AEROELASTIC SIMULATION OF FLEXIBLE FLAPPING WING BASED ON STRUCTURAL FEM AND QUASI-STEADY AERODYNAMIC MODEL

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Abstract

Developing a FWNAV is an ambitious and arduous tasks relying currently mostly on trial and error method. In order to assist these developments, a preliminary design tool evaluating the aeroelastic performance of a flapping wing is sound. Our approach, coupling a structural finite element solver to a quasi-steady aerodynamic model, is here reported along with some of its applications: the selection of an appropriate actuation strategy, the combination a DOF and a waveform, and the definition of an optimization environment based on genetic algorithm so as to design 'optimized' wing geometry. Results indicate first, that a flapping actuation is more efficient than a heaving actuation for resonant wing, and second, that a sine actuation is a good compromise too. Third, the issue raised previously by a risk reduction optimization were fixed and enable more coherent individuals to be found out. In a nutshell, our aeroelastic framework can be seen as a satisfactory preliminary design tool for FWNAVs.

Introduction

Micro air vehicles (MAV) have been for the last fifteen years strongly investigated and several solutions relying either on fixed, rotary or flapping wing for the lift generation mechanism were developed. When considering hovering as a requisite for MAV, only the latter two technologies compete and insect-like flapping wing can be seen as an attractive alternative, especially on the energy-efficiency and noise signature aspects. Those perspectives are even more valuable when considering flapping wing nano air vehicles (FWNAV), scaled down version of flapping wing MAV with wingspan below $7.5cm$. Several FWNAVs [1–3] are currently under development and several others can be expected in the upcoming years following the tendency shown with MAVs.

Until now, the development of FWNAVs relies heavily on the trial and error method, especially regarding the lift generation, to ultimately bring an airborne design. The phenomena responsible for the lift generation are related to a complex fluid-structure interaction problem. Predicting the aeroelastic response of the wing might reduce the trial and error iterations and so speed up its development. This is even more legitimate at the preliminary design stage, where important choices are made, such as the actuation strategy, that impact later on the overall FWNAV performance. Therefore an aeroelastic framework incorporating the major structural and aerodynamic features of insect flight makes sense. Our approach, coupling a structural finite element solver to a quasi-steady aerodynamic model, is reported here along with some of its applications to the preliminary design of a resonant FWNAV.

The first section of this paper is focused on the presentation of the aeroelastic framework itself, while the second and third sections present applications of the framework, respectively for the selection of an appropriate actuation strategy and for the definition of wing geometry with 'optimized' performance.

It has to be noted that the aeroelastic framework is primarily developed for resonant FWNAV so as to assist the design of our prototype [4–6], but non-resonant designs can also be investigated as well as flappinng wing MAVs.

1 Aeroelastic framework

To assist efficiently the design of a FWNAV, it is necessary to evaluate quickly and accurately various performance items such as its mass, mean lift or actuation power. For this purpose, the structural and aerodynamic phenomena occurring on an insect-like flapped wing, either rigid or flexible, have to be modeled. Several aeroelastic frameworks [7–11] of the insect flight have already been developed to predict the aeroelastic response of a flapping wing.

In order to achieve an acceptable compromise between a low computational load and an acceptable prediction of performance, our aeroelastic framework, based on the finite element method (FEM) for the structural computation, is coupled to a lower fidelity aerodynamic model. In addition, the framework allows non-linearities due to the wing large displacement to be handled along with the local effects of flexibility on the aerodynamic forces.

1.1 Framework overview

Our aeroelastic framework handles throughout several flapping motions a FE-model of the FWNAV wing, presented in section 1.2, where the aerodynamic forces are computed at each time-step using each FE kinematics and shape information by a blade-element method based aerodynamic model, presented in section 1.3. This allows the flexibility in both spanwise and chordwise direction to be taken into account. This methodology was suggested by Combes et al. [12] in their conclusion and partially supported by Thiria et al. [13] in their conclusion on the effects of flexibility: a two-step approach with first, a solid mechanics problem determining an instantaneous shape, and second, a fluid dynamics problem governed by the structural deformation. Once the aerodynamic forces computed at each FE, they are distributed and applied on each node. The aeroelastic coupling is explicit as no sub-iteration occurs within a time-step between the structural and the aerodynamic computation. However the coupling is enhanced by the computation of several consecutive flapping motions. Similarly, the overall stability of the framework is increased by starting the aeroelastic analysis with a few uncoupled flapping motions in order to reach smooth and steady kinematics data before coupling the aerodynamic model and the FE solver.

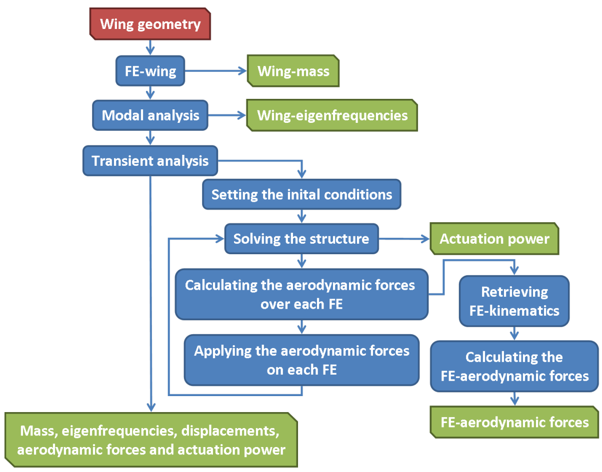


Fig. 1 Flowchart of the aeroelastic framework. The mandatory input data are indicated in red whereas the outputs are in green. Although each analysis is mandatory to compute the aeroelastic response of a wing at resonance, the framework can be adapted to non-resonant design.

Figure 1 summarizes the FE-kernel of our aeroelastic framework, where every analysis is handled autonomously once a wing geometry is given. In addition to this kernel, implemented as *Ansys* templates, a *Python* layer manages all the pre- and post-processing of the aeroelastic analysis such as modifying the template according to the analysis variables, handling the analysis files, and summarizing them so as to compute the performance of the wing.

1.2 Structural model

As biomimetism and bioinspiration is generally used for FWNAVs, our aeroelastic framework has to take into account the major mechanical characteristics of insect wings, the venation and membrane patterns in one hand, and the large displacement of a flexible structure in the other hand. Therefore beam and shell FEs, the BEAM4 and SHELL93 respectively in *Ansys*, are used to model the wing as they offer large displacement capabilities as well as 6 degrees of freedoms (DOF) at each node, making them compatible with the aerodynamic model.

As already explained, the aerodynamic model is based on a blade-element method that takes into account the local deformation of the wing. As a consequence its current implementation requires a cartesian mesh in both the spanwise and chordwise directions. Furthermore, in order to simplify the FE-model, only wing features relevant to the insect flight are considered and features such as the morphological roles of the venation and membranes pattern are neglected.

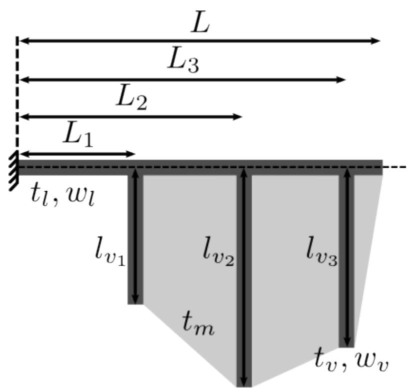


Fig. 2 Parametric geometry of the wing with our aeroelastic framework

Thus the wing is modelled by the parametric geometry defined in the figure 2, a leading edge branching out into a set of perpendicular veins. This simplification has already proved to mimic the insect wing kinematics [14] while complying with the cartesian mesh restriction and simplifying the automation of the aeroelastic framework.

1.3 Aerodynamic model

For the computation of the aerodynamic forces, a blade-element method is chosen as illustrated in the figure 3. Usually these models [15–17] consider the wing as rigid or flexible only in the spanwise direction, referred as the unidirectional flexible approach, but offer low or average computation load. Thus several issues inherent to chordwise flexibility and affecting the aerodynamic forces generation (real blade profile, effective angle of attack (AOA), difference of acceleration within the blade, position of the shedding vortices, etc.) are neglected or at best averaged. Only a few models [7, 11] consider both flexibilities, referred as the bidirectional approach, and thus offer a better characterization of both the fluid and the structure but at the cost of slightly more complicated models and more intensive computation loads. Therefore with the prospects of a resonant FWNAV, where strong spanwise and chordwise deformations are sought, and of a preliminary design tool, where a low computation load is required, a compromise between these models is needed.

Our model is here engineered so as to take advantages of the simple formulation and low computation load of Sane et al. [15], whereby only the translational and added mass components are considered, and to reformulate it as a bidirectional approach.

As illustrated in the figure 4, each FE is considered as a plate where two aerodynamic forces are acting. Those forces are calculated using local nodes-averaged kinematics data and applied in the $\xi_i \eta_i$ frame. The proper Euler-angles are used to perform each changes of coordinates from the global to the local ones and vice versa.

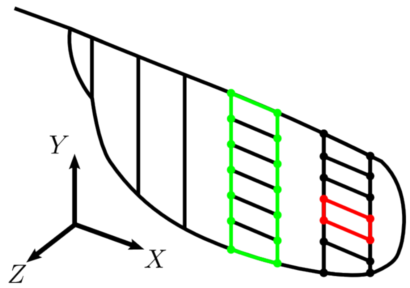


Fig. 3 Principles of the blade-element method where the wing is discretized in adjacent strips. In a rigid unidirectional approach (green), the wing deformation is either not taken into account (rigid) or only along the spanwise direction (unidirectional) and thus any chordwise deformation is at best averaged. In a bidirectional approach (red), both the spanwise and chordwise deformation are taken into account by the aerodynamic model.

1.3.1 *Translational forces*

The translational force $F_{tr,i}$ is computed as:

$$F_{tr,i} = \frac{\rho c_i s_i}{2}\left[C_L\left(\alpha_i'\right)^2 + C_D\left(\alpha_i'\right)^2\right]^{1/2} V_i\left|V_i\right| \quad (1)$$
$$\text{with} \quad \alpha_i' = \alpha_i - atan\left(\frac{\dot{y}_i}{\dot{z}_i}\right)$$
$$\text{and} \quad V_i = \vec{\dot{y}}_i + \vec{\dot{z}}_i$$

where ρ is the surrounding medium density, s_i the local span, c_i the local chord, C_L and C_D the local lift and drag coeffcients, α_i the local geometric AOA, α_i' the local effective AOA, and finally

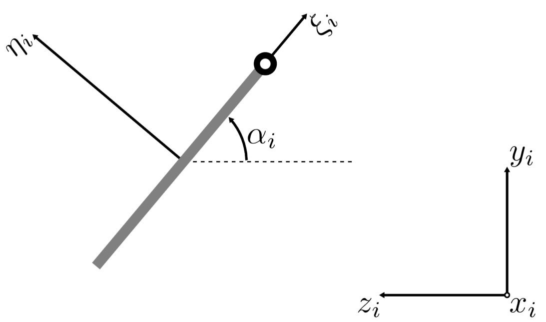


Fig. 4 Sectional view of a FE showing the reference frames necessary for the calculation of the aerodynamic forces on its. The empty black circle represents the leading edge side of the element.

V_i the 'freestream' velocity which is the vectorial addition of the local vertical y_i- and horizontal z_i-velocities. The lift C_L and drag C_D coefficients are taken from Dickinson et al. [18].

1.3.2 *Added mass forces*

The added mass are derived from the original set of equations defined by Sedov [19] for a rigid plate rotating at half-chord and modified so as to consider the flexibility. Indeed the quantity of air set into motion at the local-level, where strong acceleration might occur but only transmitted to a limited quantity of air, have to be balanced by the one set more globally at the blade-level. Thus the added mass force $F_{ad,i}$ is computed as:

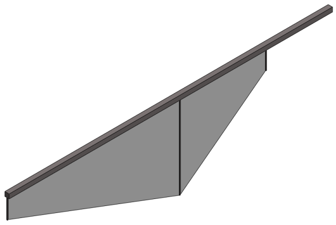
$$F_{ad,i} = -\lambda_{\eta,i}\frac{\ddot{\eta}_i}{n} \quad (2)$$
$$\text{with} \quad \lambda_{\eta,i} = \rho\pi\frac{c_i C_{blade}}{4}s_i$$

where $\ddot{\eta}_i$ is the local normal acceleration, $\lambda_{\eta,i}$ the virtual mass coefficient, n the number of FEs within the blade, and C_{blade} its mean chord. The last two parameters are here to balance the local and global accelerations and their induced effects on aerodynamic forces.

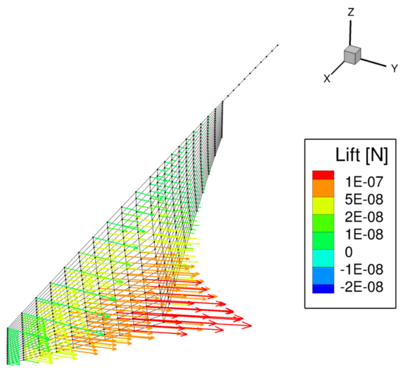
In a nutshell, our aeroelastic framework, based on FEM for the structural computation and on a blade-element method compatible with a bidirectional flexible approach, enables the evaluation of various performance items as well as the visualization of the instantaneous aerodynamic forces and wing shape as seen in figure 5. This capability is somehow practical for engineers in order to assist their mechanical feeling in the design of FWNAV. However our aeroelastic framework have to be validated and experiments are in progress with wing deformation and force measurements in vacuum and in air.

2 Choosing an actuation strategy

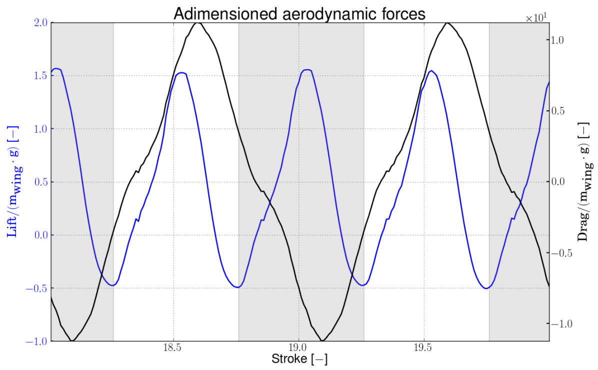
The actuation strategy is one of the core design choice as it affects the entire development by pointing in a specific direction to the engineering and reverting afterwards to another strategy might be costly and time-consuming. Currently



(a) CAD



(b) Instantaneous aeroelastic response



(c) Lift (blue) and drag (black) forces

Fig. 5 Example of computation on a wing in a pure 200μm heaving sine motion at its resonance frequency f=216.95Hz with large displacement capabilities of the FEs on. A mean lift of 4.02e-05N for a weight of 9.04e-06N is estimated. A grid of 10x20 FE per membrane is used along with 200 time-steps per stroke and 4 subtime-steps.

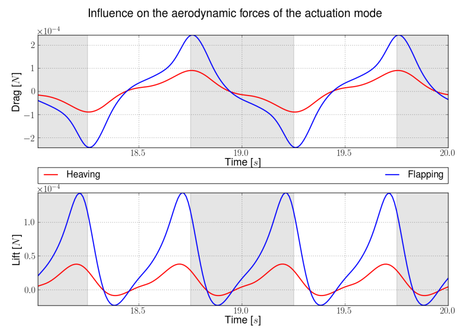
resonant [2, 3] and non-resonant [1, 7] designs of FWNAV are being developed. The conception of both designs is challenging in numerous areas, but resonant designs are slightly more challenging due to the resonance and the induced structural phenomena that have to be brought under control. In the case of our FWNAV [3, 5], the wing has to generate, thanks to its mode-shape, its own insect-like kinematics so as to generate enough aerodynamic forces for flight. Thus the higher the wing deformation, the higher might be the aerodynamic forces. However the wing deformation is strongly correlated to its actuation and it is of interest to find out what might be the best actuation strategy given the specifications of a FWNAV.

An actuation strategy is here defined as the combination of the actuation mode, which DOF is actuated, and of its kinematics, how it is actuated. Both items are discussed below in the case of a resonant design using our aeroelastic framework to evaluate the consequences of various actuation scenarios on the aerodynamic forces and on the actuation power. The idea is here to quickly estimate them and therefore calculations are made with the large displacement capability of the FE turned off so as to follow a preliminary design process of a FWNAV. The wing of the figure 5 is our benchmark along with a heaving actuation mode resulting from technical limitations of our test-bench i.e. not of our prototype.

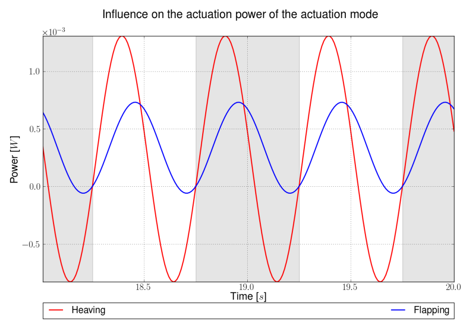
2.1 Actuation mode

In order to increase the wing deformation i.e. kinematics, the wing root can be actuated on various DOFs depending on the specifications of the FWNAV. One specification of our design is to avoid any complex mechanical link between the actuator and the wings so as to minimize the energy-losses. Therefore a single DOF actuation is here considered along with a heaving and flapping actuation mode.

The heaving and the flapping actuations are set to a pure sine actuated at the first eigenfrequency of the wing. The amplitude of the actuation is set to be respectively 200μm and 0.04rad. These amplitudes are consistent with the expected dimensions of our prototype and well below our actuator capabilities. Both actuation results are presented in the figure 6.



(a) Aerodynamic forces



(b) Actuation power

Fig. 6 Comparison between a heaving (red) and a flapping (blue) sine excitation of the wing root. Shaded areas indicates the downstroke motion. The flapping actuation proves itself to be more efficient towards lift-generation and power consumption.

Even with the small displacement assumption used in the FE computation and its associated overestimation of the aerodynamic forces, outlined by comparing the red curves of the figure 6(a) with the ones of the figure 5(c), the results are unequivocal: a flapping actuation generates more wing deformations and thus more aerodynamic forces. Indeed with a flapping actuation, the wing tip is experiencing a higher translational velocity i.e. a larger drag force, combined to a larger torsion angle inducing a larger lift force. The flapping actuation is also more efficient by having a considerably reduced peak-to-peak power consumption for almost the same mean power consumption.

To sum up, a flapping actuation proves itself to be more efficient than a heaving actuation and is therefore implemented on our prototype showing also improved kinematics.

2.2 Actuation kinematics

Similarly, results in the literature [17, 20–23] show that the actuation kinematics influences the aerodynamic forces generation. Therefore various actuation kinematics are here evaluated: a sine, a triangle and a square waveforms. The triangle and square waveforms are given respectively by the equations 3 and 4 where f_{wing} is the first eigenfrequency of the wing.

$$z(t) = \frac{\arcsin\left(0.99\sin\left(2\pi f_{wing} t\right)\right)}{\arcsin\left(0.99\right)} \tag{3}$$

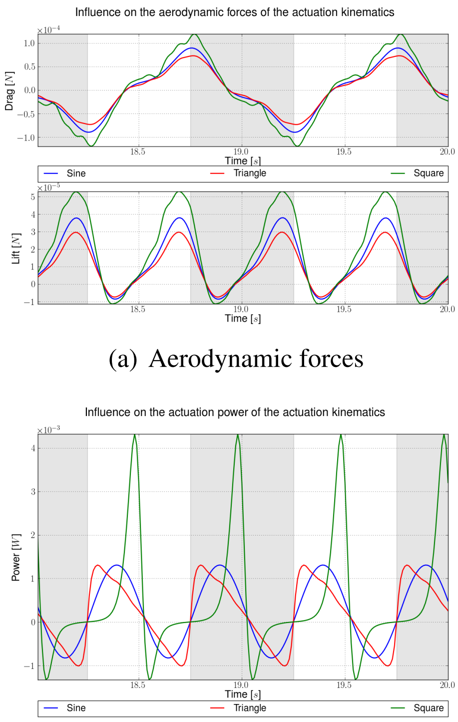
$$z(t) = \frac{\tanh\left(3 \cdot \sin\left(2\pi f_{wing} t\right)\right)}{\tanh\left(3\right)} \tag{4}$$

The results are presented in the figure 7.

When looking at the aerodynamic forces, the square waveform performs better in both drag and lift. However when the actuation power is taken into account, a strong peak to peak amplitude is observed indicating that the actuator will have to be sized for this peak consumption. Conversely the triangle waveform behaves better on actuation power but shows a worth efficiency on aerodynamic forces. Therefore the sine waveform is an adequate compromise for the actuation of a resonant wing in heaving when the energy-efficiency is mandatory and a square waveform otherwise. Other waveforms can also be investigated , if needed, by parameterizing further the waveform and playing on its symmetry for example.

3 Assisting the wing design

Once the actuation strategy is chosen, another application of our aeroelastic framework is the design of an 'optimized' wing by using the framework as a kernel and evaluating iteratively various wing geometries until one might be used as a



(a) Aerodynamic forces

(b) Actuation power

Fig. 7 Comparison between a sine, a triangle and a square signal for a heaving excitation of the wing root of 200μ at the first eigenfrequency of the wing. Shaded areas indicates the downstroke motion.

baseline for trial and error approach or for higher fidelity aeroelastic model.

As outlined by the number of insect flying in nature, local maximum or minimum are expected and thus the optimization strategy have to be sufficiently robust and efficient so as to find the global one. Despite an increasing computation load when compared to more classical iterative methods, heuristic algorithms have better chance in achieving this task accounting that our aeroelastic framework is developed so as to limit the computation load. Therefore our optimization strategy is based on a genetic algorithm (GA) and is here below presented as well as a preliminary risk reduction optimization so as to identify bottlenecks in our approach before starting complex optimization. Here again the idea is to quickly find out the performance of a wing geometry and the large displacement capability of the FEs is turned off.

3.1 Optimization environment

As explained before, our aeroelastic framework is a combination of a *Python* script and *Ansys* templates which runs autonomously and provides therefore a perfect kernel for an optimizer. In order to ease its implementation, an additional *Python* layer based on the *Pyevolve* module is used to provide GA support.

To optimize a given score function, the module generates a population of individuals from a given design space. Each individual is defined by the parametric geometry of the figure 2, evaluated in our aeroelastic framework, and scored using the output data from the framework. Once an entire population is evaluated, the module generates a new population by selecting the best individuals and combining them as well as breeding randomly new individuals to the population. The module iterates until convergence or the maximum number of populations is reached.

A downside of the GA is its computation load inherent to its random approach. Thus the behavior of the optimization process have to be estimated on a simple case so as to identify limitations in the evaluation of individuals especially in distinguishing successful from failed evaluations and by containing the computation cost to an acceptable minimum.

3.2 Risk-reduction optimization

The risk-reduction optimization was initially presented in Vanneste et al. [24] and consists of a flag-like wing, a leading with two perpendicular vein of same lengths. The individuals were scored by the mean lift adimensioned by the wing weight as given in the equation 5. The factor 1000 is here to stretch the score between each individual.

$$J = \frac{\bar{L}}{M_{wing} \cdot g} \cdot 1000 \tag{5}$$

Some stability issues were raised, especially regarding mesh size and time-step effects, resulting in misleading optimized wing. Therefore the evaluation of each individual is now improved by slightly increasing the mesh size and the number of time-substeps, and above all checking the frequency spectrum of the aerodynamic forces for inconsistencies due to a misleading structural computation. Therefore a FFT on the drag signal is achieved and normalized by its maximal amplitude. Eventually the computation is discarded if more than three frequencies contribute to more than 10% to the signal, by setting the score to 0 like in the case of negative mean lift or broken computation.

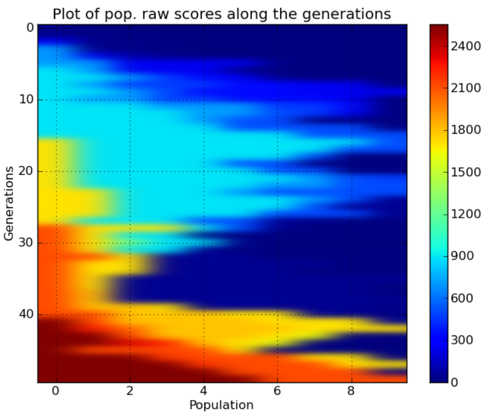
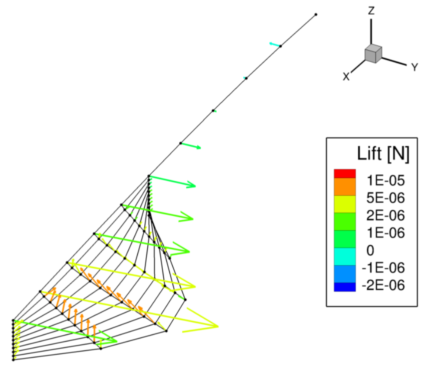


Fig. 8 Colormap of the raw score.

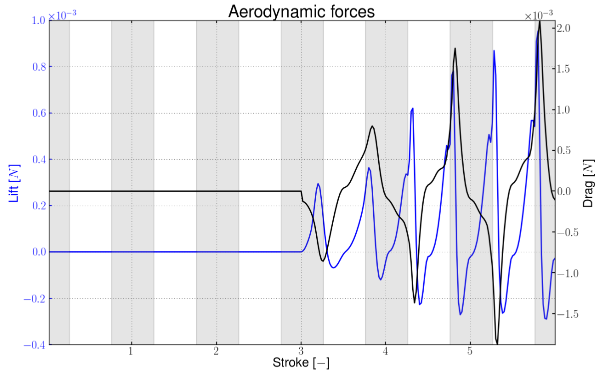
With these modifications, the flag-like optimization is relaunched as illustrated in the figure 8. The results display a more complex design space, indicated by the increased number of 0, due to our FFT filter that digs out our design space even more.

Still a best individual can be found, even if slightly unrealistic from a biomimetism point of view, and is shown in the figure 9. Its aerodynamic performance is overestimated due to the small displacement assumption made in FE computation, but the high frequency is coherent with the formulation of our aerodynamic model, where an high flapping frequency induces a higher wing velocity to which the translational forces are more sensitive than the added mass forces.

This indicates that the aerodynamic model has to be slightly modified in future works to decrease the frequency dependency. Similarly optimization with large displacement will have to be



(a) Instantaneous aeroelastic response



(b) Lift (blue) and drag (black) forces

Fig. 9 Best individual found by the optimizer after 50 generations of 10 individuals. The resonance frequency is at 759.48Hz for a pure 100μm heaving sine motion in small displacement. A mean lift of 1.45e-04N for a weight of 5.78e-05N is estimated. A grid of 5x10 FE per membrane is used along with 50 time-steps per stroke and 4 subtime-steps.

carried out in future works so as to clear up this approximation.

Conclusion

FWNAVs focus more and more interests from the scientific community and an increasing number of projects can be expected to be launched. The development from scratch of a FWNAV is an ambitious and arduous tasks relying often on trial and error approach, because well-established guidelines and know-how such as for airplane are missing. In order to assist these developments, a preliminary design tool evaluating the aeroelastic performance of a flapping wing is sound. Our approach, coupling a structural finite element solver to a quasi-steady aerodynamic model, is here reported along with some of its applications to the preliminary design of a resonant FWNAV: the selection of an appropriate actuation strategy and the definition of an optimization environment so as to design 'optimized' wing geometry.

The actuation strategy, the combination of a DOF and a waveform, is of utmost importance in the early design stages as it will bound from start the engineering solutions available to achieve an airborne design. Results indicate that a flapping actuation is more efficient in both the power consumption and aerodynamic forces generation than a heaving actuation for resonant wing and this results has been integrated in our prototype. Similarly a sine actuation proves to be a good compromise to generate aerodynamic forces while being power-efficient.

To hint an airborne-likely design, the aeroelastic framework is combined to a GA in order to scan a design space and find promising individuals that may constitute a baseline design. Before launching optimization on complex and more insect-like, a risk reduction optimization is mandatory and several issues were addressed by a better analysis on the data output from our aeroelastic framework. Once relaunched, more coherent individuals are found even if the small displacement assumption used here for the computation is misleading for the performance. Nevertheless due to the practicality, easiness and speed of our aeroelastic framework, it can be seen as a satisfactory preliminary design tool for FWNAVs. Further works will obviously focus on the experimental validation of our aeroelastic framework in small and large displacements, and then mostly on the optimization process so as to reduce further the computational cost of our aeroelastic framework and ultimately to speed up the development of our resonant FWNAV.

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Copyright Statement

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