

# BUCKLING-OPTIMIZED VARIABLE STIFFNESS LAMINATES FOR A COMPOSITE FUSELAGE WINDOW SECTION

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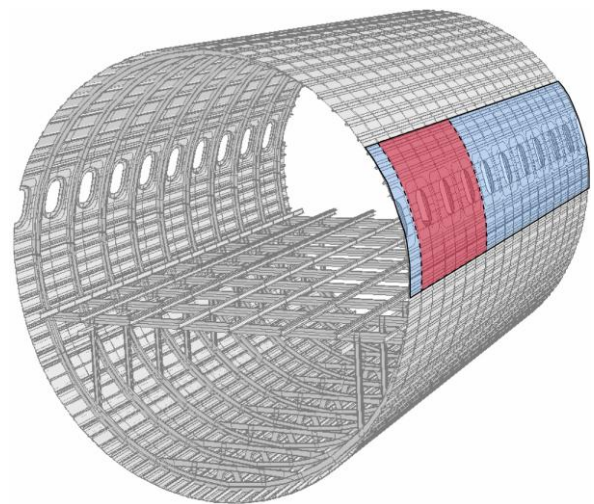
## Abstract

*Composite material offers potential compared to metals in the way the final structure can be tailored with the anisotropic material properties. In most thin planar structures in aerospace today these anisotropic properties of composite material are not fully exploited because traditional quasi isotropic laminates are used. Modern Automated Fiber Placement manufacturing capabilities enable the introduction of variable stiffness laminates on a larger scale. In this paper a two-step variable stiffness design approach is proposed. First the optimal fiber orientations are determined. From these fiber orientations the physical composite fiber tows are derived. Besides buckling optimization, as addressed in this paper, variable stiffness laminates can also be used e.g. for improvement of load transfer and morphing structures.*

## 1 Introduction

Over the last three decades the use of composite material in aerospace structures has increased significantly. The main reason for this is the better stiffness and strength to weight ratio compared to metals, but also the possibilities for further automation of composite manufacturing techniques have contributed to this increase. Currently most composite laminates used in aerospace have traditional lay-ups with fixed fiber angles of 0, +-45 and 90 degrees. The laminates are tailored for complying with strength and stiffness requirements by changing

laminates by changing the ratios and sequences of the 0, +-45 and 90 degrees plies in the laminate. For instance in the side section of the fuselage, see Figure 1, the number of +-45 degree plies is increased to improve the resistance against shear loading.



**Figure 1: Illustration of an aircraft fuselage barrel with the window section indicated in blue.**

Traditional laminates with a fixed number of unidirectional plies have constant stiffness properties throughout the laminate. In contrast, laminates with non-unidirectional plies, i.e. containing curved fibers, have varying stiffness properties throughout the laminate and are therefore usually referred to as variable stiffness laminates [2]. Various studies have shown that improvement in mechanical performance can be found for variable stiffness laminates in comparison to traditional laminates with the same thickness [2][3][4][5]. These studies were mostly based on academic cases with relatively

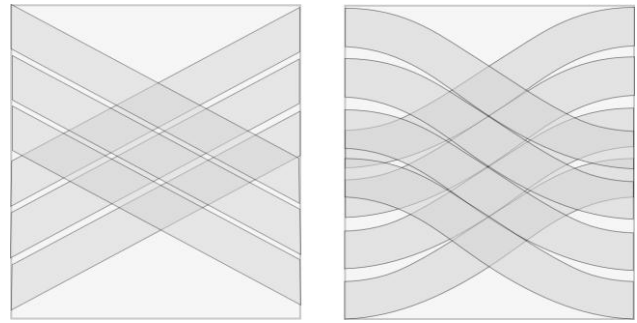
simple geometries like flat plates and circular cylinders.

Some key challenges related to more industrial application of variable stiffness laminates are for example failure criteria or manufacturability constraints, which may limit the potential optimum mechanical performance of the laminate. Therefore it is important to account for these constraints in the design process and an attempt is made to demonstrate this in the present study. Also other challenges can be mentioned, e.g. that variable stiffness laminates still have some unresolved issues for aerospace application such as certification and possibly higher costs because of the increased laminate complexity. However, this study will not address these other challenges.

The aim in the present study is to extend the design of variable stiffness laminates towards an industrial case with a more complex geometry compared to the academic cases. An optimization approach is presented for variable stiffness laminates on a ply-by-ply basis to improve the buckling load. The approach uses control points as design variables that map the local fiber orientations to the finite element mesh. This way a flexible approach is created that can be used on a variety of thin-structures. The approach is also compared with results found in literature for buckling of a flat plate.

## 2 Variable stiffness laminates

The basis for improved mechanical performance of composite structures lies in optimized use of the anisotropic properties of the laminate material. The use of traditional laminates with only unidirectional plies (illustrated in Figure 2) poses a strong limitation on the possibilities for laminate design. Allowing the use of variable stiffness laminates with arbitrary angles over the entire laminate the design space is significantly increased, see Figure 2.



**Figure 2: Illustration of composite fiber direction for a conventional constant stiffness laminate (left) and variable stiffness laminate (right).**

Variable stiffness laminates can be optimized for various applications such as bearing bypass, load changes, buckling and tailoring elastic properties (morphing structures). The focus in this paper lies on buckling performance of variable stiffness laminates, where in previous research on academic level improvement has been shown. This paper presents a generic approach for variable stiffness application on non-trivial structures. The performance improvement over traditional laminates is assessed.

Variable stiffness laminates have been investigated for around two decades. In literature these design and optimization problems are addressed in various ways. One approach is the optimization e.g. for buckling problems, through direct variation of local fiber orientations; another approach is defining the fiber orientations on the basis of principal strains for strain resistance optimization problems. The controlled variable stiffness optimization has been extensively investigated by Gürdal and co-workers [2][3][4][5] showing good improvements compared to conventional laminates.

For a common research case, buckling of a flat plate, a significant buckling load ( $\lambda$ ) improvement ranging from 35 – 67% is shown in literature by using variable stiffness (VS) laminates compared to constant stiffness (CS) laminates. In research by Lund et. al.[8] the

discrete material optimization (DMO) is used for the flat plate buckling case showing 35% improvement. In work by Setoodeh et al. [4] a generalized reciprocal approximation approach is used to define the critical buckling load using first order Taylor series expansion. In the technical report by Luraghi [9] a NURBS base-curve approach is used that uses a central curve from which parallel derivative curves are created.

In the present study the controlled variable stiffness optimization on a ply-by-ply basis is used to optimize buckling performance. In the following section the parameterization followed by the numerical approach is described.

### 2.1 Parameterization and design space discretization

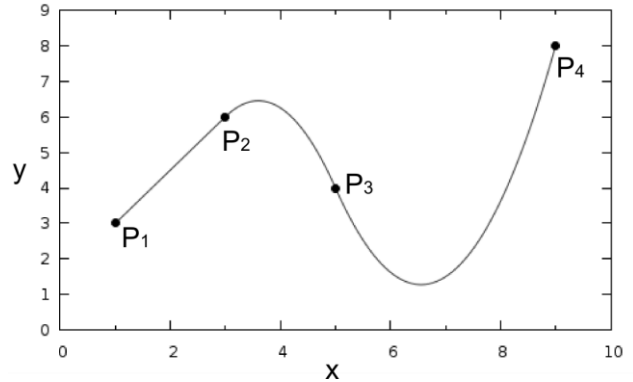
One of the aspects intensively investigated in literature of variable stiffness design and optimization is the parameterization and discretization of the design space. Variable stiffness laminates allow for a significantly larger design space compared to the traditional quasi-isotropic laminates. On the one hand the laminate stiffness can be varied in every point resulting in strongly increased potential for improvement. On the other hand, for efficient optimization the number of design variables has to be as low as possible.

For the approach in this study a ply-by-ply design space is chosen that can be seen as a compromise between a low number of optimization variables and a large design space. The basis for the analysis is a finite element model where the fiber orientation per ply can be varied in every element.

These per-element in-plane fiber orientations between -90 and 90 degrees are controlled by a very limited number of control points which are interpolated using a Catmull-Rom 2-D spline definition from which a spline surface is constructed, see equation 1 [7] and Figure 3.

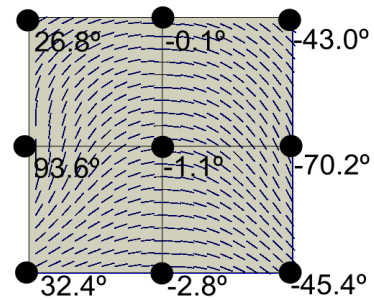
$$q(t) = 0.5 \cdot (2P_1 + (-P_0 + P_2) \cdot t + (2P_0 - 5P_1 + 4P_2 - P_3) \cdot t^2 + (-P_0 + 3P_1 - 3P_2 + P_3) \cdot t^3) \quad (1)$$

In which  $q$  is the interpolated value,  $t$  the normalized scalar along the spline and  $P_0$  to  $P_3$  the four control-points.



**Figure 3: Example of a spline curve to construct a spline surface which is used to interpolate the local ply orientations**

In the ply-by-ply variable stiffness design the control points are mapped using a surface-like approach. The spline curves in the first direction are used for the second direction where the spline function  $z$ -value represents the local ply orientation in degrees, see Figure 4.



**Figure 4: Example of a nine control point field with fiber-angle values of the control points. The vector field is interpolated from these control point locations and values using the spline surface mapping.**

With this approach a flexible mapping is achieved that can also be used for local thickness variations and for flat/curved topologies.

### 3 Numerical approach and optimization

This research is focused on the buckling problem of an aerospace structure. In the simplified case of a flat plate with composite material, symmetric and balanced layup and uniaxial compression with simple support, the general buckling behavior and critical load is determined by equation 2. [10]

$$N_x = \frac{\pi^2}{a^2} \left[ D_{11}m^2 + 2(D_{12} + 2D_{66})(AR)^2 + D_{22} \frac{(AR)^4}{m^2} \right] \quad (2)$$

Here the value of the parameter  $AR$  is the plate aspect ratio (*length/width*),  $a$  the length,  $m$  the number of half-wave present in the compression direction. The  $D_{11}$ ,  $D_{12}$ ,  $D_{22}$  and  $D_{66}$  values are the laminate bending stiffness terms. The variable stiffness laminate is used to influence the bending stiffness values by changing the local ply orientation. Another way to improve the buckling performance is to use variable stiffness laminates to unload a section of the laminate, thus reducing the local  $N_x$ .

In this research the optimization of the buckling performance of the structure is evaluated for different variable stiffness designs with the use of linear buckling analyses. Non-linear post-buckling analysis would be computationally too expensive to include in a full optimization. Instead it is chosen to perform a non-linear post-buckling analysis on the optimized structure to investigate the post-buckling response and failure load.

#### 3.1 Optimization problem formulation

For the considered design optimization problem the objective is to find the highest linear buckling value within the design space. This design space is governed by the afore mentioned orientation mapping but also by constraints. The optimization problem is formulated as:

$$\begin{aligned} \max_d \quad & J(d) \\ \text{s.t.} \quad & g(d) \leq 0, \\ & d \in [d_{\min}, d_{\max}] \end{aligned} \quad (3)$$

Where  $J$  is the scalar design objective, in this case the linear buckling load,  $g$  is a set of constraints and  $d$  is the set of design variables. The constraints  $g$  in this formulation can be used for example to incorporate laminate failure criteria or manufacturing constraints. However, currently we have incorporated simplified manufacturing considerations in the parameterization of the problem.

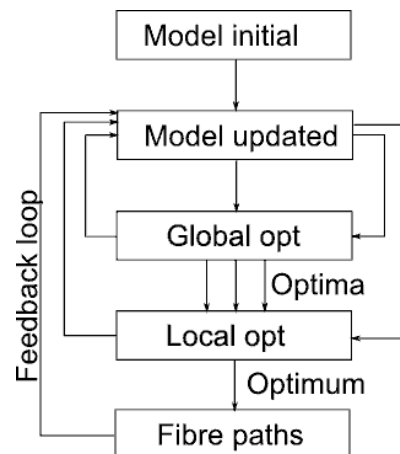
In earlier research it was found that the optimization problem is complex and hard to capture with a surrogate model. The design objective field is highly nonlinear. Therefore the proposed approach for finding the optimum design efficiently is an initial global search algorithm for finding interesting design ‘areas’.

- Latin Hypercube sampling LHS
- OpenOpt GLP (GA based)

This is followed by parallel local optimization algorithm calculations to find the local optimum. At the end two most interesting ‘areas’ are investigated with local optimization.

- Scipy fmin
- OpenOpt NLP/NSP

Note, for each design objective evaluation a linear buckling finite element simulation of the structure is performed.



**Figure 5: Overview of the optimization and feedback loop. From the global opt. a selection of optima is chosen for the local optimizer. The optimum is analyzed with fiber paths and feedback for model update.**



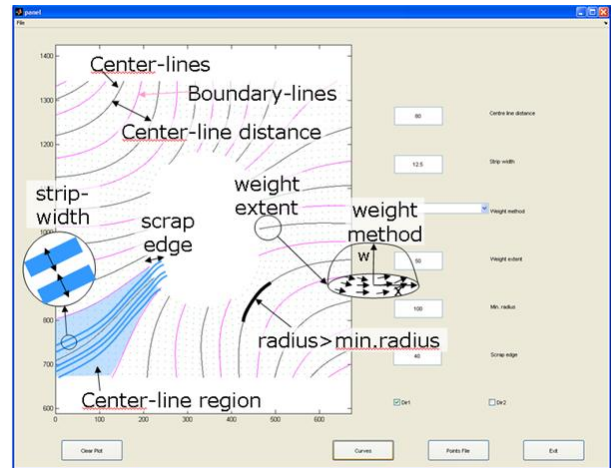
Constraints included in the optimization are the lower and upper bounds of the design variables to avoid undesired designs. Furthermore, manufacturing constraints for minimal radius of fiber curvature, which are related to the operational limitations of the advanced fiber placement machine, are accounted for. This is ensured by relatively simple geometrical calculations with the distance between the control points (i.e. in which the prescribed angles are the design variables) and the relative ply orientation variation between these two control points.

### 3.2 Fiber path extraction

The result of the optimization procedure is a set of ply-by-ply per-element fiber orientations that are spatially smooth because of the underlying spline based representation.

However, these fiber orientations are not directly useable for manufacturing because the AFP machine requires discrete tow path coordinates instead of continuous vector fields of fiber angles.

Therefore a specific translation procedure was developed that uses the optimized continuous vector fields of fiber angles as input and produces a feasible manufacturing design based on discrete paths of fiber tows, see Figure 6. Several constraints are accounted for in this procedure to ensure the manufacturing quality of the laminates. The translation procedure explores the optimized continuous vector fields and determines areas with similar fiber orientations and designates for each of these areas a so-called “center line”, which represents the local discrete tow path for this area. The fiber placed tows are extended by using the local ply orientation data in combination with a weighing method. Also the minimal radius of the discrete tow path can be set and is evaluated during the fiber tow design process. For more information regarding this routine, the reader is referred to ref. [1].



**Figure 6: View of the fiber path tool to derive a to be manufactured design from the vector-field**

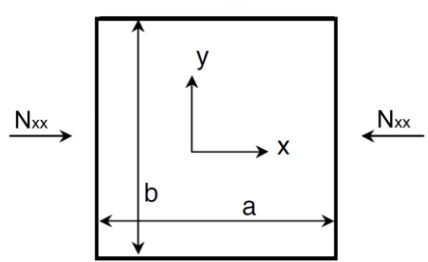
It is of main importance that there is a reasonable correlation between fiber angle vector fields of the input and the output of this fiber path translation procedure. If this correlation is low, the performance of the optimized structure will deviate significantly from the manufactured structure. Therefore a feedback loop is created that uses the fiber path output and translates it to a finite element model. This way the previously optimized structure’s performance can be compared with the manufacturing design performance.

## 4 Case studies

In this section the performance comparison applications will be described on a flat plate and a fuselage section cases. Both applications are based on buckling performance optimization by variable stiffness laminates with constant thickness and thus the same weight.

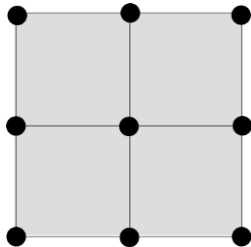
### (1) Buckling of a simple square plate

This example is used to verify the implementation of the procedure by comparison of the result with literature references. The gain in performance will be compared in a relative sense. The composite square plate is simply supported and has a thickness of 2.0 mm. In Figure 7 a schematic of the flat plate with compression load is shown.



**Figure 7: Square plate with simply supported boundary conditions and size of 500 mm ( $a$  and  $b$  values).**

For comparison the quasi-isotropic  $[45/-45/0/90]_{2s}$  and steered options are analyzed. The variable stiffness lay-up consists of four ply-pairs with symmetric laminate.



**Figure 8: Top view of the plate with the nine control points for the variable stiffness mapping**

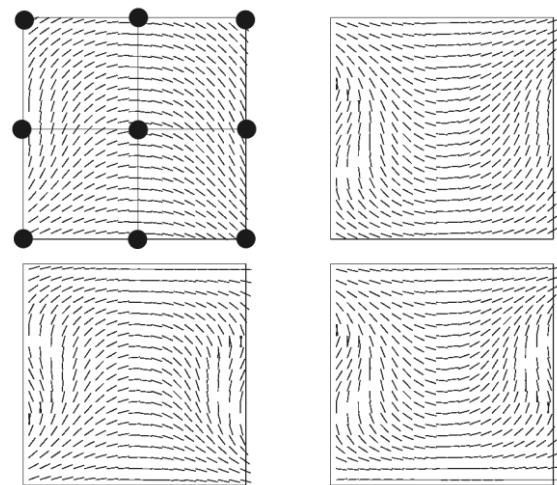
A  $3 \times 3$  grid, see Figure 8, without symmetry conditions is chosen for the control points resulting in 36 design variables for the four ply-pairs in total with a lower- and upper bound of  $\pm 90^\circ$ .

For the flat plate variable stiffness design the model allows variation of all element ply orientations in all plies. Initially a Latin Hypercube sampling of 100 design points is created for a global assessment, from which the most promising design is chosen. This is followed by the Nelder-Mead simplex optimization [6] to find the local optimal buckling load. In Table 1 the results are shown for the quasi-isotropic plate, the LHS intermediate result and the final optimized design. Clearly, the simplex optimization process yielded a significant further improvement of an additional 49% in buckling

load as compared to the LHS optimum with the quasi-isotropic (QI) as reference.

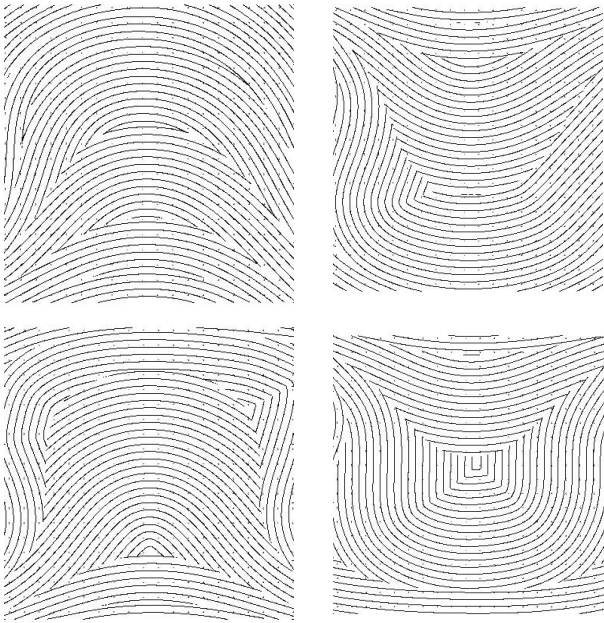
**Table 1: Comparison of quasi isotropic and variable stiffness (VS) designs. The results are normalized to the quasi-isotropic design.**

	Lay-up	Normalized lambda
Quasi-isotropic	$[45/-45/90/0]_{2s}$	1.00
Latin Hypercube	VS	1.12
Optimum	VS	1.61



**Figure 9: Optimized result of the flat plate buckling for the four ply-pairs. In the upper-left example the nine control points are shown. The orientation vector per element is shown of the optimized result**

The optimized structure with a normalized lambda for compression is shown in Figure 9. These are the element-wise orientations within the steered plies. From this vector-data the optimized fiber path design is created using the afore mentioned fiber-path extraction tool. The resulting fiber paths are shown in Figure 10. The load is applied in horizontal direction and it is clear that in the center of the plate the fibers are parallel to the load.



**Figure 10: Fiber path designs for the four ply pairs of the optimized flat plate.**

The resulting fiber path design is fed back into the finite element analysis by correlating the paths with the element’s central location. The closest path data point is used to determine the ply orientation for the element. The resulting element-wise orientations from the feedback loop are used and the flat panel analyzed again with the updated orientations. The results for the compression buckling load are shown in Table 2.

**Table 2: Comparison of compression buckling load for the feedback designs. The compression buckling load is reduced with a small percentage.**

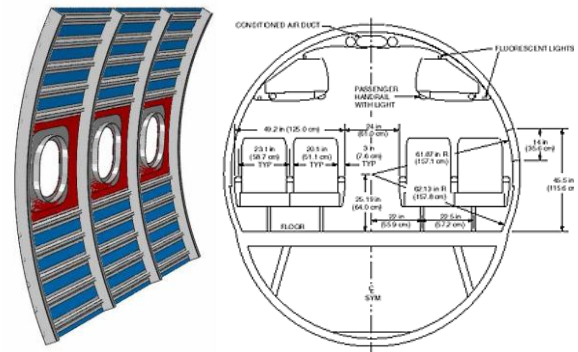
	Lay-up	Normalized lambda
Optimum	VS	1.61
Feedback of optimum	VS	1.53

From the flat plate case it can be concluded that the use of variable stiffness is of clear benefit for performance optimization. An increase in compression buckling load of 61% is achieved. In literature, similar improvements are found [4][8][9]. The feedback loop results in an updated and manufactured design with only a small reduction in performance compared to the

optimized design. The final performance increase is 53% for compression buckling of a flat plate.

**(2) Fuselage window section**

Based on the results of the first case study, the same approach is used for a more complex structure, the fuselage window section. The geometry model consists of a section of three frame pitches of a common mid-range aircraft fuselage. This includes a top and bottom stiffened panel section with lower skin thickness of 13 plies, and a window section with 24 plies and including window frames.



**Figure 11: Fuselage window section with frames, stringer and window frames on the left. The skin thickness is lower in the area indicated by blue. On the right an illustration of a fuselage cross-section**

In this case study the skin of the entire structure (i.e. in the blue and red areas in fig. 6) is included in the variable stiffness optimization approach. The geometrical properties of the fuselage section are derived from the MAAXIMUS barrel and are shown in Table 3.

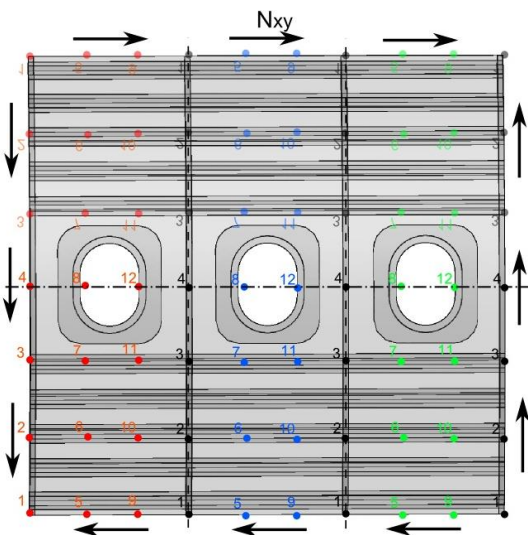
**Table 3: Geometrical and material properties for fuselage section**

Radius (r)	2150 mm
Length (l)	2012 mm
Frame type	C-frame
Frame pitch	670 mm
Stringer type	Omega/hat
Stringer pitch	185 mm
Skin thickness	1.65/2.67 mm
Composite material : E1=157GPa, E2=8.5GPa, G12=G13=G23=4.2GPa, nu=0.35, t=0.125mm	



Skin: 13 ply composite layup: [-45/45/90/0/-45/45/0/45/-45/0/90/45/-45]
VS skin: 13 ply composite layup: [ $\theta_1$ / $-\theta_1$ / $\theta_2$ / $-\theta_2$ / $\theta_3$ / $-\theta_3$ / 0 / $-\theta_3$ / $\theta_3$ / $-\theta_2$ / $\theta_2$ / $-\theta_1$ / $\theta_1$ ]
Skin: 22 ply composite layup: [-45/45/90/0/-45/45 / 0 / 45 / -45 / 0 / 90 / 90 / 0 / -45 / 45 / 0 / 45 / -45 / 0 / 90 / 45 / -45]
VS skin: 22 ply composite layup: [ $\theta_1$ / $-\theta_1$ / $\theta_2$ / $-\theta_2$ / $\theta_3$ / $-\theta_3$ / 0 / 45 / -45 / 0 / 90 / 90 / 0 / -45 / 45 / 0 / $-\theta_3$ / $\theta_3$ / $-\theta_2$ / $\theta_2$ / $-\theta_1$ / $\theta_1$ ]

Since the fuselage section consists of several bay sections, a large number of control points is needed. Since the geometry shows symmetry around the horizontal axis and a repetition of the frame sections the number of design variables for the optimization could be reduced from 216 to 36, as illustrated in Figure 12.

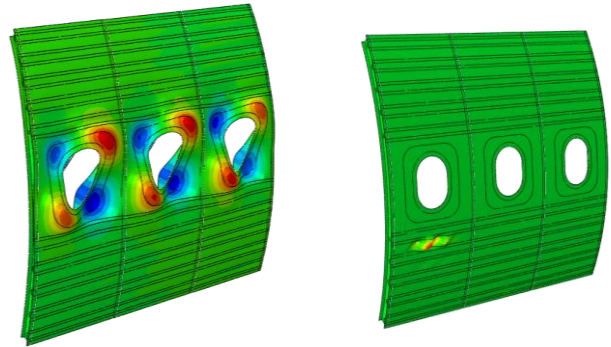


**Figure 12: Illustration of the control points used with symmetry and repetition. The shear load  $N_{xy}$  applied to the section is shown**

For the variable stiffness discretization towards the element ply orientations, the so-called 2D spline option is chosen, see Figure 12. This enables fiber variation in axial (flight) and in circumferential direction. In the next section the results of the variable stiffness optimization are shown.

Within the skin of the entire fuselage section the variable stiffness laminate is applied with the use of 36 control points in total. In the central

window section the laminate consists of 24 plies and in the stiffened panel sections 13 plies are used. The same optimization setup is used as in the flat plate case with initial Latin Hypercube sampling, followed by a simplex optimization method. The Latin Hypercube sampling optima are shown in Table 4.



**Figure 13: Common buckling modes found in the analyses, window buckling mode (left) and stiffened panel mode (right)**

The Latin Hypercube sampling for the 36 control points show an increase in the shear buckling load of 7.6%. The optimal results show smaller margins of improvement compared to the flat plate compression buckling. The improved buckling load is partly caused by load redistribution within the panel and improved out-of-plane stiffness of the window area in particular. Within the next step the two global starting positions are used to perform local optimizations. The results of these optimizations are shown in Table 4.

**Table 4: Comparison of quasi isotropic and variable stiffness (VS) designs. The results are normalized to the quasi-isotropic design.**

	Lay-up	Normalized lambda
Quasi-isotropic	Conventional	1.00
Latin Hypercube	VS	1.076
Optimum	VS	1.12

During the optimization a mode-change can be observed from window panel modes for the conventional quasi-isotropic to almost converged window- and stiffened panel modes

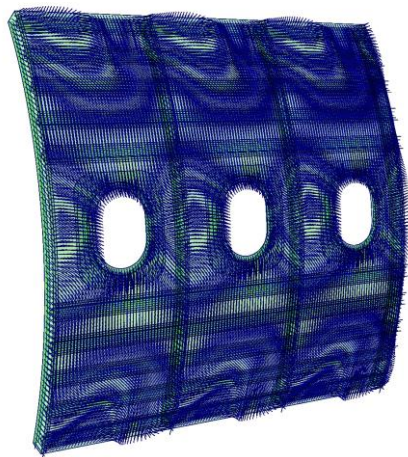


for the optimized solution. This is further illustrated in Figure 14 representing the first three modes lambda ( $\lambda$ ) calculated during the optimization. Convergence can be observed for these three buckling modes.



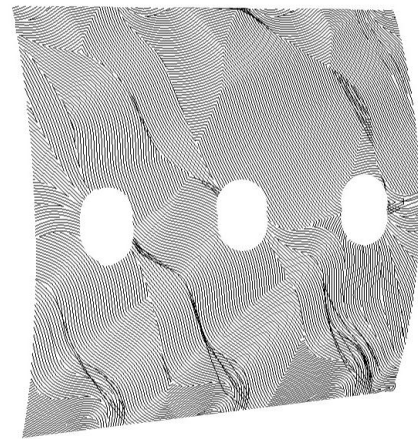
**Figure 14: Convergence of mode 1 and 2 for the second local optimization. Initially a larger deviation between the modes is present.**

In Figure 15 the variable stiffness laminate is illustrated. With this number of control points and resulting fiber orientation complexity, the ‘engineering’ reason for the improved buckling performance of the variable stiffness laminates is hard to determine.



**Figure 15: Variable stiffness orientation in the skin of the fuselage side section. The symmetry around the window frames and the repetition between the frames can be observed.**

As a final step within the optimization process the vector fields for the different plies are used to derive fiber paths. With the fiber path tool the vectors are used to derive a curved design.



**Figure 16: Fuselage window section fiber paths in the skin**

The fiber path design is mapped back onto the curved fuselage section and the orientation calculated. The results in Table 5 show a very strong decrease in performance which is caused by deviations of the feedback loop compared to the optimum. This translates to some areas, in particular in the stiffened panel section with low buckling loads compared to the optimum design.

**Table 5: Comparison of compression buckling load for the feedback designs.**

	Lay-up	Normalized lambda
Optimum	VS	1.12
Feedback	VS	0.68

The correlation between optimized design and feedback design will be further investigated in future research.

## 5 Conclusions

In this paper the results of a variable stiffness composite laminate approach is presented on a flat panel case and fuselage section. The buckling performance is optimized using a spline interpolation function describing the local varying ply orientations within the structure. This enables a large variable stiffness design space with limited amount of design variables and therefore faster optimizations. The

optimization is performed by a combined Latin Hypercube sampling and Nelder-Mead simplex local optimization.

Results for the flat panel case with 36 design variables show good correspondence with literature finding for buckling improvement for uniaxial loading with the use of variable stiffness laminates. Based on the most optimal design with limited manufacturing constraints a 61% increase in buckling load is achieved that reduces to 53% when considering manufacturing constraints by using the feedback loop.

Variable stiffness laminates are successfully introduced in a realistic aircraft fuselage topology with frames and stringers within this study. The fuselage side section skin variable stiffness laminate is created using 36 design variables that control most of the plies. An improvement of 12% in buckling performance is found when compared to constant stiffness laminate. Also converged buckling modes are observed that suggest a converged optimum. The ‘engineering’ explanation for the improvement can be found in the variation in stiffness which causes a load-redistribution in the skin section. Also the laminates bending stiffness is increased locally between frame and window frame.

Future work will involve weight minimization instead of the presented buckling performance maximization. This will be combined with the use of thickness variation of the laminate though natural overlaps by fiber steering but also discrete placement of additional plies.

## 6 Acknowledgement

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## References

[1] W.J. Vankan, W.M. van den Brink, R. Maas, Aircraft composite fuselage optimization through barrel and

panel level analyses, Eccomas Composite conference 2011, Hamburg, Germany

- [2] Z. Gürdal and R. Olmedo, In-plane response of laminates with spatially varying fibre orientations: Variable stiffness concept, *AIAA J.*, 31(4), pp. 751–758, 1993.
- [3] A. Alhajahmad, M.M. Abdalla and Z. Gürdal, Design tailoring for pressure pillowling using tow-placed steered fibers, *J. Aircraft*, 45(2), pp. 630-640, 2008.
- [4] S. Setoodeh, M.M. Abdalla and Z. Gürdal, Design of variable stiffness laminates using lamination parameters, *Composites, Part B: Engineering*, 37, pp. 301-309, 2006.
- [5] A.W. Blom, M.M. Abdalla and Z. Gürdal, Optimization of Course Locations in Fiber-Placed Panels for General Fiber Angle Distributions, *J. Compos. Sc. Technol.*, 70(4), pp. 564-570, 2010.
- [6] Nelder, J.A. and Mead, R. (1965), “A simplex method for function minimization”, *The Computer Journal*, 7, pp. 308-313 Wright, M.H. (1996), “Direct Search Methods: Once Scorned, Now Respectable”, in *Numerical Analysis 1995, Proceedings of the 1995 Dundee Biennial Conference in Numerical Analysis*, D.F. Griffiths and G.A. Watson (Eds.), Addison Wesley Longman, Harlow, UK, pp. 191-208.
- [7] Catmull, E., and Rom, R. A class of local interpolating splines. In *Computer Aided Geometric Design*, R. E. Barnhill and R. F. Reisenfeld, Eds. Academic Press, New York, 1974, pp. 317–326.
- [8] Lund E, Kuhlmeier L, Stegmann J. Buckling optimization of laminated hybrid composite shell structures using discrete material optimization. In: *6th World Congress on Structural and Multidisciplinary Optimization*, Rio de Janeiro, June 2005.
- [9] F. Luraghi, Surrogate Model-Based Design Optimization of Variable-Stiffness Composite Panels, Technical report, ONERA, 2011
- [10] C. Kassapoglou, Design and Analysis of Composite structures: with applications of aerospace structures, 2011

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