ON THE EFFECT OF DISTRIBUTED ROUGHNESS ON TRANSITION OVER ROTOR-STATOR DEVICES

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Abstract

The implications of distributed roughness on the stability of boundary-layer flows present in rotor-stator devices is to be studied, using both theoretical and experimental approaches. The project is motivated by the desire to develop passive drag-reduction techniques in aerospace applications. This paper is a summary of progress to date and focusses on the theoretical study of the von Kármán flow with distributed surface roughness.

The results presented here suggest that increasing the level of roughness causes an overall decrease in the stability of the flow, with transition seen at lower Reynolds numbers. Furthermore, the mechanism through which the flow is destabilized appears to switch from the inviscid cross-flow mode to the viscous streamwise-curvature mode. From this it is clear that surface roughness affects the flow through viscous mechanisms, as one might expect.

1 Introduction

It is now firmly established that the classic belief that surface roughness inevitably increases skin-friction drag no longer holds [1–5]. The *right sort of roughness* can reduce drag due to energetically beneficial interactions between coherent, energy-bearing eddy structures and roughness protrusions within the boundary layer. The challenge for the coming decades remains to identify what constitutes the *right sort of roughness* for any given particular type of boundary layer and for the intended specific technological application.

We investigate roughness effects on the 3D boundary-layer flow between two concentric rotating disks. Such flows are established on many types of rotating machinery. For example, the flow between co-rotating compressor or turbine disks and the flow between a turbine disk and an adjacent stationary casing can be modeled by such rotor-stator systems. The class of theoretical boundary-layer flows we focus on have previously been referred to as the *BEK* system [6]. This naming is natural as the system contains the familiar Bödewadt, Ekman and von Kármán flows at particular parameter (Rossby number) values. The boundary-layer flows are distinguished by a characteristic cross-flow component which makes the flow characteristics and transition of all such (incompressible) boundary layers closely resemble each other. For practical and theoretical reasons, the von Kármán flow (created over a disk rotating in otherwise still fluid) has served as the paradigm for studying boundary layers with cross-flow component for over six decades [7–13]. However, despite its relevance to many practical applications, roughness effects on the rotating disk and related boundary layers have received little attention.

It is recognized that previous work on the 3D BEK flows is mostly limited to flow transition over smooth surfaces only. Previous experimental studies have focused almost exclusively on disks designed to test theoretical predictions obtained for hypothetical, idealized flow conditions. These are rarely present in real-world engineering environments. Shifting focus from the idealized scenario to the real-world must be expected to yield new results of immediate practical and scientific relevance. If we can identify, for instance, how and where roughness leads to what level of energy dissipation then we may subsequently become able to control and reduce the energy dissipation rates and skin-friction drag.

It is emphasized that we study the effects of realistic, uniform distributed roughness of increasing levels and different types, and not the effects of a small number of roughness elements [14–18]. We note that the 3D flows considered here are fundamentally different from that of the essentially 2D flows over flat plates or inside tubes for which roughness effects have been studied for decades following, for example, the work of Nikuradse [19].

The beginning of the natural transition process of the BEK boundary-layer flows is signaled by the development of the well-known spiral vortices within the transition region [9–13,17,18,20–24]. Experimental studies [25] report the very interesting result that a modest roughness level decreased the number of these vortices from initially 32 for a smooth disk to 26 for the rough disk. This raises questions as to whether roughness levels above that level can reduce the number of spiral vortices further and whether transition can possibly bypass the spiral-vortex route entirely above a certain critical roughness level? This would indicate roughness promoting an entirely different transition mechanism and warrants further study since it will be associated with different energetical implications. A clarification of this observation is the initial aim of this project.

This paper summaries the beginnings of the ongoing study and focusses on the von Kármán flow.

2 Theoretical study of the modified von Kármán flow

The theoretical study necessarily begins by obtaining steady flow profiles in the presence of distributed roughness. There exists two previous studies [26, 27] that tackle the theoretical modifications to the classic von Kármán similarity solution for the flow over successively increasing roughness levels. In the extended project we follow Yoon *et al.*'s [27] formulation and generalize it to the broader BEK system with a view to conducting stability analyses at general Rossby number. However, in this paper we focus on the particular case of the modified von Kármán flow arising from surface roughness.

2.1 The steady flow profiles

The surface of the disk is described by $s^* = \delta^* \cos(2\pi r^*/\gamma^*)$, with * indicating a dimensional quantity. The quantity δ^* is the amplitude of the surface variation from its mean value, γ^* is the wavelength of the surface variation, and r^* is the distance along the disk in the radial direction. The surface function can be altered to suit any required profile by changing the value of δ^* and γ^*, along with the functional form (although we use the cos function throughout this study). The disk is considered to be rotating about its axis of symmetry at a rotation rate Ω^*. It is natural to consider this geometry in a cylindrical polar coordinate system (r^*, θ, z^*) (fixed in the stationary frame) in which the governing Navier–Stokes equations are well known. The steady-flow components in these directions are denoted (u^*, v^*, w^*).

All dimensional quantities are scaled on a characteristic length-scale given by the boundary-layer thickness, λ^*, and the velocity scale given by $\lambda^*\Omega^*$. This leads to the Reynolds number $Re = \Omega^*\lambda^{*2}/\nu^*$ which is interpreted as a measure of the spin rate. The surface function nondimensionalizes to

$$s = \delta \cos\left(\frac{2\pi r}{\gamma}\right), \qquad (1)$$

with amplitude and wavelength parameters δ and γ, respectively. These are our control parameters and are expressed in units of boundary-layer thickness as a consequence of the scalings.

Before attempting to solve the governing equations, it is necessary to transform out the surface distribution. To this end we use a new coordinate system (r, θ, η) defined by the transformation $\eta = z - s(r)$. In this modified coordinate system the flow components are transformed to be

$$U = u, \quad V = v, \quad W = -s'u + w, \qquad (2)$$

where the prime denotes differentiation with respect to r. At this stage we make the boundary-layer assumption, $Re^{-1} << 1$, and set

$$\zeta = Re^{1/2}\eta, \quad \tilde{W} = Re^{1/2}W.$$

The governing equations for the steady flow are obtained after introducing variables closely related to the von Kármán similarity variables,

$$f(r,\zeta) = \frac{U}{r}, \quad g(r,\zeta) = \frac{V}{r}, \quad h(r,\zeta) = \tilde{W}, \quad (3)$$

and are stated as

$$2f + r\frac{\partial f}{\partial r} + \frac{\partial h}{\partial \zeta} = 0, \qquad (4)$$

$$rf\frac{\partial f}{\partial r} + h\frac{\partial f}{\partial \zeta} + f^2\left(1 + r\frac{s's''}{1+s'^2}\right) = (1+s'^2)\frac{\partial^2 f}{\partial \zeta^2} + \frac{g^2}{1+s'^2}, \qquad (5)$$

$$rf\frac{\partial g}{\partial r} + h\frac{\partial g}{\partial \zeta} + fg = (1+s'^2)\frac{\partial^2 g}{\partial \zeta^2} - fg. \qquad (6)$$

These are subject to the boundary conditions

$$f(r) = h(r) = 0, \ g(r) = 1 \quad \text{at } \zeta = 0,$$
$$f(r) = g(r) = 0 \quad \text{as } \zeta \to \infty, \qquad (7)$$

which represent the no slip and quiescent fluid conditions at all radial positions in this frame of reference. Note that this system reduces to the standard von Kármán system of ODEs in ζ when $s(r) \to 0$, as would be expected.

Equations (4)–(7) can be solved to find the velocity profiles for a disk with a surface distribution parameterized by δ and γ using the commercially available NAG routines. The resulting theoretical flow profiles are dependent on the radial position r, owing to the use of $s(r)$. However, the cyclical nature of $s(r)$ means that the flow profiles have a cyclical nature themselves, characterized by the wavelength parameter γ. This is in contrast to the usual von Kármán solution, where all position dependence is contained within the similarity transformation of equation (3). However, given the small wavelengths involved in these distributions ($\gamma^* = O(\lambda^*)$), it is possible to revert back to a von Kármán type flow for the purposes of the stability analysis by using a single profile obtained from the ensemble-averaged flow from 100 flows at evenly spaced locations over one wavelength. Physically, the roughness distribution is expected to be of sufficiently small wavelength that a homogeneous flow response is to be expected across the disk, with only a larger scale spatial variation given by the similarity transformation. This approach has the significant advantage that, in this preliminary study at least, previous stability codes developed for the smooth disk [22, 23] can be used enabling a direct comparison between the results over smooth ($\delta = 0$) and rough ($\delta > 0$) disks. This approach will be taken and the relevance of this method will be the subject of experimental verification at a later date.

Figures 1 shows the results from ensemble averages of flows with $\gamma = 1$ and a range of δ to 0.3 over one wavelength from $r = 3$ (although the solution is independent of this location). For the radial flow (f-velocity profile, upper plot), roughness is seen to decrease the maximum fluid velocity within the boundary layer, i.e. roughness acts to reduce the wall jet. This is physically sensible as roughness would increase the friction holding the base of the wall jet back as it moves along the radius of the disk. For the azimuthal case (g-velocity profile, middle plot), roughness is seen to broaden the boundary layer through a thickening of this profile; again, this is physically sensible. The normal flow (h-velocity profile, lower plot) appears to increase the flow entrained into the boundary layer. However the interpretation of the normal profile requires a reversal of the transformation applied in equation (2). We note that the use of the boundary-layer assumption means that this component is actually an order of magnitude smaller than the other components and has only minor implications for the stability of the flow.

2.2 Stability analysis

Now that the steady flows for the rough disks have been obtained, the next stage in the theoretical study is to perform linear stability analysis for a range of values of parameters δ and γ. Equation (2) shows that the averaged radial- and azimuthal-velocity components are untransformed and consistent with von Kármán solutions used in previous studies of smooth disks. Furthermore, the averaged normal component can be obtained from reversing the transformation. Applying the reverse transformation enables an analysis of the steady flows using perturbation equations identical to those in previous studies [21–24, 28]. The perturbations applied to the steady flow are assumed to have normal mode form and this introduces further parameters for the disturbances: α, the radial wavenumber; β, the azimuthal wavenumber; ω, the frequency. In what follows, we conduct a spatial analysis and so assume α is complex with the imaginary part giving the spatial growth rate of disturbances. Both β and ω are assumed to be real and, in order to ensure periodicity round the disk, $n = \beta Re$ (identified as the number of vortices) must be interpreted at real integer values only.

Imposing boundary conditions on the perturbation equations ensures that the disturbances are contained within the boundary layer and forms an eigenvalue problem that is solved for particular combinations of values of α, β and ω at each parameter set (Re, δ, γ). From these we form the dispersion relation, $D(\alpha, \beta, \omega; Re, \delta, \gamma) = 0$, with the aim of studying the occurrence of convective instabilities. In each analysis the α-branches are calculated using a fourth-order Runge–Kutta integrator with Gram–Schmidt orthonormalization and a Newton–Raphson linear search procedure [21, 28].

Since we are supposing here that the flow is not absolutely unstable, it follows that in the Briggs–Bers procedure [21,28] we can reduce the imaginary part of the frequency down to zero,

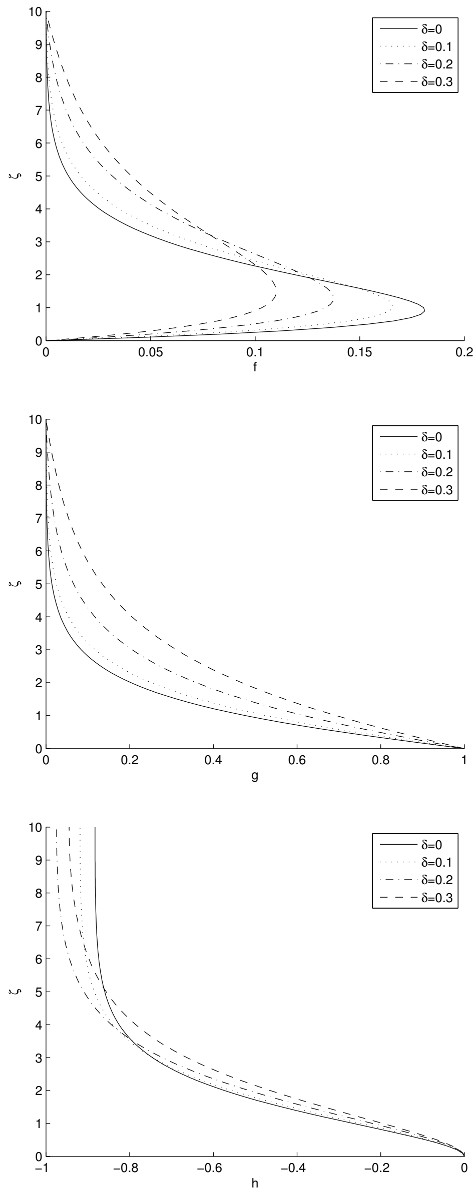

Fig. 1 Steady-flow profiles for $\gamma = 1$ and δ to 0.3

so that $\omega_i = 0$. To produce the neutral curves for convective instability a number of approaches can be taken in this stationary frame of reference. One approach is to insist that the vortices rotate at some fixed multiple of the disk surface velocity, thereby fixing the ratio ω_r/β, and then α and β are calculated using the spatial analysis. This is the approach taken here. In particular, we explicitly assume that the vortices rotate with the surface of the disk (i.e. are *stationary* relative to the rotating disk) so that $\omega_r = \beta$. This is consistent with experimental observations [25].

Two spatial branches were found to determine the convective instability characteristics for all parameter sets considered. These branches arise from the cross-flow (type I) instability mode and the streamline-curvature (type II) mode, and are identical to those discussed in related publications concerned with smooth bodies [21–24, 28, 29]. The type I mode is known to arise from the inflectional nature of the streamwise steady velocity component; the type II mode corresponds to a viscous instability associated with the way in which the outer-flow streamlines are curved by an $O(Re^{-1})$ amount close to the outer edge of the boundary layer.

Figure 2 demonstrates the results of the linear stability analysis for $\gamma = 1$ and values of δ to 0.3. The neutral curve found at $\delta = 0$ is identical to that computed in the literature for flows over smooth rotating disks. The upper lobe of each neutral curve represents the type I mode, and the lower lobe the type II mode. We see that the effect of increasing δ is to increase the critical Reynolds number of the type I lobe, furthermore the range of unstable parameters narrows - both are stabilizing effects. However, we also see that critical Reynolds numbers for the type II (lower) lobe is reduced and becomes increasingly important relative to the type I lobe with roughness. Ultimately, the effect of increased roughness is seen to be a reduction in critical Reynolds number of the most dangerous mode; roughness is therefore seen to be *destabilizing*. Similar results were obtained for $\gamma \neq 1$. The type II mode arises from viscous effects, and it is of no surprise that the surface roughness appears to influence the flow through viscous mechanisms.

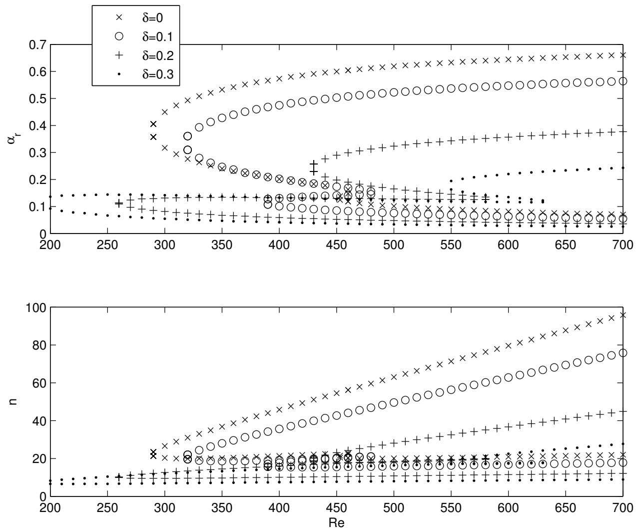

Fig. 2 Neutral curves for $\gamma = 1.0$ and various δ.

Furthermore, the lower plot of Figure 2 shows a reduction in the predicted number of vortices (associated with the disturbance wavenumber in the azimuthal direction). These preliminary results are qualitatively consistent with experimental observations [25] that the number of vortices reduces with roughness height. This gives credibility to idea that the usual spiral-vortex route to turbulence within von Kármán flows could be bypassed by distributed roughness of sufficient amplitude.

3 Experimental approach

The experimental rig and measurement techniques used in this research already exist at Warwick, and have previously been used successfully for research into the effects of compliance on transition over rotating disks [17, 18]. Our facility essentially consists of a water-filled tank (diameter 1m) housing the computer-controlled disk (diameter 0.4m), as show in Figure 3. The main measurements are carried out with a TSI IFA 300 constant-temperature hot-film anemometry system. The hot-film probe on its support traverse is fully computer controlled and can be calibrated in situ. In order to characterize the roughness of the disks we have so far employed the classic classi-

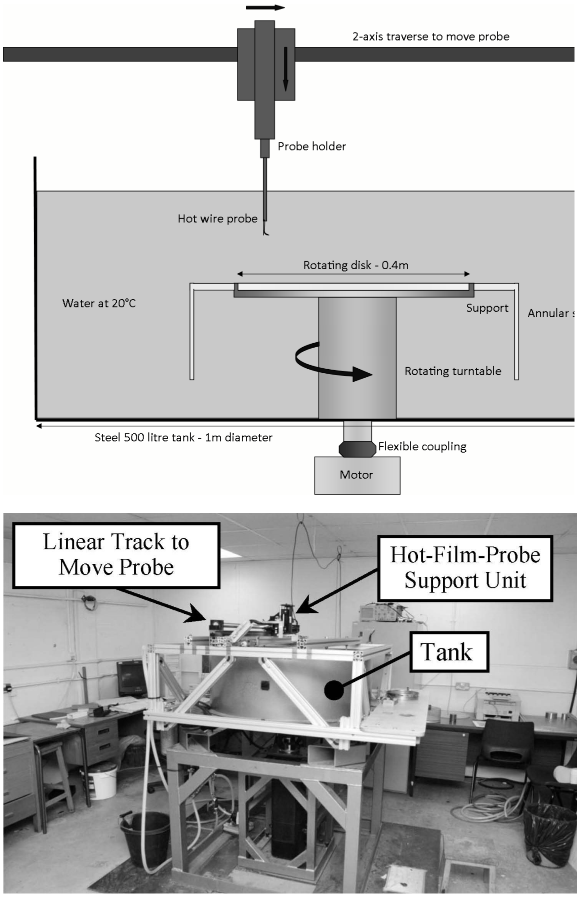

Fig. 3 The rotating-disk facility.

fication scheme of *hydraulically smooth*, *transitional* or *completely rough* used, for instance, for flat-plate boundary layers or for the flow inside tubes with rough walls. However, considering the general background [20] that led to this scheme and, in particular, the fact that it was developed for fully turbulent flow, it is by no means clear in how far it will directly translate to the (transitional) rotating-disk flow. This classic roughness-classification scheme relies on the roughness parameter, $\Theta = k_s \nu^\star/\nu$, where k_s is typical roughness height, $\nu^\star = \sqrt{\tau_0/\rho}$ is the friction velocity, and τ_0 the wall shear stress. The larger its value the more pronounced the expected influences of roughness. Walls are classified as hydraulically smooth for $\Theta < 5$ and as completely rough for $\Theta > 70$. Roughness effects appear within the transitional regime $5 \leq \Theta \leq 70$. For typical operating conditions of our rotating-disk facility we estimate that we will need roughness protrusions within, roughly, 36μm and 500μm to cover the transitional regime - provided the classic roughness scheme for 2D flows does indeed extend to the 3D rotating disk flow. If our continuing research reveals that the classic roughness-classification scheme is not suited for fully 3D flows with cross-flow component then we will attempt to adapt the scheme for this particular type of flow.

There have been many different techniques used to manufacture the necessary rough disks, including sand blasting, wire mesh, laser etching and rapid prototyping. However, the elected method was to cut shallow grooves into aluminium disks using a lathe. The grooves can be cut at a range of depths and pitches to simulate the amplitude and wavelength of the theoretical surface function. The surface roughness of these disks has been measured with our Rank-Taylor-Hobson Talysurf facility (see [17, 18]) and has been found to be consistent with the theory. The experimental study is ongoing.

4 Conclusion

This paper presents a summary of progress on an ongoing project into the impact of distributed surface roughness on BEK flows; only results from the theoretical analysis of the von Kármán flow are available at this time. In this particular case, surface roughness is seen to destabilize the boundary-layer flow, as demonstrated by a reduction in the most dangerous critical Reynolds number with increased δ. The analysis shows a change in the dominant instability mode from the inviscid cross-flow mode to the viscous streamline-curvature mode. Given that an introduction of roughness is a surface effect, one would expect the effects to be felt through the viscosity of the fluid - this has been demonstrated. Furthermore, the result that roughness acts to reduce the number of spiral vortices in the transitional region is consistent with experimental observations in the literature.

These preliminary results give credibility to idea that the usual spiral-vortex route to turbulence within von Kármán flows could be bypassed by distributed roughness of sufficient amplitude. However, the linear approach detailed in this paper is limited in that it can only realistically cope with small levels of roughness before non-linear effects become dominant and a modified theoretical approach is needed.

Future planned work involves the experimental study, not least to clarify the validity of the theoretical approach, and the broadening of the theoretical study to general flows within the BEK system. The long-term aims are towards an understanding of the flow physics needed in order to determine the *right sort of roughness*.

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