ON THE TRANSMISSION OF SOUND ACROSS A NON ISOTHERMAL BOUNDARY LAYER

L. M. B. C. Campos*, M. H. Kobayashi**
***Instituto Superior Técnico, Departamento de Engenharia Mecânica/SMA, Av. Rovisco Pais, 1, 1049–001, Lisbon, Portugal, **University of Hawaii at Manoa, Department of Mechanical Engineering, 2540 Dole Street, Honolulu, HI 96822, USA**

Keywords: *Aeroaccoustics, non-isothermal boundary-layer.*

Abstract

The transmission of sound from a source outside an non-isothermal high-speed boundary layer is considered. The sound source is assumed to lie in a uniform stream, matched to a zero velocity at the wall by a linear velocity profile. The unidirectional shear mean flow is assumed to be isentropic, but non-homentropic, so that the entropy, sound speed and temperature can vary from one streamline to the other. The condition of homenergetic flow or constant enthalpy is used to relate the sound speed to the mean flow velocity, and specify the temperature profile in the boundary layer. Compared to an homentropic boundary layer, where sound refraction is due to the shear flow alone, a non-homentropic boundary layer introduces additional refraction due to the non-uniform sound speed and associated temperature gradients. It is shown that for a high-speed, even in isentropic conditions, the non-homentropic effects of temperature gradients and non-uniform sound speed can cause significant sound attenuation, viz. for the same sound source outside the boundary layer, the acoustic pressure at the wall can be substantially reduced. This agrees qualitatively with the result of testing of propfans at high-subsonic speed, which showed significant sound attenuation in the fuselage boundary layer.

1 Introduction

The propagation of sound in shear flows is specified by a wave equation ([27], [39], [50], [48], [44], [13], [12]) which has been studied mostly by numerical and approximate analytical methods, with three motivations in mind: (i) propagation in ducts containing a shear flow, such as jet engine ducts ([56], [45], [42], [28], [53], [54], [19], [20], [36], [55], [41], [33]); (ii) effect of boundary layers, on sound near a wall, such as fuselage or cabin of an aircraft ([1], [22], [23], [47], [26]); (iii) effect of laminar shear layers on sound transmission, e.g. shear layers of a jet exhaust or wake of a control or high-lift device ([43], [25], [2], [3], [38]). The model of a shear layer as a laminar shear flow of finite width ([34]) is intermediate between a vortex sheet ([43], [46]) as a discontinuity of tangential velocity, and a irregular shear layer ([31], [5], [6], [7], [8]), which may entrain turbulence ([40], [51], [30], [29], [9], [10], [11]).

In the present paper sound propagation in laminar shear flow is considered. The simplest velocity profile is the linear shear, which may be matched to uniform streams to represent (i) a boundary layer near a flat wall, (ii) a double boundary layer in a parallel-sided duct or (iii) a shear layer between stream of different velocities. The effect of the uniform flow reduces to a Doppler effect, whereas the linear shear has a critical layer, which has been considered in the literature, sometimes implicitly, by four methods of solution: (a) in terms of parabolic cylinder functions ([24]), (b) in terms of Whittaker functions ([34], [35]), (c) in terms of confluent hypergeometric functions ([52], [37], [38]) and (d) as a linear combination of Frobenius-Fuchs series which are even and odd relative to the critical layer ([17]). These four methods address the acoustic wave equation in linear shear flow, which has two singularities: one regular, at the critical layer and another irregular, at infinity, where the mean flow velocity diverges. In the neighborhood of the regular singularity ([32], [21], [49]), the Frobenius-Fuchs method supplies a pair of linearly independent solutions in power series (in the case of a linear shear flow there is no logarithmic singularity at the critical layer). In the neighborhood of the irregular singularity ([32]), the Frobenius-Fuchs method breaks down, i.e. provides no solution at all or at most one solution; in this case the method of normal integrals or infinite determinants may be used. They are not needed for the linear shear flow, since the wave equation has only two singularities and thus the expansion about the regular singularity (i.e. the critical layer), has infinite radius of convergence (up to the irregular singularity at infinity). Note that the critical layer and other singularities of the wave equation determine the form of its solution. The latter also appear for sound propagation on homentropic shear flow with other velocity profiles, e.g. exponential [18] or hyperbolic tangent [14].

All the literature mentioned before, concerning the acoustics of unidirectional shear flow, assumes uniform sound speed; since for an unidirectional shear flow, the mean flow pressure is uniform, it follows from the equation of state, that the mean flow mass density, temperature and entropy are also uniform, i.e. all those results concerns the acoustics or stability of linear, homentropic shear flow. In order to assess the effect of not assuming an homentropic mean flow, in the present paper a linear homenergetic shear flow is considered, for which the stagnation enthalpy, not the entropy, is conserved. In this case the sound speed is related to the mean flow velocity, i.e. is no longer isothermal, i.e. it can support a temperature gradient. This introduces an extra term in the acoustic wave equation; besides, it adds another two singularities, at the critical flow conditions, where the sound speed vanishes. Thus, whereas the acoustic wave equation in a linear shear flow has two singularities in the homentropic case (treated in the literature), in the present homenergetic case it has four singularities: (i) a regular singularity at the critical layer and an irregular singularity at infinity, inherited from the low Mach number (or homentropic) case; (ii) two regular singularities at the critical flow conditions, which occur for high-speed non-homentropic mean flow.

In order to compare the homentropic and the homenergetic models, the sound field due to a time harmonic line source outside the boundary layer for an homogeneous shear flow is compared with the homenergetic case.

2 Homentropic and homenergetic mean shear flows

The wave equation for an unidirecitonal sheared mean flow where the sound speed is allowed to vary in a transverse direction can be written as [13],[12]:

$$\begin{aligned}\frac{d}{dt}\left(\frac{1}{c^2}\frac{d^2p}{dt^2}-\nabla\left(\log\rho_0\right).\nabla p-\nabla^2 p\right)\\ +2U'\frac{\partial^2}{\partial x\partial y}=0\end{aligned}\tag{1}$$

Since the mean flow properties depend only on the transverse coordinate y, i.e. the mean flow is steady and longitudinally uniform, it is convenient to use the a Fourier decomposition in time t and longitudinal coordinate x:

$$p(x,y,t)=\int_{\mathbb{R}^2}e^{i(\omega t-kx)}\mathrm{d}\omega\mathrm{d}k\tag{2}$$

where $P(y;k,\omega)$ denotes the acoustic pressure perturbation spectrum, for a wave of frequency ω and longitudinal wave number k at position y. The dependence of the acoustic pressure on the latter is generally not sinusoidal, i.e. is specified by substituting (2) in (1), viz.:

$$\begin{aligned}&(\omega - kU)P''+\\ &\quad 2[kU' + (\omega - kU)c'/c]P'+\\ &\quad (\omega - kU)[(\omega - kU)^2/c^2 - k^2]P = 0.\end{aligned} \tag{3}$$

Most of the literature on the acoustics of linear shear flows ([39], [24], [25], [34], [35], [52], [37], [38], [17]) assumes an homentropic flow, i.e. constant entropy; in this case the sound speed is constant, and the wave equation (3) reduces to the well-known form ([27], [50], [44])

$$\begin{aligned}&(\omega - kU)P'' + 2kU'P'+\\ &\quad (\omega - kU)[(\omega - kU)^2/c^2 - k^2]P = 0,\end{aligned} \tag{4}$$

which is the one considered in all of the references above. The derivation of (3) applies equally well to isentropic, non-hometropic mean flow [13], [12], [15], [16], in which case the wave equation (4) holds only at low Mach number, when the sound speed is constant. In the present paper neither the restriction to homentropic mean flow nor the restriction to low Mach number mean flow is made, so the equation (3) does not reduce to (4), i.e. the mean flow temperature is not assumed to be uniform. Thus the acoustic wave equation (3) describes the propagation of sound in a non-isothermal unidirectional shear flow, if the isentropic condition is retained, but the homentropic condition is not imposed. A temperature profile, which is consistent with isentropic, non-homentropic mean flow, i.e. allows ρ_0, T, c to vary from one streamline to the next (i.e. as function of y), is the condition ([4]) of homenergetic mean flow, i.e. constant stagnation enthalpy; this relates the sound speed $c(y)$ and mean flow velocity $U(y)$ at arbitrary streamline to the stagnation sound speed c_0 by

$$[c(y)]^2 = c_0^2 - \varepsilon^2[U(y)]^2$$

where $\varepsilon = \sqrt{(\gamma-1)/2}$. Thus the acoustic wave equation in a high-Mach number homenergetic shear flow:

$$\begin{aligned}&(\omega - kU)(c_0^2 - \varepsilon^2U^2)P''+\\ &\quad 2U'[k(c_0^2 - \varepsilon^2U^2) - \varepsilon^2U(\omega - kU)]P'+\\ &\quad (\omega - kU)[(\omega - kU)^2 - k^2(c_0^2 - \varepsilon^2U^2)]P = 0\end{aligned} \tag{5}$$

has the following singularities: (i) a critical layer where the Doppler shifted frequency vanishes:

$$0 = \omega_*(y_c) = \omega - kU(y_c) \therefore U(y_c) = \omega/k,$$

i.e. the mean flow velocity equals the acoustic phase speed calculated form the horizontal wavenumber; (ii) two critical flow points, where the sound speed vanishes:

$$0 = c(y_\pm) \therefore U(y_\pm) = \pm c_0/\varepsilon = \pm\sqrt{2/(\gamma-1)};$$

(iii) the points at infinity $y = \pm\infty$ may also be singularities.

In order to complete the specification of the wave equation (5) a linear shear flow is considered:

$$U(y) = \Omega y, \tag{6}$$

for which the vorticity is constant $\Omega = \mathrm{d}U/\mathrm{d}y =$ const and specifies the position of the critical layer, viz.

$$y_c = \omega/\Omega k,$$

which is generally distinct from the two critical flow points

$$y_\pm = \pm c_0/\varepsilon\Omega.$$

Coincidence would be possible only for $y_c = y_+$ if the phase speed has a precise relation to the stagnation sound speed

$$\frac{\omega}{k} = \frac{c_0}{\varepsilon},$$

for propagation in the positive x-direction $k > 0$; alternatively $y_c = y_-$ for propagation in the negative x-direction $k < 0$. The change of independent variable

$$\zeta := y/y_c = \Omega k y/\omega,$$

places the critical layer at the point unity $\zeta_c = 1$ and transforms the wave equation (5) to:

$$\begin{aligned}&\left(1 - \Lambda^2\zeta^2\right)(1-\zeta)T''+\\ &2\left(1 - \Lambda^2\zeta\right)T'-\\ &\alpha(1-\zeta)\left[1 - \Lambda^2\zeta^2 - \beta(1-\zeta)^2\right]T = 0,\end{aligned} \tag{7}$$

where $T(\zeta;\alpha,\beta,\Lambda) = P(y;k,\omega,c_0)$. Here the three dimensionless parameters

$$\alpha := (\omega/\Omega)^2, \quad \beta := \omega/kc_0 \quad \Lambda := \varepsilon\omega/kc_0$$

denotes, respectively, (i) the square of the ratio of wave frequency ω to mean vorticity Ω, which is smaller for larger shear flow effect; (ii) the square of the ratio of horizontal phase speed $u = \omega/k$ to sound speed c_0, viz. $\beta = u/c_0$, so that $\beta = 1$ for horizontal propagation, $\beta > 1$ for transversely propagating waves $\omega > kc_0$ and $\beta < 1$ for transversely evanescent waves; (iii) $\Lambda = 0$ for low Mach number flow $c = c_0$ or $\gamma = 1$ or $\varepsilon = 0$, so that $\Lambda \neq 0$ is a measure of high speed effects.

The change of independent variable

$$\xi = \frac{\zeta - 1}{1/\Lambda - 1}$$

shifts the regular singularities at the critical layer and critical flow points to:

$$\zeta_c, \zeta_\pm = 1, \pm\Lambda^{-1} \mapsto \xi_c, \xi_+, \xi_- = 0, 1, \frac{\Lambda+1}{\Lambda-1} := F \tag{8}$$

and leads to the differential equation

$$\xi(\xi-1)(\xi-F)R'' - 2[F - \Lambda\xi/(\Lambda-1)]R' - \alpha(1-1/\Lambda)^2\xi[(\xi-1)(\xi-F) + \beta^2\xi^2/\Lambda^2]R = 0, \tag{9}$$

where $R(\xi;\alpha,\beta,\Lambda) := T(\zeta;\alpha,\beta,\Lambda)$. The point at infinity is an irregular singulary of the wave equation.

Since the critical layer corresponds to the regular singularity $\xi = 0$ of the differential equation (9), the solution in its neighborhood can be determined by the Frobenius-Fuchs method:

$$R(\xi) = (A + B\log\xi)R_3(\xi) + B\bar{\bar{R}}_0$$

where A, B are constants of integration and the two particular integrals are: (i) of the first kind:

$$R_3(\xi) = \sum_{n=0}^{\infty} a_n(3)\xi^{n+3},$$

which vanishes at the critical layer and has recurrence formula for the coefficients, $\sigma \in \mathbb{R}$:

$$\begin{aligned} &F(n+\sigma+1)(n+\sigma-2)a_{n+1}(\sigma) = \\ &2\left[\Lambda/(\Lambda-1)\right](n+\sigma)(n+\sigma-2)a_n(\sigma) - \\ &\left[\alpha F(1-1/\Lambda)^2 - (n+\sigma-1)(n+\sigma-2)\right]a_{n-1}(\sigma) - \\ &(\alpha/\Lambda)(1-1/\Lambda)^2 \\ &\quad\left[(1+F)a_{n-2}(\sigma) - (\Lambda-1)^2(1+\beta^2/\Lambda^2)a_{n-3}(\sigma)\right] \end{aligned}$$

$$\bar{\bar{R}}_0 \sum_{n=0}^{\infty} b_n(0)\xi^n,$$

and (ii) of the second kind:

$$b_n(0) = a_n(0) + \lim_{\sigma\to 0} \sigma a_n'(\sigma).$$

3 Line source outside a boundary layer

The linear shear flow assumed before (6) could be unbounded for the homentropic case and is limited by the critical flow points (8) in the homenergetic case. In either case the linear shear flow can be matched to an uniform stream:

$$U(y) = \begin{cases} \Omega y & \text{if } y \le L \\ \Omega L := U_\infty & \text{if } y \ge L \end{cases}$$

where $L = U_\infty/\Omega$ is the boundary layer thickness and U_∞ the free stream velocity. The critical layer occurs in the boundary layer if $y_c < L$ or $\omega < \Omega kL$. The acoustic field inside the boundary layer has been calculated before and the acoustic pressure $P(y;k,\omega)$ and velocity $\sim P'(Y;k,\omega)$ are to be matched across $y = L$ to the acoustic field in the free stream, thus determining the constants of integration A, B in the general solutions. In the free stream the mean flow velocity is constant and the wave equation (3) simplifies to

$$P_\infty'' + K^2 P_\infty = S\delta(y - y_0) \tag{10}$$

where K is the vertical wavenumber in the free stream:

$$K := \sqrt{(\omega - kU_\infty)^2/c_\infty^2 - k^2},$$

and a line source of strength S was placed in the free stream at a distance y_0 from the wall. The forced solution of (10) is the first term of:

$$P_\infty(y;k,\omega) = -iS/4K \exp[iK|y-y_0|] + C_+ \exp(-iKy)$$

and the second term is an upward propagating wave of amplitude C_+, reflected from the boundary layer (because the source lies in the free stream). The source strength is chosen to be $S = i4K$ and C_+ is determined so as to satisfy a rigid wall condition. The dimensionless parameters of the solution in the boundary layer are reconsidered bearing in mind the matching to the uniform stream, viz.:

$$\alpha_\infty := (\omega/\Omega)^2 = (\omega L/U_\infty)^2, \quad \beta_\infty := \omega/Kc_\infty,$$
$$\Lambda_\infty = \varepsilon\omega/Kc_\infty = \varepsilon\beta_\infty.$$

Note the relation between the free stream and stagnation sound speeds:

$$c_0^2 = c_\infty^2 + \varepsilon^2 U_\infty^2 = c_\infty^2(1+\varepsilon^2 M_\infty^2), \ \ M_\infty := U_\infty/c_\infty$$

where M_∞ denotes the free stream Mach number. The distance from the wall is made dimensionless dividing by the boundary layer thickness:

$$z := y/L = \Omega y/U_\infty,$$

Finally, using the above non-dimensional parameters, the vertical wave number can be written as

$$K = k\sqrt{(\beta - M_\infty)^2 - 1},$$

so that it is real, i.e. waves propagate in the free stream iff $M_\infty - 1 \leq \beta_\infty \leq 1 + M_\infty$.

4 Results and Discussion

The first set of plots (Figures 1 to 4) concern sound propagation in an homenergetic shear flow. In the case (Figure 1) of a high subsonic free stream propagation in the free stream corresponds to $\beta_\infty < -0.3$ or $\beta_\infty > 1.7$. For a wave frequency equal to the vorticity $\beta_\infty = \alpha_\infty = 1$, the amplitude (Figure 1a) is almost uniform in the case of downstream propagation $\beta_\infty = 2$, and decays rapidly away from the wall in the case of upstream propagation $\beta_\infty = 4$; in the cases of evanescence in the free stream the amplitude increases slowly away from the wall for upstream $\beta_\infty = -0.1$, and oscillates for downstream propagation $\beta_\infty = 0.5$. The phase (Figure 1b) varies little in all cases, and is largest for upstream propagation $\beta_\infty = 4$ and lowest for upstream propagation $\beta_\infty = 2$, with smaller values in modulus in the evanescent cases $\beta_\infty = -0.1, 0.5$.

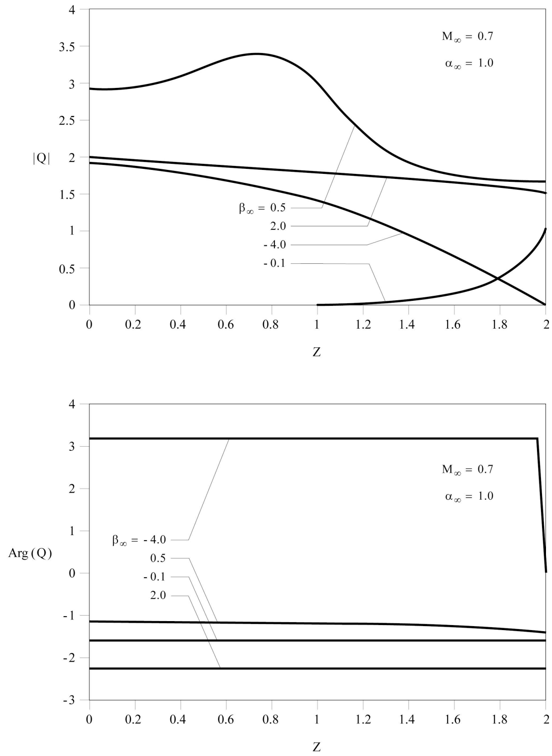

Fig. 1 Amplitude (a) and (b) phase of acoustic pressure versus distance from a rigid wall, made dimensionless dividing by the boundary layer thickness, for an homonergetic flow with linear velocity profile, matched to an uniform stream of high subsonic Mach number . Sound from a line source at a distance from the wall equal to the double of boundary layer thickness. Wave frequency equal to vorticity $\alpha_\infty = 1$, and four values of ratio of horizontal phase speed to sound speed in free stream, including positive $\beta_\infty > 0$ and negative $\beta_\infty < 0$ horizontal wavenumber k corresponding respectively to downstream and upstream propagation.

For a sonic free stream $M_\infty = 1$, the propagation range in the free stream is $\beta_\infty < 0$, $\beta_\infty > 2$. For a wave frequency much higher than the vorticity $\alpha_\infty = 10$, corresponding (Figure 2) to ray theory, there is small change in amplitude (Figure 2a) or phase (Figure 2b) for downstream evanescence $\beta_\infty = 1$; the amplitude oscillations are more marked for propagation downstream $\beta_\infty = 4$ and upstream $\beta_\infty = -0.5$, with phase jumps of π at the nodes.

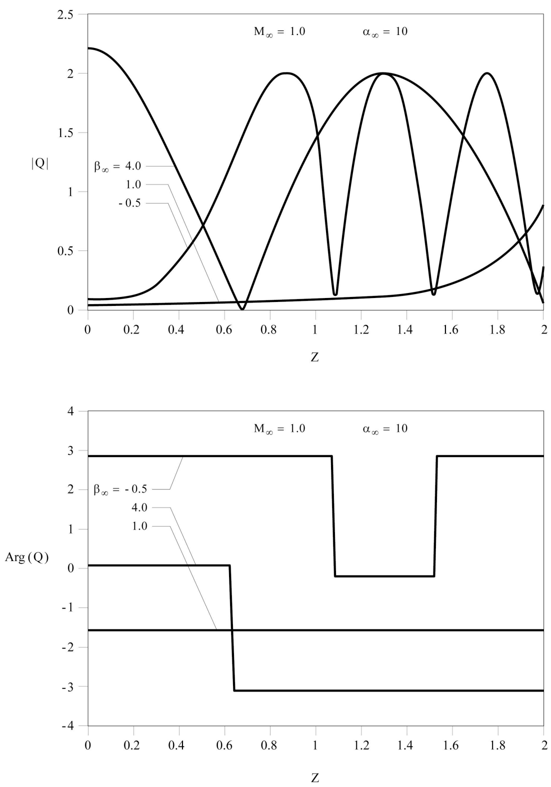

Fig. 2 As Figure 1, for sonic free stream $\beta_\infty = 1$, wave frequency much higher than vorticity $\alpha_\infty = 10$, and three values of β_∞.

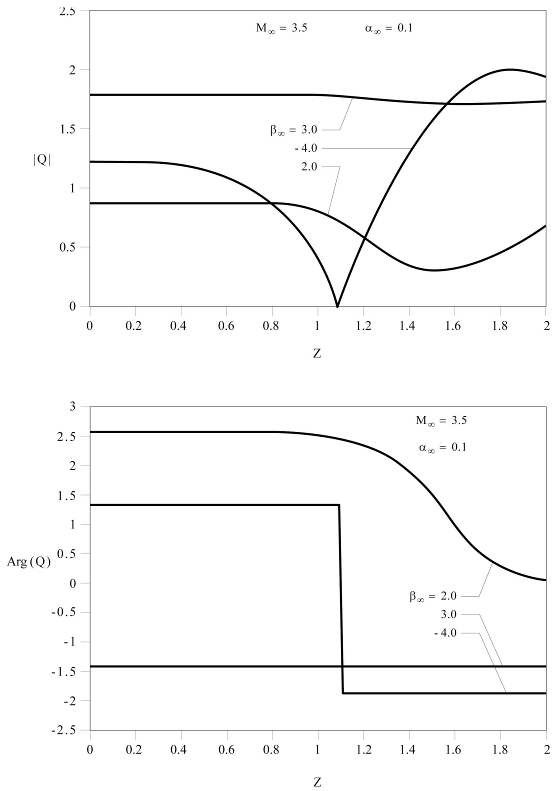

Fig. 3 As Figure 1, for supersonic free stream $M_\infty = 3.5$, wave frequency much smaller than vorticity $\alpha_\infty = 0.1$, and three values of β_∞.

For a supersonic free stream $\beta_\infty = 3.5$ the propagating range is $\beta_\infty < 2.5$ or $\beta_\infty > 4.5$. For (Figure 3) a frequency small compared with the vorticity $\alpha_\infty = 0.1$, the amplitude (Figure 3a) and phase (Figure 3b) vary little in the case of downstream evanescence $\beta_\infty = 3$. In the other case of downstream evanescence $\beta_\infty = 2$ there is significant amplitude variation and smooth phase changes. In the case of upstream propagation $\beta_\infty = -4$, the amplitude oscillation includes a node in the boundary layer, corresponding to a jump of in the otherwise constant phase.

The last two plots (Figures 4 and 5) concern a comparison of the sound field due to a line source over a rigid wall at a distance of two boundary layer thicknesses, for a boundary layer with a linear velocity profile in homentropic (dotted line) or homenergetic (solid line) conditions. Note that the homentropic boundary layer is isothermal, i.e. has a constant sound speed everywhere; the homenergetic shear flow has a sound speed and temperature which, for a linear velocity profile, decrease away from the wall leading to a negative temperature gradient. The first plot concerns a case of wave frequency equal to the vorticity $\alpha_\infty = 1$, and oblique upstream propagation $\beta_\infty = 4$, for which there is no critical level in the boundary layer (Figure 4), since $\beta_\infty > M_\infty$ implies $z_c > 1$. The amplitude (Figure 4a) is almost identical for the homentropic (S) and homenergetic (E) case at low free stream Mach number $M_\infty = 0.1$, but the difference increases with increasing Mach number $M_\infty = 0.2, 0.7, 1$, leading to very different values of the wall pressure in the supersonic case $M_\infty = 3.5$, when the acoustic pressure at the wall is much larger in the homentropic case. The reduction in sound speed in the free stream in the homenergetic case, implies that sound propagates against a sound speed increasing towards the wall, thus causing reflection and a leading to a smaller amplitude at the wall. The phase (Figure 4b) is larger for the homentropic than for the homenergetic case, with a small difference at low Mach number $M_\infty = 0.1$, and a more noticeable difference for increasing Mach number $M_\infty = 0.2, 0.7, 1$. The constant sound speed in the homentropic case leads to a larger phase shift than the sound speed decreasing into the free stream in the homenergetic case. The exception is the supersonic free stream $M_\infty = 3.5$, for which the phase is the same in the homentropic and homenergetic cases. The reason is that $M_\infty = 3.5$ is the only case in Figure 4 of evanescence $\beta_\infty < M_\infty + 1$ for $\beta_\infty = 4$. Thus for $M_\infty = 3.5$ the evanescent waves in the free stream have the same phase in the homentropic and homenergetic cases. The phases differ in the homentropic and homenergetic cases for propagation in the free stream $M_\infty + 1 < \beta_\infty + 4$ which includes all cases in Figure 4 except $M_\infty = 3.5$.

The final plot (Figure 5) concerns again wave frequency equal to the vorticity $\alpha_\infty = 1$, with the condition $\beta_\infty = M_\infty/2$ which places the critical layer $z_c = 0.5$ at the middle of the boundary layer. The amplitude (Figure 5a) of the sound field is always larger in the homentropic than in the homenergetic case; it is almost uniform in the homentropic case and has a dip in the homenergetic case. The homentropic case corresponding to constant sound speed implies sound reflection due only to the velocity gradient in the shear flow; the reflection is stronger for the homenergetic case because then it is augmented by the gradient in sound speed (or temperature). With increasing Mach number, the amplitude decreases monotonically in the homentropic case, and tends to increase in the homenergetic case. The phase (Figure 5b) differs most between the homentropic and homenergetic case for the largest Mach number, and is more uniform in the former case. The homentropic case of constant sound speed leads to a larger phase than the homenergetic case where

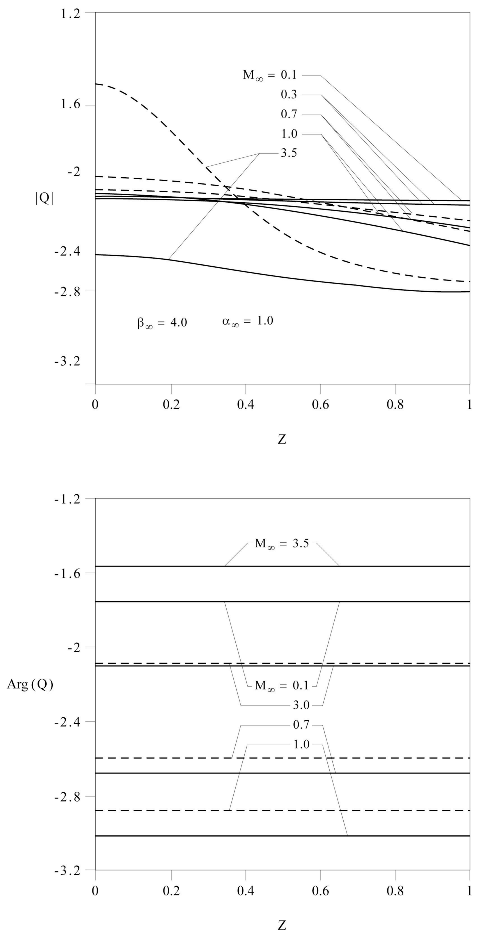

Fig. 4 As Figure 1, comparing the sound fields due to a line source for homenergetic (solid line) and homentropic (dotted line) shear flows, for low speed $M_\infty = 0.1$, incompressible $M_\infty = 0.3$, subsonic $M_\infty = 0.7$, sonic $M_\infty = 1$ and supersonic $M_\infty 3.5$ free streams. Wave frequency equal to vorticity $\alpha_\infty = 1$ and $\beta_\infty = 4$

the sound speed decays away from the wall; the effect is more noticeable for larger free stream Mach number because then the change is sound speed (or temperature gradient) is larger. The near coincidence of the homentropic and homenergetic case at low Mach number results from the sound speed being nearly constant in that case, so that the wave equation (3) simplifies to the usual form (4); as the Mach number increases, the extra terms in (3) compared with (4) play a larger role.

The sound field due to a source in a uniform stream, matched to a linear shear flow, was considered for the homentropic case [17], when the flow is isothermal. The consideration of the same unidirectional shear flow velocity profile in non-homentropic conditions, e.g. for a homenergetic profile, leads to an non-isothermal flow, with variable sound speed. The additional refraction effects are significant if the Mach number is supersonic, and lead to a noticeable reduction of acoustic pressure at the wall, for an homonergetic compared with an isothermal boundary layer. For subsonic Mach numbers the temperature gradients are small and have a small effect. The velocity profile has a significant effect, even at low Mach numbers, in the presence of a critical level.

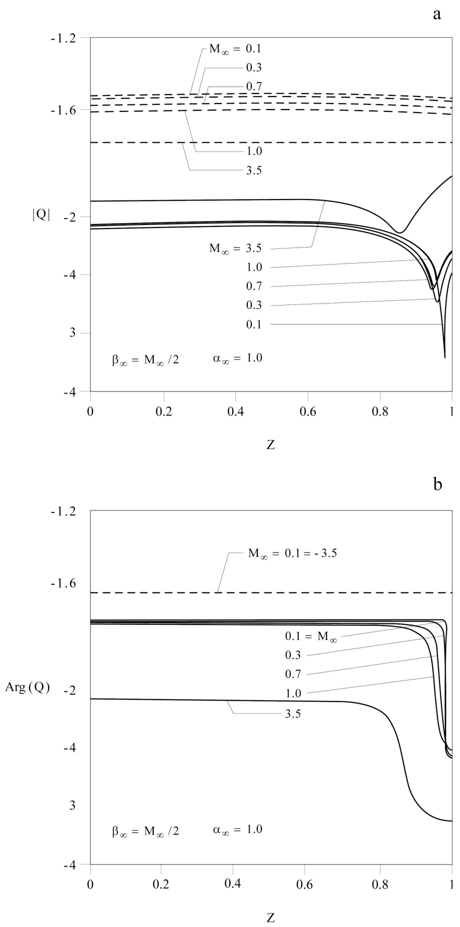

Fig. 5 As Figure 4, with the relation $\beta_\infty = M_\infty/2$ to ensure that there is a critical level in the middle of the boundary layer $y_c = L/2$.

Copyright Statement

References

[1] Almgren G. Acoustic boundary layer influence in scale model simulation of sound propagation. *Journal of Sound and Vibration*, Vol. 105, pp 321—327, 1976.

[2] Balsa T. The far-field of high-frequency convected, singularities in shear flows with application to jet noise prediction. *Journal of Fluid Mechanics*, Vol. 74, pp 193—208, 1976.

[3] Balsa T. Reffraction and shielding of sound from a source in a jet. *Journal of Fluid Mechanics*, Vol. 76, pp 443—456, 1976.

[4] Batchelor G. K. *An Introduction to Fluid Dynamics*. Cambridge University Press, Cambridge, Great Britain, 1967.

[5] Campos L. M. B. C. On the spectral broadening of sound by turbulent shear layers. part I: Scattering by moving interfaces and reffraction in turbulence. *Journal of Fluid Mechanics*, Vol. 89, pp 723—749, 1978.

[6] Campos L. M. B. C. On the spectral broadening of sound by turbulent shear layers. part II: Spectral broadening of experimetal and aircraft noise. *Journal of Fluid Mechanics*, Vol. 89, pp 751—779, 1978.

[7] Campos L. M. B. C. Sur la propagation du sons dans les ecoulements non-uniformes et non-stationaieres. *Reveu d'Acoustique*, Vol. 67, pp 217—233, 1983.

[8] Campos L. M. B. C. On waves in gases. part i: Acoustics of jets, turbulence and ducts. *Review of Modern Physics*, Vol. 58, pp 117—182, 1986.

[9] Campos L. M. B. C. Effects on acoustic fatigue loads of multiple reflection between a plate and a turbulent wake. *Acustica*, Vol. 76, pp 109—117, 1992.

[10] Campos L. M. B. C. On the correlation of

acoustic pressures induced by a turbulent wake on a nearby wall. *Acta Acustica*, Vol. 82, pp 9—17, 1996.

[11] Campos L. M. B. C. On the spectra of aerodynamic noise and aeroacoustic fatigue. *Progress in Aerospace Sciences*, Vol. 33, pp 353—389, 1997.

[12] Campos L. M. B. C. On 24 forms of the acoustic wave equation in vortical flows and dissipative media. *Reviews of Applied Mechanics*, Vol. 60, pp 291—315, 2007.

[13] Campos L. M. B. C. On 36 forms of the acoustic wave equation in potential flows and inhomogeneous media. *Reviews of Applied Mechanics*, Vol. 60, pp 149—171, 2007.

[14] Campos L. M. B. C and Kobayashi M. H. On the reflection and transmission of sound in a thick shear layer. *Journal of Fluid Mechanics*, Vol. 424, pp 303—326, 2000.

[15] Campos L. M. B. C and Kobayashi M. H. On the propagation of sound in a high-speed non-isothermal shear flow. *International Journal of Aeroacoustics*, Vol. 6, pp 93—126, 2009.

[16] Campos L. M. B. C and Kobayashi M. H. Sound transmission from a source outside a non-isothermal boundary layer. *AIAA Journal*, Vol. 48, pp 878—892, 2010.

[17] Campos L. M. B. C, Oliveira J. M. G. S, and Kobayashi M. H. On sound propagation in a linear shear flow. *Journal of Sound and Vibration*, Vol. 219, No 5, pp 739—770, 1999.

[18] Campos L. M. B. C and Serrão P. G. T. Á. On the acoustics of an exponential boundary layer. *Philosophical Transactions of the Royal Society London A*, Vol. 356, pp 2335—2378, 1998.

[19] Eversman W. Effect of boundary layer on the transmission and attenuation of sound in an acoustically treated circular duct. *Journal of the Acoustical Society of America*, Vol. 39, pp 1372—1380, 1971.

[20] Eversman W and Beckenmeyer R. J. Transmission of sound in ducts with thin shear layers: Convergence to the uniform flow case. *Journal of the Acoustical Society of America*, Vol. 52, pp 216—225, 1972.

[21] Forsyth A. R. *Treatise of Differential Equations*. Sixth edition, MacMillan, 1926.

[22] Goldstein M. E. Scattering and distortion of the unsteady motion on transversely sheared mean flows. *Journal of Sound and Vibration*, Vol. 91, pp 601—602, 1979.

[23] Goldstein M. E. High frequency sound emission from moving point miltipole sources embedded in arbitrary transversely sheared mean flows. *Journal of Sound and Vibration*, Vol. 80, No 4, pp 499—522, 1982.

[24] Goldstein M. E and Rice E. Effect of shear on duct wall impedence. *Journal of Sound and Vibration*, Vol. 30, pp 79—84, 1973.

[25] Graham E. W and Graham B. B. Effect of a sheared layer on plane waves in a fluid. *Journal of the Acoustical Society of America*, Vol. 46, pp 169—175, 1969.

[26] Hanson D. Shielding of propfan cabin noise by the fuselage boundary layer. *Journal of Sound and Vibration*, Vol. 92, pp 591—598, 1984.

[27] Haurwitz W. Zur theorie der wellenbewegungen in luft und wasser. *Veröffentlichen des Geophysik Institut von der Universität Leipzig*, Vol. 6, pp 334—364, 1932.

[28] Hersh A. S and Catton I. Effects of shear flow on sound propagation in rectangular ducts. *Journal of the Acoustical Society of America*, Vol. 50, pp 992—1003, 1971.

[29] Ho C. M and Kovasznay L. S. G. Acoustical shadowgraph. *Physics of Fluids*, Vol. 19, pp 1118—1123, 1976.

[30] Ho C. M and Kovasznay L. S. G. Propagation of a coherent acoustic wave through a turbulent shear flow. *Journal of the Acoustical Society of America*, Vol. 60, pp 40—45, 1976.

[31] Howe M. S. The attenuation of sound by a randomly irregular impedance layer. *Proceedings of the Royal Society London A*, Vol. 347, pp 513—535, 1976.

[32] Ince E. L. *Ordinary Differential Equations*. Dover, New York, 1956.

[33] Ishii S and Kakutani T. Acoustic waves in parallel shear flows in a duct. *Journal of Sound and Vibration*, Vol. 113, pp 127—139, 1987.

[34] Jones D. The scattering of sound by a simple shear layer. *Philosophical Transactions of the Royal Society London A*, Vol. 284, pp 287—328, 1977.

[35] Jones D. Acoustics of a splitter plate. *Journal of the Institute of Mathematics and its Applica-

tions, Vol. 21, pp 197—209, 1978.

[36] Ko S. H. Sound attenuation in acoustically lined circular ducts in the presence of uniform flow and shear flow. *Journal of Sound and Vibration*, Vol. 22, pp 193—210, 1972.

[37] Koutsoyannis S. P. Characterization of acoustic disturbance in linearly flows. *Journal of Sound and Vibration*, Vol. 68, pp 187—202, 1980.

[38] Koutsoyannis S. P, Karamcheti K, and Galant D. Acoustic resonances and sound scattering by a shear layer. *AIAA Journal*, Vol. 18, pp 1446—1450, 1980.

[39] Kücheman D. F. G. Störungebewegungen in einer gasströmung mit grenzschicht. *Zeitschrift für Angewandte Mathematik und Mechanik*, Vol. 18, pp 207—222, 1938.

[40] Lighthill M. J. On the energy scattered from interaction of turbulence with sound and shock waves. *Proceedings of the Cambridge Philosophical Society*, Vol. 44, pp 531—548, 1953.

[41] Mani R. Sound propagation in parallel sheared flows in ducts: the mode estimation problem. *Proceedings of the Royal Society London A*, Vol. 371, pp 393—412, 1980.

[42] Mariano S. Effect of wall shear layers on the sound attenuation in acoustically lined rectangular ducts. *Journal of Sound and Vibrations*, Vol. 19, pp 261—275, 1971.

[43] Miles J. M. On the disturbed motion of a vortex sheet. *Journal of Fluid Mechanics*, Vol. 4, pp 538—554, 1958.

[44] Möring W, Müller E, and Obermeier F. Problems in flow acoustics. *Reviews of Modern Physics*, Vol. 55, pp 707—724, 1983.

[45] Mungur P and Plumblee H. E. Propagation and attenuation of sound in a soft-walled annular duct containing a sheared flow. SP 207, NASA, 1969.

[46] Munt R. M. Interaction of sound with a subsonic jet issuing from a semi-infinite jet-pipe. *Journal of Fluid Mechanics*, Vol. 83, pp 609—620, 1977.

[47] Myers M. K and Chuang S. L. Uniform asymptotic approximations for duct acoustic modes in a thin boundary-layer flow. *AIAA Journal*, Vol. 22, pp 1234—1241, 1983.

[48] Nayfeh A, Kaiser J, and Telionis D. Acoustics of aircraft engine-duct systems. *AIAA Journal*, Vol. 13, pp 130—153, 1975.

[49] Poole E. G. C. *Linear Differential Equations*. Dover, 1935.

[50] Pridmore-Brown D. Sound propagation in a fluid flowing through an attenuating duct. *Journal of Fluid Mechanics*, Vol. 4, pp 393—406, 1958.

[51] Schmidt D. W and Tilmann P. M. Experimental study of sound phase fluctuations caused by turbulent wakes. *Journal of the Acoustical Society of America*, Vol. 47, pp 1310—1324, 1970.

[52] Scott J. N. Propagation of sound waves through a linear shear layer. *AIAA Journal*, Vol. 17, pp 237—245, 1979.

[53] Shankar P. N. Acoustic reffraction by duct shear layers. *Journal of Fluid Mechanics*, Vol. 47, pp 81—91, 1971.

[54] Shankar P. N. Acoustic reffraction and attenuation in cylindrical and annular ducts. *Journal of Sound and Vibration*, Vol. 22, pp 233—296, 1972.

[55] Swinbanks M. A. The sound field generated by a source distribution in a long duct carrying sheared flow. *Journal of Sound and Vibration*, Vol. 40, pp 51—76, 1975.

[56] Tack D. H and Lambert R. F. Influence of shear flow on sound attenuation in lined ducts. *Journal of the Acoustical Society of America*, Vol. 38, pp 655—666, 1965.