THE NEW OPTIMALITY CRITERIA FOR STRUCTURAL DESIGN WITH CONSTRAINING ELASTIC DISPLACEMENTS (NON-COMPOSITE FORWARD SWEPT WING WITHOUT DIVERGENCE)

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Abstract

In the requirements of static aeroelasticity the constrained deformations usually are the torsional deformations of along-stream wing sections, because the twist values determine angles of attack and consequently aerodynamic forces. Thus, the bending deformations have no matter; this circumstance opens new capabilities for optimization of structures swept and delta wings, for which one the bending and torsional deformations are interdependent. So, for example, loosening a trailing edge of a delta wing in a direction of span and allowing thereby large vertical displacement, it is possible to reduce the twisting of along-stream sections. Naturally weakening is necessary to implement not to the detriment of safety.

We suggest the method which have selectivity to kinds of deformations and reliably determine not only area of a structure, where it is necessary to add a material for reaching demanded stiffness, but also area where it is possible to weaken structure, for increasing of demanded (useful) deformations.

1 Basic Theory

Consider the search for a structural design with minimal volume (mass) with a generalized displacement constraint at a single point on the structure. To develop our constraint relationship, we first apply a unit generalized force (unit load) in the direction of the constrained displacement (either a displacement or a rotation such as twist). We use the Maxwell-Mohr[1] formula to calculate this constrained displacement, that called Δ. We first assume that the design is broken into m finite elements, each in a plane stress condition, so that

$$\Delta = \sum_{i=1}^{m} \int_{S_i} \frac{[R_i^*]}{E_i \delta_i} dS_i ,$$

$$[R_i^*] = [\bar{R}_{xi}(R_{xi} - \mu_i R_{yi}) + \bar{R}_{yi}(R_{yi} - \mu_i R_{xi}) + 2(1+\mu_i)\bar{R}_{xyi} R_{xyi}]. \quad (1)$$

Here $\bar{R}_i = \bar{\sigma}_i \delta_i$ represent internal forces in element i due to the unit loading; $R_i = \sigma_i \delta_i$ represents internal forces in element i due to the applied loading; δ_i is the thickness of the element; S_i is the planar area of the element; E_i and μ_i are the modulus of elasticity and Poisson's ratio of the element material.

The internal loads due to the unit generalized force and applied loading are called $\bar{R}_i$ and R_i; they are determined by the usual finite element procedure. If finite elements with a constant stress field are used, Eqn. 1 becomes:

$$\Delta = \sum_{i=1}^{m} \frac{[R_i^*] S_i}{E_i \delta_i}. \quad (2)$$

For thin-walled structures whose elements are in a plane stress condition, the volume of material is written as

$$V = \sum_{i=1}^{m} S_i \delta_i . \quad (3)$$

We minimize this volume under the condition that

$$\Delta = \Delta_o \quad (4)$$

and $$V = \sum_{i=1}^{m} S_i \delta_i \Rightarrow min \quad (5)$$

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[1] This is also referred to as the unit load or dummy load method.

where Δ_o is the given value of the generalized displacement.
The elements of the series in (2) provide the contribution of each element to the displacement, which is constrained. If the value $[R_i^*]$ is large, the displacement is largely determined by deformations of the i element. If the size $[R_i^*]$ is **negative** then reducing the material volume or modulus of elasticity will reduce the deflection. Because the unit generalized force is in the direction of the constrained deformations, a negative value of the Mohr integral shows where it is necessary to reduce the volume of an element.
For an illustration of this method, consider a cantilever beam loaded with two forces, as shown in Fig. 1. The tip bending displacement angle θ in Fig. 1 is to be constrained to be zero.

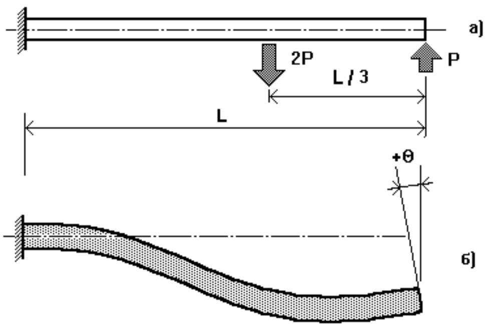

Fig. 1. Cantilever beam.

First apply a unit bending moment $\overline{M}=1$ to the end of the beam in the $+\theta$ direction and find the Mohr integral, defined in this case as

$$I_M = \int_L \frac{\overline{M}M}{EI} dz \qquad (6)$$

Let's assume for simplicity that EI = *constant*. By multiplying the distribution of the moments from the applied and unit load cases we find the distribution of Mohr integrals over the length of the beam shown in Fig. 2.
From the distributions in Fig. 2 it is seen that the reduction in bending stiffness between points 2 and 3 will cause an increase the angle θ, while the reduction of stiffness in site 1 will result in a reduction of θ.

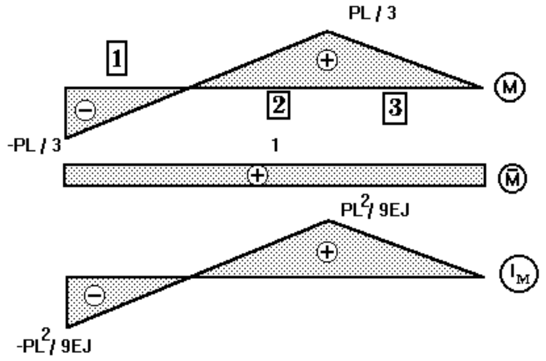

Fig. 2. Internal loads and Mohr integral values

The angle θ of the tip section can be reduced by stiffening or strengthening zone 2-3, or de-stiffening zone 1, or taking these actions simultaneously.

Negative values of the Mohr integrals always identify design zones that are can be weakened to satisfy displacement constraints.

From (2) we see that in these regions it is advisable to choose the minimum thickness allowed for strength, construction or other technological reasons. In formula (2) we collect all terms bearing a negative term so that the equation is written as

$$\Delta = \sum_{i=1}^{n} \frac{[R_i^*]\, S_i}{E_i\, \tilde{\delta}_i} - \sum_{i=n+1}^{m} \left| \frac{[R_i^*]\, S_i}{E_i\, \delta_i} \right| \qquad (7)$$

Here: δ_i is the minimum allowable thickness of elements with negative values of the Mohr integrals; $\tilde{\delta}_i$ is the thickness of elements for which the Mohr integrals are positive; n is the number of elements where the Mohr integrals are positive.
Let's define two terms

$$\Delta^+ = \sum_{i=1}^{n} \frac{[R_i^*]\, S_i}{E_i\, \tilde{\delta}_i}, \qquad (8)$$

$$\Delta^- = \sum_{i=n+1}^{m} \left| \frac{[R_i^*]\, S_i}{E_i\, \delta_i} \right|. \qquad (9)$$

Then

$$\Delta = \Delta^+ - \Delta^- . \qquad (10)$$

The condition in equation (4) will look like

$$\Delta^+ = \Delta_o + \Delta^- . \qquad (11)$$

If, in zones with negative Mohr integrals, the minimally allowable thicknesses are used, the thickness of these elements are eliminated as design variable and we find optimum distribution of a material only in zones with positive Mohr integrals. Thus, we have a task of conditional optimization: to minimize volume of a material of a design

$$\tilde{V} = \sum_{i=1}^{n} S_i \, \tilde{\delta}_i \Rightarrow min \tag{12}$$

under the condition in Eqn. 11.

We will use LaGrange multipliers to find the solution. Let's define a function

$$L = \sum_{i=1}^{n} S_i \, \tilde{\delta}_i + \lambda_1 (\Delta^+ - \Delta_o - \Delta^-), \tag{13}$$

where λ_1 is the LaGrange multiplier. The conditions of a minimum of the function are:

$$\frac{\partial L}{\partial \tilde{\delta}_i} = 1 - \lambda_1 \frac{[R_i^*]}{E_i \, \tilde{\delta}_i^2} = 0, \; i = 1,2,...n; \tag{14}$$

$$\frac{\partial L}{\partial \lambda_1} = \Delta^+ - \Delta_o - \Delta^- = 0. \tag{15}$$

From Eqns. 14 and 15 we have

$$\tilde{\delta}_i = \sqrt{\lambda_1 \frac{[R_i^*]}{E_i}} \tag{16}$$

Substituting (16) into equation (15) we find the LaGrange multiplier

$$\lambda_1 = \frac{\left\{ \sum_{i=1}^{n} S_i \sqrt{\frac{[R_i^*]}{E_i}} \right\}^2}{\left(\Delta_o + \Delta^- \right)^2} \tag{17}$$

Equation (16), accounting for formula (17), can be written as

$$\tilde{\delta}_i = \frac{\left\{ \sum_{i=1}^{n} S_i \sqrt{\frac{[R_i^*]}{E_i}} \right\}}{\left(\Delta_o + \Delta^- \right)} \cdot \sqrt{\frac{[R_i^*]}{E_i}} \quad i = 1,2,...n.. \tag{18}$$

Formula (18) defines the "law of distribution" of a material for elements of the design ensuring the constraint on the generalized displacement with internal forces $\overline{R}_i$ and R_i.

Let's calculate the required volume of the design from (3) by substitution of $\tilde{\delta}_i$ from (18) and the minimally allowable thickness δ_i.

$$V = \frac{\left\{ \sum_{i=1}^{n} S_i \sqrt{\frac{[R_i^*]}{E_i}} \right\}^2}{\left(\Delta_o + \Delta^- \right)} + \sum_{i=n+1}^{m} S_i \, \delta_i . \tag{19}$$

Let's examine formulas (18) and (19). In the presence of zones with negative Mohr integrals, that is by $\Delta^- \neq 0$, *we have an opportunity to develop a design with a constrained zero generalized displacement*: $\Delta_o = 0$. Moreover, the satisfaction of the requirement $\Delta_o < 0$ is possible, provided that $|\Delta_o| < \Delta^-$. If the zones with negative Mohr integrals are absent, we can only reduce the existing generalized displacement.

1.1 Property of the found material distribution

Expression (14), determining minimum condition of a material volume of a structure by limitation $\Delta = \Delta_0$ we represent as follows:

$$\frac{[R_i^*]}{E_i \tilde{\delta}_i^2} = \frac{1}{\lambda_1}, \tag{20}$$

or

$$\frac{[R_i^*] \, S_i}{E_i \, \tilde{\delta}_i \, \tilde{V}_i} = \frac{1}{\lambda_1} = const, \qquad i = 1,2,...n. \tag{21}$$

Substituting values $[R_i^*]$ from (1) into (21) and taking into account, that $R_i = \tilde{\delta}_i \sigma_i$, we receive

$$\frac{\tilde{\delta}_i \, S_i}{\tilde{V}_i} [\overline{\sigma}_{xi} \frac{\sigma_{xi} - \mu_i \sigma_{yi}}{E_i} + \overline{\sigma}_{yi} \frac{\sigma_{yi} - \mu_i \sigma_{xi}}{E_i} +$$

$$+ \overline{\sigma}_{xyi} \frac{2(1+\mu_i)\sigma_{xyi}}{E_i}] = \tag{22}$$

$$= \left[\overline{\sigma}_{xi} \varepsilon_{xi} + \overline{\sigma}_{yi} \varepsilon_{yi} + \overline{\sigma}_{xyi} \varepsilon_{xyi} \right] = const,$$

$i = 1,2,...n.$

From here it is visible, that the equation (14) determines the relevant property of the created in according with the formula (18) project, namely: ***the specific energy of internal forces from unit loading, on deformations from real loading should be constant in all structure elements.***

The condition (22) can also be considered as some generalization of the known

Z.Wasiutynski theorem [1] about equality of a specific potential energy of deformation in all elements most of rigid structures made from a given volume of a material.

In an optimal structure with given value of generalized displacement, in zones with positive Mohr integrals the specific energy of internal forces from unit loading on deformations from a real loading should be constant in all elements of structure.

In zones with negative Mohr integrals it is necessary to permit structures to be deformed as much as possible, for what to use in these places materials with low elasticity modulus and large allowable deformations.

2 Theory application

An aeroelastic stiffness requirement such as absence the divergence, can be posed, approximately, as a limitation on deformation. To use optimality criteria, these constraints enter as inequalities and, in the case of wings, the measure of design stiffness is taken as an elastic displacement at a finite number of selected, “characteristic” or “typical” cross-sections. The deformations of these so-called cross-sections and their aerodynamic properties for our needs are representative of the elastic twist angle $\theta(\bar{z})$ of the structure along the wing (the coordinate $\bar{z}$ is the nondimensional distance from the lifting surface root and the typical section). The rule to choose the “typical” cross-section for individual wing is out of this paper, but that method is applicable to different wing structure type including small aspect ratio wings [2].
As an example of application optimization method we design the forward swept wing structure with constraint on twist angle of the wing section placed on $\bar{z}$=0.81. We choose the constraint $\Delta_o \leq 0$ to exclude the divergence in flight. As prototype we consider the wing with the form in plane from airplane Sukhoi S-47 “Berkut” but having another sizes and made from isotropic material, see Fig. 3.

Fig. 3. Fighter S-47 “Berkut”.

The applied loading has the forward center of pressure and corresponds to a flight situation with the maximum g-loading. Loading during the optimization was defined with taking into account elastic features of wing. As initial material distribution we take the identical skin thickness equal 1mm, wall of ribs also have the thickness equal 1mm. Wall of spars have the thickness 2.5mm. Spars and ribs cap equal 10mm^2. Wing deformation with the initial material distribution is shown on Fig. 4.

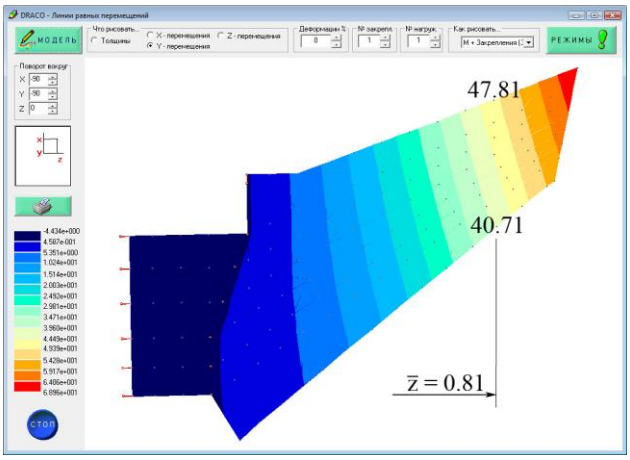

Fig. 4. The displacements of initial structure.

Optimization results are shown on Fig. 5,6,7.

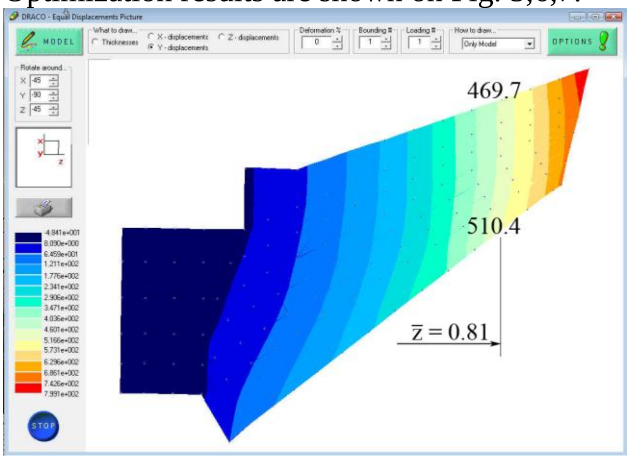

Fig. 5. The displacements of optimal structure.

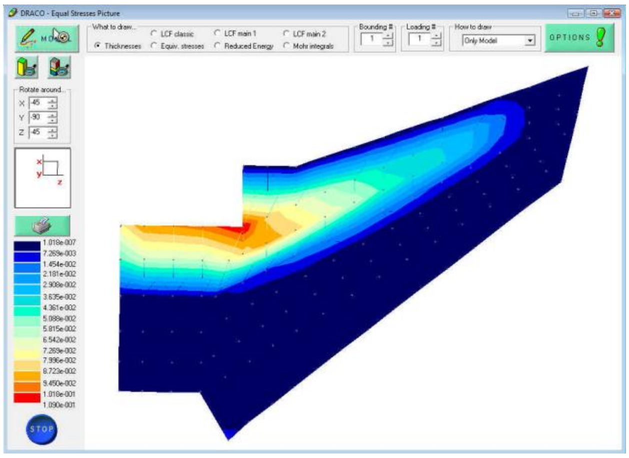

Fig. 6. Skin thickness distribution.

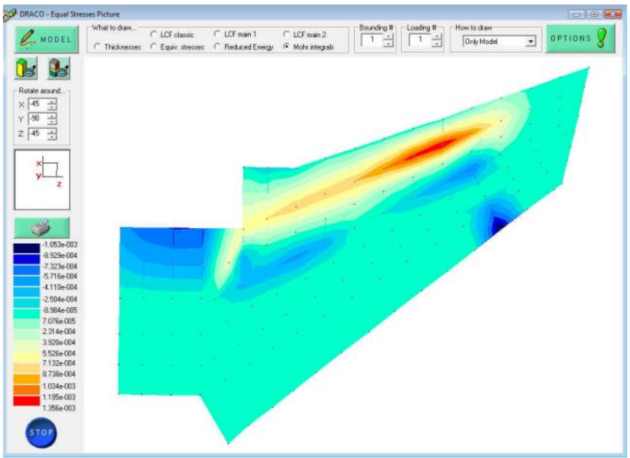

Fig. 7. Mohr integrals distribution in the skin.

It can be seen from fig. 5 that elastic twist angles $\theta(\bar{z})$ of the aerofoil are negative along the wing.

Therefore the divergence of such wing is impossible.

Fig. 6 shows that it is necessary to increase the skin thickness only at the leading edge where Mohr integrals are large positive, see fig. 7.
By the way, achieved result supposes the using in structure traditional, well known materials.

2.1 New wing structures

The new optimality criteria enable to control structure deformations at the designing stage and to develop airframes in which elastic displacements of one part structure are controlled by material distribution in another part of structure. Such control effectiveness shows Mohr integral distribution.
For airplane with high cruise speed and good takeoff and landing characteristics we suggest the combined wing shown on fig.8.

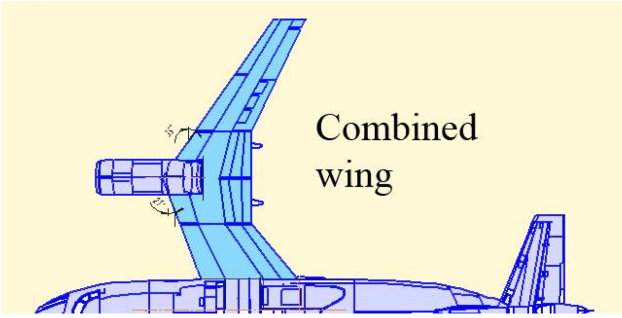

Fig. 8. The combined wing.

Here the back swept wing part controls the deformations of the forward swept part of structure and the whole structure has in flight the elastic deformation shown on fig. 9 (after optimization).

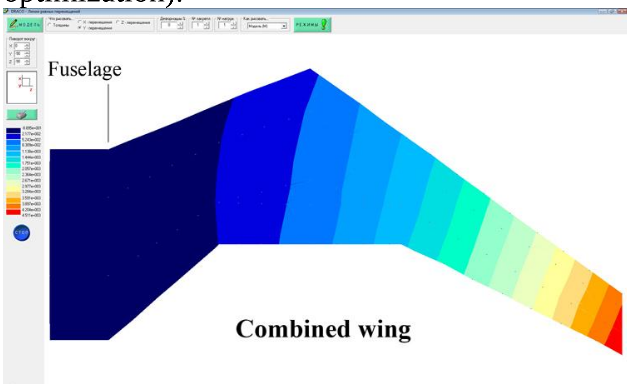

Fig. 9. The displacements of combined wing.

In particular regarding material distribution, on fig. 10 is shown the optimal skin thicknesses.

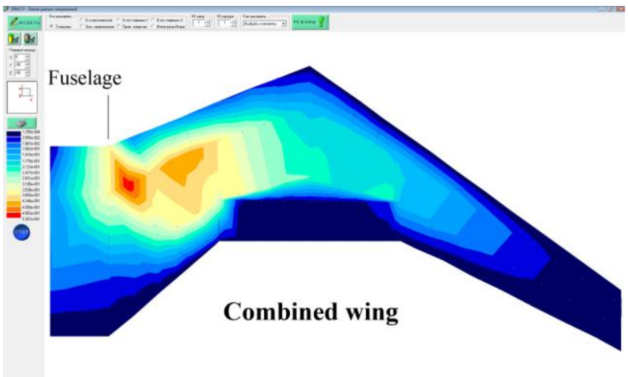

Fig. 10. Upper skin thicknesses.

Another type of combined wing is represented on fig. 11. This turbo propelled aircraft has high cruise speed (low local shock wave drug due swept parts of wing) and good takeoff and landing characteristics (due forward swept and unswept parts).

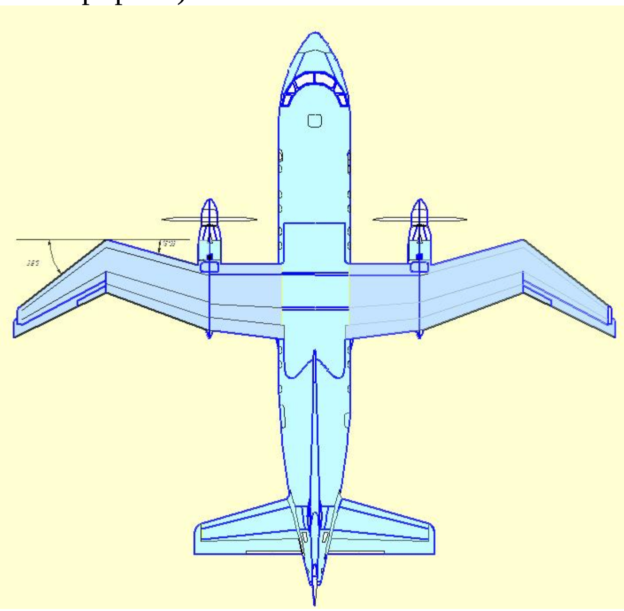

Fig. 11. Airplane with combined wing.

This wing has deformation in flight shown on fig. 12. It can be seen, that elastic twist angles of the aerofoil along the wing are zero.

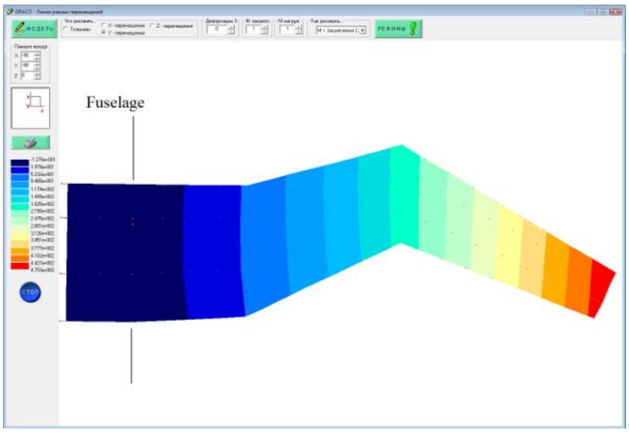

Fig. 13. Elastic displacements in flight.

Here we not implement the optimization of material distribution. These deformations meet the typical material distribution for torsion-box type wing with skin thicknesses which is shown on fig. 13.

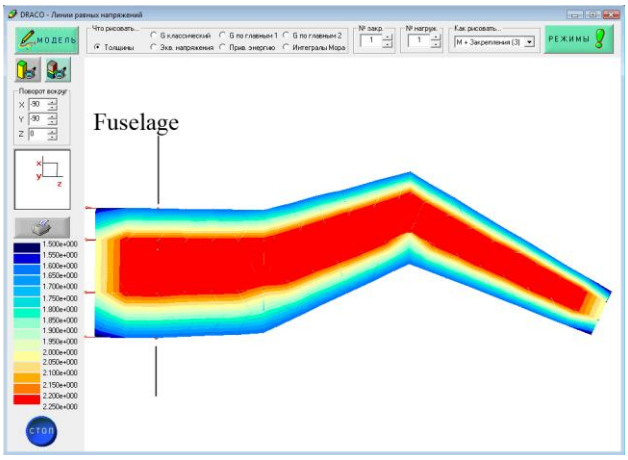

Fig. 13. Upper skin thicknesses.

3 Resume

The new optimality criteria enable to get the optimal structures with required deformations not only by increasing structure stiffness but also by decreasing some part stiffness of structure. This allows find the high effective structures.

References

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