PARAMETRIC STUDY OF RAIM PERFORMANCE USING NEARLY UNIFORMLY DISTRIBUTED REFERENCE POINTS ON THE EARTH'S SURFACE

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Abstract

Integrity is a measure for the correctness of the position solution and is crucial for safety critical applications. The integrity of a navigation system can be checked by Receiver Autonomous Integrity Monitoring (RAIM). This paper will present simulation results of a RAIM algorithm using reference points that are nearly uniformly distributed on the earth's surface.

1 Introduction

For safety critical applications such as precision landing procedures in aviation as well as precise maritime harbor applications it is important to know how much the user can rely on the navigation system. The integrity of a navigation system is a measure for the correctness of the position solution determined by the system and indicates a confidence level for using the position information. There are basically three different kinds of approaches to provide an estimate for this level of trust: GPS combined with satellite-based augmentation systems (SBAS), the Galileo integrity concept (see [5]) by means of the freely accessible Safety-of-Life service and the RAIM method (see [1], [3], [4], [6], [8]) that can be applied on the user side.
SBAS is based on differential GPS providing the user with corrections. The RAIM method is performed directly within the receiver, thus the term *autonomous* monitoring.

Unfortunately, the GPS+SBAS and the Galileo approach are not fully compatible, e.g. the Galileo integrity information cannot be used for GPS satellites in a combined GPS-Galileo satellite scenario available in the near future. Therefore, this paper will focus on the RAIM method which overcomes this limitation.

RAIM uses the residuals of an over-determined position solution, e.g. at least five satellites have to be visible to the user. It is an method that can be easily implemented since no additional hardware is needed beside the receiver. The benefits of a navigation system, which is able to monitor its own integrity at receiver level, are obvious. Furthermore, as a result of the reconstruction of the Russian GLONASS system as well as the upcoming Chinese Compass system, users will be able to use lots of satellites (GPS, Galileo, GLONASS, Compass) for navigation in the near future.

2 RAIM availability

Basically, the RAIM method consists of two parts. First we have to take a closer look at the geometry of the satellite constellation from the user's point of view in order to check if the current geometry allows the RAIM method to be applied. If this is the case, RAIM is available and during a second step a threshold can be calculated.

The first requirement for the geometry is that the number of visible satellites n is at least five in order to result in an over-determined position solution.

For every visible satellite we calculate a *slope* value that shows how an error on this satellite will affect the position error of the user. This 3d-problem will be decomposed in three 1d-problems in a user orientated coordination system. First, we calculate B as an auxiliary quantity

$$B = [I - H(H^T H)^{-1} H^T] Cov^{-1}{}_{PR} \ldots$$

$$\ldots [I - H(H^T H)^{-1} H^T]. \qquad (1)$$

This gives us the three *slope* values

$$slope_i^{East} = \sqrt{\frac{M(1,i)^2}{B(i,i)}}, \qquad (2)$$

$$slope_i^{North} = \sqrt{\frac{M(2,i)^2}{B(i,i)}}, \qquad (3)$$

$$slope_i^{Vertical} = \sqrt{\frac{M(3,i)^2}{B(i,i)}}, \qquad (4)$$

where H is the design or geometry matrix, M is the transformation of the pseudo-ranges to the user orientated coordination system and Cov_{PR} is the covariance matrix for the pseudo-range measurements.

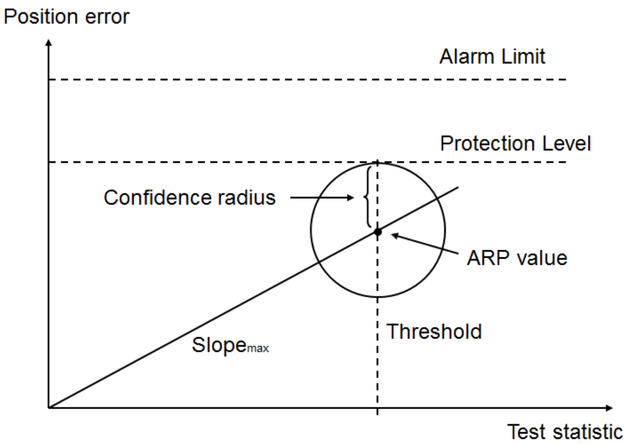

Fig. 1. Illustration of the Approximated Radial-Error Protected (ARP).

In order to use a conservative estimate we assume that an error is occurring on the pseudo-range measurement which is having the highest impact to the position solution, namely having the maximum *slope* value.

This leads us to the *Approximated Radial-Error Protected* (ARP) which is illustrated in Fig. 1 and can be calculated as follows

$$ARP^{East} = \max_i(slope_i^{East})\sqrt{T}, \qquad (5)$$

$$ARP^{North} = \max_i(slope_i^{North})\sqrt{T}, \qquad (6)$$

$$ARP^{Vertical} = \max_i(slope_i^{Vertical})\sqrt{T}. \qquad (7)$$

The threshold T needed to calculate Eq. (5)-(7) will be derived in section 3.

In the absence of measurement noise, the ARP value and the resulting protection level (see Fig. 1) will be the same. If there is noise and assuming it is normally distributed in each position component, the confidence radius R_{conf} will result to

$$R_{conf}^{East} = k(P_{md})\sigma^{East}, \qquad (8)$$

$$R_{conf}^{North} = k(P_{md})\sigma^{North}, \qquad (9)$$

$$R_{conf}^{Vertical} = k(P_{md})\sigma^{Vertical}, \qquad (10)$$

where $k(P_{md})$ is the quantile of the normal distribution that is only exceeded with the *probability of missed detection* $P_{md.}$

Overall, this gives us the horizontal protection level (HPL) and the vertical protection level (VPL)

$$HPL = \max(ARP^{East} + \ldots$$

$$\ldots R_{conf}^{East}, ARP^{North} + R_{conf}^{North})\sqrt{2}, \qquad (11)$$

$$VPL = ARP^{Vertical} + R_{conf}^{Vertical}. \qquad (12)$$

The protection levels are a function of the geometry of the satellite constellation from the user's point of view and the requirements in terms of integrity and continuity from which $P_{fa.}$ and P_{md} can be calculated.

The alarm limit (see Fig. 1) is the maximum position error that can be tolerated for a certain navigation procedure (e.g. precision approach) without triggering an alarm. Normally, one differentiates between the horizontal alarm limit (HAL) and the vertical alarm limit (VAL). An overview of alarm limits for the different flight phases provided by ICAO can be found here [2].

Finally, the satellite geometry at the user's position allows for error detection by means of RAIM if

$$HPL \leq HAL \quad and$$

$$VPL \leq VAL \quad and$$

$$n > 4.$$

If RAIM is not available a warning has to be given to the user.

3 Least Squares Residual RAIM

The Least Squares Residual (LSR) RAIM method is based on the linearized pseudo-range measurement equation given as

$$y = Hx + \varepsilon \tag{13}$$

where y is the $n\times1$ linearizes measurement vector (.e.g. differences between the measured pseudo-ranges and the ranges calculated based on the nominal satellite position), x is the 4×1 vector containing the incremental deviation with respect to the nominal state (e.g. three elements for the position components and one for the clock bias), H is the $n\times4$ design matrix between x and y and ε is the $n\times1$ error vector of the measurement which may consist of a deterministic and a random component.

To be able to detect errors and inconsistencies, one has to choose a threshold which is heavily related to the noise of the pseudo-range measurements. Saying that the RAIM method is checking the measurements with respect to self-consistency, it is obvious that the number of measurements n in Eq. (1) has to be greater than or equal to 5.

Solving Eq. (1) in a least square sense will result in

$$\hat{x} = (H^T H)^{-1} H^T y \tag{14}$$

where $\hat{x}$ is the least square estimate of x.

One measurement of consistency suggested by [7] is the least squares residual

$$w = y - \hat{y}$$

$$w = y - H\hat{x}$$

$$w = [I - H(H^T H)^{-1} H^T] y \tag{15}$$

$$w = [I - H(H^T H)^{-1} H^T](Hx + \varepsilon)$$

$$w = [I - H(H^T H)^{-1} H^T]\varepsilon \tag{16}$$

where $\hat{y}$ is the least square estimate of the pseudo-ranges and I is the $n\times n$ unit matrix.

Let us assume for the moment that all pseudo-range measurements will be independent and having the same standard derivation. Thus, each component of ε and w will be normally distributed

$$\varepsilon \sim N(0, \sigma^2 I) \tag{17}$$

$$w \sim N(0, \sigma^2 I). \tag{18}$$

As mentioned earlier, one needs to find a threshold related to noise and inconsistency of the pseudo-range measurements. A test statistic satisfying these requirements can be found by the absolute square sum of residuals, here called Sum of Squared Errors *SSE,* which is X^2-distributed with n-4 degrees of freedom according to Eq. (6)

$$SSE = \|w\|^2 = w^T w \sim X^2{}_{n-4}. \tag{19}$$

This scalar test statistic has a known probability density function under the assumptions made above.

Given a probability of false alarm P_{fa}, this allows us to calculate a threshold T for this probability density function. If the test statistic SSE exceeds the threshold T an alarm will be send to the user since an inconsistency or a significant error in the pseudo-ranges has most likely occurred.

Assuming that all the pseudo-range measurements will have the same standard derivation is a strong simplification.

Especially since it is obvious that satellites having small elevation angles will be more affected by the troposphere and also multipath effects are more likely to occur. Thus, the accuracy of the pseudo-range measurements can be approximated as a function of the elevation angle resulting in

$$SSE = \sum_{i=1}^{n} \frac{1}{\sigma_i^2} w_i^2. \qquad (20)$$

It was already stated that the test statistic SSE is X^2-distributed with parameter σ=1 and n-4 degrees of freedom. Hence, we have to find a threshold T of the X^2-distribution which is a function of the probability of false alarm P_{fa}

$$T = T(P_{fa}, n-4). \qquad (21)$$

With the given probability density function of the X^2-distribution

$$f_\nu = \frac{x^{(\nu-2)/2} e^{-x/2}}{2^{\nu/2}\Gamma(\nu/2)}, \quad x > 0, \qquad (22)$$

where ν is the number of degrees of freedom and Γ is the gamma function we have to solve Eq. (23) in order to find a suitable threshold

$$\int_0^T \frac{x^{(\nu-2)/2} e^{-x/2}}{2^{\nu/2}\Gamma(\nu/2)} dx = 1 - P_{fa}, \quad x > 0. \qquad (23)$$

There is no closed-form solution for this term, so one has to use numerical integration. However, we are able to find a closed-form solution for ν=2

$$\int_0^T \frac{1}{2} e^{-x/2} dx = 1 - p_{fa}, \quad x > 0, \qquad (24)$$

$$T = 2 \ln P_{fa}. \qquad (25)$$

By successfully performing the statistical hypothesis testing

$$SSE \le T \qquad (26)$$

the pseudo-ranges are assumed to be gross error free and can be trusted for navigation purposes.

4 Parametric study

4.1 Reference points

Most of the present RAIM simulations (see [3], [4], [6], [9]) used a constant spacing in longitude and latitude for the reference points, e.g. 5x5°, resulting in high point densities in the pole regions and therefore heavily biased overall RAIM availability. This parametric study will apply the RAIM algorithm using reference points that are nearly uniformly distributed on the earth's surface derived by an icosahedron (see Fig. 1.).

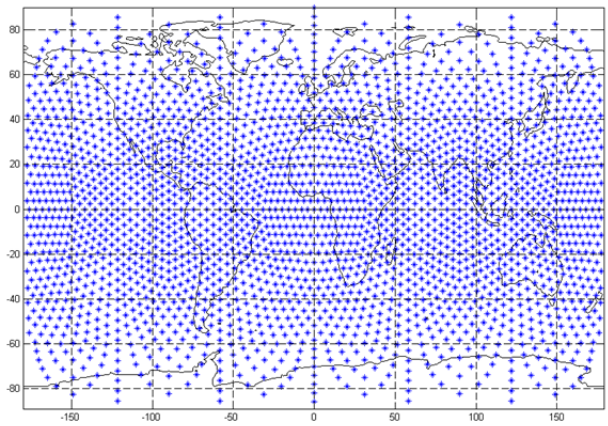

Fig. 1. Reference points for the RAIM simulation nearly uniformly distributed on the earth's surface derived by an icosahedron.

Starting point is an icosahedron with 20 identical equilateral triangular faces, 12 vertices and 30 edges. These 12 vertices are then projected on a sphere where they are uniformly distributed. Subsequently all the edges will be divided in halves and neighboring points will be connected. The newly generated vertices are again projected on the sphere. This will be iterated until a suitable density of reference points is achieved. In a final step the reference points from the sphere are projected on the earth's reference ellipsoid where they are nearly uniformly distributed since the reference ellipsoid slightly differs from a sphere.

In Fig. 1 you can see the 2562 reference points that are nearly uniformly distributed on the earth's surface derived by the method described here.

4.2 Probability of false alarm and probability of missed detection

This study is not following most of the present RAIM simulation by treating the probability of false alarm P_{fa} and the probability of missed detection P_{md} as constant values. Instead, we make use of the approach derived by [10] where P_{fa} and P_{md} is calculated from the performance requirements towards continuity and integrity as well as the likelihood of an error occurring on a single satellite.

This will lead to probabilities being a function of the number of satellites visible to the user. For example P_{fa} will vary between $7.7 \cdot 10^{-7}$ (5 GPS satellites) and $1.5 \cdot 10^{-7}$ (14 GPS, 13 Galileo and 12 Glonass satellites). P_{md} is ranging between $6.4 \cdot 10^{-3}$ (5 GPS satellites) and $1.9 \cdot 10^{-2}$ (14 GPS, 13 Galileo and 12 Glonass satellites).

4.3 Alarm limits

This parametric study is investigating whether satellite based navigation together with RAIM can meet the International Civil Aviation Organization (ICAO) requirement and thus can be used as a primary means of aircraft navigation. The first scenario investigated is APV-I (Approach Procedure with Vertical Guidance) requirement which leads to a HAL of 40m and a VAL of 50m (see [2]). The second scenario is the CAT-I precision approach resulting in a HAL of 40m and a VAL of 15m (see [2]) and therefore being more challenging.

4.4 Simulation results

4.4.1 RAIM availability for the GPS scenario

By using the 29 active GPS satellites and an elevation masking of 5° the user will have access to 9.8 satellites as an average. The GPS satellite constellation will be the same again every day from the user's perspective. Therefore, the simulation covered 10 full days.

The RAIM availability for this GPS scenario over all nearly uniformly distributed reference points and over all time steps covered by the simulation is illustrated in Fig. 2 (under the APV-I requirement: HAL 40m, VAL 50m) and in Fig. 3 (under the CAT-I requirement: HAL 40m, VAL 15m). In Fig. 2 one can see a RAIM availability of 100% for nearly all the regions except some areas between 70° and 80° latitude.

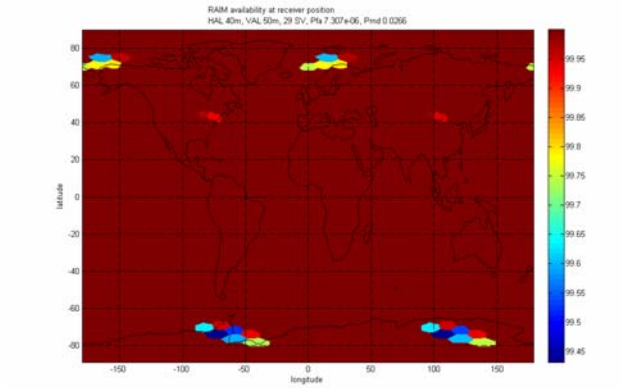

Fig. 2. RAIM availability of a 29 satellite GPS constellation over all reference points under the APV-I requirement (HAL 40m, VAL 50m).

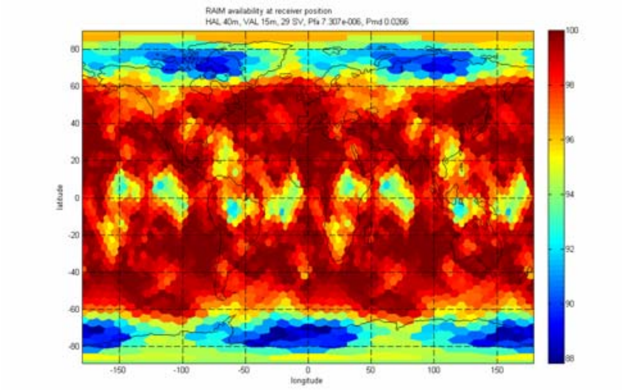

Fig. 3. RAIM availability of a 29 satellite GPS constellation over all reference points under the CAT-I requirement (HAL 40m, VAL 15m).

In addition to these contour plots, we can calculate an overall RAIM availability being the average over all reference points (e.g. potential user position on the earth's surface) and over all time steps. The big advantage of the nearly uniformly distributed reference points compared to the commonly used constant spacing in longitude and latitude is that it will not lead to a biased overall RAIM availability due to high point densities in the pole regions. It also makes sense to calculate the RAIM availably at the worst user location (e.g. the reference point with the lowest RAIM availability) in order to have a conservative estimate.

The RAIM availability under the APV-I scenario is 99,997% overall and 99.43% at the worst user location where it is 97.77% overall and 87.81% at the worst user location for the CAT-I scenario. Since the requirement towards availability for APV-I and CAT-I is between 99.0% and 99,999% according to [2], this parametric study seems to prove that monitoring the integrity using GPS satellites is always possibly under the APV-I requirement.

4.4.2 RAIM availability for the Galileo scenario

The Galileo scenario is assuming a constellation of 30 Galileo satellites (27 active and 3 spares) with an elevation masking of 10°. The elevation masking of 10° is necessary due to the Galileo transmitter antenna design. This scenario which will be available for future users would allow the user to have access to 9.15 satellites as an average. In accordance with the GPS scenario, the simulation covered 10 days after which the Galileo satellite constellation will be the same again from the user's perspective.

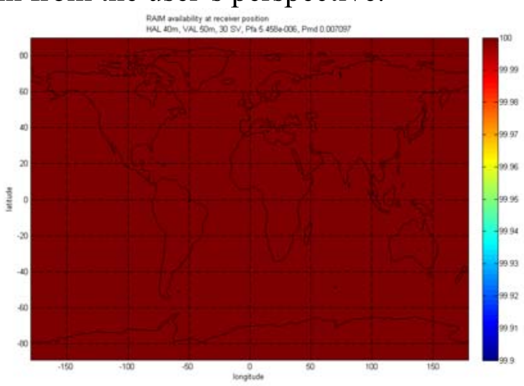

Fig. 4. RAIM availability of a 30 satellite Galileo constellation over all reference points under the APV-I requirement (HAL 40m, VAL 50m).

The RAIM availability for the Galileo scenario over all nearly uniformly distributed reference points and over all time steps covered by the simulation is illustrated in Fig. 4 (under the APV-I requirement: HAL 40m, VAL 50m) and in Fig. 5 (under the CAT-I requirement: HAL 40m, VAL 15m). It can be seen that the RAIM availability of the Galileo satellite constellation under the APV-I requirements is always 100%. This result is better than the one obtained with the GPS scenario even though we have fewer satellites visible to the user as well as higher elevation masking. Therefore, we can assume that the Galileo constellation is superior in terms of integrity.

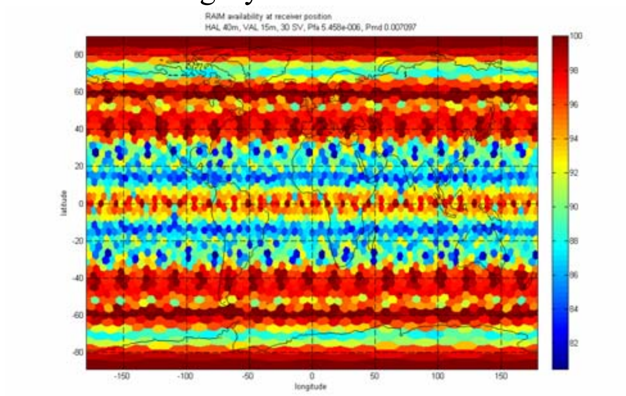

Fig. 5. RAIM availability of a 30 satellite Galileo constellation over all reference points under the CAT-I requirement (HAL 40m, VAL 15m).

In Fig. 5 one can see the RAIM availability under the CAT-I requirements which is symmetrical with respect to the equator. This is quite obvious due to the symmetry of the Galileo satellite constellation. The RAIM availability under the CAT-I scenario is 91.54% overall and 81.19% at the worst user location. The lowest RAIM availability can be expected between ±35° latitude (see Fig. 5).

4.4.3 RAIM availability for a combined GPS-Galileo-GLONASS scenario

Another graphical illustration method besides the contour plots (see Fig. 2-Fig. 5) can be obtained by looking at the probability density function of the protection levels of all reference points and over all time steps (see Fig. 6 to Fig. 9). In the two-dimensional domain defined by the vertical and horizontal protection level the probability density for a certain protection level combination is indicated by a color bar where the dark colors are referring to a high probability density. In addition, this method also allows us to display the vertical and horizontal alarm limit dividing the two-dimensional domain in four sectors. For every sector we calculated the probability of a protection level combination occurring in this sector.

In Fig. 6 one can see the probability density function of the protection level using the GPS constellation and displaying the APV-I requirement (HAL 40m, VAL 50m). The probability that both the vertical and the horizontal protection level are smaller than the respective alarm limit is 99.997% (this was already stated in section 4.4.1).

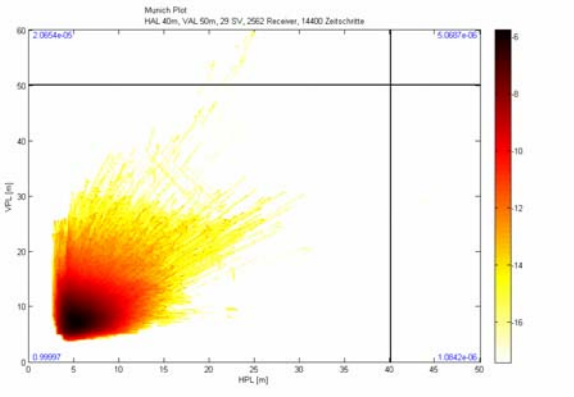

Fig. 6. Probability density function of the protection level using the GPS constellation and displaying the APV-I requirement (HAL 40m, VAL 50m).

For the Galileo constellation (see Fig. 7) one can see the scattering around the mean value is quite limited compared with the GPS constellation. Again, this is clearly showing that the geometrical constellation of the Galileo satellites has certain advantages over GPS in terms of integrity. The maximum protection levels obtained within this simulation are 17.38m in horizontal and 33.54m in vertical direction.

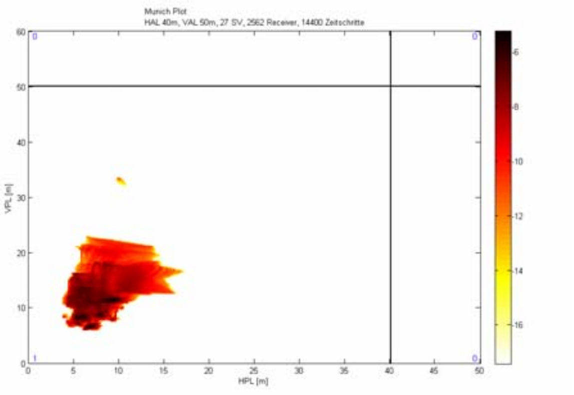

Fig. 7. Probability density function of the protection level using the Galileo constellation and displaying the APV-I requirement (HAL 40m, VAL 50m).

For a combined GPS-Galileo constellation both the APV-I and the CAT-I requirements can be met since this constellation will allow for an overall RAIM availability of 100%. Only under the CAT-I scenario the availability at the worst user location will decrease to 99.95%. The maximum protection levels obtained for this combined constellation are 12.17m in horizontal and 16.67m in vertical direction.

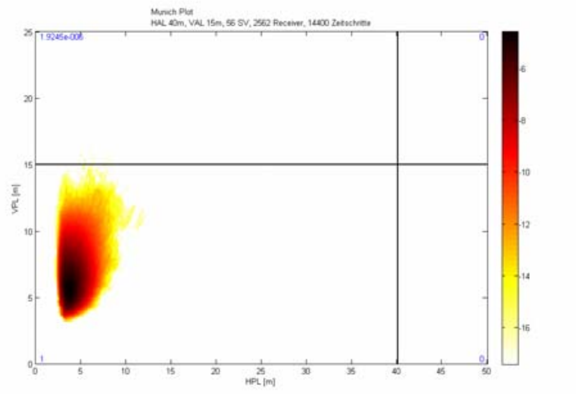

Fig. 8. Probability density function of the protection level using a combined GPS-Galileo constellation and displaying the CAT-I requirement (HAL 40m, VAL 15m).

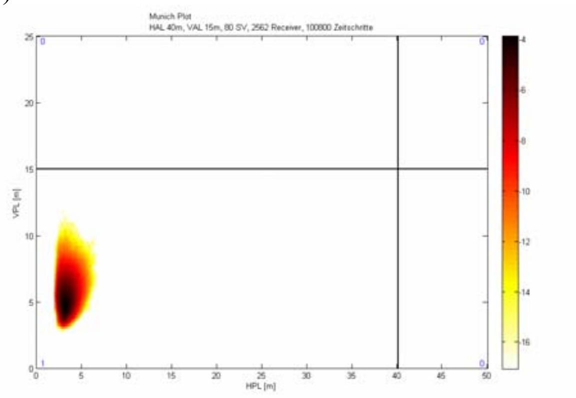

Fig. 9. Probability density function of the protection level using a combined GPS-Galileo-GLONASS constellation and displaying the CAT-I requirement (HAL 40m, VAL 15m).

The full GPS-Galileo-GLONASS constellation can be obtained by introducing a nominal GLONASS constellation consisting of 24 satellites and using an elevation masking of 5° to the simulation. Since the GLONASS satellite constellation will repeat after seven days from the user's perspective, the simulation for the GPS-Galileo-GLONASS scenario covered 70 days. This constellation consisting of 80 navigation satellites will met both the APV-I and the CAT-I requirements – overall and at the worst user location – always resulting in an availability of 100%. This can also be seen in Fig. 9 where the probability density function of the protection level using a combined GPS-Galileo-GLONASS constellation is displayed under the CAT-I requirement. The maximum protection levels obtained for this full navigation satellite constellation are 6.98m in horizontal and 12.19m in vertical direction.

5 Summary and conclusions

This parametric study calculated the RAIM availability using the following scenarios of navigation satellites to be accessible to the user:

- GPS,
- Galileo,
- GPS and Galileo,
- GPS, Galileo and GLONASS.

The reference points used for the parametric study are uniformly distributed on the earth's surface and were derived by an icosahedron. Compared to constant spacing in longitude and latitude, this method will not result in high point densities in the pole regions and therefore heavily biased overall RAIM availabilities.

This parametric study also showed some promising first results indicating that satellite based navigation using more than one satellite navigation system (e.g. GPS and Galileo or GPS, Galileo and GLONASS) can meet ICAO's APV-I and CAT-I requirements. This can lead to future developments allowing satellite navigation to be used as a primary means of aircraft navigation.

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