DEVELOPMENT AND EVALUATION OF TRAJECTORY PREDICTION MODEL

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Abstract

This paper describes a trajectory prediction model for future ATM (Air Traffic Management). The model predicts trajectories based on aircraft performance, airline operation, the navigation database, and weather forecasts. The trajectory prediction model is evaluated on accuracy, integrity, and availability for operational use. In order to attain the required accuracy, uncertainty factor analysis based on the actual flight operation environment is important. Trajectories predicted using the model are compared with recorded flight data. The first step is error factor analysis of ground speed. Error factors from the aircraft speed model and error factors from weather forecasts were analyzed for ground speed prediction. As a result, the Mach number and CAS difference between prediction and measurement were large in most cases when ground speed difference was large.

1 Introduction

Trajectory-based operation or trajectory management is considered a key capability of ATM (Air Traffic Management) in the future. Trajectory is a description of the movement of an aircraft, both in the air and on the ground, including position, time and, through calculation, speed and acceleration [1]. A trajectory is generated based on the expectation of the flight operator in consideration of various elements, such as aircraft performance and weather conditions. It is modified to avoid hazards such as bad weather and conflict with other aircraft. All flight phases, from departure to arrival, can be uniformly managed by trajectory-based operation, and in this way, flight operation efficiency can be improved.

NextGen of the United States and SESAR of Europe aim to realize trajectory-based operation [2], [3]. CARATS (Collaborative Actions for Renovation of Air Traffic Systems) is being discussed for the future in Japan [4]. Trajectory-based operation is one of its key capabilities.

Aircraft has a Flight Management System (FMS), which determines the optimum trajectory for fuel consumption and flight time. Trajectory optimization is limited to individual aircraft. It does not work for all aircraft. Aircraft equipped with the latest FMS have highly accurate trajectory control. Old aircraft do not have this function. Therefore, a ground-based trajectory prediction system for all aircraft is required for trajectory management and overall aircraft optimization.

The trajectory prediction system is evaluated on accuracy, integrity, and availability for operational use [5]. Initial 4D - 4DTRAD Concept of Operations defines the required time prediction accuracy at 30 seconds in en route airspace and 10 seconds in terminal airspace [6]. To do that, uncertainty factor analysis based on actual flight operation environment is important.

The Electronic Navigation Research Institute (ENRI) is developing a trajectory prediction model for trajectory-based operation [7]-[9]. It generates precise four-dimensional trajectories by using aircraft performance data, weather forecast data, etc. ENRI compares trajectories generated from its prediction model with operational data, and carries out prediction error analysis.

This paper describes the development and evaluation of the trajectory prediction model.

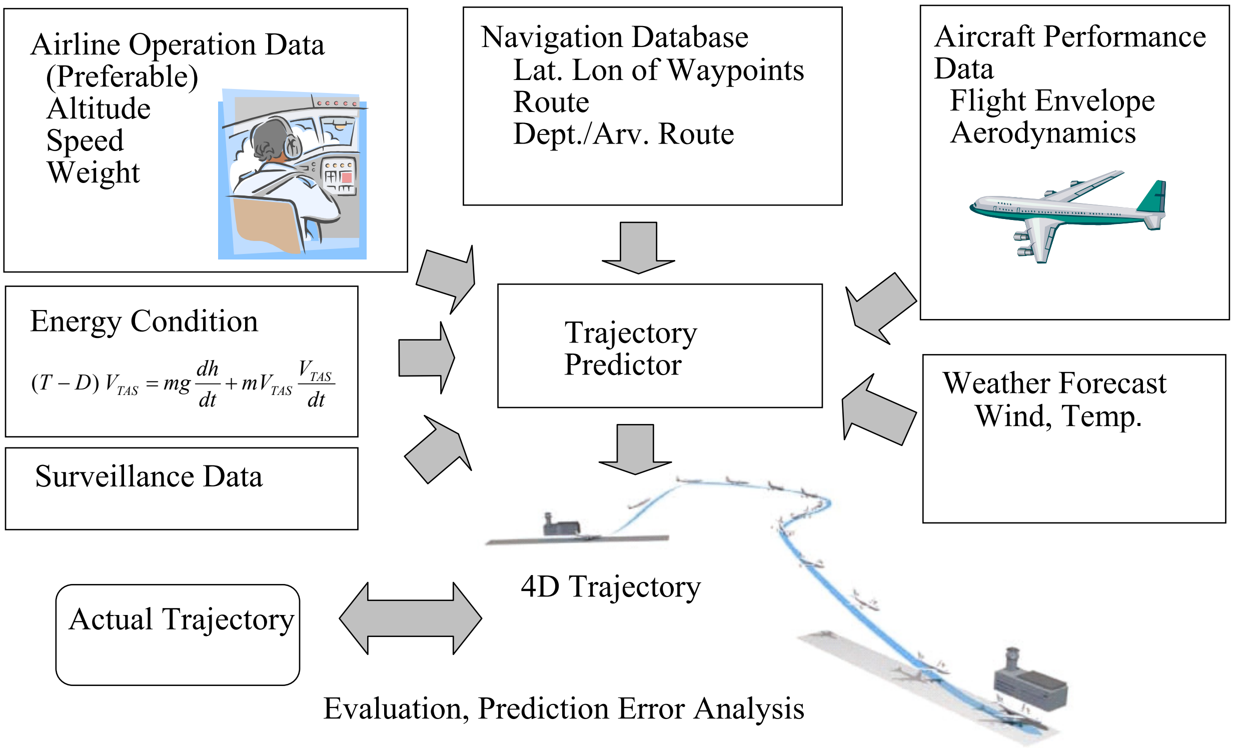

Fig. 1. Block Diagram of Trajectory Prediction Model

Firstly, the method of trajectory prediction is shown. Secondly, the paper discusses the evaluation of the trajectory prediction model, and analyzes error factors that influence trajectory prediction. Trajectories predicted by the trajectory prediction model are compared with the trajectories of operational data. To predict the crossing time at a waypoint, the accurate prediction of ground speed (GS) is important. Weather conditions influence GS. Error factors from weather forecasts and the speed model of aircraft are analyzed.

2 Method of trajectory prediction

Fig. 1 shows a block diagram of the trajectory prediction model. It uses aircraft performance data, airline operation data, a navigation database, and weather forecast data, etc. A Total Energy Model (TEM) is used as the aircraft model [10]. The TEM equates the rate of work done by forces acting on the aircraft to the rate of increase in potential and kinetic energy.

The aircraft performance data includes the flight envelope of each aircraft model (maximum speed, minimum speed, etc.), aerodynamics (wing area and drag coefficient), engine thrust, fuel consumption, etc. The airline operation data includes preferable altitude, speed, and weight in the climb, cruise, and descend phases. BADA 3.7 (Base of Aircraft Data) by EUROCONTROL is used for this data. The navigation database provides position data for routes and waypoints. Surveillance data includes an aircraft's present position and speed. It is used to monitor and update trajectories.

In order to calculate the flight time to a waypoint en route from a present position, GS and distance along the route are used. The total flight time is calculated as the sum of small segment flight times. Total flight time T_p from origin O to destination P in Fig. 2 is given by Eq. (1).

$$T_p = \int_0^S \frac{1}{V_{GND}(s)} ds \qquad (1)$$

where S is the distance from origin O to destination P, s is small segment distance, $V_{GND}(s)$ is GS at the small segment. The distance can be calculated accurately by using the RNAV (Area Navigation) route [11]. Accuracy of GS prediction is important. The

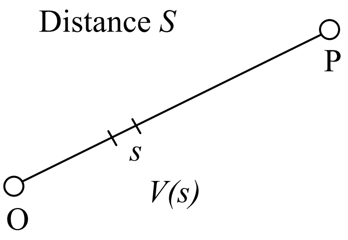

Fig. 2. Flight Time Prediction

size of the segment is decided with an aircraft's wind-influenced changes in GS.

The aircraft usually flies at a constant Indicated Air Speed or Calibrated Air Speed (CAS) and Mach number. The trajectory prediction model converts CAS or Mach number into True Air Speed (TAS). GS is calculated, taking into account the influence of wind aloft.

Eq. (2) shows the relationship between TAS V_{TAS} and CAS V_{CAS} [10].

$$V_{TAS} = \left[\frac{2P}{\mu\rho}\left\{\left(1+\frac{(P_0)_{ISA}}{P}\left[\left(1+\frac{\mu}{2}\frac{(\rho_0)_{ISA}}{(P_0)_{ISA}}V_{CAS}{}^2\right)^{1/\mu}-1\right]\right)^{\mu}-1\right\}\right]^{1/2} \quad (2)$$

where P is the pressure at altitude, ρ is the air density at altitude, γ is the isentropic expansion coefficient for air = 1.4, and $\mu = (\gamma - 1)/\gamma$. ISA stands for International Standard Atmosphere, $(P_0)_{ISA}$ is the ISA pressure at sea level = 101325 Pa, and $(\rho_0)_{ISA}$ is the ISA air density at sea level = 1.225 kg/m^3. Air pressure P and density ρ are given as functions of temperature.

$$P = (P_0)_{ISA}[T/T_0]^{-\frac{g}{K_T R}} \quad (3)$$

$$\rho = \rho_0[T/T_0]^{-\frac{g}{K_T R}-1} \quad (4)$$

where T is the temperature at altitude (K), T_0 is the temperature at sea level, ρ_0 is the air density at sea level, R is the real gas constant for air, g is gravity acceleration, K_T is the ISA temperature gradient with altitude, $-g/K_T R$ = 5.25583.

Eq. (5) shows the relationship between TAS V_{TAS} and Mach number M.

$$V_{TAS} = M\sqrt{\gamma \cdot R \cdot T} \quad (5)$$

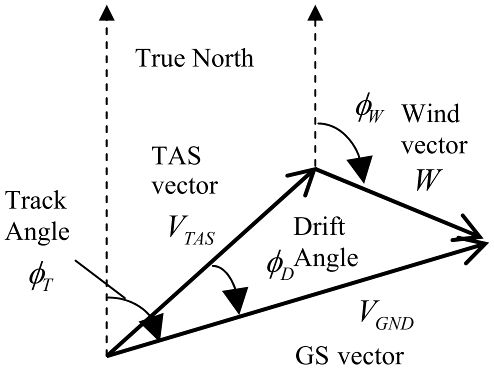

Fig. 3. GS Calculation

The transition altitude is the boundary between CAS-based operation and Mach-based operation. When the altitude is above the transition altitude, the Mach number is used to calculate TAS. On the other hand, when the altitude is below the transition altitude, CAS is used.

The GS is calculated by TAS and the wind vector (Fig. 3). GS V_{GND} is calculated by Eq. (6) [9].

$$V_{GND} = V_{TAS}\cos\phi_D + W\cos(\phi_W - \phi_T) \quad (6)$$

where W is wind speed, ϕ_W is wind direction, ϕ_T is track angle, and ϕ_D is drift angle. ϕ_D is calculated by Eq. (7).

$$\phi_D = \sin^{-1}\left(\frac{W}{V_{TAS}}\sin(\phi_W - \phi_T)\right) \quad (7)$$

TEM is used for movement calculation in the climb and descend phases.

$$(T - D)V_{TAS} = mg\frac{dh}{dt} + mV_{TAS}\frac{dV_{TAS}}{dt} \quad (8)$$

where T is thrust, D is drag, m is aircraft mass, h is altitude, and t is time. The rate of work done by forces (thrust and drag) acting on the aircraft equals the rate of increase in potential and kinetic energy. Potential energy corresponds to altitude, and kinetic energy corresponds to TAS. For example, the descent rate at a constant TAS and idle thrust is calculated by solving Eq. (8) with parameters for thrust, drag, and TAS.

Wind direction, wind speed, and temperature are calculated using weather

Table 1. Weather Forecast Model

Name	GSM (Global Scale)	MSM (Meso Scale) (GPV)
Area	Global Earth	Japan Lat. 22.4 - 47.6 deg, Lon. 120 - 150 deg
Update	6 hour	3 hour
Horizontal Grid	Lat. 0.5 deg, Lon. 0.5 deg	Lat. 0.1 deg Lon. 0.125 deg
Altitude	GSM & MSM common 1,000hPa (364 ft), 925hPa(2,500 ft), 850hPa(4,781 ft), 700hPa(9,882 ft), 600hPa(13,801ft), 500hPa(18,289 ft), 400hPa(23,574ft), 300hPa(30,065 ft), 250hPa(33,999ft), 200hPa(38,662 ft),150hPa(44,647 ft),100hPa(53,083 ft)	MSM only 975hPa(1,061 ft), 950hPa(1,773 ft), 900hPa(3,243 ft), 800hPa(6,394 ft)

forecasts by the JMA (Japan Meteorological Agency). They are defined at a three-dimensional grid point (latitude, longitude, and altitude) in the atmosphere. A numerical forecast model of GSM (Global Scale Model) or MSM (Meso Scale Model) are used. Table 1 summarizes the features of both models delivered by the JMBSC (Japan Meteorological Business Support Center).

MSM is better than GSM when the wind's position and time gradients are large. This is because the grid interval of MSM is small and the forecast update cycle is short. However, MSM doesn't include oceanic regions. MSM is used for domestic flights and GSM is used for international flights.

The grid point data is interpolated into continuous four-dimensional data, using latitude, longitude, altitude and time. Spline interpolation is used. The wind and temperature at arbitrary positions and times along a trajectory is calculated using this method.

3 Comparison of prediction and measurement

3.1 Comparison of weather

Fig. 4 through Fig. 6 show an example of a weather forecast and measurements along an aircraft's trajectory. They include wind speed, wind direction, and temperature. Fig. 7 shows the altitude profile of the aircraft. The horizontal axis is the flight time. The weather forecast data is interpolated from GSM delivered before the aircraft departure time. Forecast updates delivered after departure are good for improving prediction accuracy by reducing forecast error for long-distance flights. In order to avoid complex calculations, only pre-departure forecasts are used here. Measurement data is recorded and calculated by aircraft avionics.

The forecast of wind speed, wind direction, and temperature correspond well to measurements. The wind speed changes from 5 kt (1 kt = 0.514 m/s) to 160 kt according to the aircraft's position. The maximum rate of wind speed change is about 1.4 kt/min around 7 a.m. of flight time. This is about 43 kt in 30 minutes. A flight distance of 30 minutes is about 210 nm (1 nm = 1.852 km). Because wind speed can change to more than 100 kt, wind forecasts are important for the accurate prediction of GS. Analysis of wind change is useful for determining the size of segments accumulated for trajectory prediction.

In the wind direction of Fig. 5, the value goes up and down sharply when the angle crosses 360 degrees.

ISA defines temperature in the stratosphere at a constant -56.5 ℃, but the range of measured temperatures is around 10 ℃. The speed of sound is a function of temperature as shown in Eq. (5). In constant Mach number operation in the cruise phase, TAS changes by about 10 kt according to a temperature change of 10 ℃.

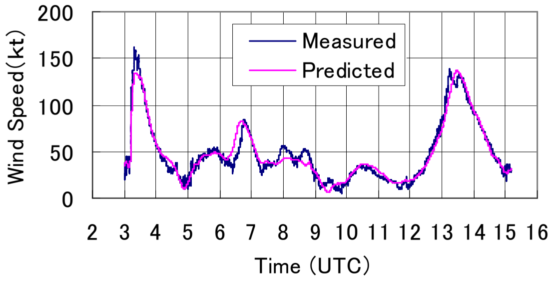

Fig. 4. Wind Speed

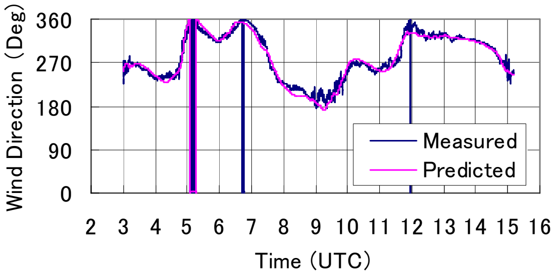

Fig. 5. Wind Direction

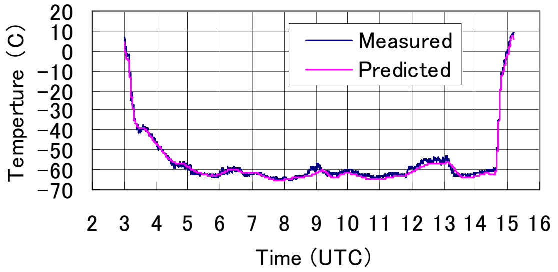

Fig. 6. Temperature

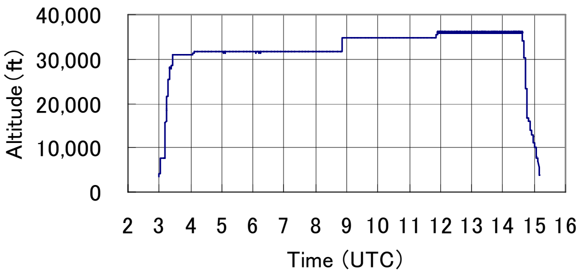

Fig. 7. Altitude

Therefore, forecast temperature, not the ISA, is useful to calculate TAS with high accuracy.

3.2 Comparison of speeds

The predicted speed of the same aircraft is compared with its measured speed. Fig. 8 through Fig. 11 show the prediction and measurements of GS, TAS, Mach number, and CAS. For the calculation of prediction data, the CAS or Mach number is obtained from the speed setting of the BADA airline operation model, and it is converted into TAS by temperature forecast. GS is calculated by adding the wind forecast. Actual altitude is used in calculating the prediction because the main purpose here is speed analysis. Measured speed is recorded by the aircraft. Though the measurement's precision is not known, it is

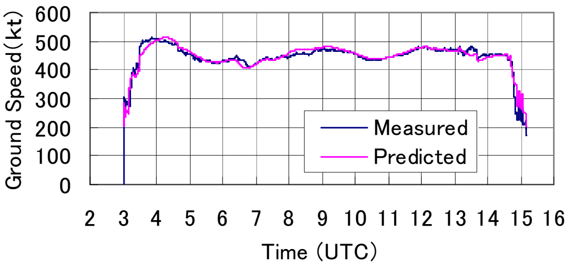

Fig. 8. Ground Speed

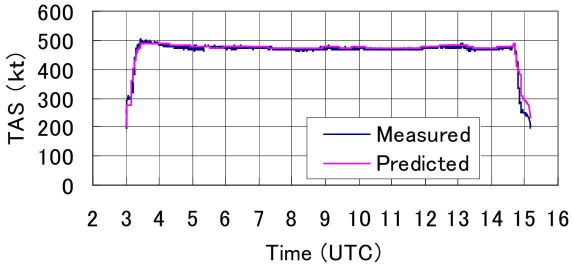

Fig. 9. True Air Speed

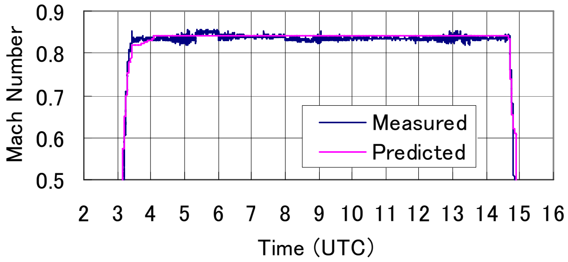

Fig. 10. Mach Number

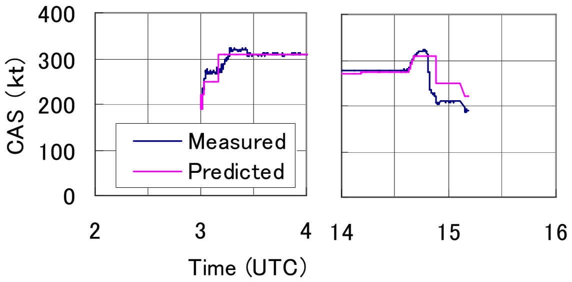

Fig. 11. Calibrated Air Speed

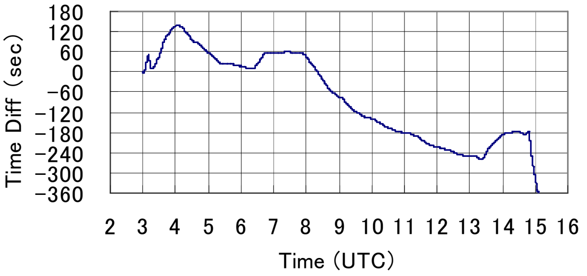

Fig. 12. Time Difference

assumed to be true because it is measured and calculated by the aircraft.

The predicted speed corresponds well to the measured speed. The GS in the cruise phase changes by about 100 kt. TAS is almost constant if the influence of wind is ignored. Aircraft usually fly on a Mach number setting in the cruise phase. The Mach number is almost constant. An aircraft flies on a CAS setting in the climb and descend phases. Predicted CAS is partially different from measurements.

Fig. 12 shows flight time error. The flight route is assumed to be fixed when calculating flight time error. Flight time error is calculated from the difference between predicted and measured GS. The difference in flight time on a small segment en route is calculated using the GS in each segment. The difference is accumulated to calculate flight time error over the entire distance. The rate of flight time error is about one minute for every hour's flight time from 9 to 12 a.m. The predicted average Mach number in the segment is 0.840, and the measured average Mach number is 0.831. The sum of the Mach number differences is the constant rate time error.

4 Analysis of error factors

4.1 Error factors of flight time prediction

As for error factors in flight time prediction, these can be divided into flight distance error and GS error. In the departure phase, there is the flight distance difference between the standard departure routes from different departure runways of an airport. In the descent phase, there is the flight distance difference between the standard arrival routes to different landing runways of a destination airport. Radar control by air traffic controllers and tactical maneuvers cause flight distance differences from the planned route in all phases. Radar control is used for short cuts or flight path extensions. The factors in speed error are divided into differences in aircraft speed model and differences in weather forecasts. These are described in the following section.

4.2 Error factors of ground speed prediction

The error factors of GS in the trajectory prediction model have been analyzed. The total derivative of GS calculation in Eq. (6) is Eq. (9) including wind speed, wind direction, TAS, and track angle.

$$\begin{aligned} dV_{GND} &= \frac{\partial V_{GND}}{\partial W} dW + \frac{\partial V_{GND}}{\partial \phi_W} d\phi_W \\ &\quad + \frac{\partial V_{GND}}{\partial V_{TAS}} dV_{TAS} + \frac{\partial V_{GND}}{\partial \phi_T} d\phi_T \end{aligned} \tag{9}$$

In Eq. (9), dV_{GND} means GS error, and for example, dW means wind speed error. $\partial V_{GND} / \partial W$ is the partial derivative coefficient of GS to wind speed. This represents the degree of influence of wind speed error on GS error. The partial derivative coefficient changes depending on the relationship between wind direction and track angle. This is large when the wind direction and track angle are parallel. On the other hand, it is small when the wind direction and track angle are perpendicular.

As TAS is expressed by temperature and CAS, Eq. (9) becomes Eq. (10). Moreover, it becomes Eq. (11) when TAS is expressed by temperature and Mach number.

$$\begin{aligned} dV_{GND} &= \frac{\partial V_{GND}}{\partial W} dW + \frac{\partial V_{GND}}{\partial \phi_W} d\phi_W \\ &\quad + \frac{\partial V_{GND}}{\partial T} dT + \frac{\partial V_{GND}}{\partial V_{CAS}} dV_{CAS} + \frac{\partial V_{GND}}{\partial \phi_T} d\phi_T \end{aligned} \tag{10}$$

$$\begin{aligned} dV_{GND} &= \frac{\partial V_{GND}}{\partial W} dW + \frac{\partial V_{GND}}{\partial \phi_W} d\phi_W \\ &\quad + \frac{\partial V_{GND}}{\partial T} dT + \frac{\partial V_{GND}}{\partial M} dM + \frac{\partial V_{GND}}{\partial \phi_T} d\phi_T \end{aligned} \tag{11}$$

In these equations, the main elements in the error factors are divided into wind speed, wind direction, temperature, track angle, and CAS or Mach number. The former three elements are prediction errors related to weather forecasts, and the latter two elements are prediction errors related to the aircraft speed model.

4.3 Influence of error factors on ground speed

Table 2 shows the results of the numeric calculation of error factors, comparing prediction and measurement in four sample aircraft. The prediction method is the same as

Table 2. Comparison of Error Factors

Sample Number	R2	R3	R20	R100
Wind Speed Ave.(kt)	0.32	1.28	1.48	-3.56
Wind Speed Dev.(kt)	5.21	4.29	7.77	8.20
Wind Direction Ave.(kt)	-0.05	0.93	-1.19	1.05
Wind Direction Dev. (kt)	4.89	4.63	4.06	5.02
Temperature Ave. (kt)	-0.60	0.03	-0.03	-0.07
Temperature Dev. (kt)	1.28	0.53	0.17	0.21
TAS Ave. (kt)	-4.38	-6.99	-3.52	1.43
TAS Dev. (kt)	10.12	9.35	6.84	17.26
Mach Ave. (kt)	-2.41	-4.32	-1.81	0.58
Mach Dev. (kt)	10.41	9.27	7.00	17.06
CAS Ave. (kt)	9.20	-2.90	-8.11	-11.75
CAS Dev. (kt)	14.16	9.67	8.18	16.49
Total Ave. (kt)	1.51	4.59	2.30	1.19
Total Dev.(kt)	14.25	12.60	11.21	19.37

Chapter 3. Measurement data is recorded by aircraft, and assumed to be true.

The average and standard deviation in Table 2 are mean values and the standard deviation of the product of the partial derivative coefficient and the total derivative of each error factor in whole trajectories. The partial derivative coefficient changes according to the aircraft's position. The numerical partial derivative is calculated in Eq. (2) through Eq. (11). Numerical partial derivatives are expressed as an explicit function. Each parameter is slightly changed, and the ratio of GS change is calculated. The wind speed average of Sample R2 is 0.32 kt, which means that error in wind speed influence on GS is 0.32 kt as average. The average represents the degree of influence for the whole trajectory, and the standard deviation represents the degree of influence in small intervals.

When the numerical values of each error factor are compared, speed model error is larger than weather forecast error in most samples. In Sample R100, the absolute wind speed average is large because there is a partial difference between the forecast and measured wind speed. Concerning the weather forecast, the influence of wind speed and wind direction is large, and the influence of temperature is small.

Table 2 shows a typical sample. More than 100 samples were analyzed by the same method. Large prediction error can be seen in some samples. Analysis of large GS error shows a large Mach number and CAS difference, or, partial large weather forecast difference. In most cases, the former is the main factor.

Analysis results estimate that the accurate prediction for an initial trajectory management of 30 seconds en route is feasible, if the speed intention of the aircraft is reflected in trajectory prediction.

The aircraft speed setting is decided in consideration of various conditions, such as aircraft weight, fuel costs, weather conditions, and delay. The speed is influenced by the cost index (CI) of FMS [12]. Analyzing the general speed setting is useful, and this is reflected in the airline operation data of the prediction model. ICAO recommend that speed changes of more than 5 % TAS from that given in the flight plan shall be reported to air traffic services unit. In the future, it will be useful to acquire the speed intention of every flight for trajectory prediction with SWIM (System Wide Information Management). Mach number and CAS information can be acquired by the data communication of the DAPs (Downlink Aircraft Parameters) function of SSR (Secondary Surveillance Radar) mode S. It is useful to take into account such surveillance information for trajectory prediction.

With regard to weather forecast, monitoring wind and temperature error is important. If error increases in some areas, trajectory prediction accuracy decreases in those areas. Currently, some aircraft have a wind information downlink function. Because of increasing weather forecast error detected by comparing forecast and measured wind, measures have been taken such as weather forecast update delivery or error margin expansion of trajectory prediction.

5 Conclusion

This paper has provided an outline and evaluation of the trajectory prediction model. Error analysis of trajectory prediction compared prediction data calculated using the prediction model with measurement data measured by aircraft. It showed aircraft speed and weather forecasts, such as wind direction, wind speed, and temperature.

Error factors due to the aircraft speed model and error factors due to weather forecasts were analyzed for GS prediction. As a result, the Mach number and CAS difference between the operational model and measurement data were large in most cases where GS difference was large. Aircraft operational model error was larger than weather forecast error.

The detailed analysis is being considered for future study when weather forecast error is large, as well as interpolation method of weather forecast in space and time. Analysis of the climb and descent phases by means of monitoring operational speed data will be the next subject.

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