ON SOUND GENERATION BY MOVING SURFACES AND CONVECTED SOURCES IN A FLOW

L.M.B.C. Campos*, F.J.P. Lau*
*** Centro de Ciência e Tecnologia Aeronáuticas e Espaciais (CCTAE), Instituto Superior Técnico (IST), 1049-001 Lisboa Codex, Portugal.**
luis.campos@ist.utl.pt; lau@ist.utl.pt

Abstract

The present paper presents a theory of sound generation by surfaces of in arbitrary motion, with two generalizations relative to the Ffowcs-William Hawkins (FWH) equation: (i) it allows for the presence of a steady, non-uniform potential flow of low Mach number; (ii) it includes the effects on the radiation field of reflections from solid surfaces, e.g. those which cause non-uniformity of the flow. The final result is a generalization of the Khirchoff integral with: (i) a retarded time modified by sound convection by the mean flow; (ii) position coordinates of observer and source modified to account for the presence of the obstacles which reflect sound waves and cause the mean flow to be non-uniform.

1 Introduction

In most of the aeroacoustics literature, the modeling of sound generation by surfaces in arbitrary motion is based on the FWH-equation [1-3], which has effectively superseded an earlier attempt at the same result [4]. Two of the most important applications of the FWH equation are propeller and rotor noise. A variety of methods have been used in connection with the noise aircraft propellers [5-34] and the noise of helicopter rotors [35-46]. In both applications a non-uniform mean flow may be generally present, requiring a generalization of the FWH equation, that assumes a medium in uniform motion. A few examples can be given of non-uniform mean flow effects: (i) for an helicopter in hover, the entrainment of ambient air by the rotor causes a non-uniform mean flow; (ii) for an helicopter in forward (or other translational) flight there is in addition an incident stream, that is uniform only far from the helicopter, even for straight and steady flight; (iii) the noise of an aircraft propeller in flight is also affected both by the aircraft speed and entrainment of ambient air, leading an non-uniform flow. Since the FWH equation assumes a medium in uniform motion, these non-uniform mean at flow effects are not included. This suggests is an extension of the FWH equation allowing (*Figure 1*) for sound generation by surfaces in arbitrary motion in a flowing medium. The effect of a steady, potential, low Mach number mean flow on sound has been shown [47-51] to be equivalent to a change in retarded time; this can be explained as the effect of convection of sound by the mean flow. Thus it may be expected that sound generation by surfaces in arbitrary motion in a steady, low Mach number potential flow, be represented by the Khirchoff integral extended in three ways: (i) with the spherical spatial decay modified by a Doppler factor due to source motion, as in the FWH equation; (ii) the effect of a mean flow, either uniform with no restriction on Mach number [52-56] or steady, potential of low Mach number [47-51] is to modify the retarded time; (iii) both in the Doppler factor and in the retarded time the position vectors of observer and source are replaced by modified position vectors incorporating the unit perturbation potential of the mean flow. This change also accounts for the scattering of sound by the obstacle(s) which caused the mean flow to become non-uniform.

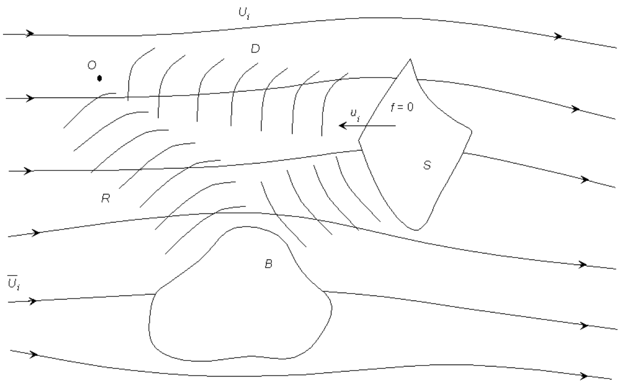

Fig. 1. Observer (O) receiving a direct (D) sound wave from sound sources(S) on a surface $f(x_i)=0$ moving with arbitrary velocity u_i in a steady non-uniform potential mean flow of low Mach number with velocity $U_i(x_j)$; the non-uniform flow is due to the introduction in an uniform stream of velocity $\overline{U}_i$ of obstacle(S) (B) that also reflect (R) sound waves towards the observer.

After explaining in introduction (§1) the need to consider sound generation by moving sources in a non-uniformly flowing medium, two developments are needed: (§2) the change in retarded time due to a steady, non-uniform potential, low Mach number mean flow [48], is obtained in a different way, that applies also to an uniform flow without restriction on Mach number;(§3) having accounted for the mean flow, as a change in retarded time into the FWH equation [3] the latter can be derived, using the properties of generalized functions [57-61] in an alternative slightly modified way. The combination of these results is the Khirchoff integral, generalized to moving sources in an steady, non-uniform low Mach number mean flow, that remains (§4) of convolution type only for a uniform flow. This will be applied subsequently to the comparison with noise of model propellers in wind tunnel tests. It serves also to generalize the acoustic reciprocity principle [62-65] to moving surfaces in non-uniform flows (§5).

2 Influence of steady, low Mach number potential flow

The convected wave equation with sources, applies to the acoustic potential:

$$\left\{(\partial/\partial t + V_i\,\partial/\partial x_i)^2 - c^2\ \partial^2/\partial {x_i}^2\right\}\Phi(x_i,t) = S(x_i,t), \tag{1}$$

in two cases: (i) a steady, non-uniform, potential flow Mach number [47-51]; (ii) an uniform flow of arbitrary March number [52-56]. It is transformed [48] to the classical wave equation:

$$\left\{\partial^2/\partial \bar{t}^{2} - c^2\partial^2/\partial {x_i}^2\right\}\Psi(x_i,\bar{t}) = S(x_I,\bar{t}), \tag{2}$$

by a change in time:

$$\bar{t} \equiv t + \phi(x_i)/c^2,\quad \Psi(x_i,\bar{t}) \equiv \Phi(x_i,t), \tag{3a,b}$$

where ϕ denotes the potential of the mean flow:

$$\phi(x_i) \equiv \int^{x_i} V_i(z_j)\,dz_i\,. \tag{4}$$

The interpretation is that convection of sound by the mean flow changes retarded time, as will be shown next.

The retarded time, defined as the difference between the time t of reception by an observer located at position x_i, of the sound emitted at time τ by a source at y_i, is given by:

$$t-\tau = \int_{y_i}^{x_i} \left[w(\xi_1)\right]^{-1} d\xi, \tag{5}$$

where $w(\xi_1)$ is the local speed of propagation, that equals the sound speed c (constant at low Mach number) plus the mean flow velocity V_i projected on the direction of propagation from observer to source:

$$w \equiv c + V_i n_i \qquad z_i \equiv x_i - y_i = |z_i|\,n_i, \tag{6a,b}$$

where n_i is a unit vector. Substituting (6a) in (5) yields the retarded time:

$$\tau - t = c^{-1}\int_{y_i}^{x_i}\left(1+M_i n_i\right)^{-1} d\xi,\quad M_i \equiv V_i/c, \tag{7a,b}$$

where the Mach vector (7b) was introduced. At low Mach number this simplifies to:

$$\tau = t - (1/c)\int_{y_i}^{x_i} (1 - V_i n_i / c)\, d\xi, \qquad (8)$$

which is integrated readily:

$$\tau = t - |x_i - y_i| / c + [\phi(x_i) - \phi(y_i)]/c^2, \qquad (9)$$

using the mean flow potential (4) at observer and source position. For the classical wave equation in a medium at rest the retarded time would be given by (l0a):

$$\begin{aligned} \tau &= \bar{t} - |x_i - y_i| / c, \\ \bar{t} &= t + [\phi(x_i) - \phi(y_i)]/c^2, \end{aligned} \qquad (10a,b)$$

and comparison of (10a) with (9) proves the time transformation (l0b). The latter (3a) differs from (l0b) in omitting [48] the term $\phi(y_i)$ on the grounds that the flow potential is defined to within an additive "constant"; however, in the case of a source distribution, this "constant" will have to be evaluated at different source points, and it is preferable to retain $\phi(y_i)$ in (9) and (l0b).

In the case of an uniform flow (5,6a) simplify to:

$$V_i = const : \tau = t - |x_i - y_i| / (c + V_i n_i), \qquad (11)$$

leading to the retarded time:

$$\tau = t - \frac{|x_i - y_i|^2 / c}{|x_i - y_i| + M_i (x_i - y_i)} \qquad (12)$$

without restriction on Mach number. At low Mach number the retarded time simplifies to:

$$M^2 << 1 : \tau = t - \frac{|x_i - y_i|}{c} + \frac{V_i (x_i - y_i)}{c^2}; \qquad (13)$$

this (13) agrees with (9) for an uniform flow, because then the flow potential is $\phi = V_i x_i$. The preceding results apply [47-51] to sound in a steady, low Mach number potential mean flow in free space. The non-uniform flow may be caused by obstacles, which also scatter sound; the effect of these obstacles as sound scatterers is represented [47,49] by replacing x_i by $X_i = x_i + \psi_i$, where ψ_i is the unit perturbation potential. This further generalization is to be pursued later (§4), since the aim next (§3) is to consider sound generation by moving surfaces.

3 Arbitrary moving surfaces as sources of sound

Sound generation by surfaces in arbitrary motion in a medium at rest is specified by the FWH equation, which has originally derived [3] using the properties of generalized functions [57-61]; these properties are used next in a somewhat different but equivalent way, considering (*Figure* 1) also a surface $f(y_i = 0)$, with interior $f < 0$ and exterior $f > 0$, so that the mass density $\rho^{(1)}$ inside and $\rho^{(2)}$ outside can be represented by the generalized function:

$$\begin{aligned} \bar{\rho} &\equiv \rho^{(1)} + \left[\rho^{(2)} - \rho^{(1)}\right] H(f) \\ &= \begin{cases} \rho^{(1)} & if \;\; f < 0, \\ \rho^{(2)} & if \;\; f > 0, \end{cases} \end{aligned} \qquad (14)$$

where H(f) is the Heaviside's unit function, viz. 0 for $f < 0$ and 1 for $f > 0$. Defining similarly the mass flux $\overline{\rho v_i}$, and using the equation of continuity on both sides of the surface:

$$\alpha = 1,2 : \quad \partial \rho^{(\alpha)} / \partial t + \partial \left\{ \rho^{(\alpha)} v_i^{(\alpha)} \right\} / \partial x_i = 0, \quad (15)$$

leads to:

$$\begin{aligned} &\partial \bar{\rho} / \partial t + \partial \left(\overline{\rho v_i} \right) / \partial x_i = \\ &\qquad [\rho] \partial H(f) / \partial t + [\rho v_i] \partial H(f) / \partial x_i , \end{aligned} \qquad (16)$$

where [....] denotes a difference across the surface, e.g.:

$$[\rho] = \rho^{(2)} - \rho^{(1)}, \quad \bar{\rho} = \rho^{(1)} + [\rho] H(f). \quad (17a,b)$$

The derivatives of the Heaviside unit function involve the Dirac delta function $\delta(f)$ viz.:

$$\frac{\partial H(f)}{\partial x_i} = \delta(f)\frac{\partial f}{\partial x_i}, \quad \frac{\partial H(f)}{\partial t} = \delta(f)\frac{\partial f}{\partial t}, \quad (18a,b)$$

where $\partial f/\partial x_i$ in (18a) is normal to the surface, and $\partial f/\partial t$ in (18b) may be interpreted by considering the motion of the surface (19a):

$$0 = df/dt = \partial f/\partial t + u_i\, \partial f/\partial x_i, \quad (19a)$$

$$u_i \equiv \left(dx^i/dt\right)_f, \quad (19b)$$

where (19b) is the velocity of an arbitrary point on the surface. From (19a) follows (20a):

$$\partial f/\partial t = -u_i\, \partial f/\partial x_i = -u_i N_i \left|\partial f/\partial x_j\right|, \quad (20a)$$

$$N_i \equiv \left(\partial f/\partial x_i\right)/\left|\partial f/\partial x\right|, \quad (20b)$$

where $u_1 N_i$ is the velocity projected on the normal (20b) to the surface.
Substituting (18a, 20a) in (16) leads to the equation of continuity valid in all space:

$$\begin{aligned}\partial\overline{\rho}/\partial t + \partial\left(\overline{\rho v_i}\right)/\partial x_i &= \\ &\left[\rho\left(v_i - u_i\right)\right]\delta(f)\,\partial f/\partial x_i.\end{aligned} \quad (21)$$

This equation (21) can also be derived by an equivalent integral method [3]. The equation of momentum on the two sides of the surface:

$$\begin{aligned}\alpha = 1,2: \partial\left\{\rho^{(a)} v_i^{(a)}\right\}/\partial t \\ + \partial\left\{v_i^{(a)} v_j^{(a)} + p_{ij}^{(a)}\right\}/\partial x_j = 0,\end{aligned} \quad (22)$$

where p_{ij} is the total stress tensor, leads by the differential method above to:

$$\begin{aligned}\partial\left(\overline{\rho v_i}\right)/\partial t + \partial\left(\overline{\rho v_i v_j} + \overline{p_{ij}}\right)/\partial x_i &= \\ \left[\rho v_i\right]\partial H(f)/\partial t \\ + \left[\rho v_i v_j + p_{ij}\right]\partial H(f)/\partial x_j.\end{aligned} \quad (23)$$

Substitution of (18a, 20a) leads to the momentum equation in all space:

$$\begin{aligned}\partial\left(\overline{\rho v_i}\right)/\partial t + \partial\left(\overline{\rho v_i v_j} + \overline{p_{ij}}\right)/\partial x_j &= \\ \left[\rho v_i\left(v_j - u_j\right) + p_{ij}\right]\delta(f)\,\partial f/\partial x_j.\end{aligned} \quad (24)$$

This equation could also or alternatively be derived by the integral method [3].
The surface is assumed to be rigid and impermeable, so that on it velocity equals that of the fluid (25a) and inside the fluid is assumed to be at rest:

$$v_i = \begin{cases} u_i & if \quad f = 0, \quad (25a) \\ 0 & if \quad f < 0; \quad (25b)\end{cases}$$

also the acoustic pressure is separated out of the total stresses:

$$p_{ij} = -p\delta_{ij} + p'_{ij}, \quad p \equiv c^2\overline{\rho}, \quad (26a,b)$$

leading to the continuity (21) and momentum (24) equations:

$$\partial\overline{\rho}/\partial t + \partial\left(\overline{\rho v_i}\right)/\partial x_i = \rho v_i\, \delta(f)\, \partial f/\partial x_i, \quad (27a)$$

$$\begin{aligned}\partial\left(\overline{\rho v_i}\right)/\partial t - \partial\left(c^2\overline{\rho}\right)/\partial x_i &= \\ -\partial\left(\overline{\rho v_i v_j} + \overline{p'_{ij}}\right)/\partial x_j \\ + p_{ij}\, \delta(f)\, \partial f/\partial x_j.\end{aligned} \quad (27b)$$

Elimination between these leads to the classical wave equation (28a):

$$\partial^2\overline{\rho}/\partial t^2 - c^2\, \partial^2\overline{\rho}/\partial x_i^2 = S\left(\vec{x}_i, t\right), \quad (28a)$$

$$\begin{aligned}S \equiv \partial^2\left(\overline{\rho v_i v_j} + p'_{ij}\right)/\partial x_i \partial x_j \\ - \partial\left\{p_{ij}\delta(f)\partial f/\partial x_j\right\}/\partial x_i \\ + \partial\left\{\rho v_i \delta(f)\partial f/\partial x_i\right\}/\partial t,\end{aligned} \quad (28b)$$

where the source S in (28b) has three terms: (i) the Lightill tensor, representing the noise of turbulence; (ii) the surface stresses, corresponding to 'loading' noise; (iii) the surface volume change, corresponding to 'thickness' noise. The former (i) are volume sources in the body of the fluid and the latter (ii-iii) source distributions on the moving surface. This equation and its solution are generalized next to include the effects sound convection by the non-uniform mean flow, and of sound scattering by acoustically-compact bodies which cause the incident stream to become non-uniform.

4 Khirchoff integral with modified retarded time and Doppler factor

The solution of the wave equation with sources (28a) is [62-65]:

$$4\,\pi\,c^2\,\rho(x_i,t) = \int S\left(y_i, t-|x_i-y_i|/c\right)|x_i-y_i|^{-1}\,d^3y_i. \quad (29)$$

using the retarded time for a medium at rest. Neglecting the volume sources represented by the Lighthill tensor, viz. the first term of the r.h.s. of (28b), the two remaining terms represent sound sources on the moving surface, leading [3], for an unstretched surface of area elements dS to:

$$4\pi\,p(x_i,t) = \int \frac{Q\left(y_i, t-|x_i-y_i|/c\right)}{|x_i-y_i| - u_i(x_i-y_i)/c}\,dS, \quad (30a)$$

where (26b) was used, and the source term is:

$$Q(y_i,\tau) = -\partial\left(p_{ij}N_j\right)/\partial y_i + \partial\left(\rho v_i N_i\right)/\partial\tau. \quad (30b)$$

In the presence (Fig. 1) of a non-uniform, steady, low Mach number potential mean flow, of potential $\phi(x_i)$, the retarded time in (30a) is changed [47] to (9):

$$4\,\pi\,p(x_i,t) = \int Q\left(y_i, t-|x_i-y_i|/c + [\phi(x_i)-\phi(y_i)]/c^2\right) \times \{|x_i-y_i| - u_i\,(x_i-y_i)/c\}^{-1} dS \quad (31)$$

The moving surfaces not only act as sources of sound, but also cause a non-uniform flow of unit potential ψ_i:

$$\phi(x_i) = \bar{V}_i\left[\psi_i(x_j) + x_i\right] = \bar{V}_i X_i, \quad (32a)$$

$$X_i \equiv x_i + \psi_i(x_j), \quad (32b)$$

where $\bar{V}_i$ is the free stream velocity far from the surfaces; the surfaces also scatter sound waves, and this is represented [47-49] by replacing x_i,y_i by X_i,Y_i given by (32b) in (31):

$$4\pi\,p(x_i,t) = \int Q\left(Y_i, t-|X_i-Y_i|/c - \bar{V}_i(X_i-Y_i)/c^2\right) \times \left\{|X_i-Y_i| - u_i(X_i-Y_i)/c\right\}^{-1} dS, \quad (33)$$

that is the final generalization; the substitution (32b) holds [47] for compact scatterers, that is rigid bodies with scale much smaller than the wavelength.
The generalized Khirchoff integral (33) thus represents: (i) sound generation by surfaces (30b) in arbitrary motion with velocity u_i in (19b); (ii) the effect of a non-uniform, steady low-Mach number potential mean flow, through the retarded time in (31); (iii) the scattering of sound by the surfaces through the modified coordinates (32b) in (33). If the source is a function of bounded variation in a finite or infinite time internal, then it can be represented respectively by a Fourier series [66-69] or a Fourier integral [70-72]. Thus there is no loss of generality in considering one term, i.e. an harmonic time dependence:

$$Q(y_i,\tau) = \bar{Q}(y_i)\exp(i\omega\tau), \quad (34)$$

so that the acoustic field is now given by:

$$4\pi\,p(x,t) = e^{i\omega t}\int \bar{Q}(Y_i)\left\{|X_i-Y_i| - u_i(X_i-Y_i)/c\right\}^{-1} \times \exp\{-i\,(\omega/c) \times \left[|X_i-Y_i| - \bar{V}_i(X_i-Y_i)/c\right]\}\,dS. \quad (35)$$

The latter is a convolution integral [73-76] of the type:

$$h*g(x_i) = \int h(y_i)\,g(x_i-y_i)\,dS, \quad (36)$$

only if X_i is a linear function of x_i in (32b), i.e. in the absence of a perturbation potential $\psi_i = 0$, when the mean flow is uniform $V_i = const$, and the flow potential $\phi(x_i) = V_i x_i$ is linear in x_i.

Thus, for an uniform mean flow, the acoustic field is given by:

$$4\pi\, p(x_i,t) = e^{i\omega t} \int Q\left(Y_i, t - \frac{|x_i - y_i|^2}{c|x_i - y_i| - \bar{V}(x_i - y_i)} \right) \times \{(x_i - y_i) - u_i(x_i - y_i)/c\}^{-1} dS, \quad (37)$$

without restriction on Mach number, since the retarded time (12) was used. In the case of a time-harmonic source (34) this becomes:

$$4\pi\, p(x_i,t) = e^{i\omega t} \int \bar{Q}(y_i) \times \exp\{-i\omega B(x_i - y_i)\} \times A(x_i - y_i) ds, \quad (38)$$

where the A-effect is due to source motion and the B-effect due to the mean flow:

$$z_i \equiv x_i - y_i: \quad 1/A(z_i) \equiv |z_i| - u_i z_i / c, \quad (39a)$$

$$B(z_i) \equiv |z_i|^2 / \{c(|z_i| + M_i z_i)\}. \quad (39b)$$

The acoustic field is then specified by the convolution integral:

$$4\pi\, p(x,t) = e^{i\omega t} \int Q * \{A \exp[-i(\omega/c)B]\}(x_i), \quad (40)$$

that has the usual properties, such as equality of the spatial derivatives at the observer and source $\partial/\partial x_i = \partial/\partial y_i$.

5 Discussion

Three types of radiation integrals have been considered: (i) the original Khirchoff integral (29), that is the solution of the classical wave equation in a medium at rest [62-65], and has been extended to a medium in uniform motion with arbitrary Mach number [52-56]; (ii) the first extension (30a), which is a solution of FWH equation (28a,b), modeling the generation of sound by sources in arbitrary motion, in a medium in uniform motion with arbitrary Mach number [3]; (ii) the second extension [47-51] to a steady, non-uniform low Mach number potential mean flow; (iii) if the non-uniform flow is due to the presence of obstacle(s) in an uniform incident stream, their effects on compact scatterers of sound is replaced by the replacement of the position vector by a modified position vector (32b), that includes the unit perturbation potential of the mean flow; (iv) the combination of the two preceding extensions (ii) and (iii) leads (Figure 1) to the most general form (33) of the Khirchoff integral, applying to sound generation by surfaces in arbitrary motion, in a steady, non-uniform low Mach number potential mean flow, due to obstacle(s) in a free stream, and including the scattering of sound by these obstacles.

The extension of classical wave theory to non-uniform flows requires two generalizations: (i) the form of the wave operator that describes sound propagation; (ii) the general integral describing the radiation of sound. There are at least 60 forms of the acoustic wave equation [51,77] generalizing the classical wave operator to potential and vertical, steady and unsteady flows, with or without viscous and thermal dissipation, for linear and non-linear waves. The extension of the Kirchoff integral as a general forced solution of the classical wave equation is available for the convected wave equation. In the case of a uniformly moving medium the extension is a Galilean transformation to a moving frame where the whole fluid is at rest, replacing local by material time derivatives, and leading from the classical to the convected wave equation. In the case of a non-uniform flow there is no uniformly moving frame where the whole fluid is at rest. It can be proved [47-51] that the convected wave equation holds for a steady potential flow at low Mach number; it no longer holds [50, 51] if any of these restrictions is removed.

The original Khirchoff integral proves the reciprocity theorem allowing interchange of source and observer positions in a medium at rest:

$$\left(\vec{x},\vec{y},\vec{V}_\infty\right)\leftrightarrow\left(\vec{y},\vec{x},-\vec{V}_\infty\right), \qquad (41)$$

and its extensions to moving media show that [47-51] the reciprocity principle is still valid if the mean flow velocity is reversed, in order to leave the retarded time unchanged; for example, if the mean flow is from observer to source, when the observer and source positions are interchanged, the direction of the mean flow has to be reversed, so that the flow is still from observer to source, and the mean flow convection effect on the retarded time is the same. For the inversion of the non-uniform mean flow over all space, it is sufficient to reverse the uniform incident free stream, as indicated in (41). The extension of the acoustic reciprocity principle, with mean flow reversal, has been shown to apply to: (i) a uniform mean flow of arbitrary Mach number; (ii) a steady, non-uniform potential mean flow of low Mach number.

The FWH equation has been applied to rotor and propeller noise mostly using numerical discretization schemes for the blades and surrounding medium and using the mean flow to calculate the magnitude of aerodynamic sound sources. It applies to moving surfaces and generalizes the earlier result for a static surface [72]. An interesting alternative would be an analytical approach, taking into account the effect of the mean flow on the amplitude and phase of sound, in uniform as well as non-uniform mean flows. In the present work these effects are included, so that they can be to an analytical theory of propeller noise, e.g. allowing for angle-of-attack effects. The noise of propeller aircraft is of greater concern at take-off and landing, when the incidence effects change both the sound sources and the radiation pattern. This is therefore an important application of the present theory. The analytical approach is well suited to the problem of propeller design synthesis: given a desired noise radiation pattern in the far-field, determine a pressure distribution in the propeller that will produce it. This is the inverse of the usual problem of determining the far-field noise from the pressure distribution on the propeller. In general the inverse problem has no unique solution, since many sound source distributions can produce the same sound field. However, if a parametric form of sound source distribution is given, the parameters can be chosen to approximate as closely as possible a desired sound field, and the solution of the problem can be made unique.

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