ESTIMATION OF AERODYNAMIC DERIVATIVES IN AIRCRAFT MODEL USING ITERATIVE LEARNING IDENTIFICATION

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Abstract

This paper presents estimation of aerodynamic derivatives in aircraft models using a system identification technique for multi-variable continuous-time state-space systems, called iterative learning identification technique. As an application of the iterative learning identification, the proposed technique is applied to estimation of the aerodynamic derivatives in a lateral linear model of aircraft. The effectiveness of the technique is discussed in numerical simulations.

1 Introduction

In design of flight control systems for aircraft, stability and control derivatives, also called aerodynamic derivatives are needed to not only analyze flight properties but also design a flight controller for satisfying flight specification. They are derived from flight dynamics and are also obtained by flight and/or wind-tunnel tests. Such tests are so hard because there are many derivatives to be obtained and the measurements have to be done in several flight conditions [1], [2]. The use of system identification [3], [4] is one of methods to reduce the burdens. Recently, a system identification technique using iterative learning control, called iterative learning identification in this paper, has been developed for system identification of continuous-time systems described by the transfer function (TF) [5] - [10]. In identification techniques that have been developed for continuous-time systems so far, the derivatives of the input and output signals are required for estimating computation [11]. Since the derivatives of the output signal are sensitive to the measurement noise, the parameters may be not accurately estimated, especially when a large measurement noise is included in the output signal. In iterative learning identification, on the other hand, the derivatives of the command signal rather than the measured output are used. The measurement noise does not directly influence the estimated parameters. An advantage is that this method is robust against insufficient excitation and sampling because data used in the updated computation are newly obtained at each iteration. Moreover, it is possible to estimate the parameters of unstable systems by incorporating a stabilizing controller in the iterative learning control system. That is, it is not very hard to estimate the parameters in closed-loop systems.

This paper presents estimation of the aerodynamic derivatives in aircraft models using the iterative learning identification technique. Sugie *et al.* [6] - [9] presented identification procedures for TF models. This paper extends the iterative learning identification technique for state-space (SS) systems with multi-inputs and multi-outputs. As an application of iterative learning identification, the technique is applied to estimation of the aerodynamic derivatives in a lateral linear model of aircraft. The effectiveness of the technique is discussed in numerical simulations.

2 Preliminaries for Identification

The system to be identified in this paper is a multi-input and multi-output SS LTI system

$$\begin{cases} \dot{x}(t) = A_p(\eta)x(t) + B_p(\eta)u(t) \\ y(t) = C_p(\eta)x(t) + D_p(\eta)u(t) + v(t) \end{cases} \quad (1)$$

where $x(t) \in \boldsymbol{R}^{n_x}$ is the state, $u(t) \in \boldsymbol{R}^{n_u}$ the input, $y(t) \in \boldsymbol{R}^{n_y}$ the output and $v(t) \in \boldsymbol{R}^{n_y}$ is the noise included in $y(t)$. Additionally, $\eta \in \boldsymbol{R}^{q}$ is a q-dimensional vector that consists of the SS parameters to be identified and is called the SS parameter vector. The transfer function from $u(t)$ to $y(t)$ is represented by

$$\begin{aligned} P(p) &\triangleq \frac{N(p)}{D(p)} \\ &\triangleq \frac{1}{D(p)} \begin{bmatrix} N_{11}(p) & \ldots & N_{1n_u}(p) \\ \vdots & & \vdots \\ N_{n_y1}(p) & \ldots & N_{n_yn_u}(p) \end{bmatrix} \end{aligned} \quad (2)$$

where p is the differential operator; that is,

$$p^l u(t) \triangleq \frac{d^l u(t)}{dt^l}. \quad (3)$$

Letting n and m_{ij} be the order of polynomials $D(p)$ and $N_{ij}(p)$, respectively, they are expressed as follows.

$$D(p) \triangleq p^n + a_{n-1}p^{n-1} + \cdots + a_0 \quad (4)$$

$$N_{ij}(p) \triangleq b_{ij,m_{ij}} p^{m_{ij}} + \cdots + b_{ij,0} \quad (5)$$
$$(i = 1, \cdots, n_y, \ j = 1, \cdots, n_u)$$

Here, a_i $(i = 0, \cdots, n-1)$ and $b_{ij,l}$ $(i = 1, \cdots, n_y, \ j = 1, \cdots, n_u, \ l = 0, \cdots, m_{ij})$ are coefficients of $D(p)$ and $N_{ij}(p)$, respectively, and are called the TF parameters. Vectors with respect to the coefficients of $D(p)$ and $N_{ij}(p)$ are defined as follows.

$$\theta_a \triangleq [a_0 \ \cdots \ a_{n-1}]^T \in \boldsymbol{R}^{n}, \ \ \theta_{a1} \triangleq [\theta_a^T \ 1]^T \in \boldsymbol{R}^{n+1},$$
$$\theta_{b_{ij}} \triangleq [b_{ij,0} \ \cdots \ b_{ij,m_{ij}}]^T \in \boldsymbol{R}^{m_{ij}}. \quad (6)$$

Collecting $\theta_{b_{ij}}$ $(i = 1, \cdots, n_y, j = 1, \cdots, n_u)$, θ_b is defined as

$$\theta_b \triangleq [\theta_{b_{11}}^T \ \cdots \ \theta_{b_{1n_u}}^T \ \ \cdots \ \ \theta_{b_{n_y1}}^T \ \cdots \ \theta_{b_{n_yn_u}}^T]^T \in \boldsymbol{R}^{m} \quad (7)$$

where

$$m_i \triangleq \sum_{j=1}^{n_u} m_{ij} + n_u,$$
$$m \triangleq \sum_{i=1}^{n_y} m_i = \sum_{i=1}^{n_y}\sum_{j=1}^{n_u} m_{ij} + n_y n_u. \quad (8)$$

Then, the TF parameter vector is defined as

$$\theta \triangleq [\theta_b^T \ \ \theta_a^T]^T \in \boldsymbol{R}^{m+n}. \quad (9)$$

It is supposed in this paper that Eq. (1) is the minimal realization; that is, $n = n_x$. Moreover, the number of SS parameters is not greater than that of the TF parameters; that is,

$$q \leq m + n \quad (10)$$

In iterative learning identification, n_u command signals are needed to generate the reference for the output and the controlled input. They are given as the elements of the following vector

$$h(t) \triangleq [h_1(t) \ \cdots \ h_{n_u}(t)]^T \in \boldsymbol{R}^{n_u}. \quad (11)$$

Here, $h_j(t)$ $(j = 1, \cdots, n_u)$ are smooth and are differentiable by n times. The following vector, which consists of $h_j(t)$ and its derivatives, is defined as

$$\phi_{j,l}(t) \triangleq [h_j(t) \ \frac{dh_j(t)}{dt} \ \cdots \ \frac{d^l h_j(t)}{dt^l}]^T \in \boldsymbol{R}^{l+1} \quad (12)$$

where the range of l is given by $0 \leq l \leq n$. Using $\phi_{j,l}(t)$ $(j = 1, \cdots, n_u)$ in the iterative learning control system, whose structure is given by a tracking control system and will be shown in the following section, $y(t)$ is measured at a specified time interval. The SS parameter vector is updated so as to reduce the response error. That is, iterative learning identification estimates the SS parameters by performing tracking control and updating the SS parameter vector iteratively. In the

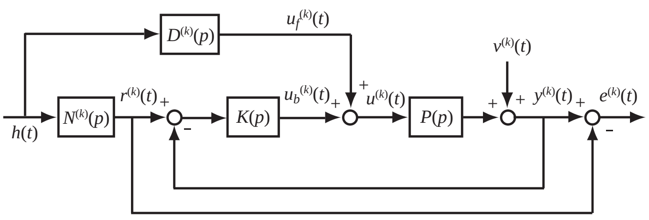

Fig. 1 Iterative learning control system for identification.

following sections, for convenience, the iteration number is denoted as k. The signal vectors, estimated parameters and polynomials of the transfer functions at the k-th iteration are denoted as $(\cdot)^{(k)}$. Their true values are denoted as $(\cdot)^{*}$.

3 Response Error and Parameter Update

3.1 Iterative learning control system for identification

Figure 1 shows an iterative learning control system for identification. Here, $K(p)$ is an $n_u \times n_y$ feedback controller for stabilization (it does not matter whether the structure of $K(p)$ is known or not); $u_b^{(k)}(t) \in R^{n_u}$ is the k-th iteration feedback input; $u_f^{(k)}(t) \in R^{n_u}$ is the k-th iteration feedforward input generated by feeding the command $h(t)$ into the k-th iteration estimated denominator polynomial $D^{(k)}(p)$; $r^{(k)}(t) \in R^{n_y}$ is the k-th iteration reference for $y^{(k)}(t)$ and is generated by feeding the command $h(t)$ into the k-th iteration estimated numerator polynomial $N^{(k)}(p)$. The response error, denoted as $e^{(k)}(t) \in R^{n_y}$, is defined as the difference between $y^{(k)}(t)$ and $r^{(k)}(t)$. The SS parameters to be identified are updated so as to reduce the response error at each iteration. When $P(p)$ to be identified is stable, $K(p)$ may be omitted.

The signals in Fig. 1 are written as

$$u_b^{(k)}(t) \triangleq K(p)(r^{(k)}(t) - y^{(k)}(t)), \tag{13}$$

$$u_f^{(k)}(t) \triangleq D^{(k)}(p)h(t) = \Phi_{a1}(t)\theta_{a1}^{(k)}, \tag{14}$$

$$r^{(k)}(t) \triangleq N^{(k)}(p)h(t) = \Phi_b(t)\theta_b^{(k)}, \tag{15}$$

where

$$\Phi_{a1}(t) \triangleq \begin{bmatrix} \phi_{1,n}^T(t) \\ \vdots \\ \phi_{n_u,n}^T(t) \end{bmatrix}, \quad \Phi_a(t) \triangleq \begin{bmatrix} \phi_{1,n-1}^T(t) \\ \vdots \\ \phi_{n_u,n-1}^T(t) \end{bmatrix},$$

$$\Phi_b(t) \triangleq \begin{bmatrix} \phi_{b1}^T(t) & & 0 \\ & \ddots & \\ 0 & & \phi_{bn_y}^T(t) \end{bmatrix} \tag{16}$$

$$\phi_{bi}^T(t) \triangleq [\phi_{1,m_{i1}}^T(t) \ \cdots \ \phi_{n_u,m_{in_u}}^T(t)] \tag{17}$$
$$(i = 1, \cdots, n_y)$$

Then, the response error at the k-th iteration is given by

$$\begin{aligned} e^{(k)}(t) &\triangleq y^{(k)}(t) - r^{(k)}(t) \\ &= Y(p)u_f^{(k)}(t) - S(p)r^{(k)}(t) + S(p)v^{(k)}(t) \end{aligned} \tag{18}$$

where

$$S(p) \triangleq (I_{n_y} + P(p)K(p))^{-1}, \tag{19}$$

$$Y(p) \triangleq (I_{n_y} + P(p)K(p))^{-1}P(p). \tag{20}$$

When $v^{(k)}(t) = 0$, $e^{(k)}(t) = 0$, it indicates that the superscript (k) is changed into $*$ in Eq. (18). The following equation holds.

$$\begin{aligned} y^{*}(t) &\triangleq Y(p)u_f^{*}(t) = S(p)\frac{N^{*}(p)}{D^{*}(p)}D^{*}(p)h(t) \\ &= S(p)r^{*}(t) \end{aligned} \tag{21}$$

Using Eqs. (13)-(21), $e^{(k)}(t)$ is written as

$$\begin{aligned} e^{(k)}(t) &= Y(p)\Phi_a(t)(\theta_a^{(k)} - \theta_a^{*}) \\ &\quad - S(p)\Phi_b(t)(\theta_b^{(k)} - \theta_b^{*}) + S(p)v^{(k)}(t) \end{aligned} \tag{22}$$

Equation (22) explicitly represents the dependence of the TF parameters $\theta_a^{(k)}$ and $\theta_b^{(k)}$ to the response error $e^{(k)}(t)$. That is, reducing $e^{(k)}(t)$ corresponds to approaching $\theta_a^{(k)}$ and $\theta_b^{(k)}$ to θ_a^{*} and θ_b^{*}, respectively. To realize this, the response error is first represented by the sampled data whose number is sufficiently greater than that of TF parameters and is then projected onto the subspace of the parameters.

3.2 Vector representation by sampled data

To treat sampled data compactly, as an example, the following vector denoted by the boldface is defined for the output $y^{(k)}(t)$,

$$\mathbf{y}^{(k)} \triangleq \begin{bmatrix} y^{(k)}(0) \\ \vdots \\ y^{(k)}(NT_s) \end{bmatrix} \in R^{n_y(N+1)} \tag{23}$$

where N is the number of sampled data and T_s is the sampling time. Vectors denoted by the boldface are also defined for other signals. Furthermore, the following matrices are defined for $\Phi_{a1}(t)$, $\Phi_a(t)$ and $\Phi_b(t)$, respectively.

$$\Gamma_{a1} \triangleq \begin{bmatrix} \Phi_{a1}(0) \\ \vdots \\ \Phi_{a1}(NT_s) \end{bmatrix}, \quad \Gamma_a \triangleq \begin{bmatrix} \Phi_a(0) \\ \vdots \\ \Phi_a(NT_s) \end{bmatrix}, \quad \Gamma_b \triangleq \begin{bmatrix} \Phi_b(0) \\ \vdots \\ \Phi_b(NT_s) \end{bmatrix} \tag{24}$$

If N is given as $N \gg n+m$, the following rank condition almost holds.

$$\mathrm{rank}\Gamma_a = n, \quad \mathrm{rank}\Gamma_b = m \tag{25}$$

Letting $g_S(t) \in R^{n_y \times n_y}$ and $g_Y(t) \in R^{n_y \times n_u}$ be the impulse response matrices, respectively, where $S(p)$ and $Y(p)$ are discretized by the 0-th order hold with the sampling time T_s, the following matrices are defined with respect to $g_S(t)$ and $g_Y(t)$.

$$G_S \triangleq \begin{bmatrix} g_S(0) & 0 & \dots & 0 \\ g_S(T_s) & g_S(0) & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ g_S(NT_s) & g_S((N-1)T_s) & \dots & g_S(0) \end{bmatrix} \tag{26}$$

$$G_Y \triangleq \begin{bmatrix} g_Y(0) & 0 & \dots & 0 \\ g_Y(T_s) & g_Y(0) & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ g_Y(NT_s) & g_Y((N-1)T_s) & \dots & g_Y(0) \end{bmatrix} \tag{27}$$

Then, Eq. (22) can be approximated in terms of the sampled data vector.

$$\begin{aligned} \mathbf{e}^{(k)} &\simeq G_Y\Gamma_a(\theta_a^{(k)} - \theta_a^*) - G_S\Gamma_b(\theta_b^{(k)} - \theta_b^*) + G_S\mathbf{v}^{(k)} \\ &= \Lambda(\theta^{(k)} - \theta^*) + G_S\mathbf{v}^{(k)} \end{aligned} \tag{28}$$

where

$$\Lambda \triangleq [-G_S\Gamma_b \;\; G_Y\Gamma_a] \in R^{n_y(N+1)\times(m+n)} \tag{29}$$

3.3 SS parameter and response error

Equation (28) is the relationship between the TF parameter vector θ and the response error. This is extended to the relationship between the SS parameter vector η and the response error in this section. The basic idea is that θ is regarded as a function with respect to η. The first-order approximation of θ^* around $\eta^{(k)}$ is given by

$$\theta^* \simeq \theta^{(k)} + \Psi^{(k)}(\eta^* - \eta^{(k)}) \tag{30}$$

where

$$\Psi^{(k)} \triangleq \frac{\partial\theta(\eta^{(k)})}{\partial\eta^T}. \tag{31}$$

Substituting Eq. (30) into Eq. (28), $\mathbf{e}^{(k)}$ is written as

$$\mathbf{e}^{(k)} = \Lambda\Psi^{(k)}(\eta^{(k)} - \eta^*) + G_S\mathbf{v}^{(k)}. \tag{32}$$

Pre-multiplying both sides of Eq. (32) by $(\Lambda\Psi^{(k)})^T$, we have

$$\varepsilon^{(k)} = M^{(k)}(\eta^{(k)} - \eta^*) + \nu^{(k)} \tag{33}$$

where

$$\varepsilon^{(k)} \triangleq (\Lambda\Psi^{(k)})^T \mathbf{e}^{(k)}, \ \ \nu^{(k)} \triangleq (\Lambda\Psi^{(k)})^T G_S \mathbf{v}^{(k)}, \\ M^{(k)} \triangleq (\Lambda\Psi^{(k)})^T (\Lambda\Psi^{(k)}).$$

If the following condition holds,

$$\mathrm{rank}(\Lambda\Psi^{(k)}) = q \ \ ({}^\forall k \geq 0) \tag{34}$$

$M^{(k)}$ is nonsingular. When $\nu^{(k)} = 0$, the following relation holds.

$$\varepsilon^{(k)} \to 0 \ \Leftrightarrow \ \eta^{(k)} \to \eta^* \tag{35}$$

Thus, the SS parameter vector $\eta^{(k)}$ is identifiable by reducing $\varepsilon^{(k)}$; that is, $\mathbf{e}^{(k)}$ to zero. Equation (34) indicates that the persistently exciting (PE) condition [3], [4] is satisfied in the proposed technique.

3.4 Updated law of SS parameter

This section shows the updated law of the SS parameters in iterative learning identification. The updated law of $\eta^{(k)}$ is given by

$$\eta^{(k+1)} = \eta^{(k)} + H^{(k)}\varepsilon^{(k)} \tag{36}$$

where

$$H^{(k)} = -\alpha^{(k)} {M^{(k)}}^{-1} \tag{37}$$

where $\alpha^{(k)}$ is a non-decreasing gain with respect to the iteration number k and is given by

$$\alpha^{(k)} : \ \underline{\alpha} \to \overline{\alpha} \ \ (0 < \underline{\alpha} \leq \overline{\alpha} < 1). \tag{38}$$

Substituting Eqs. (33) and (37) into Eq. (36), we have

$$\eta^{(k+1)} = (1 - \alpha^{(k)})\eta^{(k)} + \alpha^{(k)}(\eta^* - {M^{(k)}}^{-1}\nu^{(k)}). \tag{39}$$

When $\nu^{(k)} = 0$, the following convergence condition is guaranteed.

$$k \to \infty \ : \ \eta^{(k)} \to \eta^* \tag{40}$$

Equation (38) is a technique to improve convergence of the parameter [10].

3.5 Procedures of iterative learning identification

To summarize section 3, the procedures of iterative learning identification are as follows.

Step 1: The SS parameter η to be identified is defined. Construct the iterative learning control system as shown in Fig. 1. If the system is unstable, provide a stabilizing controller $K(p)$. Otherwise, $K(p)$ may be omitted.

Step 2: Obtain the impulse response matrices $g_S(t)$ and $g_Y(t)$ $(t = 0, T_s, \cdots, NT_s)$. Construct G_S and G_Y defined as Eqs. (26) and (27). Set $k = 1$

Step 3: Perform tracking control; that is, feed the command signal $h(t)$ in Fig. 1. Measure $y^{(k)}(t)$ and $r^{(k)}(t)$ $(t = 0, T_s, \cdots, NT_s)$. Obtain $e^{(k)}(t)$.

Step 4: Update $\eta^{(k)}$ using Eq. (36).

Step 5: If iteration continues, set $k+1 \to k$ and go to Step 3. Otherwise, stop.

In Step 2, if the impulse response matrices $g_S(t)$ and $g_Y(t)$ is not able to obtain precisely; that is, the initial state cannot be set to zero, they may be estimated by the least square method.

4 Estimation of Aerodynamic Derivatives in an Aircraft Model

The proposed identification technique is applied to estimation of aerodynamic derivatives in a lateral linear model of aircraft in this section. The SS representation of the lateral motion of aircraft is given in the form of Eq. (1) where the state and input vectors x and u are given by [12]

$$x \triangleq [\beta \ \ \phi \ \ p \ \ r]^T, \ \ u \triangleq [\delta_a \ \ \delta_r]^T. \tag{41}$$

Here, x consists of the side slip angle β, the roll angle ϕ, the roll rate p (not the differential operator of the transfer function here) and the yaw rate r. In addition, u consists of the aileron deflection angle δ_a and the rudder deflection angle δ_r.

These variables represent the deviation from the equilibria. A_p and B_p are given as

$$A_p = E^{-1}F, \;\; B_p = E^{-1}G, \qquad (42)$$

where

$$E \triangleq \begin{bmatrix} V_a & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -I_{xz}/I_{xx} \\ 0 & 0 & -I_{xz}/I_{zz} & 1 \end{bmatrix}, \; G \triangleq \begin{bmatrix} 0 & Y_{\delta_r} \\ 0 & 0 \\ L_{\delta_a} & L_{\delta_r} \\ 0 & N_{\delta_r} \end{bmatrix},$$

$$F \triangleq \begin{bmatrix} Y_\beta & g\cos\Theta_0 & Y_p & Y_r - V_a \\ 0 & 0 & 1 & \tan\Theta_0 \\ L_\beta & 0 & L_p & L_r \\ N_\beta & 0 & N_p & N_r \end{bmatrix}.$$

In matrices E and F, V_a is the flight velocity and Θ_0 is the pitch angle at the equilibrium. I_{xx} and I_{zz} are the moments of inertia in the x-axis and z-axis, respectively. I_{xz} is the product of inertia. Y_β, Y_p, etc. are the aerodynamic derivatives to be identified. In the following subsections, the SS parameter vector η is constructed by the aerodynamic derivatives that are assigned in advance. The output y is defined by the following two cases:

(O1) Two outputs:

$$y \triangleq [\beta \;\; \phi]^T \qquad (43)$$

(O2) Four outputs:

$$y \triangleq [\beta \;\; \phi \;\; p \;\; r]^T = x \qquad (44)$$

The aircraft considered in this study is from Ref. [13]). The flight conditions are given by the altitude $H = 4{,}000$ [m] and the flight velocity $V_a = 100$ [m/s].

4.1 Case 1

η is constructed by

$$\eta = [L_\beta \;\; L_p \;\; N_\beta]^T \in R^3. \qquad (45)$$

The true value of η is

$$\eta^* = [-1.874 \;\; -0.971 \;\; 1.061]^T. \qquad (46)$$

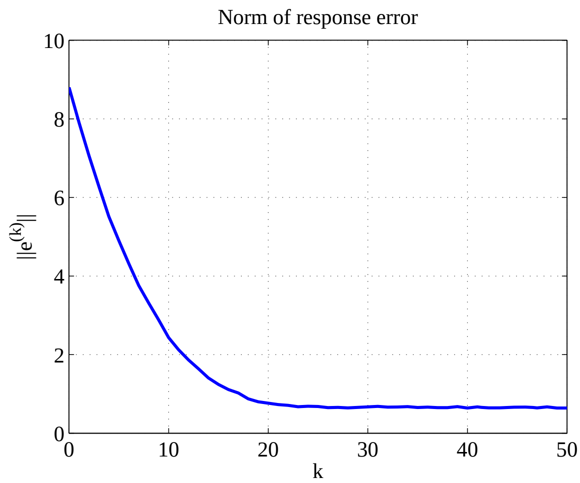

Fig. 2 Norm of response error (Case 1).

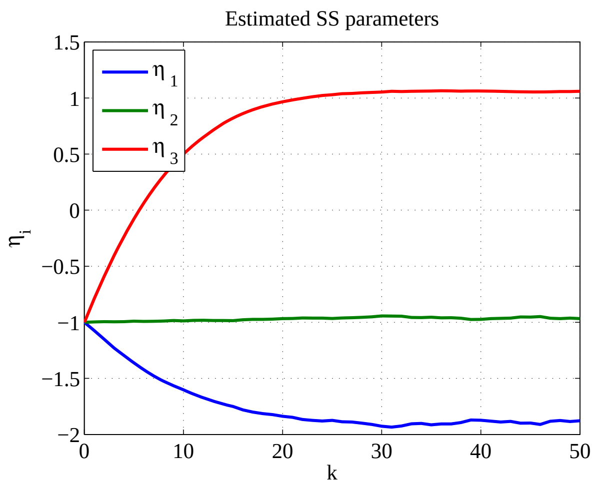

Fig. 3 Estimated SS parameters (Case 1).

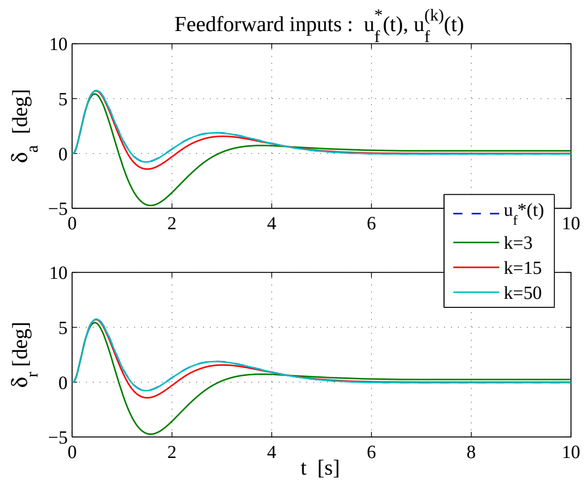

Fig. 4 Feedforward input (Case 1).

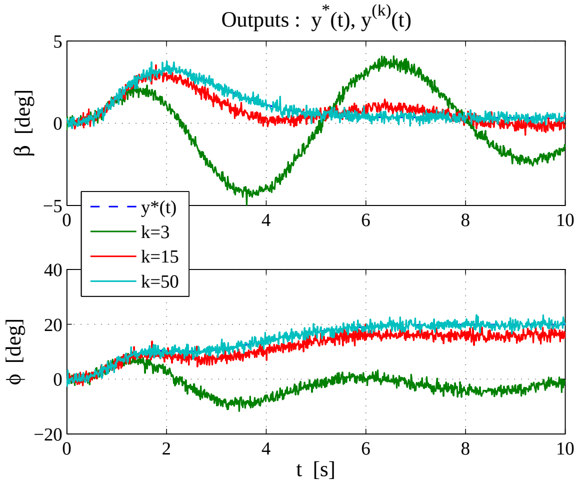

Fig. 5 Output (Case 1).

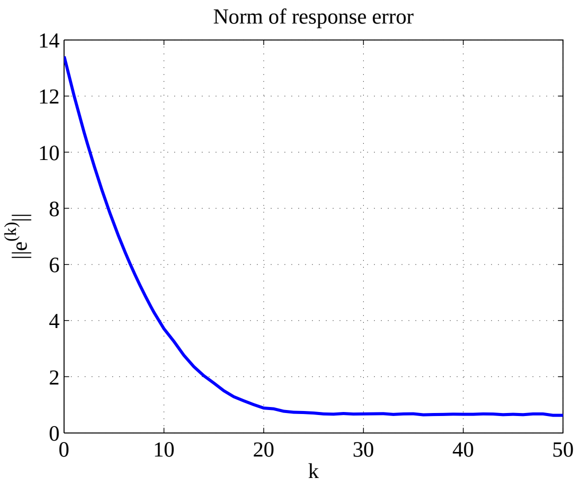

Fig. 6 Norm of response error (Case 2).

The output y is given by (O1). The command signal vector $h(t) \in \boldsymbol{R}^2$ is given by a two-dimensional step response

$$h(t) = \frac{2^6}{(p+2)^6} I_2 w(t) \tag{47}$$

where $w(t) \in \boldsymbol{R}^2$ is given by the two-dimensional unit step input $w(t) \triangleq [1 \ \ 1]^T$. The measurement noise $v^{(k)}(t) \in \boldsymbol{R}^2$ is given by the white noise whose noise signal ratio (NSR) is 20 %, where NSR is defined as

$$\mathrm{NSR} \triangleq \frac{||v^{(k)}(t)||}{||y^{(k)}(t)||}. \tag{48}$$

Since the system with Eq. (4) is stable in the flight conditions $H = 4,000$ [m] and $V_a = 100$ [m/s], the stabilizing controller $K(p)$ is omitted. The sampling time is given by $T_s = 0.01$ [s] and the number of the sampled data is $N = 1,000$. The lower and upper bounds of $\alpha^{(k)}$ in Eq. (38) are given by $\underline{\alpha} = 0.7$ and $\overline{\alpha} = 0.9$. The initial SS parameter vector is given by $\eta^{(0)} = [-1 \ -1 \ -1]^T$.

Figures 2 and 3 show the norm of the response error $\mathbf{e}^{(k)}$ and the estimated SS parameters η_i $(i = 1,2,3)$, respectively, for fifty iterations. The response error is monotonously decreased and the SS parameters asymptotically converge for $k \geq 25$. At $k = 50$, the SS parameter vector is obtained as

$$\eta^{(50)} = [-1.878 \ \ -0.967 \ \ 1.060]^T. \tag{49}$$

The error of the estimated SS parameters is less than 0.5 % of the true values. Figures 4 and 5 show the feedforward input $u_f^{(k)}(t)$ and the output $y^{(k)}(t)$ at $k = 3, 15$ and 50, respectively. When increasing the iteration number, the responses approach the ones whose SS parameters are true values. The responses of $u_f^{(50)}(t)$ and $y^{(50)}(t)$ almost coincide with $u_f^*(t)$ and $y^*(t)$, respectively.

4.2 Case 2

η is constructed by

$$\eta = [N_r \ \ L_{\delta_a} \ \ N_{\delta_r}]^T \ \in \boldsymbol{R}^3 \tag{50}$$

which includes the derivatives in matrix B_p. The output is given by (O1) and $h(t)$ is given by the two-dimensional step response. The lower and upper bounds of $\alpha^{(k)}$ in Eq. (38) are given by $\underline{\alpha} = 0.7$ and $\overline{\alpha} = 0.9$. The initial SS parameter vector is given by $\eta^{(0)} = [-1 \ 1 \ -1]^T$. The true values of η are

$$\eta^* = [-0.211 \ \ 4.5397 \ \ -0.7199]^T. \tag{51}$$

Figures 6 and 7 show the norm of the response error and the SS parameters, respectively, for fifty iterations. In addition, in this case, the response error is monotonously decreased and the SS parameters asymptotically converge for $k \geq 25$. At $k = 50$, the SS parameter vector is obtained as

$$\eta^{(50)} = [-0.210 \ \ 4.554 \ \ -0.731]^T. \tag{52}$$

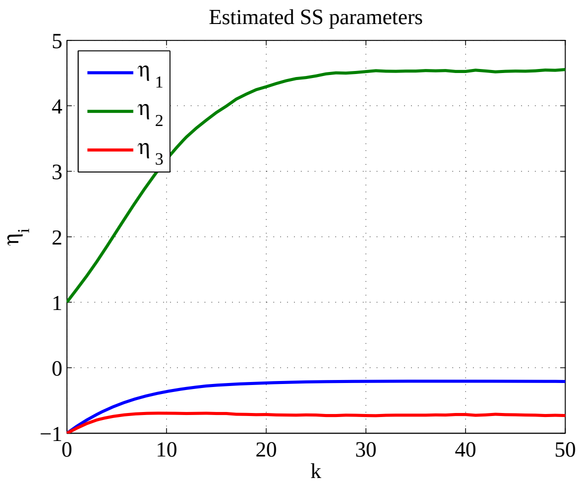

Fig. 7 Estimated SS parameters (Case 2).

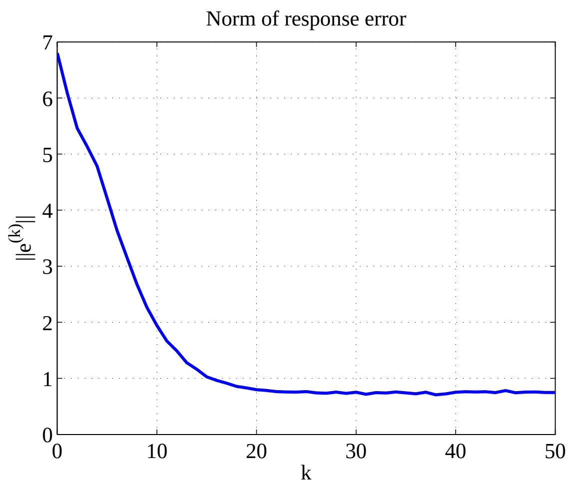

Fig. 8 Norm of response error (Case 3).

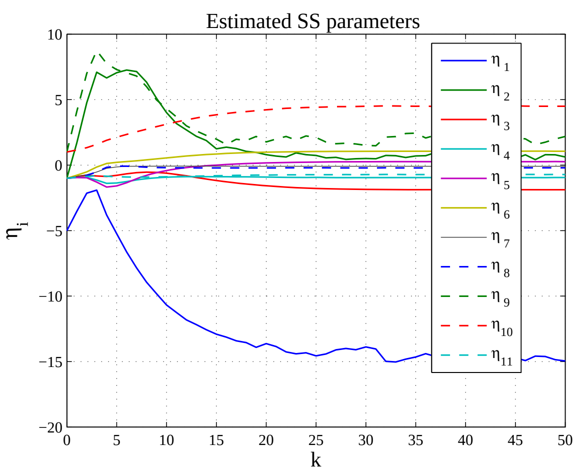

Fig. 9 Estimated SS parameters (Case 3).

4.3 Case 3

η is constructed by

$$\eta = [Y_\beta \;\; Y_r \;\; L_\beta \;\; L_p \;\; L_r \;\; N_\beta \;\; N_p \;\; N_r \;\; Y_{\delta_r} \;\; L_{\delta_a} \;\; N_{\delta_r}]^T \in \boldsymbol{R}^{11}. \quad (53)$$

The true values of η is

$$\eta^* = [-15.57 \; 0.8346 \; -1.874 \; -0.9709 \; 0.2640 \; 1.061 \; -0.0894 \; -0.211 \; 3.139 \; 4.541 \; -0.7199]^T. \quad (54)$$

When the output is given by (O1), the updated law of η, Eq. (36), fails because $M^{(k)}$ becomes singular; that is, the rank condition of $\Lambda\Psi^{(k)}$, Eq. (34), is not satisfied. When the output is changed to (O2), the rank condition is satisfied. Figures 8 - 9 show the norm of the response error and the SS parameters, respectively. At $k = 50$, the SS parameter vector is obtained as

$$\eta^{(50)} = [-14.96 \; 0.6261 \; -1.882 \; -0.9497 \; 0.2652 \; 1.060 \; -0.0916 \; -0.2099 \; 2.190 \; 4.500 \; -0.7172]^T. \quad (55)$$

The error of the estimated SS parameters is acceptable except that for η_2 $(= Y_r)$ and η_9 $(= Y_{\delta_r})$.

5 Concluding Remarks

This paper has presented estimation of the aerodynamic derivatives in aircraft models using a system identification technique for multi-variable continuous-time state-space systems using iterative learning control, called iterative learning identification technique. As an application of the iterative learning identification, the technique was applied to estimation of the aerodynamic derivatives in a lateral linear model of aircraft. The effectiveness of the technique was demonstrated in numerical simulations.

When the number of parameters to be identified increased, computational problems emerged and the accuracy of the estimated parameters decreased. Although these were avoided by increasing the number of output in the numerical simulation, more effective techniques should be considered. Furthermore, some aerodynamic derivatives were not precisely estimated because they were not sensitive to the response error. These problems will be improved in future research.

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