MANEUVERING TARGET TRACKING BASED ON THE EKF

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Abstract

A nonlinear system model of maneuvering target detection was introduced. To estimate offset angles of pitch and azimuth, relative velocity of missile and target, and distance information while detecting maneuvering target, extended Kalman filter algorithm was introduced. Simulations of nominal trajectory and extreme trajectory shows that the proposed system model and filter design approach can deal with partial information missing problems in the real flight condition and maintain good tracking accuracy for certain time even in extreme blocking situation.

1 Introduction

In the maneuvering target tracking, the dynamic estimation of the target movement is obtained with Kalman filtering in the linear Gaussian case on the minimum mean square error criteria [1]. In the real system, however, the measurement data is non-linearly related with the target movement parameters in many scenario, which leads to the inapplicability of the Kalman filtering algorithm. The common strategy is the transformation of the nonlinear filtering problem to the approximate linear filtering problem with the linearization technique, and seeks the second-best filtering algorithm solution to the original non-linear filtering problem. The linearization technique is the Taylor series expansion, and the obtained filtering technique is Extended Kalman filtering(EKF).

The modeling of target is the foundation of filtering algorithm. The studies show that Jerk model[2,3] has steady-state certainty errors problem when tracking the target with step acceleration rate. Singer model[4] using colored noise model, it is more practical than white noise in modeling maneuvering acceleration. Kendtrick describes maneuvering target normal acceleration as an asymmetric distribution time-correlated stochastic process[5]. A ‘current’ statistical model of maneuvering target[6] is proposed by Hongren Zhou using nonzero mean value and modified Rayleigh distribution to characterize maneuvering acceleration characteristics.

In this paper, the mathematical model of the missile's angle, range and velocity tracking system is established based on the target movement process, and then, the filtering model for the estimation of the angle error, the relative velocity and range between the missile and the target is deducted on the Extended Kalman filtering algorithm. The system model is used in the estimation of the target acceleration and the missile target line of sight. The algorithm discussed hereafter is used in the missile to form the range gate and the velocity gate, to drive the antenna slave system, and finally to achieve the target accurate tracking and to complete the missile optimum guidance.

2 State Equation and Measurement Equation Establishment

2.1 Angle Error Modeling

The Radar seeker measures the azimuth angle error and the elevation angle error of the target, which are usually very small in the tracking mode. Then the system angle error equation can be written as:

$$\begin{aligned}\dot{\varepsilon} &= \omega_{dry}^{r} - \omega_{day}^{a} + \eta\omega_{dax}^{a} \\ \dot{\eta} &= \omega_{drz}^{r} - \omega_{daz}^{a} - \varepsilon\omega_{dax}^{a}\end{aligned} \tag{1}$$

Where subscript and superscript r, d, a denote respectively the line of sight coordinate, the ground coordinate, and the missile antenna coordinate. In the equation ω_{dry}^{r} denote, for example, r coordinate rotation angular velocity's projection component in y axis of the line of sight coordinate.

2.2 Angular Velocity Modeling

Based on the relationship between the missile and the target in the ground coordinate (fig.1), and in consideration that the angular

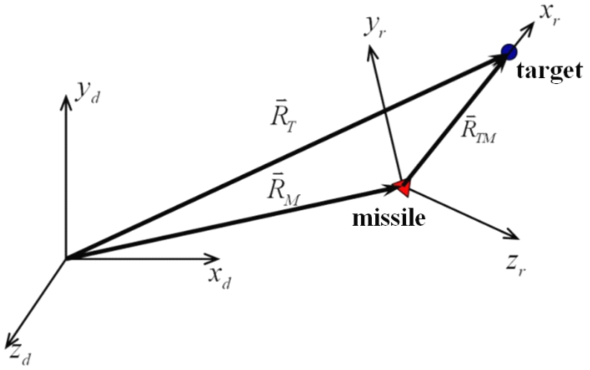

Fig. 1. The Target And Missile In The Ground Coordinate

velocities of the target line of sight and the electrical axis of the missile antenna are approximately equal, we obtain the angular velocity dynamic equation:

$$\begin{aligned}\ddot{R}_{TM} &= R_{TM}\left[\left(\omega_{drz}^{r}\right)^{2} + \left(\omega_{dry}^{r}\right)^{2}\right] + \left(nT_{dx}^{r} - nM_{dx}^{a}\right) \\ \dot{\omega}_{dry}^{r} &= -2\frac{\dot{R}_{TM}}{R_{TM}}\omega_{dry}^{r} - \frac{\left(nT_{dz}^{r} - nM_{dz}^{a}\right)}{R_{TM}} + \omega_{dax}^{a}\omega_{drz}^{r} \\ \dot{\omega}_{drz}^{r} &= -2\frac{\dot{R}_{TM}}{R_{TM}}\omega_{drz}^{r} + \frac{\left(nT_{dy}^{r} - nM_{dy}^{a}\right)}{R_{TM}} - \omega_{dax}^{a}\omega_{dry}^{r}\end{aligned} \tag{2}$$

Where R_{TM} is the range between the target and the missile， nT is the acceleration of the target, nM is the acceleration of missile, the superscript r and a denote respectively the line of sight coordinate and the antenna coordinate, subscript x y z denote respectively the three components in the coordinate.

2.3 Target Acceleration Modeling

The projection of the target acceleration vector nT_{d}^{r} in the line of sight coordinate can be expressed as:

$$nT_{di}^{r} = n\tilde{T}_{di}^{r} + \varsigma_{ti} \tag{3}$$

Where nT_{di}^{r} is acceleration component in x, y, z axes, $n\tilde{T}_{di}^{r}$ is the "current" average value of the acceleration, which is assumed as constant in the sampling term, and ς_{ti} is the random acceleration:

$$\dot{\varsigma}_{ti} = -\frac{1}{\tau_i}\varsigma_{ti} + wnT_i \tag{4}$$

Where wnT_i is zero mean value white noise, τ_i is the acceleration correlation time constant.

From the above, we obtain:

$$\begin{aligned}n\dot{T}_{di}^{r} = \dot{\varsigma}_{ti} &= -\frac{1}{\tau_i}\left(nT_{di}^{r} - n\tilde{T}_{di}^{r}\right) + wnT_i \\ &= -\frac{1}{\tau_i}nT_{di}^{r} + \left(\frac{1}{\tau_i}n\tilde{T}_{di}^{r} + wnT_i\right)\end{aligned} \tag{5}$$

2.4 State Equation Modeling

Defining a new coordinate r', we want to obtain the target acceleration component in y_r' and z_r' axis. From Coriolis equation，we have:

$$\begin{aligned}n\dot{T}_{dy}^{r'} &= n\dot{T}_{dy}^{r} - \omega_{drx}^{r}nT_{dz}^{r} \\ n\dot{T}_{dz}^{r'} &= n\dot{T}_{dz}^{r} + \omega_{drx}^{r}nT_{dy}^{r}\end{aligned} \tag{6}$$

Put the equation (5) in the equation (6) and rearrange it, then we obtain:

$$\begin{aligned}n\dot{T}_{dy}^{r} &= -\frac{1}{\tau_{dy}^{r}}nT_{dy}^{r} + \omega_{drx}^{r}nT_{dz}^{r} + \left(\frac{1}{\tau_{dy}^{r}}n\tilde{T}_{dy}^{r} + wnT_{dy}^{r}\right) \\ n\dot{T}_{dz}^{r} &= -\frac{1}{\tau_{dz}^{r}}nT_{dz}^{r} - \omega_{drx}^{r}nT_{dy}^{r} + \left(\frac{1}{\tau_{dz}^{r}}n\tilde{T}_{dz}^{r} + wnT_{dz}^{r}\right)\end{aligned} \tag{7}$$

Since the two angle errors are small:

$$\omega_{drx}^{r} = \omega_{dax}^{a} + \eta\omega_{day}^{a} - \varepsilon\omega_{daz}^{a} \approx \omega_{dax}^{a} \tag{8}$$

We finally get：

$$
\begin{aligned}
n\dot{T}^r_{dy} &= -\frac{1}{\tau^r_{dy}} nT^r_{dy} + \omega^a_{dax} nT^r_{dz} + \left(\frac{1}{\tau^r_{dy}} n\tilde{T}^r_{dy} + wnT^r_{dy}\right) \\
n\dot{T}^r_{dz} &= -\frac{1}{\tau^r_{dz}} nT^r_{dz} - \omega^a_{dax} nT^r_{dy} + \left(\frac{1}{\tau^r_{dz}} n\tilde{T}^r_{dz} + wnT^r_{dz}\right) \\
n\dot{T}^r_{dx} &= -\frac{1}{\tau^r_{dx}} nT^r_{dx} + \left(\frac{1}{\tau^r_{dx}} n\tilde{T}^r_{dx} + wnT^r_{dx}\right)
\end{aligned} \quad (9)
$$

Until now we have all differential equation needed to form the system state equation for the maneuvering target tracking.

3 Extended Kalman Filtering

The state equation and measurement equation of the discrete time non-linear system are[7]:

$$x_{k+1} = f_k(x_{k,} w_k) = \varphi x_k + \Gamma w \quad (10)$$

$$z_k = h_k(x_k, v_k) = h(x) + v \quad (11)$$

For the discussion convenience, assume that the state equation is linear function and the measurement equation non-linear equation. If the state estimation value $x_{k-1/k-1}$ and estimation errors covariance matrix $p_{k-1/k-1}$ at the time k-1 have been obtained, the step of the first order EKF algorithm is:

(1) Compute the target state $x_{k/k-1}$ according to the state equation;

(2) Carry out the Taylor series expansion of the non-linear measurement equation $h(x)$ at $x_{k/k-1}$ and neglect the non-liner terms:

$$
\begin{aligned}
h(x) &\approx h(x_{k/k-1}) + \frac{\partial h}{\partial x}\Big|_{x=x_{k/k-1}} (x - x_{k/k-1}) \\
&= h(x_{k/k-1}) + H(x_{k/k-1})(x - x_{k/k-1})
\end{aligned} \quad (12)
$$

(3) Substitute (12) into (11) and get the non-linear model:

$$
\begin{aligned}
z = & H(x_{k/k-1})x + h(x_{k/k-1}) \\
& -H(x_{k/k-1})x_{k/k-1} + v
\end{aligned} \quad (13)
$$

(4) Complete state filtering , and update the gain matrix and the errors covariance matrix:

Gain matrix:

$$K = P_{K/K-1}H^T(HP_{K/K-1}H^T + R)^{-1} \quad (14)$$

State filtering:

$$x_{k/k} = x_{k/k-1} + K\left[z - z(x_{k/k-1})\right] \quad (15)$$

errors covariance:

$$P_{K/K} = (I - KH)P_{K/K-1}(I - KH)^T + KRK^T \quad (16)$$

where $H(x_{k/k-1}) = \frac{\partial h}{\partial x}\Big|_{x=x_{k/k-1}}$, which is the Jacobian matrix of $h(x)$.

Since the measurement matrix is replaced by Jacobian matrix H, the gain K computed by (14) is not optimum. Therefore, equation (16) must be used for the update of covariance matrix in order to prevent filtering divergent.

4 Filter Design

The filter provides the missile seeker with the initial information of the target before it is operational and able to deliver the measurement output. The periodic eclipse of the observation may occur due to the overlaps of the transmission signal and the received signal even though the radar seeker has been switched on. The observability of the target changes periodically along with the eclipse period. When the radar seeker is in the range detection and tracking mode, the eclipse effect is gotten rid of, and the target observation remains stable until the distance between the missile and the target reaches the minimum measurable range of the seeker. It is known from the above that the filter will undergo four kinds of observation: the angle only observation, the angle and velocity observation, the all dimension observation and the naughty observation. Different observation requires different filtering observation equation. It is necessary, therefore, to deduce the different equation and build the different filter for each of the possible observation.

4.1 All Dimension Observation

All dimension observation means that all target information of the angle, velocity and range are able to be measured. In this situation the discrete time dynamic models of the subsystem are:

For the angle track subsystem,

$$
\begin{aligned}
X_1(k+1) = & \phi_1(k+1,k)X_1(k) \\
& +G_1(k+1,k)U_1(k) \\
& +\Gamma_1(k+1,k)W_1(k)
\end{aligned} \quad (17)
$$

$$Z_1(k) = H_1(k)X_1(k) + V_1(k) \quad (18)$$

where:

$$\phi_1(k+1,k)=e^{A_1(t)T} \cong I+A_1(t)T+\frac{1}{2}A_1^2(t)T^2 \tag{19}$$

$$G(k+1,k)=\int_0^T e^{A_1(T-\tau)}B(\tau)d\tau \cong [T+\frac{1}{2}A_1(k)T^2]B(k) \tag{20}$$

$$\Gamma(k+1,k)=\int_0^T e^{A_1(T-\tau)}d\tau \cong T+\frac{1}{2}A_1(k)T^2 \tag{21}$$

For the range and velocity track subsystem

$$X_2(k+1)=\phi_2(k+1,k)X_2(k) +G_2(k+1,k)U_2(k) +\Gamma_2(k+1,k)W_2(k) \tag{22}$$

$$Z_2(k)=H_2(k)X_2(k)+V_2(k) \tag{23}$$

Filtering algorithm equations are:

for state one-step prediction,

$$\hat{X}_{k/k-1}=\phi(k,k-1)\hat{X}_{k-1/k-1} +G(k,k-1)U_{k-1} \tag{24}$$

for state estimation,

$$\hat{X}_{k/k}=\hat{X}_{k/k-1}+K_k(Z_k-H_k\hat{X}_{k/k-1}) \tag{25}$$

for the filter gain,

$$K_k=P_{k/k-1}H_k^T(H_kP_{k/k-1}H_k^T+R_k)^{-1} \tag{26}$$

for the mean square error of the one-step prediction,

$$P_{k/k-1}=\phi(k,k-1)P_{k-1/k-1}\phi^T(k,k-1) +\Gamma(k,k-1)Q_{k-1}\Gamma^T(k,k-1) \tag{27}$$

for the mean square error of the state estimation,

$$P_{k/k}=(I-K_kH_k)P_{k/k-1}(I-K_kH_k)^T +K_kR_kK_k^T \tag{28}$$

4.2 Angle and Velocity Observation

When the range gate of the radar seeker is unlocked or the missile target range is below the minimum range measurement limitation, there will be no range information available for the observation equation. The angle and velocity measurement can still be carried on, however, in the angle tracking loop and the range velocity tracking loop, and the corresponding state equations are the same as those in section 4.1. Another way of estimating the range information is to establish a sole velocity tracking subsystem:

$$\begin{bmatrix}\dot{v}_r\\ n\dot{T}_{dx}^r\end{bmatrix}=\begin{bmatrix}0 & 1\\ 0 & -\frac{1}{\tau_{dx}^r}\end{bmatrix}\begin{bmatrix}v_r\\ nT_{dx}^r\end{bmatrix} +\begin{bmatrix}-nM_{dx}^a\\ \frac{1}{\tau_{dx}^r}n\tilde{T}_{dx}^r\end{bmatrix}+\begin{bmatrix}0\\ wnT_{dx}^r\end{bmatrix} \tag{29}$$

The range can be obtained via the integration of the estimated velocity.

4.3 Angle Only Observation

In the angle only observation, there will be no range and velocity information available in the system observation equation. The range and velocity are generated through the filter state estimation. In the process the angle information is coupled with the range and velocity information through the state matrix, and the range and velocity information are corrected with the angle information. The system state equation and measurement equation in this situation are:

$$\dot{X}=AX+U+W \tag{30}$$

$$Z=HX+V \tag{31}$$

Where,

$$X=\left[\varepsilon,\omega_{dry}^r,nT_{dz}^r,\eta,\omega_{drz}^r,nT_{dy}^r,R_{TM},v_r,nT_{dx}^r\right]^T,$$

$$A=\begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix},$$

$$W=\left[0,0,wnT_{dz}^r,0,0,wnT_{dy}^r,0,0,wnT_{dx}^r\right]^T,$$

$$H=\begin{bmatrix}k_\varepsilon & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & k_\eta & 0 & 0 & 0 & 0 & 0\end{bmatrix},$$

$$V=\begin{bmatrix}w_\varepsilon & w_\eta\end{bmatrix}^T.$$

4.4 Naughty Observation

There is no observation equation in this situation and the filtering issues are reduced to the prediction issues. The filter obtains the estimation of all dimension observation through extrapolation in order to facilitate the seeker reacquiring the target as soon as the eclipse ends. The extrapolating computation equations based upon Kalman filtering are:

$$\hat{X}_{k/k-1} = \phi(k,k-1)\hat{X}_{k-1/k-1} + U_{k-1} \tag{32}$$

$$\hat{X}_{k/k} = \hat{X}_{k/k-1} \tag{33}$$

$$K_k = P_{k/k-1} H_k^T (H_k P_{k/k-1} H_k^T + R_k)^{-1} \tag{34}$$

$$P_{k/k-1} = \phi(k,k-1) P_{k-1/k-1} \phi^T(k,k-1) + \Gamma(k,k-1) Q_{k-1} \Gamma^T(k,k-1) \tag{35}$$

$$P_{k/k} = (I - K_k H_k) P_{k/k-1} (I - K_k H_k)^T + K_k R_k K_k^T \tag{36}$$

5 Simulation

5.1 Nominal Trajectory Simulation

Fig.3 and fig.4 indicate that the filter can accurately estimate the angle error in the nominal engagement scenario. After the angle

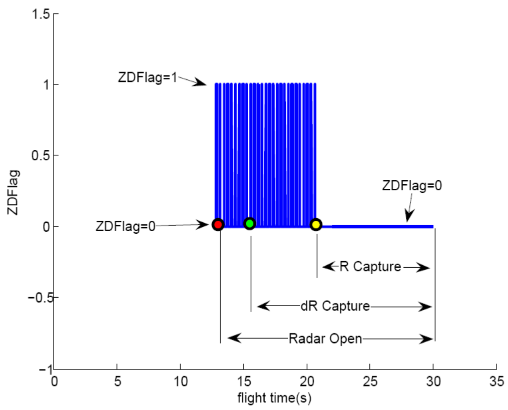

Fig. 2. The Event Time Sequence Of The Engagement

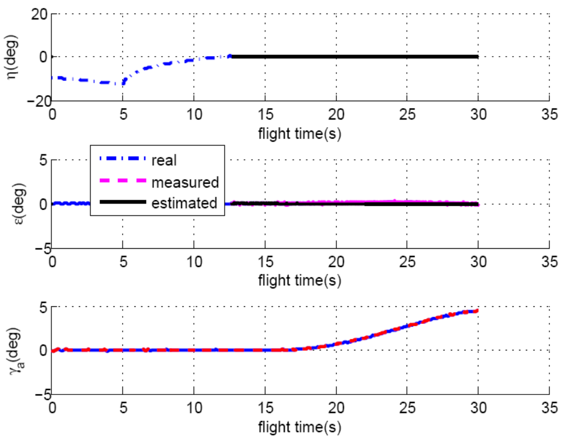

Fig. 3. Comparison Of The Angle Real Value, Measured Value And Estimated Value

tracking loop stable locked and before the velocity loop locked on, the filter provides the target position and velocity estimation with acceptable accuracy. After the velocity loop lock-on the range estimation undergoes further convergence until the range acquisition and the eclipse elimination.

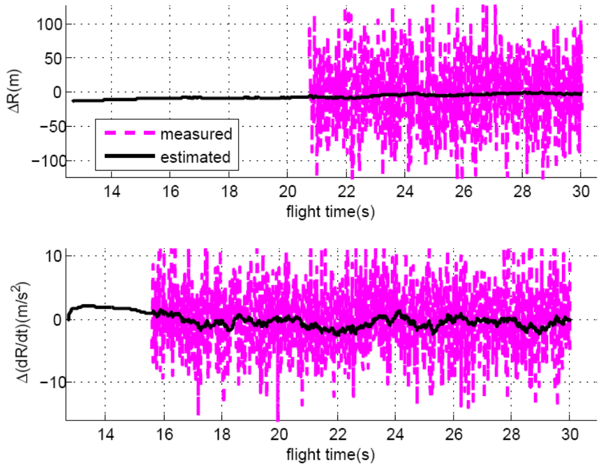

Fig. 4. Comparison Of The Measurement Error And The Estimation Error Of The Range And Velocity

Fig.5 gives the target acceleration estimation in the nominal trajectory. It shows

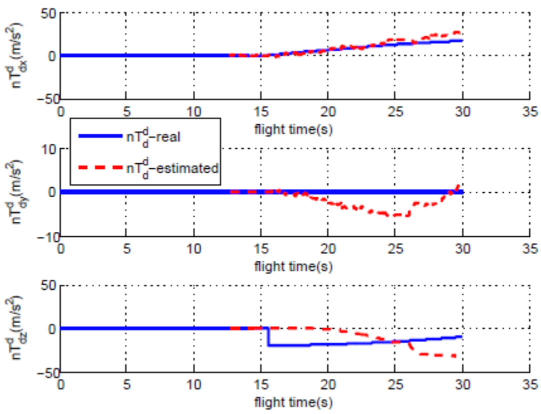

Fig. 5. ComparisOn Between The Real Value And Estimation Of The Acceleration

that good estimation in line of sight has been achieved owing to the correction by the relative velocity. Acceleration vertical to the line of sight is, however, converging relatively slow.

5.2 Extreme Trajectory Simulation

Fig.6 to fig.8 indicates that the eclipse effect increases the overshoot of the acceleration estimation and the convergent time of the range and velocity. It imposes little effect on the angle estimation, and the overall estimation is still convergent.

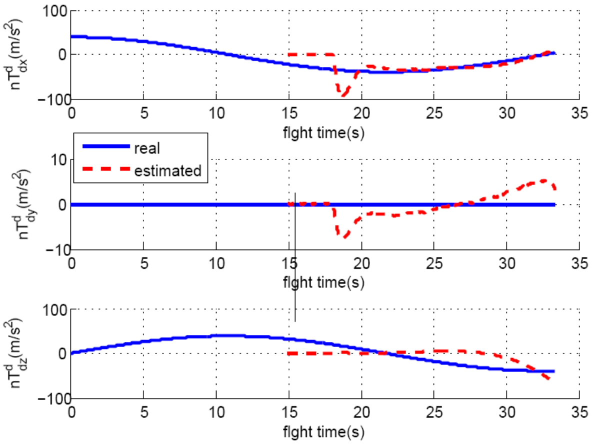

Fig. 6. The Acceleration Estimation Of The Target

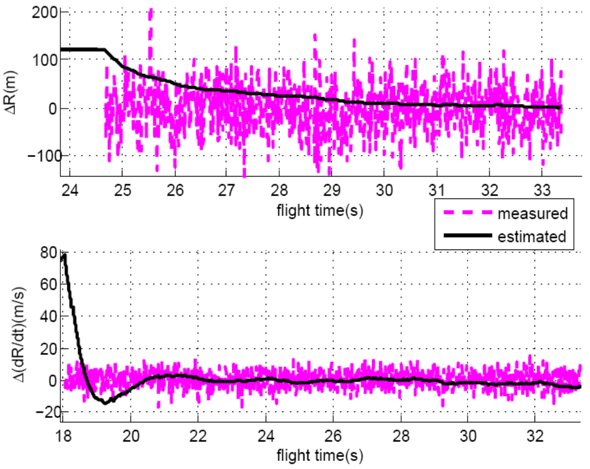

Fig. 7. The Estimation Of Range And Velocity

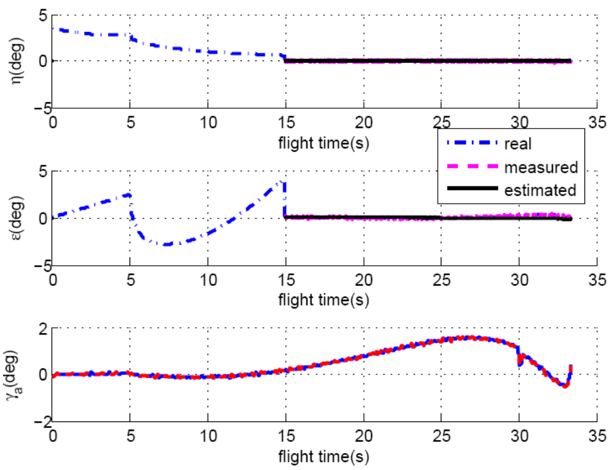

Fig. 8. The Angle Estimation

6 Conclusion

A non-linear system model suitable for the high maneuverability target tracking has been established based upon the missile target kenimatic principle and the target current statistical models. The filtering model for the estimation of the angle error, the relative velocity and range has been deduced with the extended Kalman filtering algorithm. The system model and the filter design are able to deal with the partial information missing problem in the real flight situation. In the extreme situation of the eclipse a relatively high tracking accuracy has been achieved. The simulation of nominal and extreme trajectories also indicates that the filtering algorithm is highly reliable and robust.

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