MODELING OF FLUID CONDITION OF AIRCRAFT HYDRAULIC SYSTEM

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Abstract

In the paper has been proposed the state change model of the working fluid in a hydraulic system of the aircraft in between overhaul periods. The model has been developed on the basis of Markov processes and is intended for determination of the working fluid cleanliness with the application proactive technologies of maintenance to increase the life time of aircraft hydraulic system components.

1 Introduction

Modern development of aircraft is associated with enhancing the reliability and serviceability of the main onboard systems. Consequently, the functionality, for instance, of hydraulic, oil and fuel (fluid) systems is significantly expanded, which leads to a complication of their structure, increase the reliability requirements for units working on the impact of high dynamic loads and external factors. During the operation the aircraft is subjected to constant exposure factors influencing the technical condition of its units and accessories. Simultaneously we can observe the change in working parameters of the structural elements and deterioration in the onboard systems functionality in general.

Experience accumulated in the process of aviation accidents investigation proves the fact that the presence of any hidden flaw in the system as hazard or risk factor may result in its becoming the reason for the negative phenomenon. At present the problem of the hidden flaw detection can be solved utilizing advanced maintenance techniques. One of the most perspective trends worldwide is implementation of proactive maintenance technologies based on acquisition and processing of information on current technical condition in real time which permits to identify the moment of hazardous system deterioration in proper time.

For successful implementation of proactive aircraft maintenance technology, you must have the appropriate methods, means of diagnosis and onboard systems condition monitoring and technologies for their realization allowing to obtain the necessary information in the "real" time. One of the most essential tasks is enhancing the quality and credibility of the control monitoring the condition of all the vital systems of the aircraft, including the hydraulic one which is undoubtedly crucial in control processes. It is known [1 - 4] that efficiency of the aircraft hydraulic system depends considerably on stability of:

- fluid pressure;
- fluid temperature;
- fluid flow;
- fluid vibration frequency;
- fluid viscosity;
- fluid anti-corrosion properties;
- fluid anti-wear properties;
- fluid anti-foam properties;
- fluid antioxidant properties;
- fluid density;
- fluid bulk modulus;
- fluid surface tension coefficient;
- fluid evaporation;
- fluid contamination.

The latter factor influences the stability of the others to a great extent. The aim of the given article is to state the scientific results of replacing an adequate physical model by mathematical formulation of the typical relation between contamination parameters values of the working fluid in an aircraft onboard systems. By contamination we mean all the heterogeneous components present in the fluid such as gases, foreign liquids, solid (mechanical) particles and microorganisms. If we don't take into account the specific character of some phenomena connected with infinitesimal processes behavior (a stochastic process, called Markov, with a multitude of X states) and if we take only time characteristics of different primitive actions, it is possible to describe all the above mentioned components as a complex and investigate them by means of a mathematical tool – a queuing theory.

2 Mathematical models

As a hypothesis about a working fluid contamination as a continuous process let us implement a continuous Markov process called a diffusion process [5]. Let us then consider the hydraulic fluid state change from the vantage point of diffusion processes occurring in it. A property inherent to all kinds of contamination is growth. Thus, regardless of the type of working fluid contaminants (gases, water molecules, mechanical particles or microorganisms), it is desirable to build up a theory describing the foreign contaminants growth process. As the nature of growth processes is extremely sophisticated, it is essential to build up such models of different complexity levels. First, let us consider a model enabling to identify the number of fluid contaminants at different time moments. At this approach an integral stochastic variable *X(t)* will be used to denote the number of contaminants at a moment *t*, while a postulated contamination growth mechanism will be expressed in terms of probability of determination of simple events taking place during short time periods. Then the contamination level change can be described with a probability process

$$\{X(t),\ t\geq 0\},$$

which, supposedly, is a Markov one.

To build up a model of working fluid contamination flows transferring a hydrosystem from one state to another, we shall use limit theorems of both cumulative and thinning flows. The cumulative flow of the incoming contaminants $\lambda_\Sigma(t)$ as gases $\lambda_G(t)$, water molecules $\lambda_W(t)$, mechanical particles $\lambda_M(t)$ or microorganisms $\lambda_B(t)$ in terms of spatial and time continuity will be expressed in the following model (figure 1) describing the rate of the working fluid contamination in real time.

In figure 1, it is seen that if we add up five contamination generation flows, the cumulative flow rate will be determined in the following way:

$$\lambda_\Sigma(t) = \sum_{i=1}^{n} \lambda_i(t),$$

where $\lambda_i(t)$ is a contamination flow into the working fluid background space.

In such a case, part of this contamination is filtered, dissolved or sludged with the total rate $\mu_i(t)$.

The resultant contamination flow with filtration taken into account will be described as

$$\vartheta_{ij} = \lambda_\Sigma(t) - \mu_\Sigma(t).$$

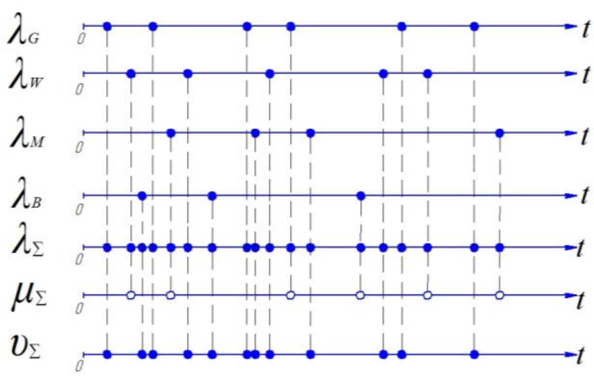

Fig.1 Hydraulic fluid contamination model

Possibility of the evaluation of the current condition of the working fluid per se, i.e. the knowledge of the total working fluid contamination generation rate at each point in time, affects considerably the operating life of the hydraulic system units and assemblies in general.

The process of working fluid contamination and its filtration, with contamination and filtration rates depending upon time, are characterized by the following system of differential-difference equations (1):

$$\left.\begin{aligned} &\frac{dP_x(t)}{dt}=\mu(t)(x-1)P_{x-1}(t)-\\ &-\big(\mu(t)+\lambda(t)\big)xP_x(t)+\\ &+\lambda(t)(x+1)P_{x+1}(t),\ x=1,2,....,\\ &\frac{dP_0(t)}{dt}=\lambda(t)P_1(t), \end{aligned}\right\} \qquad (1)$$

where $P_x(t)=\mathscr{P}\{X(t)=x\}$,$x=0,1,\ldots$, and stochastic variable $X(t)$ is the level of working fluid contamination at the time moment t. The solution of the task satisfying the initial conditions

$$P_x(0)=\begin{cases}1 \text{ for } x=1,\\ 0 \text{ for } x\neq 1,\end{cases}$$

is

$$\left.\begin{aligned} &P_x(t)=[1-\alpha(t)][1-\beta(t)]\times\\ &\times[\beta(t)]^{x-1},\ x=1,2,....,\\ &P_0(t)=\alpha(t), \end{aligned}\right\} \qquad (2)$$

where

$$\begin{aligned} &\alpha(t)=1-\frac{e^{-\gamma(t)}}{\omega(t)},\\ &\beta(t)=1-\frac{1}{\omega(t)},\\ &\gamma(t)=\int_0^t[\lambda(\tau)-\mu(\tau)]d\tau,\\ &\omega(t)=e^{-\gamma(t)}\left[1+\int_0^t\lambda(\tau)e^{\gamma(\tau)}d\tau\right]. \end{aligned} \qquad (3)$$

From this point for the given functions $\lambda(t)$ and $\mu(t)$ we will get exact expressions for the mean value and variance $X(t)$. In a special case, when $\lambda(t)$ and $\mu(t)$ do not depend on time, we will get

$$\mathcal{E}\{X(t)\}=e^{(\mu-\lambda)t}, \qquad (4)$$

$$\mathscr{D}^2\{X(t)\}=\frac{\mu+\lambda}{\mu-\lambda}e^{(\mu-\lambda)t}\left(e^{(\mu-\lambda)t}-1\right).$$

From the equation (4) it is possible to get average size impurities for the process of contamination (or filtration) of the working fluid assuming that λ (or μ) equals null. Thus, for the process of the working fluid contamination:

$$\mathcal{E}\{X(t)\}=e^{\mu\cdot t}.$$

It should be noted that this expression is equal to the solution of the differential equation

$$\frac{dx(t)}{dt}=\mu x(t),\ \lambda>0,\ x(0)=x_0,$$

which describes a simple deterministic process of contamination. Here $x(t)$ is not an integral stochastic variable, but a continuous deterministic function of time. The same can be applied for the process of filtration. In this case

$$\mathcal{E}\{X(t)\}=x_0e^{-\lambda\cdot t}, \quad X(0)=x_0,$$

and the differential equation for the deterministic filtration process is

$$\frac{dx(t)}{dt}=-\lambda x(t), \quad \lambda\rangle 0, \quad x(0)=x_0.$$

Utilizing equations (2) and (3), it is feasible to get the following expression for the working fluid probability state at the time moment t:

$$P_0(t)=\frac{\int_0^t\lambda(\tau)e^{\gamma(\tau)}d\tau}{1+\int_0^t\lambda(\tau)e^{\gamma(\tau)}d\tau}. \qquad (5)$$

Provided $X(0) = x_0$, due to the independence of the x_0 population development, probability of the complete working fluid purification is obtained by raising the expression (5) to the power x_0. In case $\lambda(t) = \lambda$ and $\mu(t) = \mu$, from the expression (5) a conclusion can be made that

$$P_0(t)=\frac{\lambda e^{(\mu-\lambda)t}-\lambda}{\mu e^{(\mu-\lambda)t}-\lambda}. \qquad (6)$$

Consequently, with $t\to\infty$ the probability of the complete working fluid purification is equal to

$$\lim_{t\to\infty} P_0(t) = \begin{cases} 1 \text{ with } \lambda \le \mu, \\ \dfrac{\mu}{\lambda} \text{ with } \lambda > \mu. \end{cases} \quad (7)$$

Let us denote the conditional probability that at the time moment $t_j = t_i + \tau$ the level of working fluid contamination will be at the x_j state, if at the t_i time moment it was at the state x_i, through the expression $P_{i,j}(t_i, \tau)$.

In expressions [1, 3, 6] it has been determined that the working fluid cleanliness for the whole period of service varies in the range from 2nd to 13th levels of purity according to Russian State Standard 17216–2001. Let us relatively divide the given range of working fluid cleanliness levels according to four main hydraulic system condition types: initial, regular, pre-failure and failure. The transition of working fluid from one state to another occurs under the influence of Poisson flow of the contaminant particles with the rate $\vartheta_{ij} = \lambda_{\Sigma}(t) - \mu(t)$ (figure 2).

Consequently, if the contamination flow is a Poisson one, transferring the working fluid from one state to another, the state probabilities are determined by simple differential equations. Let us give an example determining the probabilities $P_0(t)$, $P_1(t)$, $P_2(t)$ and $P_3(t)$ of the fact that the working fluid will be in the states x_0, x_1, x_2 и x_3 respectively at any moment of the time t. Let us consider the moment of time t implying the change Δt. In this case the probability $P_0(t+\Delta t)$ is the probability of the fact that at the moment of time $(t+\Delta t)$ the fluid is in the initial state x_0. This event may have four outcomes:

A – the fluid at the moment of time t was in the initial state x_0 and during Δt time it hasn't changed the state;

B – the fluid at the moment of time t was in a regular state x_1 and during Δt time has made a transition to the initial state x_0;

C – the fluid at the moment of time t was in a pre-failure state x_2 and during Δt time has made a transition into the initial state x_0;

D – the fluid at the moment of time t was in a failure state x_3 and during Δt time has made a transition into the initial state x_0.

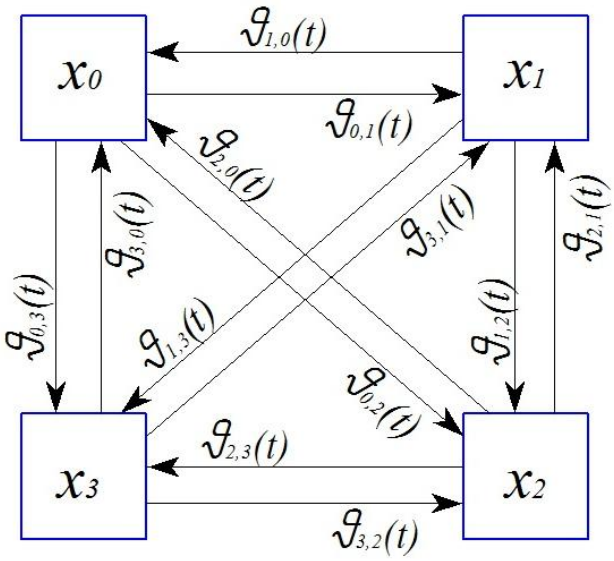

Fig. 2 Model of fluid contamination changes

Taking into consideration all the possible transitions for each state it is feasible to get a system of ordinary linear differential equations. From the experience of the aircraft maintenance we have determined the following change patterns of the working fluid state.

- The fluid contamination level changes gradually throughout the extended period of time. Thus, it is possible to claim that in most cases of the state transition with the rates $\vartheta_{0,2}(t)$, $\vartheta_{1,3}(t)$ and $\vartheta_{0,3}(t)$ will tend to zero, i.e. the transitions should not be considered.
- According to research [6], the fluid contamination level under the action of filters can get lower only in the boundaries of two proximate states, i.e. the state with rates $\vartheta_{2,0}(t)$ and $\vartheta_{3,1}(t)$ may not be considered in the first approximation.
- It is highly unlikely that the fluid being in the state x_3 which leads to failure of the hydraulic system as a whole, can perform the transition to the initial state x_0.

Thus, taking into account the above-mentioned patterns it is possible to note the final variant of the differential equations system describing the model of the working fluid state change:

$$\left.\begin{aligned}
\frac{dP_0(t)}{dt} &= -(\vartheta_{0,1}(t))P_0(t)+\vartheta_{1,0}(t)P_1(t);\\
\frac{dP_1(t)}{dt} &= -(\vartheta_{1,0}(t)+\vartheta_{1,2}(t))P_1(t)+\vartheta_{0,1}(t)P_0(t)+\vartheta_{2,1}(t)P_2(t);\\
\frac{dP_2(t)}{dt} &= -(\vartheta_{2,1}(t)+\vartheta_{2,3}(t))P_2(t)+\vartheta_{1,2}(t)P_1(t)+\vartheta_{3,2}(t)P_3(t);\\
\frac{dP_3(t)}{dt} &= -\vartheta_{3,2}(t)P_3(t)+\vartheta_{2,3}(t)P_2(t).
\end{aligned}\right\} \quad (8)$$

For the system integration (8) let us introduce the initial conditions: $P_0(0)$, $P_1(0)$, $P_2(0)$ and $P_3(0)$, with additional hypotheses made:

$$\left.\begin{aligned}
&0 \le P_k(0) \le 1;\\
&\sum_{k=1}^{n} P_k(0) = 1.
\end{aligned}\right\} \quad (9)$$

It should be noted that according to the research in the law of the working fluid contamination in proper aircraft maintenance conditions, it was established that the contamination densification process progresses quite slowly. So, up to some moment of time t_n it is possible to assume that the process is likely to be stationary ($\Delta t = 0$) and can be described using *the theory of death process*, when the value of any probability P_{k+1} can be expressed through all the preceding ones [5]:

$$P_{k+1} = \frac{\vartheta_k}{\vartheta_{k+1}} P_k = P_0 \prod_{n=0}^{k} \frac{\vartheta_n}{\vartheta_{n+1}} \quad (10)$$

with n being the number of state transitions.

We should define the moment of time t_n using in-line control sensors as part of the preventive maintenance in real time.

Let us find the common solution to the system of differential equations (8). We shall introduce a set of working fluid transition probabilities P_{ij} from state i to state j (P_{ij} is called transition probabilities). For a stationary process $\frac{dP_i(t)}{dt} = 0 \Rightarrow P_i = const\ with\ t \to \infty$.

Then the system of differential equations (8) accounting for (9) will be:

$$\left.\begin{aligned}
&P_{0,1}\cdot P_0 = P_{1,0}\cdot P_1;\\
&(P_{1,0}+P_{1,2})P_1 = P_{0,1}\cdot P_0 + P_{2,1}\cdot P_2;\\
&(P_{2,1}+P_{2,3})P_2 = P_{1,2}\cdot P_1 + P_{3,2}\cdot P_3;\\
&P_{3,2}\cdot P_3(t) = P_{2,3}\cdot P_2;\\
&P_0+P_1+P_2+P_3 = 1.
\end{aligned}\right\} \quad (11)$$

In this case $P_{ij}(t) = 1 - e^{-\vartheta_{ij} t} \approx \vartheta_{ij}\Delta t$, with ϑ_{ij} being the transition probability density. From this it can be deduced that $\vartheta_{ij} = \frac{P_{ij}}{\Delta t}$. with $\Delta t \to 0 \Rightarrow P_{ij} = \vartheta_{ij}$.

On the basis of statistical data it is possible to determine the probability of the working fluid being in the k – state: $P_k = \frac{n_k}{n_\Sigma}$. Here n_k is the number of working fluid being in the k state for the time period Δt, n_Σ is the total number of working fluid state transitions for the time period Δt.

The solution of the equation set (11) was obtained applying the formulae (9), (10) with the given densities of the ϑ_{ij} transition probabilities.

The value of the ϑ_{ij} transition probabilities is simple: ϑ is the inverse value of the average time of working fluid being in the given state. Values of ϑ are determined on the basis of statistical data of the aircraft maintenance for the given time period.

3 Experimental set-up

Developed model was studied under laboratory conditions at the training airfield of Samara State Aerospace University by means of modern technology National Instruments. Select National Instruments technology as a means of implementing this model was due to the simplicity and efficiency of its use in the software environment LabVIEW. As an integrated sensor of fluid contamination control was used sensor "FLOW", developed in the laboratory "Radio-electronic devices and methods of diagnostics of aircraft systems" of Samara State Aerospace University figure 3. As an input and output devices was selected platform «ComactDAQ», which provides a flexible hardware solution for the development of various systems for the collection and control signals from the USB port on the base program complex LabVIEW.

Fig. 3 Integrated sensor of fluid contamination control

The sensor is mounted in a hydraulic line and connected to the input-output device. Information on the number density of solids by size groups and dispersed solids particles formed in the flow sensor and into the input-output in the form of a random sequence of bell-shaped pulses whose amplitude is related to the quadratic dependence on the size (diameter) particles. Input-output analyzes formed by a sequence of pulses and results in the issuance of digital or analog form on the LCD.

4 Conclusions and future work

As a result of research carried out to establish a mathematical model describing the working fluid contamination, we have:

1. determined the working fluid contamination indices;
2. defined the set of contaminants influencing the working fluid state;
3. described the contamination process influencing the working fluid state as a Markov process;
4. proposed a mathematical model determining the change patterns of the working fluid state under the action of diffusion processes of its contamination.

The obtained model has become a basis for the development of methodology for working fluid state analysis and control within the context of preventive aircraft maintenance.

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