STREAMWISE AND CROSSFLOW INSTABILITIES ON INCLINED CIRCULAR CYLINDERS

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Abstract

*Observations of streamwise and crossflow instabilities on swept circular cylinders over a range of inclinations are presented. By considering sweep angles through the full range between the two extremes studied in the literature, we relate the streamwise vorticity of the unswept case (Kestin & Wood, J. Fluid Mech. **44**) to the more aggressive crossflow instability at high sweep angles (Poll, J. Fluid Mech. **150**). Measurements made on the unswept cylinder confirm earlier predictions in the literature, providing a firm basis for referencing the new measurements of vortical behavior on cylinders with general sweep angle. The study has implications for turbine blading.*

1   Introduction

Previous investigations in the literature have revealed streamwise vortices and "streaky structures" on flat plates [1] and on the suction surface of compressor blades [2, 3]. This organized vortex system tends to increase heat transfer to the blade surface and also makes the flow and heat transfer difficult to predict. A detailed understanding of the origin of these structures therefore has implications for the design of turbine blades, and this is the motivation for this study.

Turbine blades with subsonic inlet velocities usually have a leading edge that is quite blunt, and frequently circular. The result is that the flows near to the leading edge region can be modeled (to the first approximation at least) by the flow past an affine cylindrical body. The circular cylinder in crossflow is therefore an important canonical case and an understanding of this relatively abstract flow is crucial to the ultimate understanding of the real flows over turbine blades.

Previous investigations of such vortical structures present over circular cylinders are limited to the two extremes of sweep angle. Kestin & Wood [4] demonstrated the existence of streamwise vortices on an unswept (zero sweep) circular cylinder, using both experimental and theoretical approaches. Poll [5] experimentally examined the behavior of crossflow vortices on a highly swept cylinder. The Kestin & Wood theory may be regarded as the limiting case and it is interesting to work from that to consider the stability of the flow over a wide range of sweep angles, towards those considered by Poll. However, the introduction of sweep brings a wider range of instabilities into consideration, the most prominent being the crossflow instability resulting from the inflectional behavior of the three-dimensional boundary layer. It is clear that the vortices observed by Kestin & Wood and those observed by Poll arise from different mechanisms.

The lowest sweep angle considered by Poll was $55°$, and data in the useful range of sweep up to $50°$ are virtually non-existent in the literature, although the experiments of Dagenhart *et al.* [6] and Kohama [7] are of interest. There are quite substantial differences between the streamwise vortices observed by Kestin & Wood and the crossflow vortices observed by Poll and it has never been clear how, and where, the streamwise
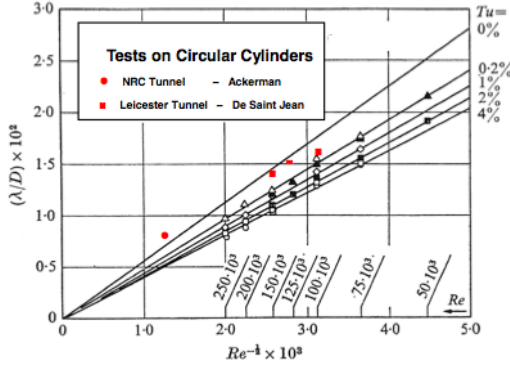
Fig. 1 Measurements of spanwise wavelength on circular cylinders and comparison with Kestin & Wood's predictions.

vorticity changes to crossflow vorticity. However it is worth remarking that Kohama gives an intriguing photograph showing two instability modes simultaneously. A likely interpretation is that one mode is the remnant of the type of streamwise vorticity observed by Kestin & Wood at zero sweep, the other coarser mode being a vigorous crossflow instability. Bridging the gap between Kestin & Wood's and Poll's sets of results is the aim of this present investigation.

In this paper, experimental observations of low speed flows are reported and attendant theories for the spanwise wavelength of the vortical structures developed as a function of sweep angle. Note that the designations of streaks, stationary structures and streamwise vortices are used interchangeably.

2 The unswept benchmark

Kestin & Wood's [4] original investigation on unswept cylinders predicted a theoretical value of spanwise wavelength between vortex pairs, λ, across a cylinder of diameter, D, given by:

$$\lambda = 1.79\pi D Re^{-0.5} \qquad (1)$$

This result is represented by the Tu = 0% line in Fig. 1. Kestin & Wood also undertook experimental work on circular cylinders which provided the results for non-zero turbulence levels, also shown in the figure.

In order to confirm the suitability of the Kestin & Wood theory as a basis for examining non-zero sweep, further experimental work was undertaken on unswept circular cylinders. In 2002 Ackerman [8] performed surface flow visualizations on a 37.26 mm diameter cylinder at a free stream Mach Number of 0.5. Streamwise streaks were observed before and after a separation bubble. This experiment was performed at the relatively high Reynolds number of 675,000 and provided a point for comparison with the Kestin & Wood prediction, as shown in Fig. 1.

More recently testing was undertaken on a 152 mm diameter aluminum cylinder in the University of Leicester low speed research tunnel at three Reynolds numbers. Surface flow visualization has provided a further three points on the Kestin & Wood plot in Fig. 1.

All four results are in reasonable agreement with the Kestin & Wood theory and are taken as confirmation of the theory for the case of the unswept circular cylinder.

3 The swept case

In all that follows, the sweep angle, Λ, is defined as the angle between the normal to the inflow and the cylinder axis. This is consistent with the definition used by Lewis & Hill [9], for example.

3.1 Analytical approach to modeling

In an attempt to generalize Kestin & Wood's prediction of vortex spacing, Equation (1), to non-zero sweep angle, two alternative approaches have been taken. The first approach considers that, in the swept case, the projection of the cylinder surface on the streamwise plane is an ellipse, leading to a modification of the effective local diameter, D. The second approach considers the growth in boundary-layer thickness through the introduction of a flow component parallel to the leading edge of the cylinder.

Under the first approach, an expression for the local effective diameter D can be derived in terms of the sweep angle and used to modify Kestin & Wood's original estimate. This is done

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by considering the local curvature at points on the notional ellipse formed by approaching the circular cylinder at an oblique angle. The resulting expression is given in terms of the sweep angle, the streamwise distance from the center of the notional ellipse, x, and the ellipse major axis, $2a$:

$$D = (1/\cos^2\Lambda)(1 - ((x/a)\sin\Lambda)^2)^{3/2}.$$

This modifies Kestin & Wood's original estimate, λ_0, to a function of sweep that is parameterized by the location x/a at which the effective radius is taken:

$$\lambda/\lambda_0 = (1/\cos\Lambda)(1 - ((x/a)\sin\Lambda)^2)^{3/4}.$$

Treating x/a as a fitting parameter, the best fit to the experimental data is obtained for $x/a = 0$, for which

$$\lambda = \frac{\lambda_0}{\cos\Lambda} = \frac{1.79\pi D}{Re^{0.5}\cos\Lambda}. \tag{2}$$

This is indicated by the "Theory" curve in Fig. 2. The location $x/a = 0$ corresponds to the 50% chord location on the cylinder. However, the applicability of Kestin & Wood's underlying stability analysis at locations away from the leading edge is questionable, and careful consideration of the steady inner flow at each location is required to confirm this result. This is the subject of ongoing work.

Equation (2) can be independently derived from the second approach which follows the formulation as outlined by Obrist [10] and is performed at the leading edge, consistent with Kestin & Wood's analysis. The method is based on an asymptotic match between an outer potential flow region and an inner flow region at the leading edge. The outer region of the flow is characterized by a modified form of Kestin & Wood's Reynolds number and by the potential flow around the cylinder. The inner region is the stagnation point flow at the leading edge of the cylinder, which is given by a swept Hiemenz flow. The thickening of the boundary layer with increased sweep can be determined and then used to modify Kestin & Wood's estimate appropriately; this results in Equation (2). Implicit in this approach is the assumption that the swept

Hiemenz flow leads to the same wall shear rate as the unswept flow. Consideration of the swept flows studied by Obrist, for example, and the unswept flow of Kestin & Wood implies this to be true. Schlichting's [11] numerical estimate of the wall shear is therefore expected to apply for all sweep angles and this simple modification to Kestin & Wood's result can be made.

In spite of the restrictive assumptions underpinning either approach, Equation (2) is reached in both cases. This is also in agreement with the traditional Cosine Rule used to predict sweep effects on airfoils. Bursnall & Loftin [12] showed this approach to be valid for subcritical flows only and that the critical Reynolds number decreases with increasing sweep.

3.2 Transverse spacing

Testing was carried out in the University of Leicester wind tunnel over a range of relatively low Reynolds numbers from 132,000 to 175,000 and over the range of sweep angles from 0 to 60.1°. Poll [5] had found that at a Reynolds number below 339,000 streaks were only visible quite late on the surface. In the current measurements streaks were faint, but visible and consistent much further forward. It was found that, if care was taken with the surface coating and the optical techniques, streaks were visible in all cases. Each count of streak spacings was performed at least three times and suitable average spacings were recorded. In all cases it was possible to make consistent and repeatable measurements of streak spacings and angles.

Care was taken to check that the new wind tunnel results and the published results of Poll were quoted and normalized in the same way. Here they are both normalized using Equation (1) as a reference. This normalization was used because Kestin & Wood's theoretical result is widely accessible and represents very closely a regression line fit through our own experimental results for unswept cylinders. Furthermore the relationship arises naturally from the theoretical approaches taken in §3.1. The results for the range of sweep angles are presented in Fig.
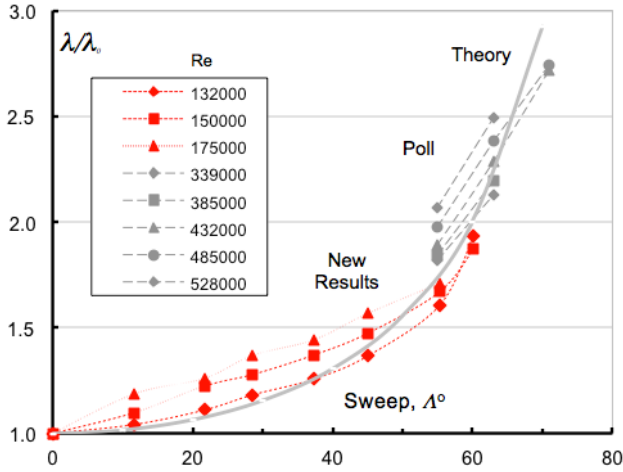
Fig. 2 Lateral spacing between streaks, normalized by Kestin & Wood's theory (Eq. 1).

Fig. 3 Normalized streak spacings, corrected for $\cos\Lambda$ term of Eq. (2).

2. They appear to be self consistent and also compatible with Poll's results obtained at higher Reynolds numbers. The theoretical curve from Equation (2) is also plotted and demonstrates reasonable agreement with both the Poll data and the new data.

This agreement is tested further in Fig. 3 by normalizing by the theoretical result and plotting $(\lambda/\lambda_0)\cos\Lambda$. Fig. 3 would seem to reflect a significant additional Reynolds number effect beyond the first-order correction. The new data are compatible with the Poll data. Poll's data suggest a similar but even stronger additional Reynolds number effect at intermediate sweep angles. Although the maximum variation from unity of these results is no more than 20%, the bulge in the region of 30 degrees of sweep and the sharp rise in Poll's results as his sweep angle is reduced to $55°$, suggest the potential for mode changes at intermediate sweep angles. Since the zero sweep results pertain to contra-rotating vortex pairs and the Poll results reflect the vigorous co-rotating vortex regime prevalent at high sweep angles, such a change is anticipated.

Full data sets were obtained at $55°$ sweep, by Poll and from the current tests. These are of particular value in determining consistency and the lateral sweep spacings are plotted as a function of Reynolds number in Fig. 4. Despite the
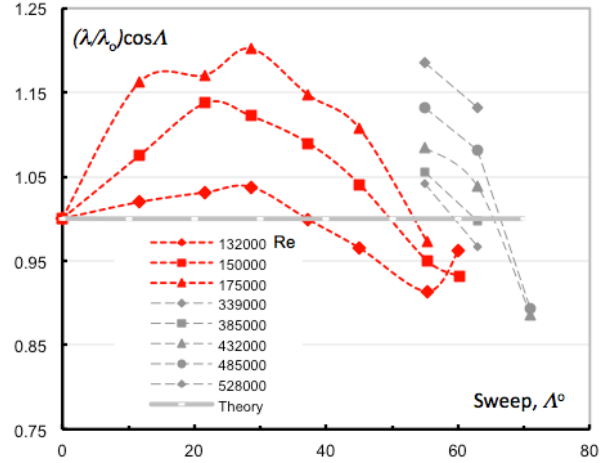
large gap in the intermediate Reynolds number range the results obtained from both investigations fall nicely onto a common relaxation line. This would seem to confirm that the present results are quite compatible with those of Poll.

In addition to lateral spacing between streaks, Poll presented some results on angular orientation of streaks, $\varepsilon°$. For swept cylinders the streaks are not linear from the leading edge but follow a somewhat S-shaped trajectory. However, in most cases, it is reasonable to take them as linear as the apogee of the cylinder is approached. Poll made his measurements in that region, which approaches the laminar separation and boundary-
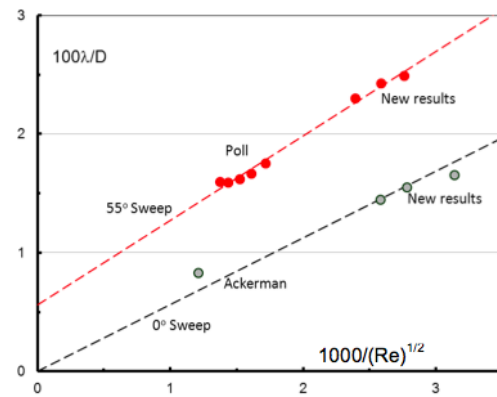

Fig. 4 Reynolds number effects on streak spacing of Poll data and present data.
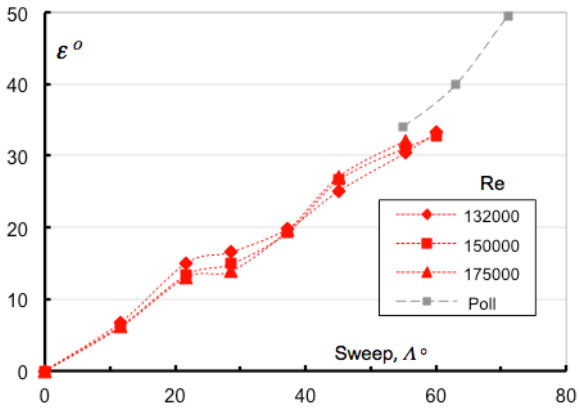
Fig. 5 Angle between streaks and normal to cylinder axis, for present results and those of Poll.

layer transition regions. For consistency, measurements from the new results are measured in the same way and are given in Fig. 5. Again it can be said that there is generally good consistency between the present results and those of Poll.

4   Conclusion

Experimental work has confirmed the suitability of the zero-sweep Kestin & Wood theory as a basis for predicting streamwise streaks and vortical structures on unswept circular cylinders and turbine blades. Testing was undertaken on a normal 38 mm diameter cylinder and a normal 152 mm diameter cylinder. The results confirm those of Kestin & Wood.

Although the Kestin & Wood work is related to unswept circular cylinders, it can also provide an excellent benchmark for sweep effects on turbine blading. Experimental work, confirming the zero-sweep results, gives a reference for subsequent work over a wide range of sweep angles. Published data on streamwise and crossflow vortices in the useful sweep range of up to 55° have been virtually non-existent. Testing on a circular cylinder has been undertaken by the authors over a range of sweep angles from zero to 61° giving results for lateral spacing and angular orientation of the vortical streaks. The results cover

the range between zero sweep and high sweep angles, and are consistent with those of Poll at high sweep angles. First order theories for circular cylinders predict the effects of sweep surprisingly well. It is the small discrepancies between theory and flow visualization that are likely to point to the occurrence of changes in the instability mode.

It is clear that the conventional view of two-dimensional laminar boundary layers following blunt leading edges is not realistic. Such boundary layers need to be treated three dimensionally, particularly when sweep is present. This calls for a sufficiently fine spanwise spacing that streamwise vortical structures are resolved. Application of computational methods to these problems is likely to be very expensive. It is hoped that until such computations become feasible for turbine design purposes the work reported here might be useful. Furthermore, a direct extension of Kestin & Wood's original stability analysis to the three-dimensional boundary layer arising from non-zero sweep is planned; this is hoped to complement the work presented in this paper.

The change in dominant instability mode with sweep angle has interesting parallels to observations of the transitional region in the boundary-layer flows over rotating cones. In particular, the experimental studies of cones with slender half-angles (rotating in either still fluid or uniform axial flows) by Kobayashi and co-workers [13, 14] show the existence of pairs of contra-rotating vortices. The vortices are known to arise from a dynamic instability induced by the centrifugal force of the flow field, and are qualitatively similar to observations of unswept cylinders. However, as the half-angle is increased, visualizations clearly show that these vortices change from pairs of contra-rotating vortices to co-rotating crossflow vortices, as observed on rotating disks, for example. These are qualitatively similar to the vortices observed over cylinders with sufficient sweep. The change in dominant mechanism is seen to occur over cones with half-angles in the range of 40 to 50 degrees. This is comparable to the apparent critical sweep angle for the change in dominant mode over swept cylinders, as reported by Kohama [7].

Although the rotating system clearly represents a fundamentally different system to the stationary, swept cylinder, the parallels drawn between high-level experimental observations of the two systems mean that the ongoing analysis of the rotating-cone system by Garrett and co-workers [15–17] could provide useful insights into the swept-cylinder problem.

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