NONLINEAR AERODYNAMIC MODELING OF UNSTABLE AIRCRAFT USING FLIGHT TEST

Roger Larsson*,, Martin Enqvist****
***Saab AB,**Linköping University**
roger.gunnar.larsson@saabgroup.com; maren@isy.liu.se;

Keywords: *Aerodynamics, Nonlinear, Unstable, Identification, Flight test*

Abstract

This paper presents five direct identification methods for estimation of nonlinear aerodynamics of unstable aircraft. This type of identification is hard under the best of circumstances. In the context of system identification, direct means that no knowledge of the stabilizing flight control system (FCS) is used. This makes the methods more general and they could thus be used easily for different aircraft.

JAS 39 Gripen is designed to be subsonic pitch unstable and supersonic pitch stable to gain performance. For maneuvering close to trim conditions, the aerodynamics can be considered linear, but for aggressive, high angle-of-attack maneuvering and flight at transonic speeds where aerodynamic shocks are present there will be nonlinearities. This leads to the need for a flight control system that can handle these complexities. The FCS of JAS 39 Gripen is gain scheduled and has many different flight modes. In order to design the control laws, high quality simulation models are needed. This in turn makes system identification an important task.

Here five methods that can be used to estimate nonlinear aerodynamic characteristics from flight test data will be presented. The first method that we will discuss here is a parameterized observer (PO) approach where the observer gain is added to the unknown parameters to be identified. This gives a simple but fast method. The second and third approaches are the Extended (EKF) and the Unscented (UKF) Kalman filter, both nonlinear versions of the ordinary Kalman filter. These three first methods are versions of the prediction error method and involve iterative minimization of a cost function using a Levenberg-Marquardt optimization procedure. The fourth method add the unknown parameters as new states with zero dynamics and the state vector in this augmented system (AUG) is then estimated with an EKF in a single run. All these four methods rely on the possibility to predict the system output.

The fifth method is a bit different. It uses a constrained Levenberg-Marquardt (CLM) optimization procedure to minimize a Lagrangian function which does not depend on simulation of the system at all. Therefore the instability is not a problem. Instead the method puts constrains on every time sample.

In this paper, results are given for both simulations and a real flight test in the transonic envelop.

Fig. 1 JAS 39 Gripen test aircraft.

1 Introduction

In the early years of motorized flight, the Wright brothers flew their Flyer I manually. Later, analysis [5] shows that this aircraft had both unstable and nonlinear aerodynamic pitch characteristics. Fortunately, the time to double was long enough for the brothers to handle. As design of aircraft led to higher speeds it was necessary to turn to inherently pitch stable solutions. Today's highly maneuverable fighter aircraft are designed to be unstable in the pitch plane in order to gain performance. The speed has increased so much that supersonic flight is not uncommon. Near transonic speed, i.e. close to the speed of sound, nonlinearities can occur due to aerodynamic shocks moving over the aircraft. The instability and nonlinearity have made the modern fighter aircraft dependent on control systems that aid the pilot in flying the aircraft. In order to design the control laws, high quality simulation models are needed. A modern fighter aircraft such as JAS 39 Gripen has a very complex flight control system which is gain scheduled and handles many different flight modes. It is therefore desirable to be able to use a direct identification method on flight test data, i.e. to be able to identify the system without any knowledge of the control system.

Several books on the subject of aircraft identification have been published recently [11, 16, 7]. However, most work in this field has focused either on unstable and linear [2] or stable and nonlinear systems [9, 1]. There are some papers that mention both nonlinear and unstable system, like [8] and [12], but where the methods are different from the ones presented in this paper.

Here five different methods have been used. These methods have been applied to both simulated data and real flight test data.

2 Methods

In this paper, the following discrete-time state-space representation of a nonlinear output-error model is used

$$x_{k+1} = f(x_k, u_k; \theta), \tag{1a}$$
$$y_k = h(x_k, u_k; \theta) + e_k, \tag{1b}$$

where $x_k \equiv x(kT_s)$ is a $n \times 1$ state vector with sample time T_s, u_k is a $m \times 1$ input vector and y_k is a $p \times 1$ output vector, θ is the unknown parameter vector to be identified and e_k is white output noise with zero mean and covariance matrix R.

2.1 Prediction Error Methods

The first three methods use a predictor of the output in (1) that can be written as

$$\hat{x}_{k+1}(\theta) = f(\hat{x}_k(\theta), u_k; \theta) + K_k(\theta)\varepsilon_k(\theta), \tag{2a}$$
$$\hat{y}_k(\theta) = h(\hat{x}_k(\theta), u_k; \theta), \tag{2b}$$
$$\varepsilon_k(\theta) = y_k - \hat{y}_k(\theta). \tag{2c}$$

The prediction error can be used to define a scalar cost function

$$V_N(\theta, Z^N) = \frac{1}{N}\sum_{k=1}^{N}\frac{1}{2}\varepsilon_k(\theta)^T\varepsilon_k(\theta), \tag{3}$$

where Z^N represents the N input-output measurements. This cost function can be minimized to obtain an estimate of θ

$$\hat{\theta} = \arg\min_{\theta} V_N(\theta, Z^N). \tag{4}$$

This is in [6] called a prediction-error method (PEM). In order to use PEM, a stable predictor is required. The choice of the predictor is not obvious if the system is unstable and nonlinear. Here three approaches for calculating the observer gain $K_k(\theta)$ in (2a) are suggested. It is assumed that all states are measured and that there is no direct influence of the inputs on the outputs leading to that (2b) can be rewritten as

$$\hat{y}_k(\theta) = C\hat{x}_k(\theta), \tag{5}$$

where C is the identity matrix.

Parametrized observer (PO) approach: This is a simple approach, commonly used for linear cases, where the observer gain $K_k(\theta)$ is added to the parameters to be estimated.

$$\theta = \begin{bmatrix} \theta_f \\ \theta_K \end{bmatrix}, \tag{6}$$

where θ_f are the parameters that appear in f and $\theta_K = vec(K_k(\theta))$ is a vector containing the observer gain parameters. By applying this approach the optimization process used for solving (4) will find a time-invariant $K_k(\theta)$ that minimizes the cost function (3).

Extended Kalman Filter (EKF) approach: The EKF is an extension of the Kalman Filter [10] to nonlinear systems. If the system was linear and all noise signals Gaussian distributed then the Kalman Filter will minimize the mean square error of the estimated states ($\hat{x}_k$) giving optimal predictions. For the nonlinear case using the EKF, the main idea is to compute $K_k(\theta)$ at each time step using a linearized model. This linearization is performed by computing the partial derivatives of f with respect to x and u evaluated in $\hat{x}_k$ and u_k, giving the matrices $A_k(\theta)$ and $B_k(\theta)$, respectively. This gives the following gain

$$S_k(\theta) = [CP^{xx}_{k|k}(\theta)C^T + R], \tag{7a}$$
$$K_k(\theta) = P^{xx}_{k|k-1}C^T S_k^{-1}(\theta), \tag{7b}$$
$$P^{xx}_{k|k}(\theta) = (I - K_k(\theta)C)P^{xx}_{k|k-1}, \tag{7c}$$
$$P^{xx}_{k|k-1}(\theta) = A_k(\theta)P^{xx}_{k-1|k-1}A_k^T + Q, \tag{7d}$$

where the predicted covariance matrix $P^{xx}_{k|k}$ represents the uncertainties of the state prediction. The full theory of the EKF can be found in [14].

This approach is a nonlinear extension of the methods described in [15] which showed that for a linear system the parameters could be estimated exactly for an unstable output-error structure.

Unscented Kalman Filter (UKF) approach: The EKF is sometimes said to have problems with highly nonlinear functions because only the mean is propagated through the nonlinearity. An alternative is to use the Unscented Kalman filter [13], where both the mean and covariance is propagated through the nonlinearity. This is done by using a so-called unscented transformation where deterministically chosen points, sigma points, are used to represent both the mean and covariance. In this case the gain is

$$K_k(\theta) = P^{xy}_{k|k-1}(\theta)\,(P^{yy}_{k|k-1}(\theta))^{-1}, \tag{8a}$$
$$P^{xx}_{k|k}(\theta) = P^{xx}_{k|k-1}(\theta) - K_k(\theta)P^{yy}_{k|k-1}(\theta)K_k^T(\theta), \tag{8b}$$

where $P^{yy}_{k|k-1}(\theta)$, $P^{xx}_{k|k-1}(\theta)$ and $P^{xy}_{k|k-1}(\theta)$ are calculated using the sigma points.

Tuning parameters: Unlike the PO approach, the two Kalman filter approaches include tuning parameters that have to be set by the user. This is undesirable since some prior knowledge of how to set these parameters is required leading to a subjective part of the identification. The parameters are the covariance Q of the process noise, the covariance R of the measurement noise and the initial state covariance P_0. The matrices Q and R work in pairs so when Q is larger than R one relies more on the measurements. For the linear case the choice of the state variance P_0 is not critical because the convergence properties are well understood and it is not difficult to get the filter to converge. There is no proof of a similar property for the nonlinear case. Here one can only hope that the filter will converge. For the analysis presented in this paper P_0 is chosen as the identity matrix.

Other subjective inputs that applies to all above approaches are the initial values for the states x_0 and parameters θ_0. For the initial states one can use the initial measurement which should not be to far from the true value. This can be done since it is assumed that all states are measured in a linear way. The initial parameter vector θ_0 is an initialization of the PEM and will affect how good the initial estimates using the different Kalman filters as well as the parametrized observer are.

2.2 State estimation method

Here a method that uses the unknown parameters as states is given.

Augmented state (AUG) approach: This approach is commonly used in the navigation community when treating uncertain parameters. These parameters are added to the model as static states, i.e., states that do not vary with time.

$$\bar{x}_k = \begin{bmatrix} x_k \\ \theta_k \end{bmatrix}. \tag{9}$$

This gives rise to the following augmented state-space equation that should be used instead of (1a)

$$\bar{x}_{k+1} = \begin{bmatrix} x_{k+1} \\ \theta_{k+1} \end{bmatrix} = \begin{bmatrix} f(x_k, u_k; \theta_k) \\ w_{\theta k} \end{bmatrix}, \tag{10}$$

where $w_{\theta k}$ is a small zero mean artificial noise term with a covariance matrix that can be used to tune the estimator. Here, the same method to estimate the observer gain $K_k(\theta)$ as in the EKF approach is used, but for the model (10) instead of (1a). The theory behind this approach can be found in [4].

Tuning parameters: Since The EKF is used the same tuning for Q and R as described earlier applies. The same goes for x_0 and P_0. The covariance of the artificial noise term $w_{\theta k}$ can, as mentioned above, be used as a tuning parameter.

2.3 Parameter and state optimization method

The fifth method differ from the previous four in the way that it does not depend on simulation of the system at all. Therefore the system instability is not an issue.

Constrained Levenberg-Marquardt (CLM) approach: This approach uses a constrained Levenberg-Marquardt optimization procedure to a Lagrangian function. Instead of augmenting the states as in the AUG approach it augments the parameter vector θ with all time samples

$$\vartheta = [x_0^T \dots x_{N-1}^T \; \theta^T]^T, \tag{11}$$

and (1a) is reformulated as $F(\vartheta) = 0$ where

$$F(\vartheta) = \begin{bmatrix} f(x_0, u_0) - x_1 \\ f(x_1, u_1) - x_2 \\ \vdots \\ f(x_{N-1}, u_{N-1}) - x_N \end{bmatrix}. \tag{12}$$

The CLM approach involves minimizing (3) using (2) with $K_k(\theta) = 0$ and the additional constraints given by $F(\vartheta) = 0$. The inclusion of these constraints makes CLM related to collocation methods. A collocation method is a numerical method where a parametrized function is used together with a number of points, collocation points, where a differential/integral equation has to be satisfied. Here the state samples are forced to satisfy (1a).

The constrained optimization problem can be formulated as iteratively solving the linear system

$$\begin{bmatrix} J_1^T J_1 + \lambda_{LM}^2 I_{n_\vartheta, n_\vartheta} & J_2^T \\ J_2 & 0 \end{bmatrix} \begin{bmatrix} \delta\vartheta \\ \lambda \end{bmatrix} = \begin{bmatrix} -J_1^T \varepsilon \\ -F \end{bmatrix}, \tag{13}$$

where

$$J_1 = \frac{\partial \varepsilon}{\partial \vartheta}, \quad J_2 = \frac{\partial F}{\partial \vartheta}. \tag{14}$$

Here, the vector $\delta\vartheta$ contains additive increments to to the augmented parameter vector (11) and λ is a vector containing the Lagrangian multipliers.

The drawback is that the Karush-Kuhn-Tucker matrix, containing J_1 and J_2, in (13) to be inverted grows with the number of time samples used. This matrix is however sparse so efficient inversion methods can be used. The method is described in [3].

Tuning parameter: Also this method has a tuning parameter, the regularization parameter λ_{LM}. This is used to improve the rank properties of the KKT matrix and it thus affects the possibility to solve the system. This parameter has to be chosen carefully.

3 Analysis

For a highly maneuverable aircraft, such as the JAS 39 Gripen, the aerodynamic forces and moments can become nonlinear during aggressive maneuvering. Also, flight in the transonic region, i.e., near the speed of sound, leads to nonlinear effects that come from movements of aerodynamic shocks. How these shocks move depend on the speed as well as the aircraft attitude, i.e., angle-of-attack. The problem to be analyzed in this paper is aircraft system identification at transonic speed.

3.1 Physical model

The physical model used for flight simulations is based on rigid body mechanics, originating from Newton's second law, treating all forces and moments acting on the aircraft. The main contribution in this model come from aerodynamics, propulsion and inertia. Focusing on a pure pitch motion, the simplified 2-DOF equations of motion are given as

$$mV\dot{\alpha} = N_{Aero} + N_{Thrust}, \tag{15a}$$
$$J_{yy}\dot{q} = M_{Aero} + M_{Thrust}, \tag{15b}$$

where the left hand sides of (15) are the total force and moment of inertia in the pitch plane, m, J_{yy} and V being the aircraft mass, moment of inertia and velocity, respectively. These are considered constant in the test case since their variations are limited during the performed maneuver. N_{Aero} and M_{Aero} are the aerodynamic force and moment to be estimated, N_{Thrust} and M_{Thrust} come from the engine thrust and are in the present case small in the pitch plane and can therefore be neglected. The aerodynamic components depend on variables such as the states, angle of attack (α) and pitch angle velocity (q), and the input in form of control surface deflections of the elevator (δ_e), canard (δ_c) and leading edge flaps (δ_{LE}). The definitions and signs are shown in Fig. 2.

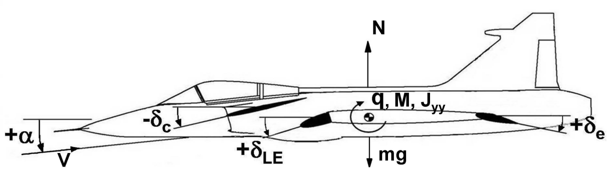

Fig. 2 Definition of the variables.

3.2 Estimation model

To get an estimation model, the equations of motion (15) are rewritten as

$$\dot{\alpha}_t = \frac{1}{mV}\cdot N_{Aero}(\alpha_t, q_t, \delta_{e,t}, \delta_{c,t}, \delta_{LE,t}), \tag{16a}$$
$$\dot{q}_t = \frac{1}{J_{yy}}\cdot M_{Aero}(\alpha_t, q_t, \delta_{e,t}, \delta_{c,t}, \delta_{LE,t}), \tag{16b}$$
$$y_t = \begin{bmatrix} \alpha_t & q_t \end{bmatrix}^T, \tag{16c}$$

and then, turning into discrete time, Euler's method for the derivatives are used

$$\dot{\alpha}_k = \frac{\alpha_{k+1} - \alpha_k}{T_s}, \tag{17a}$$
$$\dot{q}_k = \frac{q_{k+1} - q_k}{T_s}, \tag{17b}$$

where T_s is the sample time and k is the sample corresponding to the time t. This results in the following nonlinear state-space description

$$x_{k+1} = f(x_k, u_k), \tag{18a}$$
$$y_k = Cx_k + e_k. \tag{18b}$$

It is assumed that only the pitch stability, i.e, the pitching moment as a function of the angle of attack, is nonlinear and that all other relations are linear. This gives the following simplified state-space equation

$$x_{k+1} = a(x_k) + Bu_k, \tag{19a}$$
$$y_k = Cx_k + e_k, \tag{19b}$$

where the state and input vectors are $x_k = \begin{bmatrix}\alpha_K & q_k\end{bmatrix}^T$, $u(t) = \begin{bmatrix}\delta_{e,k} & \delta_{c,k} & \delta_{LE,k}\end{bmatrix}^T$ and the system matrices are given as

$$a(x_k) = \begin{bmatrix} N_\alpha \alpha(t) & N_q q(t) \\ f(\alpha(t)) & M_q q(t) \end{bmatrix}, \tag{20a}$$
$$B = \begin{bmatrix} N_{\delta_e} & N_{\delta_c} & N_{\delta_{LE}} \\ M_{\delta_e} & M_{\delta_c} & M_{\delta_{LE}} \end{bmatrix}, \tag{20b}$$
$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{20c}$$

The Ns and Ms are scaled partial derivatives of the aerodynamic force and moment with respect to the states and inputs, respectively. The scaling includes V, m, J_{yy} and the dynamic pressure, q_a, reference wing area, S and reference wing chord, $\bar{c}$. $f(\alpha)$ is the nonlinear aerodynamic pitch stability model function which is built up as a piecewise affine function, similarly to the stucture of the present aerodynamic model for JAS 39 Gripen, with break-points positioned at $\alpha(i) = \alpha_{min}, \alpha_{min}+1, \ldots, \alpha_{max}$ (see Fig. 3).

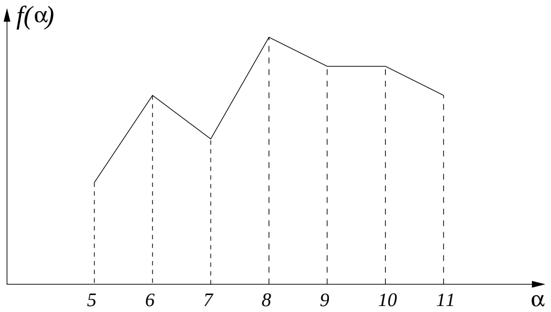

Fig. 3 Example of a piecewise affine function with $\alpha_{min} = 5$ and $\alpha_{max} = 11$.

This gives the following piecewise function for $\alpha_i < \alpha < \alpha_{i+1}$

$$f(\alpha) = \frac{f(\alpha_{i+1}) - f(\alpha_i)}{\alpha_{i+1} - \alpha_i} \cdot (\alpha - \alpha_i) + f(\alpha_i). \quad (21)$$

All Ns, Ms and break-points $f(\alpha(i))$ are put into the parameter vector θ. Thus, the total parametrized model is given by:

$$x_{k+1} = \begin{bmatrix} \theta_1 \alpha_k & \theta_2 q_k \\ f(\alpha_k, \theta_{10}, \ldots, \theta_{21}) & \theta_3 q_k \end{bmatrix} + \quad (22a)$$

$$\begin{bmatrix} \theta_4 & \theta_5 & \theta_6 \\ \theta_7 & \theta_8 & \theta_9 \end{bmatrix} \begin{bmatrix} \delta_{e,k} \\ \delta_{c,k} \\ \delta_{LE,k} \end{bmatrix}, \quad (22b)$$

$$y_k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x_k + e_k. \quad (22c)$$

3.3 Identification on simulated data

In this section, the five different methods will be analysed based on simulated data. For this a Simulink® model has been developed. It is based on the present aerodynamic model for JAS 39 Gripen under the same conditions used for the identification of the real flight test data. A simplification has been made in that the control law moves the leading edge flap in full correlation with the angle-of-attack. Therefore the leading edge parameters have been removed. The input and output for a noisy simulation is shown in Figure 4.

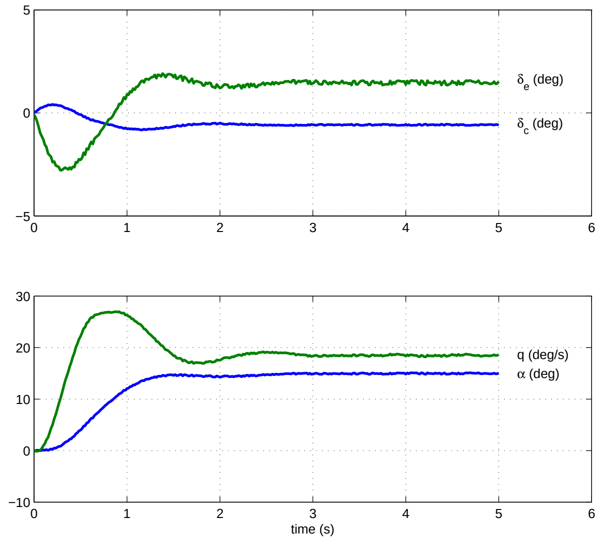

Fig. 4 Simulated noisy input and output data

Noise-free simulation The first test of the methods is to run them on noise-free data. This is done to see if there some deficiencies in the methods. The true values have been used as initial guess of the parameters. As can be seen in Figure 5 and Table 1 and 2, all methods are close to the true model. It should be noted that the UKF method is the odd one out, all other methods are more or less on top of each other.

Table 1 Aero derivatives, noise free data.

	Present	PO	EKF	UKF
N_α	0.9804	0.9813	0.9813	0.9814
N_q	0.0163	0.0159	0.0159	0.0159
M_q	0.9790	0.9769	0.9767	0.9897
N_{δ_e}	-0.0051	-0.0084	-0.0085	-0.0085
N_{δ_c}	0.0005	0.0013	0.0014	0.0014
M_{δ_e}	-0.5182	-0.5010	-0.5009	-0.5368
M_{δ_c}	0.1376	0.1335	0.1335	0.1423

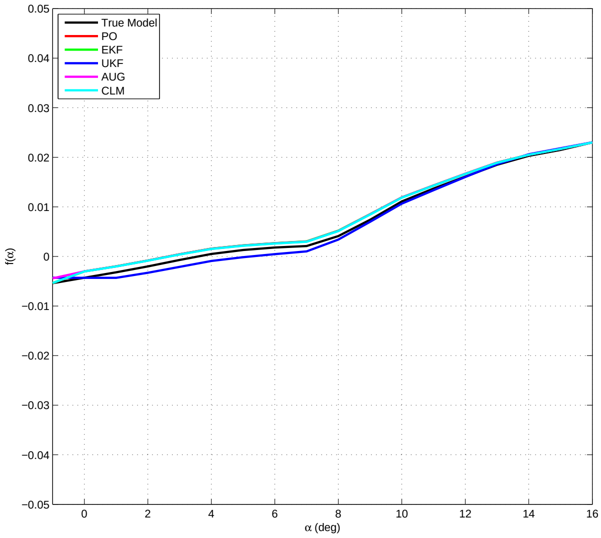

Fig. 5 Pitching moment for noise free data.

Table 2 Aero derivatives, noise free data cont..

	Present	AUG	CLM	Note
N_α	0.9804	0.9813	0.9813	
N_q	0.0163	0.0159	0.0159	
M_q	0.9790	0.9769	0.9770	
N_{δ_e}	-0.0051	-0.0085	-0.0084	
N_{δ_c}	0.0005	0.0013	0.0013	
M_{δ_e}	-0.5182	-0.5013	-0.5014	
M_{δ_c}	0.1376	0.1336	0.1336	

Noisy simulation Here zero mean measurement noises with a standard deviation of $0.15(deg)$ in angle-of-attack and $0.15(deg/s)$ in pitch angular velocity have been added to the simulated outputs. As can be seen in Figure 6 and Table 3 and 4, all methods are affected by the noise, but the PO, EKF and CLM methods still seem to still give acceptable results. Both the AUG and the UKF methods get a distorted nonlinearity. The wiggeling of the estimated function at the end points in Figure 6 is probably due to few samples at those angles-of-attack.

Initial guess offset It is not probable that the initial guess of the parameters will be the exact truth even if a lot of work has been done to get them as good as possible before the first flight. Therefore, it would be good if the methods could handle biased initialisation. To investigate how the proposed methods are affected, an initial offset of 10% of the derivatives and a linearisation of the nonlinearity have been looked at. The result is shown in Figure 7, Table 5 and 6. As can be seen the result looks very similar to that of the noisy data with the exception for the CLM approach which now has a bias on the nonlinearity, though the slope is similar to the true curve.

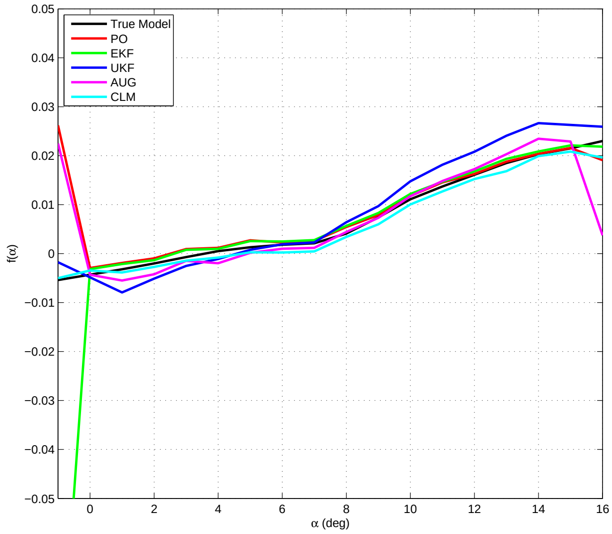

Fig. 6 Pitching moment for noisy data.

Table 3 Aero derivatives, noisy data.

	Present	PO	EKF	UKF
N_α	0.9804	0.9812	0.9821	0.9842
N_q	0.0163	0.0160	0.0158	0.0148
M_q	0.9790	0.9771	0.9765	0.9720
N_{δ_e}	-0.0051	-0.0073	-0.0097	-0.0180
N_{δ_c}	0.0005	0.0047	0.0159	0.0153
M_{δ_e}	-0.5182	-0.4989	-0.5096	-0.6157
M_{δ_c}	0.1376	0.1311	0.1360	0.1677

Table 4 Aero derivatives, noisy data cont..

	Present	AUG	CLM	Note
N_α	0.9804	0.9814	0.9797	
N_q	0.0163	0.0160	0.0166	
M_q	0.9790	0.9780	0.9809	
N_{δ_e}	-0.0051	-0.0069	-0.0036	
N_{δ_c}	0.0005	0.0192	0.0004	
M_{δ_e}	-0.5182	-0.5634	-0.5183	
M_{δ_c}	0.1376	0.1524	0.1383	

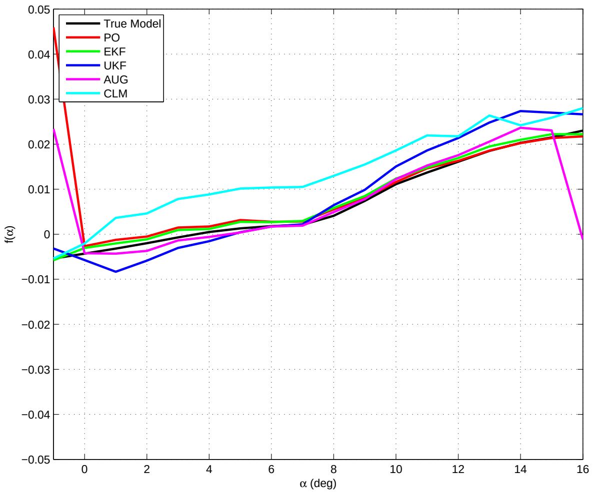

Fig. 7 Pitching moment for initial offset of θ_0.

Table 5 Aero derivatives, offset data.

	Present	PO	EKF	UKF
N_α	0.9804	0.9813	0.9877	0.9843
N_q	0.0163	0.0159	0.0086	0.0146
M_q	0.9790	0.9762	0.9757	0.9712
N_{δ_e}	-0.0051	-0.0077	-0.0618	-0.0196
N_{δ_c}	0.0005	0.0021	-0.2024	0.0061
M_{δ_e}	-0.5182	-0.4942	-0.5111	-0.6350
M_{δ_c}	0.1376	0.1037	0.1196	0.1600

Table 6 Aero derivatives, offset data cont..

	Present	AUG	CLM	Note
N_α	0.9804	0.9918	0.9802	
N_q	0.0163	0.0160	0.0165	
M_q	0.9790	0.9769	0.9613	
N_{δ_e}	-0.0051	-0.0078	-0.0051	
N_{δ_c}	0.0005	0.0256	0.0028	sign
M_{δ_e}	-0.5182	-0.5567	-0.4653	
M_{δ_c}	0.1376	0.1452	0.1212	

3.4 Identification on flight test data

The five methods have been evaluated on data from a flight test where a wind-up turn is performed. A wind-up turn is a flight maneuver where an initial roll of 90 degrees is performed followed by an almost pure, high angle of attack, pitching maneuver at almost constant speed. The identification is based on data collected after the initial roll has been performed. The sample frequency is 60 Hz ($T_s = 1/60\,(s)$) and the data set contains approximately 300 measurements shown in Figure 8. The estimation result, based on this data set, is shown in Figure 9 and Table 7 and 8. All the parameters were initialized using the values from the present model. As can be seen, all methods capture the nonlinearity around $\alpha = 7\,(deg)$ and the slope of the curve. It is interesting to se that the methods predict that the nonlinearity should be more aggressive, i.e., the curve slope should change more abruptly, than what is in the present model. There is some wiggeling in the curves at higher angles-of-attack. This can be due to the fact that there was few data for angles-of-attack between 12 to 14 degrees. Comparing the different approaches, it is interesting to note that the PO and CLM approaches give a closer resemblance to the present model which has been been built up from numerical calculations, wind tunnel tests and flight tests during a period of more than 30 years.

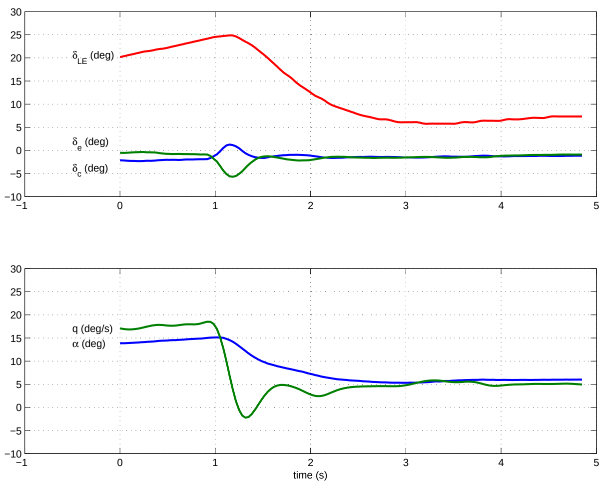

Fig. 8 Input and output data from flight test.

4 Conclusions

Five approaches for direct system identification of unstable nonlinear systems have been presented. Three of the methods are variations of the prediction error method (PEM). These are the parametrized observer (PO) approach and two approaches based on the Kalman filter, the extended kalman filter (EKF) and the uncented Kalman filter (UKF). One approach

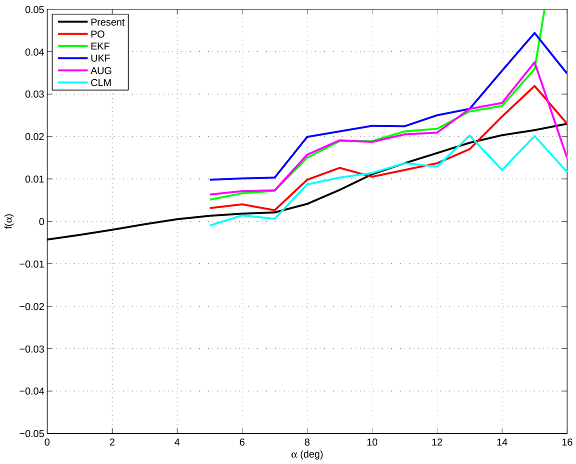

Fig. 9 Pitching moment as a function of α.

Table 7 Aerodynamic derivatives.

	Present	PO	EKF	UKF
N_α	0.9804	0.9975	0.9984	0.9930
N_q	0.0163	0.0125	0.0116	0.0135
M_q	0.9790	0.8948	0.9095	0.8867
N_{δ_e}	-0.0051	-0.0011	-0.0063	-0.0087
N_{δ_c}	0.0005	0.0072	0.0048	0.0013
$N_{\delta_{LE}}$	0.0001	-0.0068	-0.0078	-0.0058
M_{δ_e}	-0.5182	-0.5512	-0.4837	-0.4528
M_{δ_c}	0.1376	0.1730	0.1617	0.2279
$M_{\delta_{LE}}$	-0.0031	-0.0305	-0.0488	-0.0467

Table 8 Aerodynamic derivatives cont..

	Present	AUG	CLM	Note
N_α	0.9804	0.9930	0.9962	
N_q	0.0163	0.0134	0.0131	
M_q	0.9790	0.9040	0.9370	
N_{δ_e}	-0.0051	-0.0090	-0.0030	
N_{δ_c}	0.0005	0.0017	0.0013	
$N_{\delta_{LE}}$	0.0001	-0.0057	-0.0071	sign
M_{δ_e}	-0.5182	-0.4828	-0.4968	
M_{δ_c}	0.1376	0.1851	0.1242	
$M_{\delta_{LE}}$	-0.0031	-0.0454	-0.0334	

is a state estimation method, the augmented system approach (AUG) using the extended Kalman filter. The fifth method is a parameter and state estimation method, the constrained Levenberg-Marquardt (CLM) approach.

These methods have been validated on simulated data from an unstable nonlinear system and tested for noise sensitivity and initial value offsets. From these tests one can conclude that the PO and EKF approaches seem most robust.

The approaches have also been tested on real data from a flight test near the speed of sound. Here, the PO and CLM approaches show promising results since a good resemblance to the present aerodynamic model was found. The other methods show some biases in the results compared to the present aerodynamic model for the JAS 39 Gripen.

All in all, the PO approach seems to do the best job for the studied cases.

5 Acknowledgements

This research has been performed within the Vinnova industry excellence center LINK-SIC. Flight test data have been provided by Saab AB.

References

[1] A, C, Paris and O, Alaverdi *Nonlinear Aerodynamic Model Extraction from Flight-Test Data for the S-3B Viking*, Journal of Aircraft, Vol. 42, No. 1, 2005

[2] A, Ghaffari, J, Roshanian and M, Tayefi *Time-Varying Transfer Function Extraction of an Unstable Launch Vehicle via Closed-Loop Identification*, Aerospace Science and Tehnology, 11, 2007

[3] A, V, Mulders et. al. *Two Nonlinear Optimization Methods for Black Box Identification Compared*, 15th IFAC Symposium on System Identification, 2009

[4] D, Simon *Optimal State Estimation: Kalman, H_∞ and Nonlinear Approaches*, John Wiley & Sons, Inc, 2006

[5] F, E, C, Culick *What the Wright Brothers Did and Did Not Understand About Flight Mechanics - In Modern Terms*, 37th AIAA/ASME/SAE/SAEE Joint Propulsion Conference and Exhibit, 2001

[6] L, Ljung *System Identification, Theory for the User*, Prentice Hall PTR, 2nd Edition, 1999

[7] M, B, Tischler and R, K, Remple *Aircraft and Rotorcraft System Identification, Engineering Methods with Flight Test Examples* American Institute of Aeronautics and Astronautics Inc. 1st Edition 2006

[8] M, P, Horton *Real-Time Identification of Missile Aerodynamics Using a Linearised Kalman Filter Aided by an Artificial Neural Network*, IEE Proc.-Control Theory Application, Vol. 144, No. 4, 1997

[9] P, D, Bruce and M, G, Kellett *Modelling and Identification of Nonlinear Aerodynamic Functions Using B-Splines*, UKACC International Conference on Control'98, IEE Conference Publication, No. 455, 1998

[10] R, E, Kalman *A New Approach to Linear Filtering and Prediction Problems*, ASME-Journal of Basic Engineering, 82 (Series D), 1960

[11] R, V, Jategaonkar *Flight Vehicle System Identification, A Time Domain Methodology*, American Institute of Aeronautics and Astronautics Inc., 1st Edition, 2006

[12] R, V, Jategaonkar and F, Thielecke *Aircraft Parameter Estimation - A Tool for Development of Aerodynamic Databases*, *Sādhanā*,Vol. 25, Part 2, 2000

[13] S, J, Julier and J, K, Uhlmann *A New Extension of the Kalman Filter to Nonlinear Systems*, Int. Symp. Aerospace/Defense Sensing, Simul. and Controls 3, 1997

[14] T, Kailath, A, H, Sayed and B, Hassibi *Linear Estimation*, Prentice Hall PTR, 1st Edition, 2000

[15] U, Forssell and L, Ljung *Identification of Unstable Systems Using Output Error and Box-Jenkins Model Structures*, IEEE Transactions on Automatic Control, vol. 45, 2000

[16] V, Klein and E, A, Morelli *Aircraft System Identification, Theory and Practice*, American Institute of Aeronautics and Astronautics Inc., 1st Edition, 2006

5.1 Copyright Statement