SIZE EFFECT OF FAILURE MODES IN COMPOSITE LAMINATES UNDER COMPRESSION

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Abstract

The size effect of interfacial failure and buckling in composite laminates under uniaxial compression are investigated in this work. It is found that interfacial failure by excessive separation (normal mode) dominates for short composite laminates; as the length/thickness of a composite laminate increases, shear failure in the interface dominates, accompanied by buckling of the laminate. When the interface of a composite laminate fails, corresponding critical load to trigger buckling decreases. The size effect of failure modes in composite laminates is divided into six regions governed by the length to thickness ratio of composite laminates. The critical length to thickness ratios of each region are estimated using both theoretical analysis and finite element simulation. At the end, a failure mode map based on stress and length/thickness ratio relation is constructed.

1 General Introduction

Structural failure in composite materials, including interfacial cracking and buckling, is of primary concern for engineers working in this field. Broad applications of composites in aerospace, automotive, and many other green energy projects necessitate our further investigation of failure behavior in composites. Previous efforts are focused on the buckling of composite laminates, i.e., development of composite plate buckling [1,2], post-buckling problems [3,4], and finite element simulations on post-buckling behaviors in composite materials [5-7].

Recent success of cohesive zone modeling (Barenblatt [8] and Dugdale [9]) for interfacial failure and crack propagation further our understanding on interfacial failure in composite materials. An important characteristic of this methodology for modeling fracture initiation and crack propagation is that macroscopic fracture criteria based on elastic or elastic–plastic analyses, such as K_{IC} or J_{IC}, are characterized by the local traction–separation relation. Propagation of cracks between different layers in composite materials will be automatically determined by local deformation status if cohesive elements are embedded initially. There are several model cases of interfacial failure in composites, including fiber/matrix interface [10,11] and delamination [12-16], cohesive zone length effect (Turon[17] and Harper[18]), local buckling (Hu[19]), and post-buckling behaviors as well (Hwang[20]and Cappello[21]).

In this paper, we investigate the failure mode transition of composite laminate plate with different length over thickness ratio and subjected to uniaxial compression. The dimensionless length is divided into six regions according to the interfacial damage patterns and the buckling behaviors. The finite element method is used to simulate the interfacial damage evolution in each region. Based on FEM results and theoretical analysis, we obtain the critical lengths in each region. At the end, a failure stress-critical length curve is obtained, which differentiates interface damage behaviors and buckling mode at each length scale.

2 The finite element method analysis

A 2D plane strain model (Fig.1) is used to study the different interface damages corresponding to the failure mode transition of composite laminates subjected to the uniaxial compression. The thickness of the composite laminates is h=10mm. The interface is described by the cohesive element. Material properties are listed in table 1. Element types for the composite laminates and the interface are S4 and COH4 respectively. The mesh size ranges from 0.1mm to 1mm for different length l.

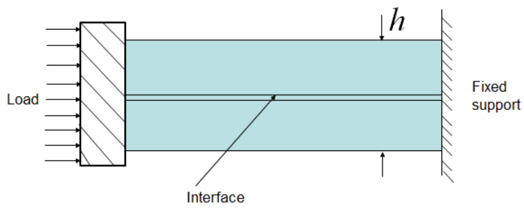


Fig.1 2D plane strain model

Table 1 The material properties

Composite laminates		Interface	
E_{11}(GPa)	135	K_{nn} (KN/mm)	90
E_{22}, E_{33}(GPa)	8.8	K_{ss} (KN/mm)	30
v	0.3	σ_f(MPa)	10
G_{12} ,G_{13}(GPa)	4.47	τ_f(MPa)	13.15
G_{23}(GPa)	3.5	G_I(mJ/mm^2)	0.252
σ_y(MPa)	800	G_{II}(mJ/mm^2)	0.665
		Criterion	quads stress

Four different length/thickness ratios R_L=h/l= 2, 7.5, 20, and 45 are chosen to represent four failure modes, respectively:

Mode (a). Interfacial normal damage with single hole,;

Mode (b). Interfacial normal damage with double holes,;

Mode (c). Interfacial shear damage firstly, and then the composite laminates is buckling,;

Mode (d). Composite laminates is buckling firstly, and then the interfacial shear damage.

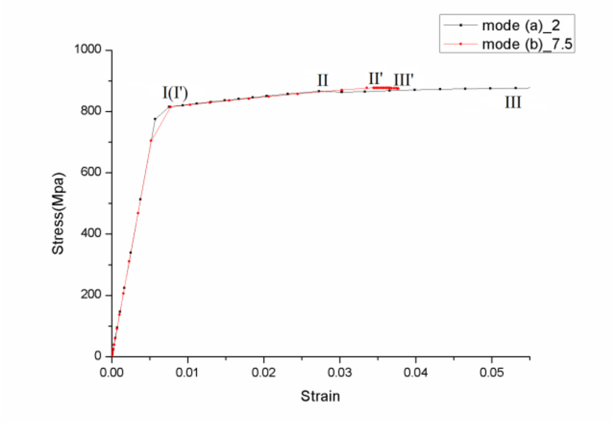


Fig. 2 Stress—strain curves of failure mode (a) and (b)

The relationship between stress and strain of failure mode (a) and (b) are showed in Fig.2. The interface would be damaged by normal separation. When the strain reaches at point I(I'), stress increases slowly for small amount of plastic deformation. When the stress approaches maximum stress at point II(II'), interfacial normal strength is reached and the interfacial cracks occur. The crack would stop the propagation until the stress reaches point III(III'). The changes from point II(II') to point III(III') and their corresponding deformation are showed in Fig.3 and Fig.4.

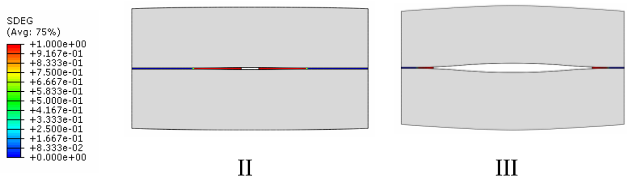


Fig. 3 Deformation of whole model at point II and III in failure mode (a)

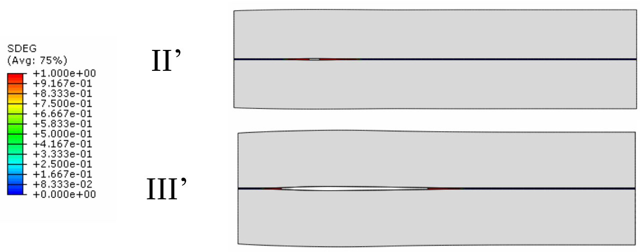


Fig. 3 Deformation of left half model at point II' and III' in failure mode (b)

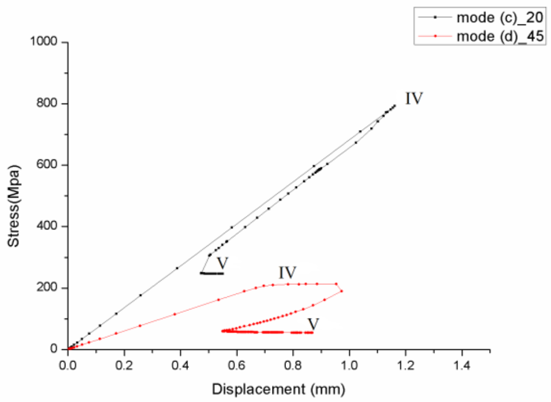
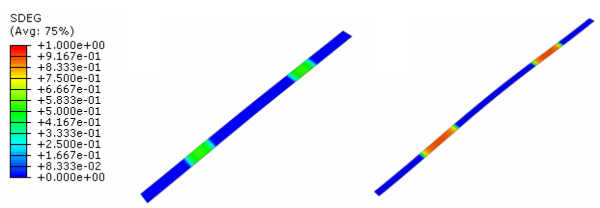


Fig. 5 Stress-strain curves of failure mode (c) and (d)

The stress-strain curves of failure mode (c) and (d) are showed in Fig.5. The interface would be damaged by excessive shearing. There are two main differences between these two failure modes: only elastic deformation is observed in mode (d); there is a cusp in the IV phase in mode (c), but a line in the IV' phase in mode (d), which means the interface is shear damaged first and the stress decrease rapidly in mode (c), while the composite laminates buckle first in mode (d). On the other hand, there are two similarities of failure mode (c) and (d): the positions of interfacial damage are same that are located at $x=l/4$ and $3l/4$, when the shear stress reaches the interfacial shear strength as shown in Fig.6. The composite laminates buckling loads at the end phase V(V') of mode (c) and (d) are a quarter of maximum loads. These two positions could be theoretically given in the next section.



Mode (c) Mode (d)

Fig.6 The location of interfacial damage: mode (c) and (d)

We study the maximum loading stress of composite laminates with length/thickness ratio R_L and the results are summarized in Fig.7. It could be divided into three groups:

When R_L<15, those belong to the failure mode (a) and (b), where interfacial normal damages dominate. The maximum loading stresses σ_c approximate 872*MPa*.

When R_L=20 and 22, those belong to the failure mode (c). The interface is shear damaged firstly and then buckling. There is a transitional region from the interfacial shear damage to normal damage. The stresses are lower than the fit curve of elastic buckling stress, because of the plastic deformation in the laminates.

When R_L=25,30,45 and 60, those belong to the failure mode (d), where composite laminates buckle firstly and followed by interface shear damage. It is an elastic buckling, and function $\sigma = n / R_L^2$ can be used to fit the relationship between damage stresses and lengths.

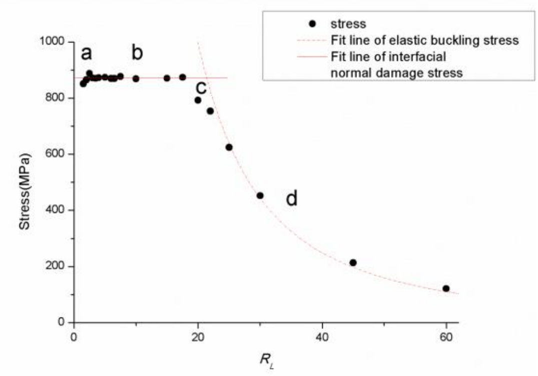
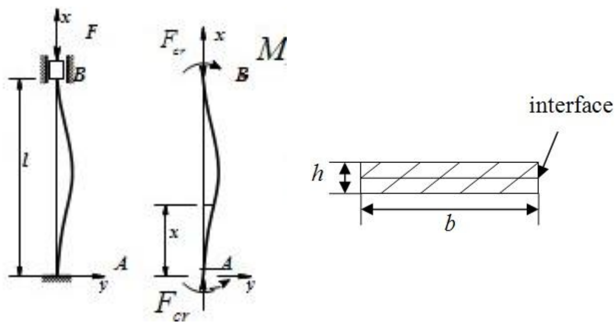


Fig.7 Relationship between loading stress and R_L

3. Theoretical analysis of different failure modes

We consider a composite laminates under uniaxial compression with thickness *h* length *l* and each end fixed support (Fig.8).



(a) Loading condition (b) Cross section

Fig.8 Sketch of laminate plate under compression

The critical length/thickness ratios are defined as: R_{Ld} is the critical length/thickness ratios between mode (a) and (b); R_{Lc} is the critical length/thickness ratios between mode (b) and (c); R_{Ls} is the critical length/thickness ratios between mode (c) and (d).

3.1 Interfacial failure of mode (a) and (b)

The interface would be damaged in normal mode if $R_L < R_{Lc}$, which due to the Poisson's effect and the frictional effect. Under uniaxial compression, we consider that there is a half-wavelength at each end of laminate; the wavelength is marked as λ. Then we use Eq.(1) to simulate the curve of the laminates when the length l is much greater than wavelength λ and the interaction between each end can be neglected. Fig.9 shows a half-wavelength in each end of laminates, the real line represents the original state, and the dotted line indicates the deformation after the load is applied. As a result, the interfacial normal damage with double holes will happen for this case.

$$y_1 \approx \begin{cases} A\cdot\sin\left(\frac{2\pi x}{\lambda}\right) & \left[x\in\left(0,\frac{1}{2}\lambda\right)\right] \\ 0 & \left[l > x \geq \frac{1}{2}\lambda\right] \end{cases} \quad y_2 \approx \begin{cases} A\cdot\sin\left[\frac{2\pi(l-x)}{\lambda}\right] & \left[x\in\left(l-\frac{1}{2}\lambda, l\right)\right] \\ 0 & \left[0 < x \leq l-\frac{1}{2}\lambda\right] \end{cases} \tag{1}$$

When the length l is not much greater than wavelength λ, interaction should be considered. Let $y=y_1+y_2=A$, the critical length/thickness ratios R_{Ld} to distinguish normal damaged single hole failure and double holes failure is calculated in Eq.(2),

$$R_{Ld} = \frac{5\lambda}{6h} \tag{2}.$$

The interfacial normal damage with double hole will happen when $R_L \in (R_{Ld}, R_{Lc})$. The interfacial normal damage with single hole will happen when $R_L \in (0, R_{Ld})$.

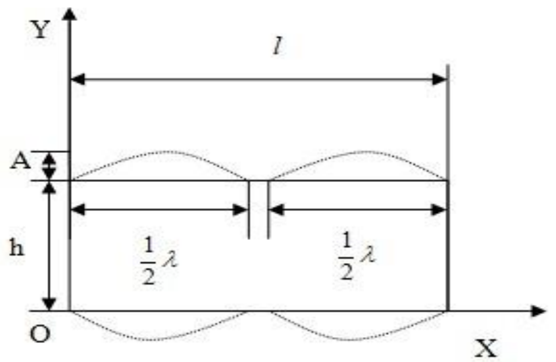


Fig.9 The original state and deformation pattern

To estimate the R_{Ld} need finding the wave-length λ. The distances between the hole and the nearer end are recorded in Fig.10 through FEM simulation. The result is $\lambda/4 \approx 7.34mm$, the $R_{Ld} = 5\lambda/6h = 2.45$ is obtained by Eq.(2). As a result, when R_L=1.5 and 2, the distances approximate the half of the lengths l/2. But when length is large enough, the distance is close to a constant, which is defined as λ/4.

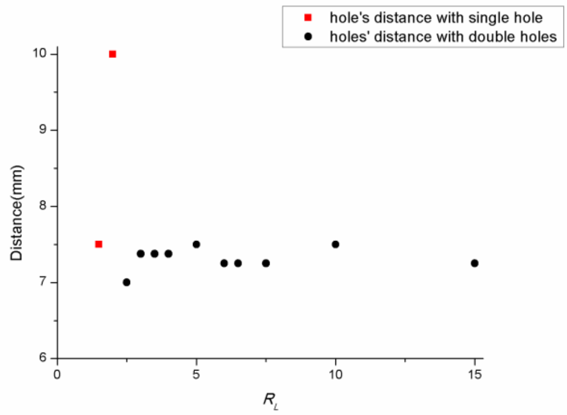


Fig.10 Hole position - length/thickness ratio

3.2 Interfacial failure of mode (c) and (d)

When the load is near to the critical buckling load, shear stress rapidly increases in the cross section. According to the Euler theory, the critical buckling load F_{cr} is written as:

$$F_{cr} = \frac{4\pi^2 E' I}{l^2} \tag{3}$$

In Eq.(3), E' is equivalent elastic modulus as $E=E'/(1-v^2_{12})$, and $I=bh^3/12$ is the inertia moment. When the laminates buckle, the bending appears in the composite laminates. There is a pair of restrict moment M_e in each end. So the moment M in laminates are obtained [22]

$$M = M_e \cos\frac{2\pi x}{l} \tag{4}$$

The shear stress is then calculated to be

$$\tau = -\frac{\pi M_e}{l I_z}\left(\frac{h^2}{4} - y^2\right)\sin\left(\frac{2\pi x}{l}\right) \tag{5}$$

As a result, the maximum shear stress $\tau_{\max}$ occurs at y=0, x=l/4 or 3l/4 and is given as

$$\tau_{\max} = \frac{3\pi M_e}{lhb} \tag{6}$$

Because the restrict moment M_e is written as

$$M_e = F_{cr} \bullet \omega_{l/4,3l/4} \tag{7}$$

Now Eq.(7) is obtained as

$$\omega_{l/4,3l/4} = \frac{l^3 \tau_{\max}}{\pi^3 E' h^2} \tag{8}$$

and $0 < \left|\omega_{l/4,3l/4}\right| < l/4$. The limit length l_{limit} of shear damage is derived from Eq.(8)

$$l_{\lim it} = \frac{\pi h}{2}\sqrt{\frac{\pi E'}{\tau_{\max}}} \tag{9}$$

The limit length/thickness ratio R_{Llim} could be written as:

$$R_{L\lim} = \frac{\pi}{2}\sqrt{\frac{\pi E'}{\tau_{\max}}} \tag{10}$$

The interfacial shear stress could not reach to the shear strength when $R_L > R_{Llim}$, based on the small deformation assumption. As $R_L < R_{Lliim}$, the damage process is divided into failure mode (c) and (d):

Mode (c): Composite laminate's interface is shear damaged firstly, and then laminates buckle. When the load approaches the critical buckling force F_{cr}, the laminates would be bended slightly. If the shear stress reaches the interfacial shear strength, the interface would be damaged and the composite laminates would be delaminated, so the load decreases rapidly. The composite laminates will again maintain balance when the load reach to the critical post-buckling load F_{cr}' which is calculated in Eq.(11).

$$F_{cr}' = \frac{4\pi^2 E'}{l^2} \times \frac{I}{8} \times 2 = \frac{F_{cr}}{4} \tag{11}$$

In this case, the plastic zone occurs in composite laminates, so the upper limit length of this damage mode is l_s. The lower limit length l_c of this damage mode is hard to be estimated because l_c is the controlled by interfacial failure in both shear and normal damage. The lower limit length/thickness ratios R_{Lc} of this damage mode can be approximated as

$$R_{Lc} = \pi\sqrt{\frac{E'}{3\sigma_c}} \tag{12}$$

It is assumed here that the shear stress dominates the composite laminate's interface failure, when $R_L \in \left(R_{Lc}, R_{Ls}\right)$.

Mode (d): Composite laminates buckle firstly, and then interface suffers the shear damage. When the load reach to the critical buckling load F_{cr}, the laminates enter into post-buckling stage, the deflection ω increases continually until the shear stress reach the interfacial shear strength. So the interface would be damaged and the composite laminates be delaminated. The load decrease from the critical buckling load F_{cr} to the critical post-buckling load F_{cr}'. Since it is pure elastic buckling, the critical length l_s corresponding to this damage mode can be estimated, and the length/thickness ratio is

$$R_{Ls} = \pi\sqrt{\frac{E'}{3\sigma_s}} \tag{13}$$

According to the material properties in Tab.1 and FEM results, each critical length could be calculated.

$$R_{Lc} = \pi\sqrt{\frac{E'}{3\sigma_c}} = 23.7$$

$$R_{Ls} = \pi\sqrt{\frac{E'}{3\sigma_s}} = 24.7$$

$$R_{L\lim} = \frac{l_{\lim it}}{h} = \frac{\pi}{2}\sqrt{\frac{\pi E'}{\tau_{\max}}} = 285.1$$

Finally, the critical length/thickness ratios $R_{Ld}=2.45$ and $R_{Ls}=24.7$ is very close to our finite element method simulate results. However the transitional length/thickness ratios $R_{Lc}=23.7$ is larger than the simulated result, for the failure stress σ_c =872MPa is a maximum theoretical result in the failure mode (c) and it is largely related to the material interface and small perturbation during buckling.

4. Conclusions

Through both theoretical analysis and finite element simulations, we divide the four failure mode of composite laminates into three groups by length/thickness ratio, which would induce different interfacial damage and buckling behaviors:

Failure mode (a) and (b) $R_L \in \left(0, R_{Lc}\right)$: the interface would be normal damaged. It contains two failure modes: (a) normal damage with single hole; (b) normal damage with double holes. The critical length/thickness ratio between mode (a) and (b) is R_{Ld};

Failure mode (c) $R_L \in (R_{Lc}, R_{Ls})$: the interface would be shear damaged firstly, and then the whole composite laminates buckle;

Failure mode (d) $R_L \in (R_{Ls}, R_{L\lim})$, the composite laminates would buckle firstly, and then the interface be shear damaged.

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