MULTI-OBJECTIVE AERODYNAMIC OPTIMIZATION DESIGN OF MULTI-ELEMENT AIRFOIL USING KRIGING MODEL

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Abstract

The Kriging-based multi-objective optimization design method for multi-element airfoil is developed in this paper. by introducing the Navier-Stocks solver with S-A turbulence model, the zonal patched grids and the genetic algorithm, the lift coefficient under the landing condition is maximized with the moment coefficient as the constraints. In order to reduce the computational time and cost, Kriging model is applied to the process of the optimization design. EI method is introduced to give additional sample points for improving the accuracy of the model. A single and multi-objective optimization design for the position of multi-element airfoil is carried out. Results show that this method can be an attractive design tool for multi-element airfoil.

1 Introduction

High lift design become more and more important for the development of a modern aircraft. high-lift system has great effect on landing/ take-off performances of the airplane, especially for the security of a transport aircraft It was found that relative small changes in the aerodynamic performance of the high lift system can produce large payoffs in airplane weight and performance, this is why high-lift systems and their aerodynamic characteristics remain in the forefront of aerospace research[1].

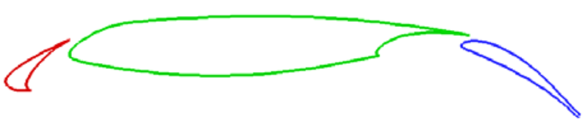
Because of its complex configuration and flow phenomena such as turbulent flow separation, transition, confluent boundary layers, wakes, etc, traditionally, wind tunnel test is only the method for high lift design, as the development of Computational Fluid Dynamic (CFD) and computing power, it can provide the revolutionary way for the high lift design. Recently, Methods using CFD with the solution of Reynolds averaged Navier-Stokes equation have found wide application in high-lift design, due to their capability to capture the viscous flow features of complex high-lift flows. Many research works have been carried out: S. Eyi [2] has accomplished the design optimization which has used an incompressible Navier-Stokes flow solver, a chimera overlaid grid system, and a constrained numerical optimizer. Sangho Kirn[3] has employed the adjoint-based Navier-Stokes design and optimization method for two-dimensional multi-element high-lift configurations. Many other people [4-6]has also put in their efforts to build high-lift design capabilities based on the Navier-Stokes technology utilizing different grid systems, different turbulence models and different optimization algorithm. During the design process, optimization algorithm is the most important for obtaining the good results, many kind of different methods have been utilized by the researches, such as gradient based optimization, non-gradient based optimization which contain genetic algorithm with or without surrogate model. Among these optimization methods, surrogate model-based optimization method has gained more and more attention because of their high efficiency and utility. The surrogate models can be used to replace the complex and time consuming experiments or numerical simulation of the optimization problems for saving lots of time during the optimization process.

In this study, the Kriging-based optimization design method for multi-element airfoil is developed. For flow analysis, a 2D structure Navier-Stokes solver is adopted to guarantee the precision and correction of flow calculation. The zonal patched grids around multi-element airfoil are produced automatically and efficiently. Genetic algorithm is used as the optimizer. In order to reduce the computational time and cost, Kriging model is employed in the process of the optimization design. To improve the accuracy of Kriging model, EI method is introduced to give additional sample points, as a result, the new and more accurate Kriging model is formed, finally this more accurate Kriging model is applied to the optimization. The structure of this paper is described as follow: firstly ,the components of optimization design process including the Kriging model, flow solver, grid generation, and the flowchart of design produce are introduced, and then the method is applied to single and multi-objective optimization design of three-element airfoil .

2 Airfoil configuration

The three-element airfoil which is the wing section of civil airplane is adopted as a baseline airfoil in the optimization process. The airfoil configuration is shown in fig 1. The airfoil has a slat, a main wing and a flap deflected at 40 degree. The flap and slat are retracted into the main element in fig1(a) in the cruise station, and in the fig 1(b) both section unfold.

(a)



(b)

Fig 1 Baseline airfoil configuration

3 Kriging model

The Kriging model expresses the relation between response of the system and variables as

$$y(\vec{x}) = f(\vec{x}) + z(\vec{x}) \qquad (1)$$

Where $y(\vec{x})$ is the unknown Kriging model, $f(\vec{x})$ is the known function dependent on $\vec{x}$, it provides a global model. $z(\vec{x})$ is stochastic process, whose average is zero but variance isn't, represents the local deviation from the global model. The covariance of $z(\vec{x})$ expresses as

$$Cov\left[Z(\vec{x}^i), Z(\vec{x}^j)\right] = \sigma^2 \mathbf{R}\left[R(\vec{x}^i, \vec{x}^j)\right] \qquad (2)$$

where $\mathbf{R}$ denotes the correlation matrix, $R(\vec{x}^i, \vec{x}^j)$ expresses the correlation function between any two sample points $\vec{x}^i$ and $\vec{x}^j$, there are a number of correlation functions at present, such as exponential function, Gaussian function and spline function, the Gaussian function will be applied in this paper, which expresses as

$$R(\vec{x}^i, \vec{x}^j) = \exp[-\sum_{k=1}^{n} \theta_k \left|x_k^i - x_k^j\right|^2] \qquad (3)$$

where $\theta_k (k = 1,...n)$ denotes the unknown correlation parameters, $\vec{x}_k^i$ and $\vec{x}_k^j$ are the kth components of $\vec{x}^i$ and $\vec{x}^j$ respectively. A constant global model is denoted, then equation (1) becomes

$$y(\vec{x}) = \beta + z(\vec{x}) \qquad (4)$$

The predictor of the approximate model could be written as

$$\hat{y}(\vec{x}) = \hat{\beta} + \vec{r}^T(\vec{x})\mathbf{R}^{-1}(Y_s - \vec{f}\hat{\beta}) \qquad (5)$$

where Y_s is the response matrix of samples, $\vec{f}$ is a column vector whose elements are all 1, $\mathbf{R}$ denotes the correlation matrix

$$\mathbf{R} = \begin{pmatrix} R(\vec{x}^1, \vec{x}^1) & \dots & R(\vec{x}^1, \vec{x}^n) \\ \vdots & \ddots & \vdots \\ R(\vec{x}^n, \vec{x}^1) & \cdots & R(\vec{x}^n, \vec{x}^n) \end{pmatrix} \qquad (6)$$

$\vec{r}(\vec{x})$ denotes the correlation vector between the sample point and the predicting point, which is

$$\vec{r}(\vec{x}) = [R(\vec{x}, \vec{x}^1), R(\vec{x}, \vec{x}^2), ...R(\vec{x}, \vec{x}^n)]^T$$

The unknown constant β in Eq.(4) can be obtained using the least square method

$$\hat{\beta} = (\vec{f}^T \mathbf{R}^{-1} \vec{f})^{-1} \vec{f}^T \mathbf{R}^{-1} Y_s \qquad (7)$$

The variance can be obtained as follows:

$$\hat{\sigma}^2 = \frac{(Y_s - \vec{f}\hat{\beta})^T \mathbf{R}^{-1} (Y_s - \vec{f}\hat{\beta})}{N} \qquad (8)$$

The parameter $\vec{\theta}$ in Eq.(3) can be estimated by maximizing the following maximum likelihood function

$$MaxF(\vec{\theta}) = -\frac{N \ln(\hat{\sigma}^2) + \ln|\mathbf{R}|}{2} \quad (\theta \geq 0) \qquad (9)$$

For each $\vec{\theta}$, we can got an interpolation model, the final Kriging model is obtained through finding the optimum $\vec{\theta}$ which maximize the likelihood function.

The accuracy of the predictor $\hat{y}(\vec{x})$ depends on the distance from the prediction point $\vec{x}$ to the sample points, the closer point $\vec{x}$ to the sample points, the less error of $\hat{y}(\vec{x})$ is. The root mean square error (RMSE) is expressed as follow:

$$s = \sqrt{s^2(\vec{x})} = \hat{\sigma}^2 \left[1 - \vec{r}^T \mathbf{R}^{-1} \vec{r} + \frac{(1 - \vec{1}^T \mathbf{R}^{-1} \vec{r})^2}{\vec{1}^T \mathbf{R}^{-1} \vec{1}} \right] \qquad (10)$$

4 Optimization process

In this section, the overall design procedure is outlined. An optimization design process can be modularized into several components such as design of experiment method, Kriging model, the flow solver, grid generation, and the optimization algorithm. Each component is significant for the efficiency and precision of design method. After defining the suitable design variables and cost function, which are typically based on aerodynamic performance, the design procedure can be described as follow. Firstly a number of sample points are generated by Latin hypercube sampling, and then the Kriging model is constructed, finally genetic algorithm is used to search the optimum solution. During the optimization process, the EI method is added to the initial Kriging-based optimization design algorithm. The optimum point obtained by optimization algorithm is also added to the initial sample points, hence N+m+1 points are added at a time, then the Kriging model is reconstructed. The flowchart of design process using the Kriging model-based optimization design algorithm is shown in fig 2.

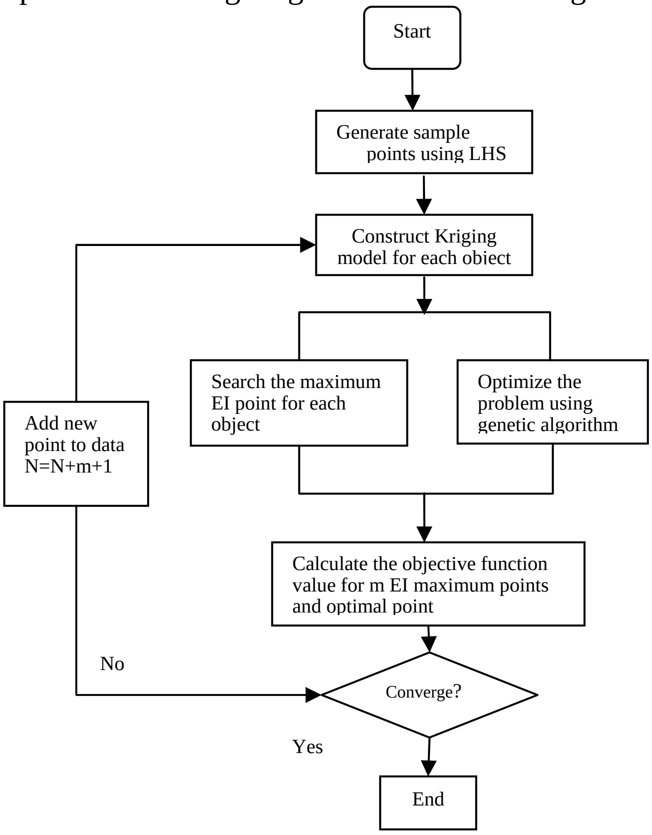


Fig 2 the flowchart of design process

4.1 Grid generation[10] and Flow solver

The zonal patched grids around multi-element airfoil are generated when the airfoil geometry shape is varied during the optimization process. C-type grids are produced on each element's body and in their wakes at first, O-type grids are given in the outmost area, an algebra method is used to produce the initial grids in each area. Finally the girds are optimized by elliptical differential equation method. The zonal patched grids around multi-element airfoils are produced automatically and efficiently. The grid topology is shown in fig 3.

The Navier-Stokes equations based flow solver is employed in the multi-element airfoil calculation. During the flow solution process, finite volume method is used to discretize the governing equations, Roe's scheme is utilized to discretize the inviscid flux vector, the central difference method is used to discretize the viscous flux vector in N-S equation, and the S-A turbulence model is employed to calculate the turbulent viscosity, implicit time stepping schemes is utilized, typical convergence acceleration techniques like multi-grid is also applied.

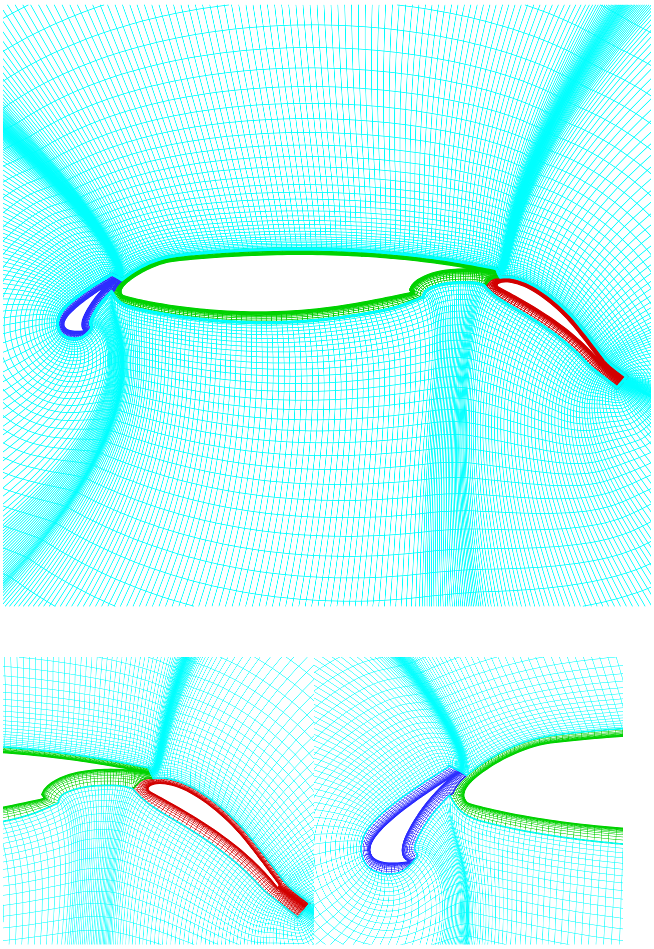


Fig 3 Grid topology

4.2 Design variables and objective function

In the optimization process, the objective function is defined as the maximum lift coefficient. The overlap, the gap, and the deflection angle of multi-element airfoil including slat and flap are used as design variables, each design variable in limited as follows:

$$1\%c \le gap_{slat} \le 3\%c \qquad 1\%c \le gap_{flap} \le 3\%c$$

$$-1\%c \le overlap_{slat} \le 1\%c \qquad 18^{\circ} \le \delta_{slat} \le 25^{\circ}$$

$$-1\%c \le overlap_{flap} \le 3\%c \qquad 35^{\circ} \le \delta_{flap} \le 40^{\circ}$$

where c is the chord length of airfoil when flap and slat are retracted into the main element.

5 Results

5.1 Single-objective optimization

The single objective optimization design is performed using Kriging-based algorithm at angle of attack of 4 degree which is responding to the landing condition. The baseline of sec2 is taken as the initial design. Number of cells is about 28,000. Mach number is set to 0.2 and Reynolds number is set to 6×10^{6}. The constraint is the moment coefficient which is not increased 1.03 times than that of the initial airfoil. The iteration is terminated when the change of EImax searched by the algorithm is little or the maximum iteration is achieved.

Table 1 indicates the design variables change between the initial and optimal airfoil. It is found that the design variables have great change in addition to the deflection of flap. Figure 4 illustrated the comparison of the airfoil configuration between the optimal and initial airfoil. The results of single objective optimization design are shown in table 2. A 6.17% lift increase has achieved and drag is reduced by 15.23%. The design produces a large drag reduction compared to the lift increase. The aerodynamic characteristic predicted by Kriging model is nearly the same as that predicted by N-S equation, but the CPU time is only a little as its N-S counterpart.

Table 1 the design variables change between the initial and optimal airfoil

Design variables	Initial	optimal
gap_{slat}	0.0209	0.0157
$overlap_{slat}$	0.0025	-0.0084
δ_{slat} (°)	25	18.335
gap_{flap}	0.02	0.0147
$overlap_{flap}$	0.01	0.0229
δ_{flap} (°)	40	39.059

Table 2 the result of single objective optimization design

	C_l	C_d	C_m	C_l/C_d
Initial	2.64	0.0854	1.258	30.85
Optimal(Kriging)	2.80	0.0705	1.283	39.72
Optimal（NS）	2.80	0.0724	1.282	38.64
Change(△)	6.17%	-15.23%	1.88%	25.24%

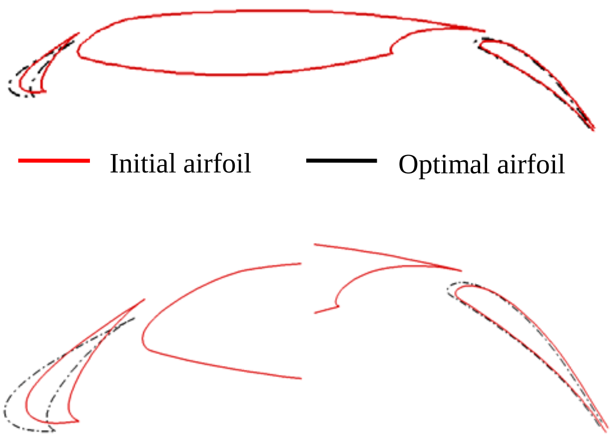


Fig 4 Comparison of geometry between the optimal and initial airfoil

5.2 Multi-objective optimization

In general, the airfoil through the single point design indicates poor off-design performance. In order to solve this problem, the present optimization method is applied to the multi-element airfoil optimization that maximizes the lift coefficient with two objective functions. The secondary design point is chosen at angle of attack of 20 degree, which corresponds to the stall angle of the initial airfoil. Mach number is set to 0.2 and Reynolds number is set to $6{\times}10^6$. The objectives and constraints are described as follow:

Maximize: $\omega C_{l1} + (1-\omega) C_{l2}$

Subjected to $1.03 * |C_{m1}| > |C_{m\_initial1}|$

$$1.03 * |C_{m2}| > |C_{m\_initial2}|$$

Where ω is the weighting factor. In this study, ω is chosen as 0.5. Table 3 shows the design variables change between the initial and optimal airfoil. It is found that the design variables have great change in addition to the deflection of slat which is only reduced by 0.063. Figure 5 illustrated the comparison of the airfoil configuration between the optimal and initial airfoil. Table 4 and table 5 is the summary of the aerodynamic coefficients of the designed airfoil at two different design points. A 2.47% lift increase is achieved at design point one, and the lift coefficient at design point two is improved by 1.04%. Unfortunately, the moment coefficient has both increased, but both of that obey the design constraints, it indicates that the improvements are relative smaller than those of the single-objective design cases. The aerodynamic characteristic predicted by Kriging model is nearly the same as that predicted by N-S equation. The comparison of pressure coefficient distribution between the optimal and initial airfoil is shown in fig 6, it is found that the suction peak of main wing is increased, which lead to the improvement of lift coefficient at two condition.

Table 3 the design variables change between the initial and optimal airfoil

Design variables	Initial	optimal
gap_{slat}	0.0209	0.0265
$overlap_{slat}$	0.0025	-0.0014
δ_{slat} (°)	25	24.937
gap_{flap}	0.02	0.0168
$overlap_{flap}$	0.01	0.0054
δ_{flap} (°)	40	38.933

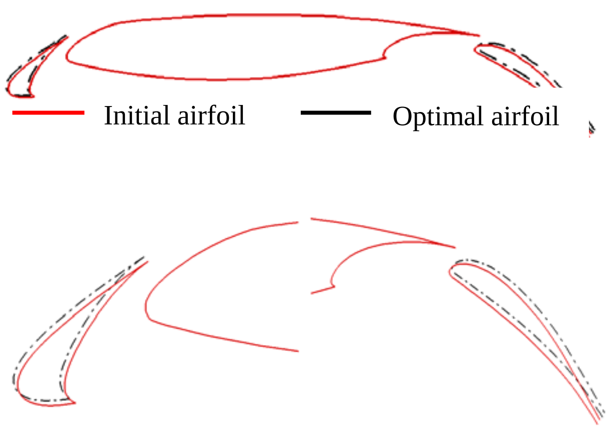


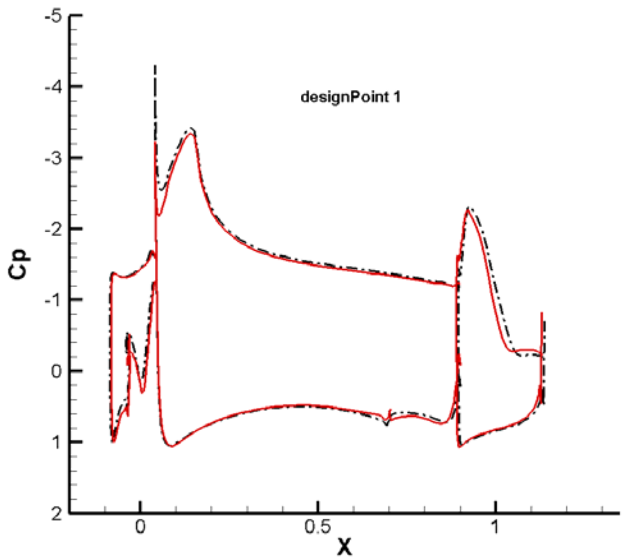
Fig 5 Comparison of geometry between the optimal and initial airfoil

Table 4 the result of multi-objective optimization design(designpoint1)

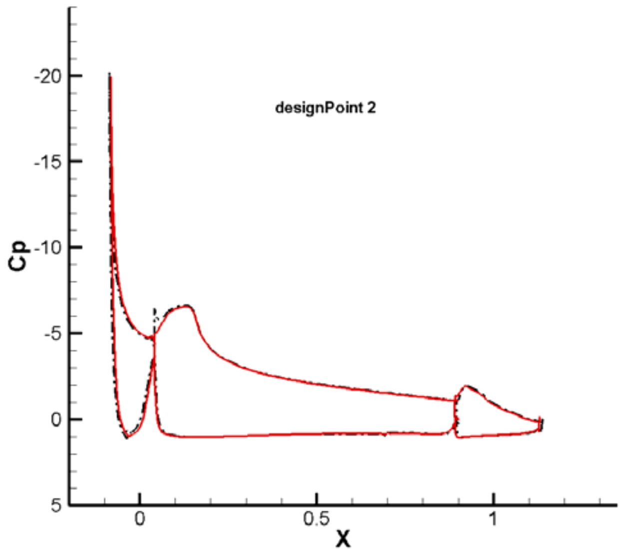
	C_l	C_d	C_m	C_l/C_d
Initial	2.63	0.0854	1.258	30.84
Optimal(Kriging)	2.70	0.0805	1.269	33.61
Optimal（NS）	2.70	0.0815	1.295	33.12
Change（△）	2.47%	-4.56%	2.99%	7.37%

Table 5 the result of multi-objective optimization design(designpoint 2)

	C_l	C_d	C_m	C_l/C_d
Initial	4.38	0.1535	1.467	28.57
Optimal(Kriging)	4.43	0.1529	1.497	28.98
Optimal（NS）	4.43	0.1555	1.498	28.49
Change（△）	1.04%	1.33%	2.08%	-0.29%



(a)design point 1



(b)design point 2

Fig 6 Comparison of pressure coefficient distribution between the optimal and initial airfoil

6 Conclusion

In this study, the Kriging-based multi-objective optimization design method for multi-element airfoil is developed. LHS is used to produce the sample points, Kriging model is utilized to replace the normal expensive solver during the optimization process. In order to improve the accuracy of the Kriging model, EI method is employed to give additional sample points. In this method, by introducing the Navier-Stocks solver with S-A turbulence model, the zonal patched grids which are generated around multi-element airfoils automatically, and the genetic algorithm, the lift coefficient under the landing condition is maximized with the moment coefficient as the constraints.

The results of single and multi-objective optimization design of multi-element airfoil indicate that the lift coefficient can be increased when the constraints are satisfied. But the improvements of the multi-objective design cases are relative smaller than those of the single-objective design cases. The Kriging model obtains very similar aerodynamic characteristic compared to N-S equation, with significant performance improvement, but the CPU time are only a little as its N-S counterpart for single and multi-objective optimization, respectively. The improvement of aerodynamic performance shows this method can be an attractive design tool for the development of multi-element airfoil.

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