RESEARCH ON OPTIMIZING ASCENT TRAJECTORY OF AERO-SPACE PLANE BASED ON COMBINED CYCLE PROPULSION

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Abstract

The ASP (Aero-Space Plane) ascent trajectory optimization is a difficult problem due to the complexity of operating the combined cycle propulsion and the absence of the enough and exact aerodynamic characteristic data during the process of calculating the dynamics model in order to achieving cargo delivery to LEO orbits.

In this paper, to acquire the optimizing ascent trajectory of a hypothetical ASP, it is established that the flight dynamics model of the ASP which can take off and land horizontally; The method of optimizing the ASP's ascent trajectory is designed by adjusting some parameters such as the flight-path angle, the angle of attack, and the cycle work models of the turbine and rocket-ramjet combined, etc. Finally, the optimizing simulation of the ascent trajectory has been finished based on the aerodynamics and propulsion data which produced in other projects, while it is obtained that the primary results about the optimizing research on the ascent trajectory.

It is very important for the ASP that the flight process from takeoff to launch payload, because the ASP will consume a majority of the fuel in the flight progress. The trajectory of the ASP in a certain flight process is the most primary factor which affects those parameters consisting of the takeoff total weight, the ratio of thrust to weight, the ratio of lift to drag, the engine's thrust force, and the engine's fuel consuming amount per unit time, etc. Therefore, it is an indispensable approach to determine those parameters for the ASP's concept design that the optimizing research on the ASP's ascent trajectory.

Nomenclature

Y	aerodynamic lift force
D	aerodynamic drag force
T	thrust force
F_x	horizontal force acting on vehicle
F_z	vertical force acting on vehicle
G	gravity
H	altitude
Ma	mach number
S	reference area
C_Y	lift coefficient
C_D	drag coefficient
q_c	dynamic pressure
g	acceleration due to gravity
m	mass of the ASP vehicle
a_x	horizontal acceleration acting on vehicle
a_z	vertical acceleration acting on vehicle
f	ground friction drag force
α	angle of attack
φ	flight-path angle

1 Introduction

The calculation of the ASP's optimal ascent trajectories is a challenge due to the high performance requirements (the altitude and velocity will be usually required to achieve 100 kilometers and mach number 10, respectively), the complicated vehicle characteristics (the hypersonic speed, the thermal effects, and the interaction between the propulsion system, the aerodynamics, and the structural dynamics, etc.), especially due to the air-breathing engines (turbine, ramjet, and scramjet, etc.), the various constraints (the payload, the takeoff weight, and the time consumption, etc.) and the significant passage through atmospheric regimes (troposphere, stratosphere, and mesosphere, etc.) before final ascent to the orbit or sub-orbit.

Nevertheless it is desirable to include into this hard optimization process simultaneously some phases. First, it is fixed that the takeoff weight and the payload separation conditions (the altitude and velocity); Second, an ascent optimization can be carried out by adjusting alternately the flight path angle, the angle of attack, and the propulsion modes of the ASP so that the vehicle can achieve an ascent flight with a positive acceleration; Finally, the total fuel consuming amount will be figured out and verified whether the ratio of the empty weight (the vehicle weight excluding the fuel and payload weight) to the takeoff weight(or the ratio of the fuel total weight to the takeoff weight) belongs to an reasonable extent. The approach presented here is especially suited for the development phase of a future Aero-Space Plane.

The development of an MDO strategy for solving a Two Stage Launch Vehicle (TSLV) conceptual design problem is presented in [1]. Applying realistic models for the vehicle dynamics and the unsteady heat transfer, the trajectory optimization is used to construct solutions for minimizing the fuel consumption and reducing the thermal loads in [2]. A heuristic technique based on a genetic algorithm is applied to calculate minimum-fuel-minimum-heat ascent control settings for an energy state model of an aerospace plane in [3]. Preliminary Multidisciplinary Optimization techniques have been developed to assist with the rapid modeling of options in order to obtain the best solutions for the constraints established in [4]. A simultaneous stage separation and trajectory optimization to a realistic model of a hypersonic two-stage space vehicle system is presented in [5].

The paper is organized as follows. Section 2 introduces the conceptual design and mission profile of the ASP. The method of optimizing the ASP's ascent trajectory based on the simplified dynamics model is presented in Section 3. Section 4 talks about the results and discussion of a typical simulation example. Section 5 summarizes the conclusions.

2 The Conceptual ASP

A hypothetical example of the ASP is shown in Fig.1. The ASP, with the Combined Cycle Propulsion consisting of four Turbine engines and six Rocket-Ramjet engines, can take off and land horizontally, is able to achieve both hypersonic (mach number is 6 to 8) flight in Near-Space (altitude is 30 to 100 kilometers) and cargo delivery to LEO and Polar orbits.

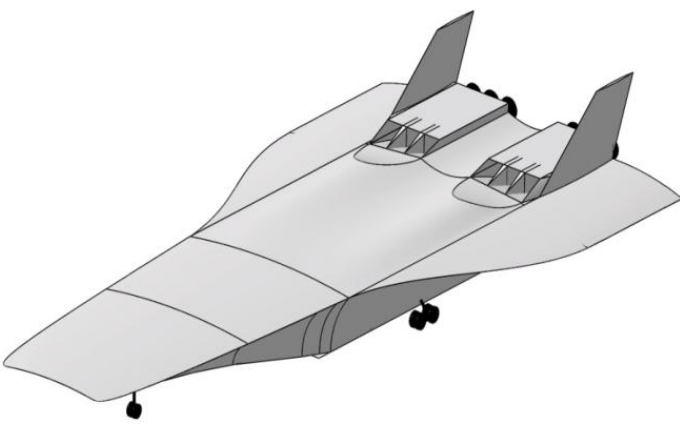

Fig.1 The Conceptual ASP

A typical mission profile is shown in Fig.2. By using those turbine engines, the ASP is capable of flying through the altitude range from 0 to 11 kilometers at the mach number which is less than 2, while the trajectory is shown by the orange line in Fig.2; Depended on those Rocket/Ramjet engines, the ASP is capable of flying through the altitude range from 11 to 100 kilometers at supersonic (mach number is 2 to 5) and hypersonic (mach number is 5 to 8), and the trajectory is shown by the red line in Fig.2; After launching the payload, the ASP can glide to the altitude which is approximately 11 kilometers without power (the trajectory is shown by the green line in Fig.2).Then, it will fly return to the airport where the ASP took off (the trajectory is also shown by the orange line in Fig.2). After separating from the ASP, the payload vehicle will achieve its orbit by the own engine (the trajectory is shown by the blue broken line in Fig.2).

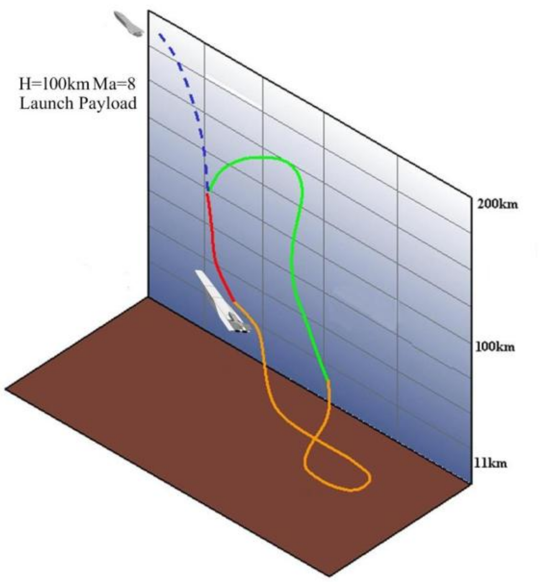

Fig.2 Typical Mission Profile of the ASP

3 Method of Optimizing ASP Ascent Trajectory

3.1 Modeling of Vehicle Dynamics

The dynamics model of the ASP has been designed based on the analysis of the forces acting on the ASP which has a flight in three phases of the ground slide, the takeoff, and the ascent flight. In Fig.3 and Fig.4, it is shown that the forces acting on the ASP which is sliding aground and taking off on the airstrip, respectively; In Fig.5, it is shown that the forces acting on the ASP which is flying in the air.

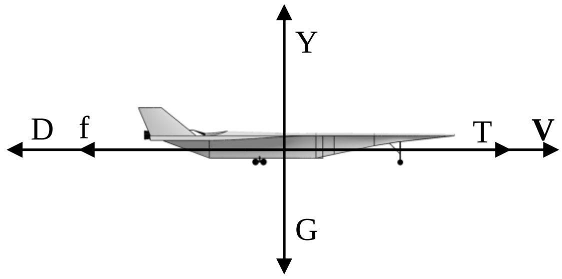

Fig.3 Forces Acting on the ASP Sliding on the Airstrip

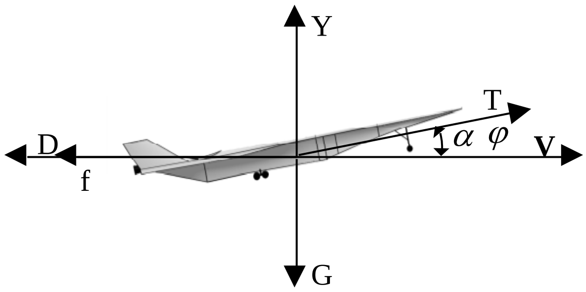

Fig.4 Forces Acting on the ASP taking off on the Airstrip

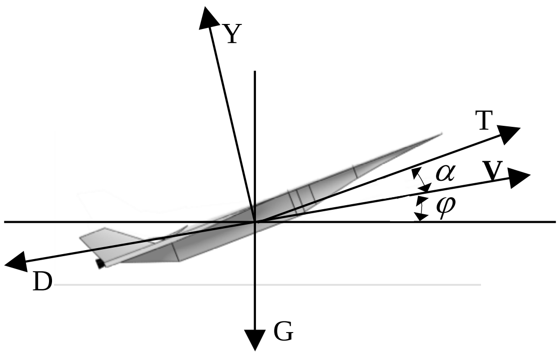

Fig.5 Forces Acting on the ASP Flying in the Air

In Fig.5, when the angle of attack α is set to equal to the flight-path angle φ, the acting force states in Fig.5 might be transformed into another acting force states in Fig.4 (except the ground friction drag f); Similarly, in Fig.4, when the angle of attack α and flight-path angle

φ are set to equal to 0, the acting force states(in Fig.4) might be transformed into another acting force states (in Fig.3); Thereby, if the ground friction drag f is transitorily ignored, the acting force states in Fig.3, Fig.4 and Fig.5 may be actually unified to the acting force states in Fig.5. To acquire an optimizing hypothetical ASP, the ascent trajectory progress always need to be repeatedly calculated. The preceding work can't provide enough data (aerodynamic coefficient and moment of inertia, etc.) for calculating the complicated nonlinear motion equations. Moreover, it is not indispensable that the accurate ascent trajectory parameters can be acquired during the progress of designing the original ASP. Thereby, the ASP's dynamics model has been simplified to make it easier to employ in this paper.

The ASP's dynamics model of the ascent phase, which is illustrated as follows:

$$F_x = T\cos(\alpha+\varphi) - D\cos\varphi - Y\sin\varphi$$

$$F_z = T\sin(\alpha+\varphi) + Y\cos\alpha - D\sin\varphi - G$$

$$G = mg \quad a_x = \frac{F_x}{m} \quad a_z = \frac{F_z}{m}$$

$$V_{x1} = V_1\cos\varphi \quad V_{z1} = V_1\sin\varphi$$

$$V_{x2} = V_{x1} + a_x \times \Delta t$$

$$V_{z2} = V_{z1} + a_z \times \Delta t$$

$$\varphi = \arctan\left(\frac{V_{z2}}{V_{x2}}\right)$$

$$V_2 = \sqrt{V_{x2}{}^2 + V_{z2}{}^2}$$

$$\Delta l = V_{x1} \times \Delta t + 0.5 \times a_x \times (\Delta t)^2$$

$$\Delta h = V_{z1} \times \Delta t + 0.5 \times a_z \times (\Delta t)^2$$

$$\Delta m = q_h \times \Delta t$$

$$Y = 0.5 q_c S C_Y(Ma, \alpha)$$

$$D = 0.5 q_c S C_D(Ma, \alpha)$$

$$T = T(PM, H, Ma)$$

$$PM = \{pm0, pm1, pm2, pm3, pm4, pm5\}$$

$$T(pm0) = T(0) = (Turbine, T_{\max})$$

$$T(pm1) = T(1) = (Turbine, T_{\text{enhanced}})$$

$$T(pm2) = T(2) = ((Turbine, T_{\text{enhanced}}) + (Ramjet, 0.5T_{ram-full}))$$

$$T(pm3) = T(3) = (Ramjet, T_{ram-full})$$

$$T(pm4) = T(4) = ((Ramjet, T_{ram-full}) + (Rocket, 0.5T_{rock-full}))$$

$$T(pm5) = T(5) = (Rocket, T_{rock-full})$$

Where Δt is the certain interval; V_1 and V_2 is defined as the initial velocity and terminal velocity over the time period of the certain interval Δt, respectively; V_{x1} and V_{x2} is defined as the horizontal portion of initial velocity and terminal velocity, respectively; V_{z1} and V_{z2} is defined as the vertical portion of initial velocity and terminal velocity, respectively; Δl and Δh is defined as the level distance and altitude over the time period of the certain interval Δt, respectively; Δm is the fuel consuming amount of the propulsion system in the certain interval Δt and q_h is the fuel consuming rate per unit time; Both C_Y and C_D are the function of mach number Ma and angle of attack α; T is the function of altitude H, mach number Ma and propulsion mode PM.

Where the propulsion mode PM is an integer set consisting of five elements ($pm0$, $pm1$, $pm2$, $pm3$, $pm4$, and $pm5$); $pm0$, $pm1$, $pm2$, $pm3$, $pm4$, and $pm5$ is set to equal to 0, 1, 2, 3, 4, and 5, respectively;

$T(0)$ and $T(1)$ is represented as Propulsion Mode which Turbine engines are providing thrust force for the ASP by Maximal Mode and Enhance Mode, respectively;

$T(2)$ is represented as Propulsion Mode which Turbine and Ramjet/Rocket engines are jointly providing thrust force for the ASP by Enhance Turbine Mode and Half Ramjet Mode;

$T(3)$ is represented as Propulsion Mode which Ramjet/Rocket engines are providing thrust force for the ASP by Full Ramjet Mode;

$T(4)$ is represented as Propulsion Mode which Ramjet/Rocket engines are jointly providing thrust force for the ASP by the Full Ramjet Mode and Half Rocket Mode;

$T(5)$ is represented as Propulsion Mode which Ramjet/Rocket engines are providing thrust force for the ASP by Full Rocket Mode.

3.2 Policy of Trajectory Optimization

It is the trajectory optimization task to find the minimal value of the ASP's takeoff weight which can satisfy the certain launch mission such as achieving 1000 kilograms cargo delivery to 200 kilometers LEO orbits or the certain payload separating condition (for example, the altitude and velocity achieve 100kilometers and mach number 8, respectively) by the iterative calculation to select a special flight trajectory.

For achieving the minimal takeoff weight of the ASP, the principal object of the trajectory optimization is confirmed as minimizing the fuel consumption and the thrust demand.

The total fuel consuming amount G_f is defined as the sum of all Δm during the entire flight process. The fuel weight coefficient k_f is defined as the ratio of G_f to G_z (the takeoff weight). G_F is defined as the acquirable minimal G_f which may meet the requirement that the ASP achieves the certain payload separating condition (the actual flight altitude H and mach number Ma achieves the payload's launch altitude H_L and mach number Ma_L, respectively) by flying along the special flight trajectory and k_f does not exceed the reasonable fuel ratio k_F based on some interrelated experiences.

It is adopted as the approach for minimizing the fuel consumption that the thrust demanded is designed to achieve as low as possible during the entire flight process. However, the relation between the thrust demanded and the flight trajectory may be bridged by adjusting the flight path angle φ and the angle of attack α.

Therefore, during the entire flight process consisting of three phases (the ground slide, the takeoff, and the ascent flight, etc.), in every Δt, one step trajectory optimization can be carried out by adjusting alternately the path flight angle, the attack angle, and the propulsion modes of the ASP so that the vehicle can achieve an acceleration flight.

The ASP has the initial flight altitude H_0, mach number Ma_0 and mass m in the certain Δt. First, the thrust of engines T is fixed by selecting the propulsion modes PM as low as possible; Second, the path flight angle φ is fixed as large as possible; Third, under the certain T and φ, the attack angle α is selected as small as possible and the horizontal acceleration a_x and vertical acceleration a_z acting on vehicle are calculated by the dynamics model of the ASP formulated in the preceding chapter ; Finally, it is verified whether the value of a_x and a_z is positive. When a_x and a_z is positive, one step trajectory optimization is accomplished and the ASP will be allowed to have a flight in Δt by adopting the select $T_{\min}$, $\varphi_{\max}$, and $\alpha_{\min}$; Otherwise the new α is selected by increase a fixed amount and a newly calculation of a_x and a_z is carried out until a_x and a_z is positive or α achieves maximum.

When α achieves maximum, but a_x and a_z is still negative or zero, the new φ is selected by decrease a fixed amount and a newly calculation of a_x and a_z is carried out until a_x and a_z is positive or α achieves maximum(α is selected from small to large in the same range).

When φ achieves minimum, but a_x and a_z is still negative or zero, the new T is selected by increase a grade PM and a newly calculation of a_x and a_z is carried out until a_x and a_z is positive or the PM achieves maximum(φ is selected from large to small in the same range, while α is selected from small to large in the same range).

When PM achieves maximum, but a_x and a_z is still negative or zero, one step trajectory optimization is broken down and it is proved that one or some of the ASP's primary parameters (such as the takeoff weight G_z, the ratio of thrust to weight T/W, and the reference wing area S, etc.) must be modified repeatedly until the ASP concept can satisfy the requirement of achieving an ascent flight.

When the ASP achieves to the certain payload separating condition (the payload's launch altitude H_L and mach number Ma_L) by a series of one step trajectory optimizations, the total fuel consuming amount will be figured out and verified if the ratio of the fuel total weight to the takeoff weight belongs to an reasonable extent. If the verifying result is positive, the entire trajectory optimization is accomplished and the optimal value of the ASP's primary parameters is obtained; Otherwise, it is indicated that the ASP's primary parameters still need to be modified repeatedly until the entire trajectory optimization is accomplished.

Moreover, it is also an optimizing object that the level distance should be as short as possible when the ASP achieves to the certain payload separating condition. L_d is defined as the sum of all Δl during the entire flight process. L_D is defined as the acquirable minimal L_d which may meet the requirement that the ASP achieves the certain payload separating condition. But L_D is the secondary object and is merely verified after the entire trajectory optimization has been accomplished.

The policy of selecting the optimal trajectory is formulated as follows:

$$G_f = \sum \Delta m, k_f = {G_f}/{G_z}$$

$$G_F = \min G_f((H \geq H_L, Ma \geq Ma_L) : k_f \leq k_F)$$

$$L_d = \sum \Delta l$$

$$L_D = \min L_d((H \geq H_L, Ma \geq Ma_L))$$

$$T_{\min} = T(\min PM)((H_0, Ma_0) : a_x \geq 0, a_z \geq 0)$$

$$\varphi_{\max} = \max \varphi((H_0, Ma_0, T_{\min}) : \alpha_{\min} \geq 0)$$

$$\alpha_{\min} = \min \alpha((H_0, Ma_0, T_{\min}) : a_x \geq 0; a_z \geq 0)$$

3.3 Architecture of Trajectory Optimization

The trajectory optimization algorithm based on the preceding policy works as in Fig.6:

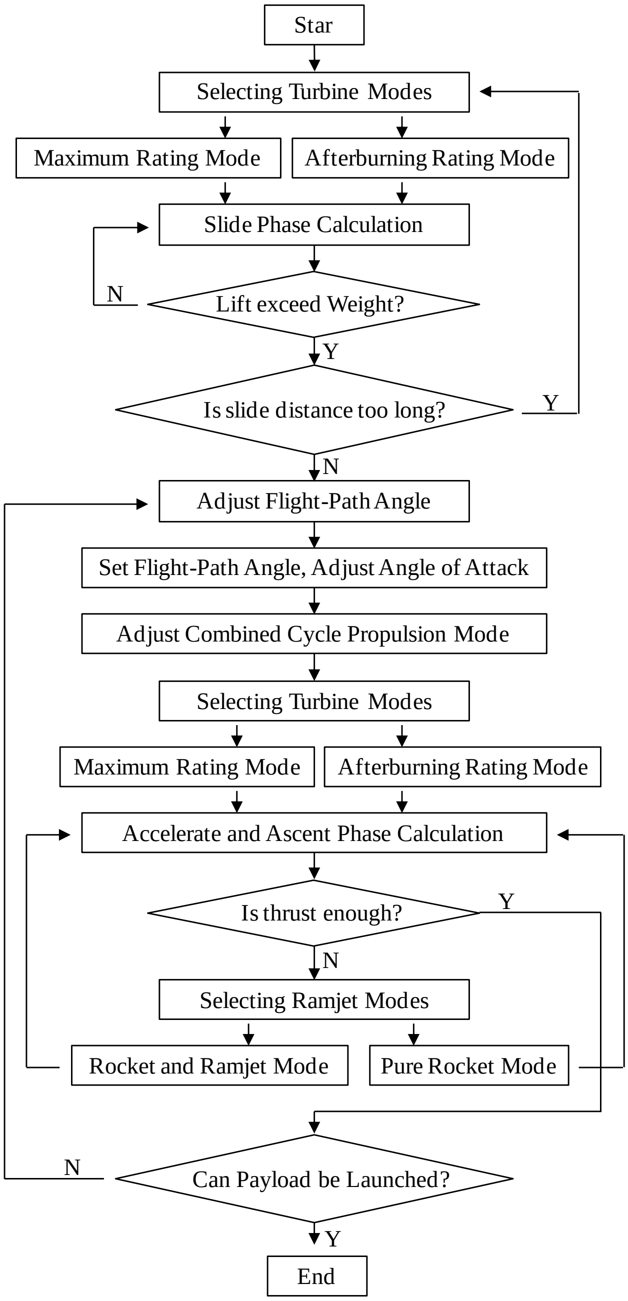

Fig.6 Architecture of Optimizing Calculation

In this paper, the trajectory optimization algorithm process (in Fig.6), which is embodied as follows:

ASP_ Trajectory – Optimizing ( )

- Initialize (H_L, Ma_L) based on mission
- Initialize $G_{payload}$ based on mission
- Introduce (C_Y, C_D) from databases
- Repeat (for Trajectory Optimization)
  - Initialize G_z in an experiential range
  - Select $(T/W, S)$ in an experiential range
  - Introduce $T(PM)$ from databases
  - Repeat (for one step optimization)
    - * Repeat (for Slide Phase optimization)
    - * Chose Turbine PM
    - * Calculation dynamics model of ASP
    - * until $(Y \ge G, and, L_d \le L_{slide})$
    - * Repeat (for Ascent optimization)
    - * Chose Combined Propulsion PM
    - * Adjust (φ, α)
    - * Calculation dynamics model of ASP
    - * until $(a_x \ge 0, a_z \ge 0)$
  - until $(H \ge H_L, Ma \ge Ma_L)$
- until $(H \ge H_L, Ma \ge Ma_L, k_f \le k_F)$
- return $(\min G_z, k_f, optimal\_trajectory)$

4 Results and Discussion

As the simulation states of the trajectory optimization, the range of 50,000 to 150,000 kilograms is considered for the takeoff weight of the ASP and is divided into 100 select-value steps by 1000 kilograms; The range of 0 to 50 degrees is considered for the flight path angle of the ASP and is divided into 50 select-value steps by one degree; The range of -2 to 20 degrees is considered for the attack angle of the ASP and is divided into 45 select-value steps by 0.5 degree; The aerodynamic characteristic (lift and drag coefficient) and combined cycle propulsion characteristic (thrust and fuel consumption) datum of the ASP is obtained from the special databases achieved by some other research projects; The fuel weight coefficient k_f is carefully set to be 60%.

The results of the trajectory optimization present that the ASP with the takeoff weight of 70,000 kilograms is able to satisfy the launch mission of achieving 1000 kilograms cargo delivery to 200 kilometers LEO orbits or the 15,000 kilograms payload separating condition (altitude and velocity achieve 100kilometers and mach number 8, respectively) by utilizing the Combined Cycle Propulsion consisting of four Turbine engines and six Rocket-Ramjet engines.

The simulation results for the 70,000 kilograms ASP are presented in the following:

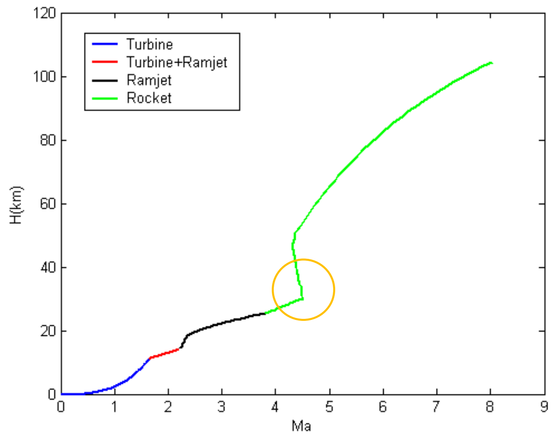

Fig.7 Altitude-Mach Number Graph of Ascent Course

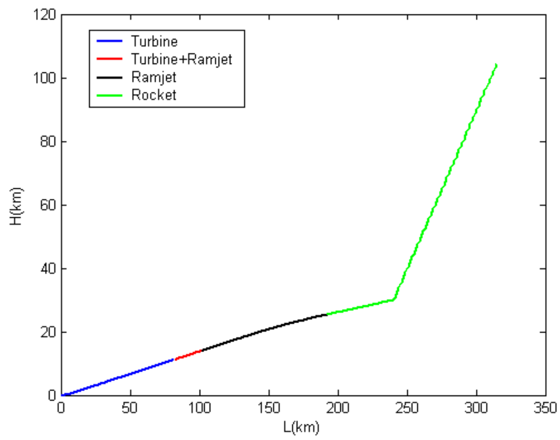

Fig.8 Altitude-Level Distance Graph of Ascent Course

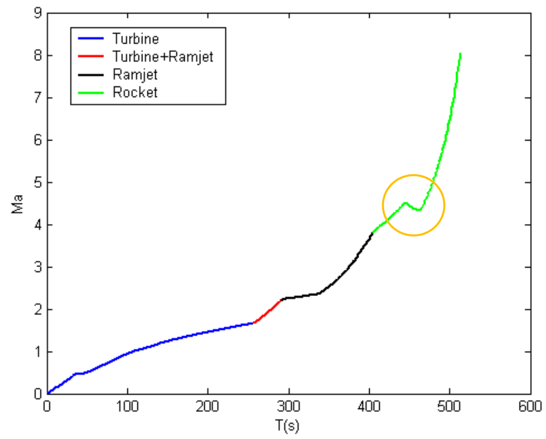

Fig.9 Time History of Mach Number of Ascent Course

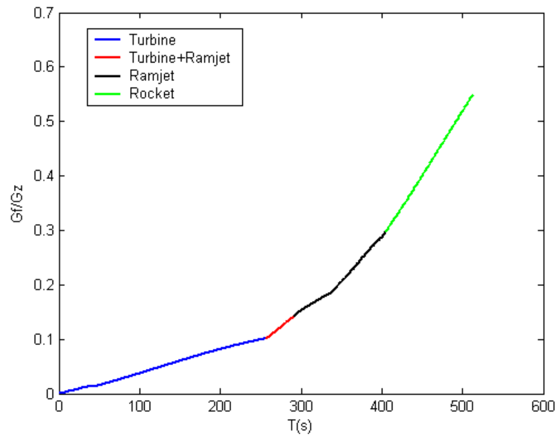

Fig.12 Time History of Fuel Consumption Rate of Ascent Course

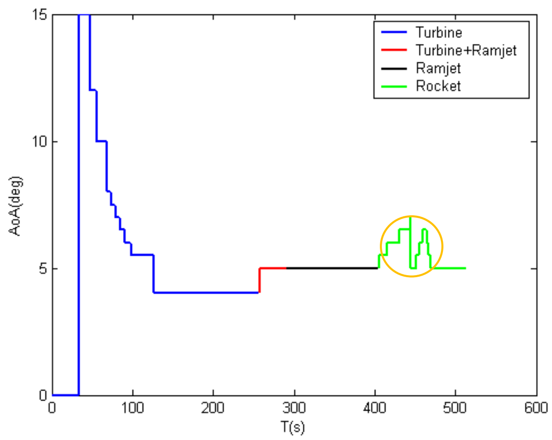

Fig.10 Time History of Attack Angle of Ascent Course

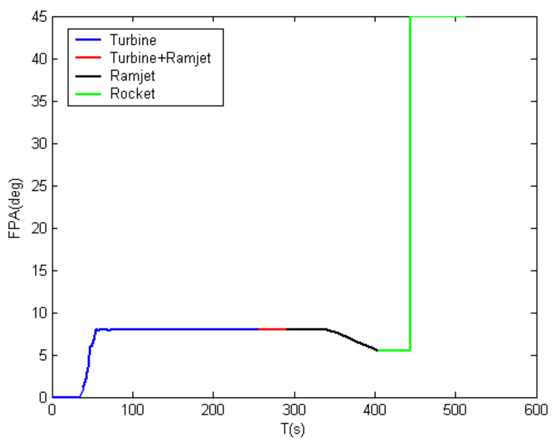

Fig.11 Time History of Flight-Path Angle of Ascent Course

Fig.7 shows the altitude-mach number graph of the ascent course; Fig.8 shows the altitude-level distance graph of the ascent course; In Figs. 9-12, the time histories for the mach number *Ma* and the attack angle *AoA* as well as for the flight-path angle *FPA* and the fuel consumption rate G_f / G_z are shown.

The relation between the altitude and the mach number is presented in Fig.7. It is obvious that the ASP can only flight to the state of the about 30 kilometers altitude and mach number 4 by the air-breathing propulsion modes (Turbine, Turbine+Ramjet, and Ramjet). Moreover, the mach number appears the visible oscillation near the mach number 4.5.

The relation between the altitude and the level distance is presented in Fig.8. There is a rapid ascent at the about 30kilometers altitude due to the use of the full rocket thrust. The level distance will exceed 300kilometers when the ASP achieves to the state of launching the payload.

The mach number time history is presented in Fig.9. The mach number appears a small oscillation near the mach number 4.5 and the time of 440 to 480 seconds. It takes the ASP about 500 seconds to fly through the entire ascent trajectory.

The angle of attack time history is shown in Fig.10. The ASP has flown by the attack angle of about 5 degrees during a majority of the ascent course. However, the ASP has to achieve flight by employing the large attack angle of 10 to 15 degrees so that the ASP's lift is enough to take off. Moreover, the attack angle appears a small undulation of about 1 to 2 degrees near the time of about 420 to 480 seconds.

The flight-path angle time history is shown in Fig.11. The ASP has flown by the flight-path angle of about 8 degrees during a majority of the ascent course. However, the flight-path angle rises rapidly to 45 degrees because the ratio of thrust to weight of the APS is suddenly augmented by using the full rocket thrust.

The fuel consumption rate shown (in Fig.12) is about 55% (less than the prescriptive value of 60%) when the ASP achieves to the state of launching the payload.

The primary parameters (Time T, Altitude H, Mach number Ma, Level distance L, Total weight G, and Propulsion Mode) of the ASP's ascent flight course are shown in Table.1.

Table 1. Primary Parameters of Ascent Course

T (s)	H (km)	Ma	L (km)	G (kg)	Propulsion Mode
34	0.000	0.439	2.425	69146	Turbine
258	11.296	1.674	83.335	62793	Turbine + Ramjet
292	14.078	2.209	102.582	59937	Ramjet
354	20.030	2.585	144.877	55146	Ramjet
406	25.427	3.829	193.086	49028	Rocket
444	29.897	4.500	240191	42973	Rocket
465	50.578	4.371	260.927	39473	Rocket
475	60.944	4.792	271.331	37806	Rocket
495	82.969	6.034	293.437	34473	Rocket
513	104.528	8.031	315.067	31473	Rocket

Table 1 shows the results for the optimal flight trajectory. The ASP can't take off until the mach number achieves 0.439 (the value is very large when comparing the ASP with the general fighter) due to the inherent aerodynamic characteristic of the kind of vehicle. The ASP may achieve the state of the 25.427 kilometers altitude and mach number 3.829 by the air-breathing propulsion modes. It takes the ASP (pushed by the rocket engines) 107 seconds to achieve the state of launching the payload (the 104.528 kilometers altitude and mach number 8.031).

It is noteworthy case that the mach number appears oscillation from 4.500 to 4.371 and next to 4.792 during the ascent flight course from 444 to 475 seconds (in Fig. 9 and Table 1). During the course from 444 to 475 seconds, the ASP has flown through the 30 kilometers altitude limit in which the increase of the flight velocity is slower than that of the acoustic velocity. Thereby, when the flight velocity of the ASP increases, the mach number of the ASP will decrease on the contrary.

5 Conclusions

In this paper, the optimization of an ascent trajectory is considered with the objective to minimize the fuel consumption and to reduce the thrust demand for the ASP equipped with a turbo/ramjet/rocket engines combination. For minimizing the fuel consumption, the thrust demanded is designed to achieve as low as possible during the entire flight process by adjusting the path flight angle and the attack angle. Because the preceding work can't provide enough data for calculating the complicated nonlinear motion equations, the ASP's dynamics model has been simplified and formulated.

By the optimizing simulation in this paper, it has been proved that the ASP, with the Combined Cycle Propulsion consisting of Turbine and Rocket/Ramjet engines, can carry 15,000 kilograms payload to a point of 100 kilometers altitude and Mach 8. The above result benefits from the optimizing research on the ASP's ascent trajectory by adjusting the flight-path angle, and the angle of attack, and the modes of the Combined Cycle Propulsion.

In this paper, the optimizing simulation adopts the time step of one second so that some crucial details in the flight process can be perceived. For example, in Fig.7, when the ASP arrives at the 30 kilometers altitude, the mach number begins to diminish until the altitude exceeds 50 kilometers. The detail is very helpful to analyze the flight capability and to estimate the rationality of the ASP's design parameters.

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