\dot{\hat{\theta}}_1 = \dot{\tilde{\theta}}_1 = -\Gamma\tilde{x}_1|x_1|,\ \dot{\hat{\theta}}_2 = \dot{\tilde{\theta}}_2 = -\Gamma\tilde{x}_2\|x_2\|_\infty \quad (32)$$

$$\dot{\hat{\sigma}}_1 = \dot{\tilde{\sigma}}_1 = -\Gamma\tilde{x}_1,\quad \dot{\hat{\sigma}}_2 = \dot{\tilde{\sigma}}_2 = -\Gamma\tilde{x}_2 \quad (33)$$

$$\dot{\hat{\omega}} = \dot{\tilde{\omega}} = -\Gamma\tilde{x}_2 g_2 u \quad (34)$$

where Γ represents the adaptation gain. In the conventional adaptive control, the increase of the adaptation gain introduces the chattering effect. However, L1 adaptive control relieves this limitation by introducing a low pass filter rejecting the high frequency signal induced by high adaptation gain.

3.4 Control Law Design

For convenience, let new residuals be defined as follows:

$$z_1 = x_1 - x_{1d},\qquad z_2 = x_2 - x_{2d} \quad (35)$$

Then, the residual dynamics can be expressed as:

$$\dot{z}_1 = f_1 + x_2 + \hat{\theta}_1|x_1| + \hat{\sigma}_1 - \dot{x}_{1d} \quad (36)$$

$$\dot{z}_2 = f_2 + g_2\hat{\omega}u + \hat{\theta}_2\|x\|_\infty + \hat{\sigma}_2 - \dot{x}_{2d} \quad (37)$$

where x_{1d} and x_{2d} are the desired state values. We consider the following Lyapunov function to derive the outer loop control law in the backstepping methodology.

$$V_2 = \frac{1}{2}z_1^2 \quad (38)$$

The time-derivative of V_2 can be determined by using Eq. (36) as follows:

$$\dot{V}_2 = z_1\left(f_1 + x_2 + \hat{\theta}_1|x_1| + \hat{\sigma}_1 - \dot{x}_{1d}\right) \quad (39)$$

In order to enforce $\dot{V}_2 < 0$, the desired value of x_2 is chosen as:

$$x_{2d} = -K_1 z_1 - \left(f_1 + \hat{\theta}_1|x_1| + \hat{\sigma}_1\right) + \dot{x}_{1d} \quad (40)$$

In order to design the inner loop control law, the following Lyapunov function is introduced.

$$V_3 = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 \quad (41)$$

After taking time-derivative of V_3 and substituting Eqs. (36) and (37) into $\dot{V}_3$ gives the following result.

$$\dot{V}_3 = z_1\left(-K_1 z_1 + z_2\right) + z_2\left(f_2 + g_2\hat{\omega}u + \hat{\theta}_2\left\|x_2\right\|_\infty + \hat{\sigma}_2 - \dot{x}_{2d}\right) \tag{42}$$

From Eq. (42), we can obtain the inner loop control law that satisfies $\dot{V}_3 < 0$ as follows:

$$u = \frac{1}{g_2\hat{\omega}}\left(-K_2 z_2 - z_1 - f_2 - \hat{\theta}_2\left\|x_2\right\|_\infty - \hat{\sigma}_2 + \dot{x}_{2d}\right) \tag{43}$$

In Eqs. (40) and (43), K_1 and K_2 represents the control gain, which are identical values of the state predictor.

If the adaptation gains increase, the adaptation parameters of $\hat{\theta}_1$, $\hat{\theta}_2$, $\hat{\sigma}_1$, $\hat{\sigma}_2$, and $\hat{\omega}$ contain the high frequency signals which cause the chattering effect during the transient phase. In L1 adaptive control methodology, low pass filters are introduced in the inner loop and the outer loop control law in order to cutoff the high frequency signal. Therefore, the uses of the high adaptation gains are possible in this method.

By introducing a low pass filter, the outer loop control law can be obtained as follows:

$$x_{2d} = -K_1 z_1 - \hat{\eta}_{1C} + \dot{x}_{1d} \tag{44}$$

where $\hat{\eta}_{1C} \triangleq C_1(s)\hat{\eta}_1$ in the frequency domain. A second-order low pass filter $C_1(s)$ and the variable $\hat{\eta}_1$ are defined as follows:

$$C_1(s) = \frac{\omega_{LF}^2}{s^2 + 2\zeta_{LF}\omega_{LF}s + \omega_{LF}^2} \tag{45}$$

$$\hat{\eta}_1 = f_1 + \hat{\theta}_1\left|x_1\right| + \hat{\sigma}_1 \tag{46}$$

where ω_{LF} and ζ_{LF} represent the design parameters of low pass filter.

In a similar way, the inner loop control law is modified by using a first-order low pass filter.

$$u_c(s) = C_2(s)u \tag{47}$$

where

$$C_2(s) = \frac{g_2\hat{\omega}k}{s + g_2\hat{\omega}k} \tag{48}$$

$$u = \frac{1}{g_2\hat{\omega}}\left(-K_2 z_2 - z_1 - f_2 - \hat{\theta}_2\left\|x_2\right\|_\infty - \hat{\sigma}_2 + \dot{x}_{2d}\right) \tag{49}$$

where k represents the design parameter of a first-order low pass filter.

Fig. 2 shows the overall configuration of the proposed controller.

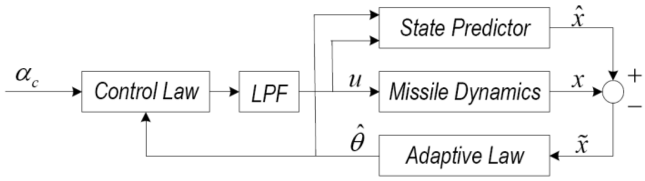

Fig. 2 The configuration of controller

4 Simulation Results

In order to demonstrate the performance of the proposed controller, numbers of simulations are carried out. In these simulations, a second-order actuator model with $\omega_{act} = 200$ rad/s, $\zeta_{act} = 0.7$, and $\dot{\delta} = 450$ deg/s is considered. The controller parameters are chosen as $K_1 = K_2 = 15$ and $\Gamma = 100000$. The low pass filter parameters are defined as $\omega_{LF} = 200$ rad/s, $\zeta_{LF} = 0.7$, and $k = 5$.

4.1 Case 1: Step Input Command

In this simulation, the proposed controller is tested with a step input command. Figs. 3 and 4 show the step input response of angle-of-attack and the fin deflection angle in the nominal case. Figs. 5 and 6 provide the simulation results under the presence of 30% model uncertainties during the flight. The results indicate that the proposed controller can provide a good tracking performance even in the presence of the model uncertainties.

4.2 Case 2: Agile Turn Scenario

In this simulation, the applicability of the proposed method is determined under an agile turn scenario (i.e., 180 degree heading reversal turn). The angle-of-attack command [10] which is obtained from the trajectory optimization to achieve the terminal velocity after agile turn is used. Figs. 7 and 8 give the angle-of-attack response and the fin deflection angle during the agile turn. The results show that the proposed controller can maintain a sound tracking performance. Therefore, the proposed method can be applied to the challenging issues of the agile missile systems.

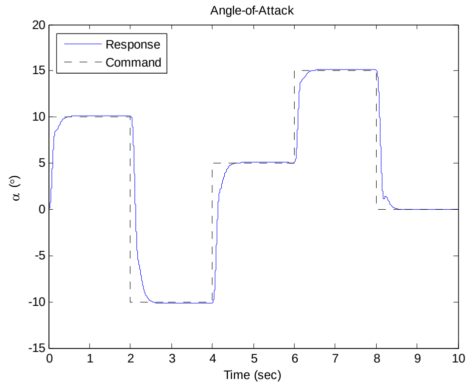

Fig. 3 Angle-of-attack (nominal)

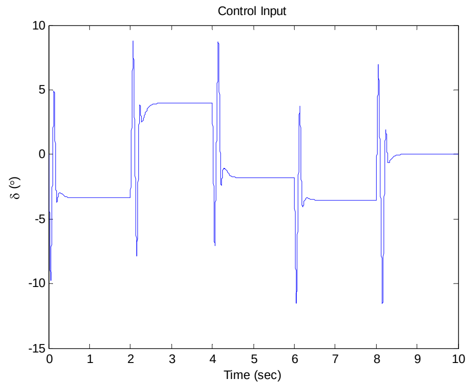

Fig. 4 Fin deflection angle (nominal)

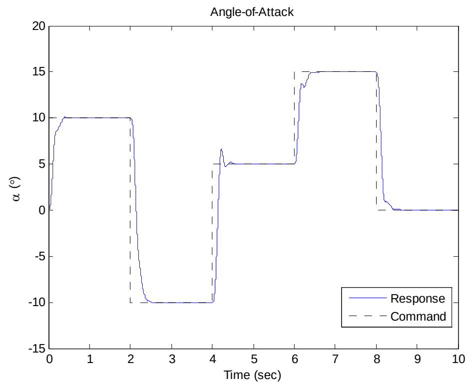

Fig. 5 Angle-of-attack (30% uncertainty)

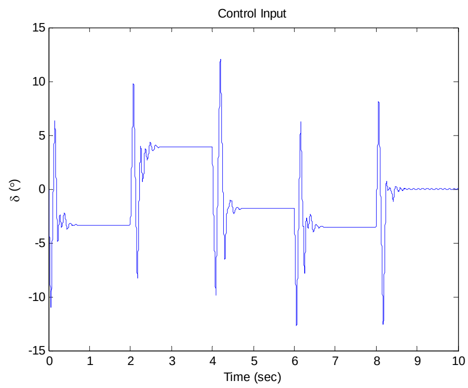

Fig. 6 Fin deflection angle(30% uncertainty)

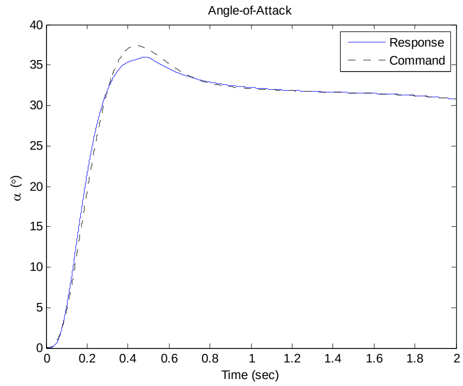

Fig. 7 Angle-of-attack (agile turn)

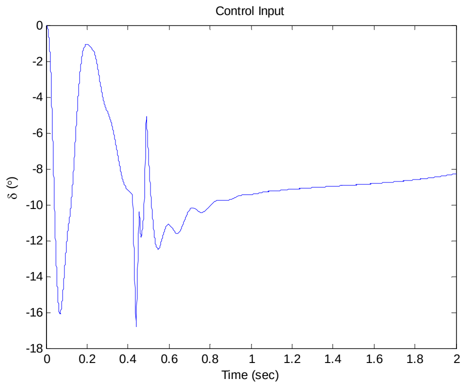

Fig. 8 Fin deflection angle (agile turn)

5 Conclusion

In this paper, we propose the agile missile autopilot based on backstepping control in conjunction with L1 adaptive scheme. In the proposed method, the commanding of angle-of-attack was considered for accomplishing a fast turn of the missile's heading angle. The simulation results indicated that the proposed controller can provide the sound performance under the presence of the model uncertainties in high angle-of-attack regime. It can also be applicable to the agile turn maneuver.

Acknowledgments

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