WING-BODY OPTIMIZATION BASED ON MULTI-FIDELITY SURROGATE MODEL

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Abstract

This paper focuses upon the efficient surrogate model algorithm for expensive simulation-based design optimization problems. Co-kriging method is used to develop a multi-fidelity surrogate model using two independent datasets. To achieve this objective, wing-body problem is taken as an example of application for high-dimensional complex design problem. In addition, a simple sampling analysis is used to demonstrate the characteristics of co-kriging multi-fidelity surrogate model based on the defined criteria. A drag reduction optimization is carried on using genetic algorithm based on the co-kriging surrogate model. The results are compared with kriging model based optimization. It is shown that the integration of multi-fidelity surrogate model into evolution algorithm provides an efficient framework for design and analysis of expensive simulation-based design optimization problems.

1 Introduction

The use of long running expensive computer simulations in design leads to a fundamental problem when trying to compare and contrast various competing options: there are never sufficient resources to analyze all of the combinations of variables that one would wish. This problem is particularly acute when using optimization schemes. Many studies have carried out on the surrogate model approach to resolve this problem[1-3]. Surrogate model, so called "model of models", can express the relationship between design variables and performances more clearly with simple structure and high computational efficiency, and have been widely used in design space exploration and optimization design[4]. Surrogate model's approximation accuracy is directly related to the number of samples and the complexity of the real function. For multi-dimensional problems, a large number of samples are needed to obtain reasonable approximation accuracy. The computation of current surrogate models, such as polynomial response surface[5], radial basis functions[6], neural networks[7] and Kriging[8-10], etc, are still too huge. Thus, there is a great need of high precision surrogate models with much less computation cost.

Recent surrogate model researches mainly focus on using additional design information to enhance the prediction accuracy of surrogate model, such as gradient information, information of other surrogate models and low-fidelity information, etc. Gradient information can effectively improve the predictive power of surrogate model. W. Liu [11] effectively enhanced the predictability of Kriging model using gradient information; Van Keulen and Vervenne [12] have presented promising results for a gradient enhanced weighted least squares (WLS) method. Using information of other surrogate models is called integration model, also known as multilayer model. To combine variety of surrogate models, methods of optimizing the weight coefficients of the model[13] or using the mean sum of each surrogate model[14] are commonly used. Surrogate model correction methods use a correction formulation to reduce the prediction error, and correction methods are divided into zero; first; second and higher-order correction method according to the use of correction functions. Zhang Dehu's studies have shown that second-order correction method has good applicability[15]. Use of low-fidelity information is known as variable-fidelity model, also known as multi-fidelity model or variable-complexity model, it usually build a relation model between low and high fidelity model to enhance the prediction ability of surrogate model. The applied research of multi-fidelity models has recently attracted wide attention because of its engineering applicability[16,17,18].

The main objective of the present study is the development of an efficient multi-fidelity surrogate model for wing-body shape optimization design that overcomes the deficiencies of the traditional surrogate method. Based on two independent high, low fidelity samples (the high fidelity samples is much smaller), a surrogate of wing-body aerodynamic characteristics is constructed using co-kriging method[19]. The new multi-fidelity surrogate model is applied to the drag reduction shape optimization design of an wing-body at transonic flow conditions using a Genetic Algorithm as optimizer. The efficiency and characteristics of the optimum shape are compared with those obtained from kriging surrogate model.

2 Multi-fidelity surrogate model

2.1 Correction method

In multi-fidelity surrogate-based methods, the accuracy of a surrogate building for high-fidelity model can be enhanced by a greater quantity of low-fidelity data. To make use of the low-fidelity data, we must formulate some form of correction process which models the ratios or differences between the low-fidelity model and high-fidelity model. Since computer codes are deterministic, and therefore not subject to measurement error, the usual measures of uncertainty derived from least-square residuals have no obvious meaning.

Assuming our high-fidelity model has values y_e at points X_e, and the low-fidelity model has values y_c at points X_c. The formulation of a correction process is simplified if the high-fidelity sample locations coincide with a subset of the low-fidelity sample locations ($X_e \subset X_c$). The correction process will usually take the form

$$y_e = Z_\rho y_c + Z_d \qquad (2.1)$$

With $Z_d = 0$, Z_ρ can take the form of any approximation model fitted to $y_e / y_c(X_e)$. Likewise, with $Z_\rho = 1$, Z_d can take the form of an approximation fitted to $y_e - y_c(X_e)$. These processes are then used to correct y_c when making predictions of the high-fidelity function y_e. These processes need the ratios or difference between multi-fidelity datasets, which means that the using of information included in low-fidelity model is limited by the high-fidelity model.

2.2 Co-kriging method

The Co-kriging method used in this paper is considered as a natural extension to the popular method of Kriging. Kriging method is a statistical prediction of a function at untried inputs. It requires fitting the correlation parameters of the model to each sample distribution by solving an optimization problem using maximum likelihood estimation. Co-kriging approximate the high-fidelity model using the formula as follows

$$Z_e(x) = \rho Z_c(x) + Z_d(x) \qquad (2.2)$$

Where $Z_c(x)$ denotes a kriging model of the low-fidelity function and $Z_d(x)$ a kriging model of the difference between low-fidelity function and high-fidelity function. Using two independent sets of multi-fidelity data, where the high-fidelity model has n_e samples, and low-fidelity model has n_c samples.

The co-variance between sample points can be described as

$$\mathrm{cov}[Z(\mathbf{x}^{(i)}), Z(\mathbf{x}^{(j)})] = \sigma^2 R_{ij} \qquad (2.3)$$

Where R_{ij} is the matrix of correlation between samples, which is determined by a Spatial Correlation Function (SCF).

$$R_{ij} = SCF(x^{(i)}, x^{(j)}) = \prod_k scf_k(|x_k^j - x_k^i|) \\ = \exp(-\sum_{k=1}^{n_v} \theta_k \left\| x_k^{(j)} - x_k^{(i)} \right\|^{p_k}) \tag{2.4}$$

The order of the correlation matrix depends only on the number of samples n_e and n_c and not on the number of variables. And the matrix is dense symmetric positive definite with ones along diagonal and become ill-conditioned when samples are too close. The SCF can be any function reflecting the characteristics of the output function. Here the exponential function is adopted.

As with kriging, the value at a point in the whole design space is treated as if it were the realization of a stochastic process, and the complete covariance matrix is thus

$$C = \begin{pmatrix} \sigma_c^{\ 2} R_c(X_c, X_c) & \rho\sigma_c^{\ 2} R_c(X_c, X_e) \\ \rho\sigma_c^{\ 2} R_c(X_e, X_c) & \rho^2\sigma_c^{\ 2} R_c(X_e, X_e) + \sigma_d^{\ 2} R_d(X_e, X_e) \end{pmatrix}$$

The notation $\mathbf{R}_c(\mathbf{X}_c, \mathbf{X}_e)$ denotes a matrix of correlations between the data $\mathbf{X}_c$ and $\mathbf{X}_e$. And there have more correlation parameters ($\theta_c, \theta_d, p_c, p_d$ and the scaling parameter ρ) need to be fitte. As the low-fidelity dataset is independent of the high-fidelity dataset, we can find the approximate of θ_c, p_c using the same way as kriging does. In order to estimate θ_d, p_d and ρ, we first define

$$\mathbf{d} = \mathbf{y}_e - \rho \mathbf{y}_c(\mathbf{X}_e) \tag{2.5}$$

Where $\mathbf{y}_c(\mathbf{X}_e)$ are the values of $\mathbf{y}_c$ at locations common to those of $\mathbf{X}_e$, then we can estimate θ_d, p_d and ρ using the kriging way. And the cokriging prediction of the high-fidelity function is given by

$$\hat{y}_e(\mathbf{x}) = \hat{\mu} + \mathbf{c}^T \mathbf{C}(\mathbf{y} - \mathbf{f}\hat{\beta}) \tag{2.6}$$

Where $\hat{\mu} = (\mathbf{f}^T \mathbf{C}^{-1} \mathbf{f})^{-1} \mathbf{f}^T \mathbf{C}^{-1} \mathbf{y}$, $\mathbf{f}$ is a column vector of ones with dimension n_s, and

$$\mathbf{c} = \begin{pmatrix} \hat{\rho}\hat{\sigma}_c^{\ 2} \mathbf{R}_c(\mathbf{X}_c, \mathbf{x}) \\ \hat{\rho}^2\hat{\sigma}_c^{\ 2} \mathbf{R}_c(\mathbf{X}_e, \mathbf{x}) + \hat{\sigma}_d^{\ 2} \mathbf{R}_d(\mathbf{X}_e, \mathbf{x}) \end{pmatrix} \tag{2.7}$$

The estimated MSE in this prediction is calculated as

$$s^2(\mathbf{x}) = \hat{\rho}^2 \hat{\sigma}_c^{\ 2} + \hat{\sigma}_d^{\ 2} - \mathbf{c}^T \mathbf{C}^{-1} \mathbf{c} + \frac{\mathbf{f} - \mathbf{f}^T \mathbf{C}^{-1} \mathbf{c}}{\mathbf{f}^T \mathbf{C}^{-1} \mathbf{f}} \tag{2.8}$$

2.3 Characteristics of co-kriging multi-fidelity surrogate model

A surrogate model only needs a sample distribution to approximate a function, thus it must concentrate as much information as possible, more sample data usually means better approximate accuracy. A typical, traditional design of a large modern transport aircraft wing-body example is used to demonstrate the characteristics of the co-kriging multi-fidelity surrogate model. Design parameters are a combination of 12 variables Class-Shape-Transformation (CST) method of three wing sections, root, kink and wingtip respectively. The aerodynamic coefficients of the wing-body at $M_\infty = 0.785$, $\alpha = 2.4^\circ$, $\mathrm{Re} = 25 \times 10^6$ are surrogated using kriging method and co-kriging method respectively.

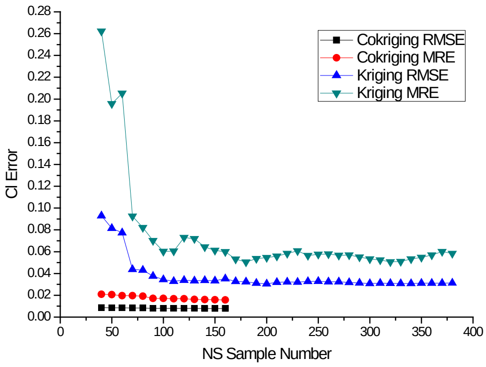
Aerodynamic analysis of wing-body is carried out using RANS equation numerical method as high-fidelity model and of full potential equation coupled with boundary layer numerical method as low-fidelity model. The number of multi-block structure grid for high-fidelity simulation is about 1.7 million, and it takes forty minutes to run a high-fidelity evaluation on a computer with Intel i7 970 in parallel mode. Meanwhile, the low-fidelity model takes only four seconds to run an aerodynamic evaluation.

The most concerned characteristic of surrogate model is the prediction ability of true functions at off-sample locations. The root mean square error (RMSE) and max relative error (MRE) of a separate validation dataset is chosen as criteria of surrogate model's accuracy.

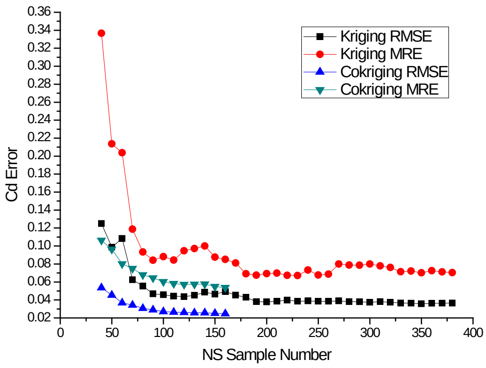
$$\text{RMSE} = \sqrt{\frac{1}{M}\sum_{i=1}^{M}\left(\frac{f(\mathbf{x}) - \hat{f}(\mathbf{x})}{\hat{f}(\mathbf{x})}\right)^2} \quad (2.9)$$

$$\text{MRE} = \underset{0<i<n}{MAX}\left(\frac{f(\mathbf{x}_i) - \hat{f}(\mathbf{x}_i)}{\hat{f}(\mathbf{x}_i)}\right) \quad (2.10)$$

Both the high-fidelity and low-fidelity sample sets were done using Latin Hypercube method and the same high-fidelity data set was used for the kriging surrogate modeling techniques. An array of different sized sample sets was used to compare the robustness and efficiency of the surrogate models for sparse to dense data sets. Figure 2 shows the comparison of approximate error between the co-kriging multi-fidelity surrogate model and kriging model. The co-kriging multi-fidelity surrogate model used in this paper which is enhanced by 400 low-fidelity data can get much better approximation than kriging model with much less high-fidelity data, and it can get a much faster sample convergence.



(a) Lift coefficient



(b) Drag coefficient

Fig. 1. The approximate error comparison

3 Wing-body aerodynamic optimization

3.1 Gennetic algorithms

Among optimization algorithms, gradient-based methods are well-known techniques that seek to find the optimum by calculating local gradient information. Although gradient-based methods are superior to non-gradient-based techniques in a local search, the optimum obtained from these methods may not be the global one, especially for aerodynamic designs[20]. Alternatively, Genetic Algorithms (GAs) are more likely to find a global optimum and are therefore attractive for aerodynamic design optimization[21]. In the present study a real coded Genetic Algorithm is applied to the optimization of wing-body configuration described in section 2.3.

The setup of objective function is minimizing the drag coefficient and expecting lift coefficient remain 0.54 as much as possible. A penalty function is used to limit the airfoil thickness in order to avoid impractical shapes and design parameters are bounded to create reasonable shapes. The mathematical model of the optimization design problem is illustrated as follows:

$$Max \quad \frac{1.0}{Cd + (Cl - 0.54)^2},$$
$$\text{s.t.} \quad \bar{c}_{\max\, root} \geq 0.15$$
$$\bar{c}_{\max\, mid} \geq 0.11$$
$$\bar{c}_{\max\, tip} \geq 0.10$$

C_d and C_l are the wing-body's drag coefficient and lift coefficient, $\bar{c}_{\max\, root}$, $\bar{c}_{\max\, mid}$ and $\bar{c}_{\max\, tip}$ are the maximum thickness of the wing root, kink and tip airfoils. The population of genetic algorithm is 30 and total evolution generation is 100.

3.2 Surrogate based optimization

Using surrogate models in optimization design can greatly improve the computational efficiency. The kriging model and cokriging model are used as aerodynamic analysis tools to carry on surrogate based drag reduction optimization design respectively. The surrogate based optimization process used in our work is shown in Figure 1. Surrogate model is updated after each genetic generation, best points are chosen to validate surrogate model's accuracy until predictive error RMSE ≤ 3% . For efficiency consideration, genetic algorithm is used to optimize the correlation parameters of co-kriging multi-fidelity surrogate model at the constructing step; at the updating step, the validated points are added to sample datasets and pattern search method is used to improve the constructed co-kriging multi-fidelity surrogate model's accuracy.

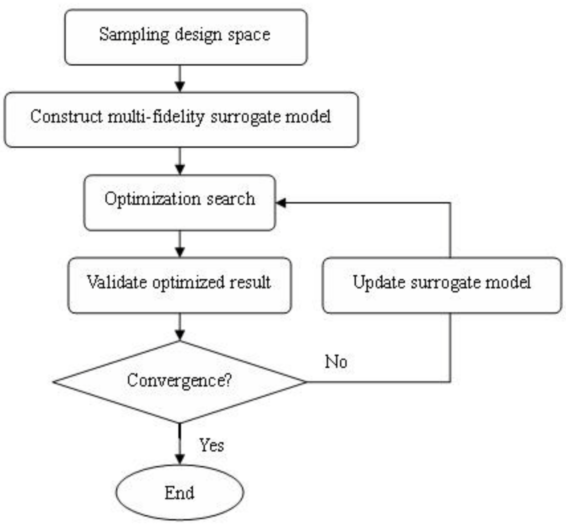


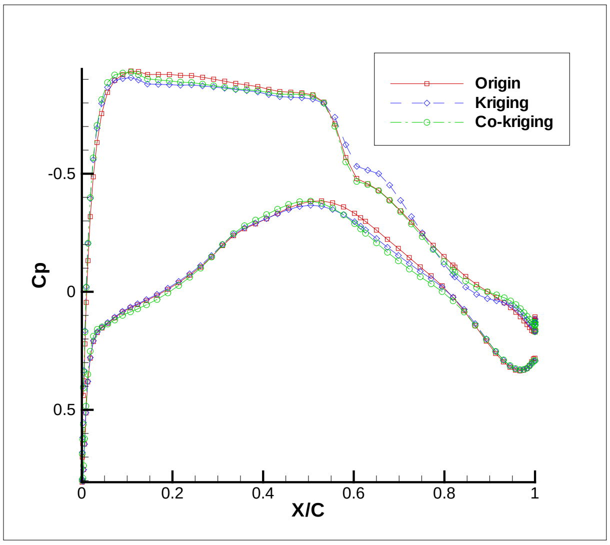
Fig. 2. Main steps of the multi-fidelity surrogate model based optimization process

3.3 Results

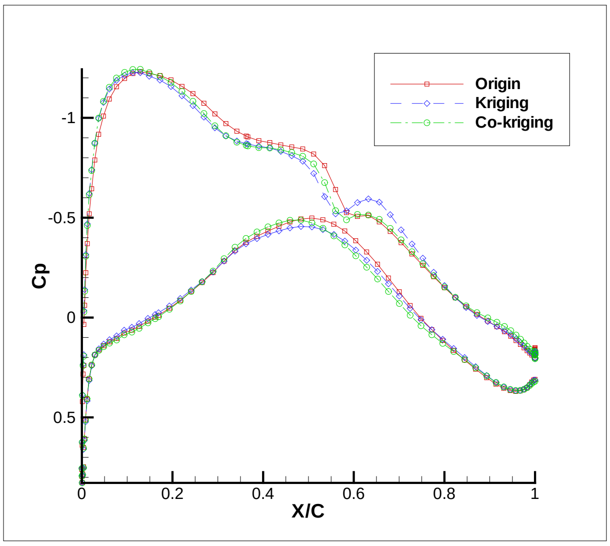
To show the effect of different surrogate methods in the optimum shape design and its influence on the convergence behavior of surrogate based optimization, kriging and co-kriging based optimization design are carried out respectively. The aerodynamic coefficient of initial and optimum wing-bodies using different surrogate models is shown in Table 1. Because co-kriging surrogate model has more accurate prediction, the optimum wing-body's lift coefficient using co-kriging surrogate model is closest to the expectation. The drag coefficient is reduced by 9 counts using kriging based GA optimization design， and co-kriging based optimization got 18 counts drag reduction. According to computational cost, kriging model requires more number of high-fidelity samples for its prediction is less accuracy; co-kriging model based optimization calls n_{he}=203 high-fidelity evaluations (including initial 160 samples), and kriging model based optimization calls n_{he}=325 high-fidelity evaluations (including initial 200 samples). Fig.3 shows the chord-wise pressure distributions of initial and optimum wing-bodies using kriging and co-kriging surrogate model. It can be seen that the performance of co-kriging model is comprehensively superior to kriging model. It indicates that co-kriging surrogate model building with little high-fidelity data and large low-fidelity data is more suitable for multi-dimensional complex engineering design optimization problems.

Table. 1. Aerodynamic coefficient of initial and optimum wing-bodies using different surrogate models

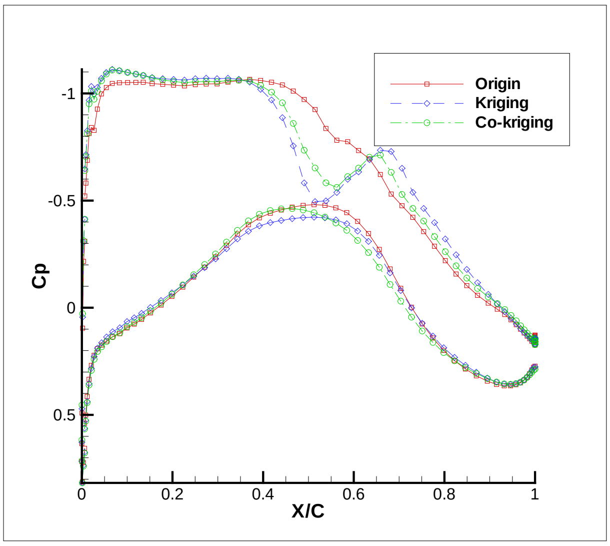
	initial	Kriging	%Diff	Co-kriging	%Diff
n_{he}		325		203	
C_l	0.5400	0.5390	-0.19%	0.5394	-0.11%
C_m	-0.111	-0.108	-2.7%	-0.101	-6.31%
C_d	0.0280	0.0271	-3.2%	0.0262	-6.43%



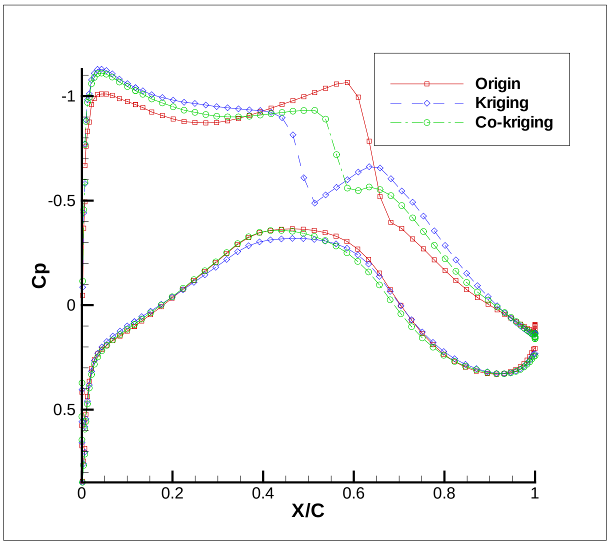
(a) y/b = 0.136



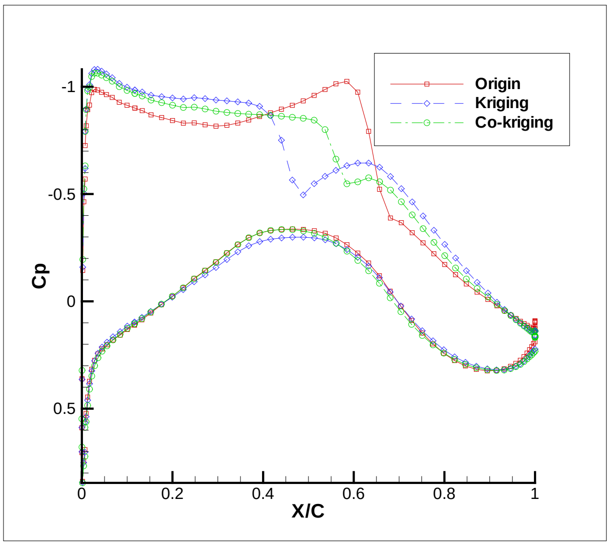
(b) y/b = 0.252



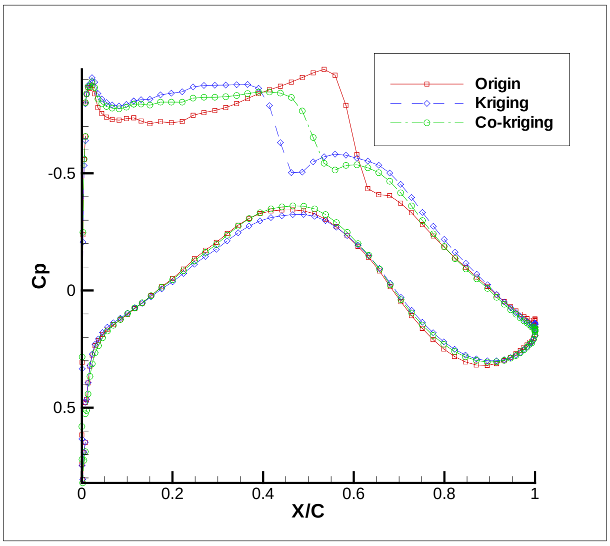
(c) y/b = 0.371



(d) y/b = 0.546



(e) y/b = 0.720



(f) y/b = 0.894

Fig. 3. Chord-wise pressure distributions of initial and optimum wing-bodies using kriging and co-kriging surrogate model

4 Conclusion

An efficient co-kriging multi-fidelity surrogate model were introduced based upon the flow characteristics of transonic viscous flow. The effect of sample size in surrogate model building and its convergence rate were investigated. A Genetic Algorithm was used as the surrogate based optimization method and the shape of a typical, traditional design of a large modern transport aircraft wing-body was optimized to achieve the minimum Cd with minimum change of Cl at specified flow conditions. The optimization results of co-kriging multi-fidelity surrogate model were compared with that of kriging model. The co-kriging multi-fidelity surrogate model provides more accurate in predicting the aerodynamic coefficienct of wing-body with smaller high-fidelity sample size, thereby reducing the computational cost of optimization for long running expensive simulation-based design problems. In addition, it was shown that a better surrogate model can improve the convergence rate of the surrogate based optimization algorithm.

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