COMBINED SCALAR AND VECTOR VELOCITY POTENTIAL FOR UNSTEADY AERODYNAMICS IN ACOUSTO-AEROELASTICITY

Harijono Djojodihardjo
Universitas Al-Azhar Indonesia and Universiti Putra Malaysia
harijono@djojodihardjo.com

Keywords: *unsteady aerodynamics, acousto-aeroelasticity, fluid structure interaction; boundaryelement method*

Abstract

Following a series of development in BE/FE Method and Modeling of structural-acoustic interaction, effort is focused on the use of combined scalar and vector potential approach on the unsteady aerodynamic part of the integrated computational scheme for acousto-aeroelastic problem. The theory for linearized unsteady aerodynamics computation for compressible flow is here developed by integrating the wake vortex particle influence in the general compressible potential flow unsteady aerodynamic solution.

1 Introduction

A series of work has been carried out by the author and colleagues [1][2] in the coupled BE/FE Method and Modeling of structural-acoustic interactions. The work carried out thus far has been focused on the formulation of the basic problem of acoustic excitation and vibration of elastic structure in a coupled fluid-elastic-structure interaction. The approach consists of three components. The first is the formulation of the acoustic field governed by the Helmholtz equation subject to the Sommerfeld radiation condition for the basic acoustic problem without solid boundaries (Dowling & Ffowcs-Williams[3], Wröbel[4], Norton[5]). The interface between the acoustic domain and the surface of the structure poses a particular boundary condition. Boundary element method has been utilized for solving the governing Helmholtz equation subject to the boundary conditions for the calculation of the acoustic pressure on the interface boundary. The second component deals with the structural dynamic problem, which is formulated using finite element approach. The third component involves the calculation of unsteady aerodynamic loading on the structure following one of the well established lifting surface methods, such as DLM or DPM methods. All these three components are then integrated into an acoustic-aerodynamic-structure coupling, which is formulated using coupled BEM/FEM techniques. The acoustic loading on the structure is calculated on the component of the boundary of the acoustic domain, which coincides with the structural surface as defined by the problem. The integrated procedure has been developed in an in-house MATLAB based computational program, and has shown promising results. As an example, the scheme described in [2] allows the direct or hydrodynamic part of the acoustic influence on the aeroelastic stability problem investigated which produced results in qualitative agreement with well established analytical and experimental results of Huang [6].

More recent results of Lu and Huang [7] and Nagai et al [8] have pointed out that hydraulic analogy alone is not sufficient to exhibit the mechanism of the acoustic-aeroelastic interaction. The conversion and amplification of the incident acoustic wave into shedding vortices from the trailing edge, known as the *trailing-edge receptivity,* has been identified as having more significant effects to the acoustic-aeroelastic interaction. The strength of the vortices shed from the trailing edge in response to the incident acoustic waves is a signature representing the effectiveness of the acoustic excitation. With such motivation, the present work is devoted to carefully look and account for the wake vortices in formulating the additional influence of the acoustic disturbance to the aerodynamic field associated with the boundary conditions at the trailing edge.

In this conjunction, techniques that capitalize on wake vortices (Winckelmans and Leonard [9], Opoku, Triatos, Nitzsche and Voutsinas [10], Leroy, Buron, Devinant [11], and Eldredge, Colonius and Leonard [12], Willis, Peraire and White [13] and Willis [14] will be reviewed. In particular, the present work will in part capitalize on the utilization of vector velocity potential to account for vortices associated with wake evolution introduced by Willis et al [13][14]. For this purpose, the flow of interest can be assumed to be a potential one. The evolution of the lifting surface trailing vorticity is modeled using an unsteady time advancing vortex particle approach, as utilized in earlier work by Djojodihardjo & Widnall [15]. The novel combined potential flow-vortex particle approach allows an automatic generation and evolution of the domain vorticity. However, since in the present work interest is focused on "steady" oscillatory flow, much of the development here will also follow the theoretical foundation laid out by Morino colleagues [16]-[18]. The BEM potential flow solver has been developed similar to that developed by Holström [19]. The domain whose velocity influence is considered is derived from both potential flow solution as well as a distribution of vorticity.

2 Formulation of the Problem

2.1. The Governing Fluid-Dynamic Equation and the Boundary Integral Solution

The present development focuses on higher Reynolds number flows prevailing in most aerodynamic applications, and hence can follow the general theory of the unsteady compressible potential flow around lifting bodies having arbitrary configurations that was developed by Morino ([16]-[17], among others). Without losing generalities, the present work considers only the subsonic flow . Applying the Green function method for the governing gas-dynamics equation of potential flow which assumes the form of wave equation yields the Huygens' Principle (or Kirchhoff s formula), which is the Green theorem for the wave equation. Looking into linearized flow situation, in which the velocity potential Φ can be represented by the free stream velocity potential with free-stream velocity of U_∞ oriented in the positive x-direction in a Cartesian coordinate system traveling with the body. It is convenient to consider the surface S_{B+W} surrounding the body and the wake to be moving with respect to the frame of reference. Then, the perturbation potential ϕ for a flow having free-stream velocity U_∞, in the direction of the positive x-axis is given by[20]:

$$\nabla^2\phi-\left(\frac{1}{a_\infty}\right)^2\left[\frac{\partial}{\partial t}+\mathbf{U}_\infty\frac{\partial}{\partial x}\right]^2\phi=F \tag{1}$$

where F is the contribution of non-linear terms. The contribution of the nonlinear terms term will be neglected in the present work, and Eq. (1) reduces to

$$\nabla^2\phi-\left(\frac{1}{a_\infty}\right)^2\left[\frac{\partial}{\partial t}+\mathbf{U}_\infty\frac{\partial}{\partial x}\right]^2\phi=0 \tag{2}$$

Let the surface of the lifting body B be described by:

$$S(\mathbf{x},t)=S(x,y,z,t)=0 \tag{3}$$

where $\mathbf{x}=(x,y,z)$ (4)

Then the boundary conditions on the body are given by $\dfrac{DS}{Dt}=0$ or

$$\frac{\partial\phi}{\partial\mathbf{n}}=-\frac{1}{|\nabla S|}\left(\frac{\partial S}{\partial t}+U_\infty\mathbf{i}\bullet\nabla S\right) \tag{5a}$$

Further simplification is made by considering the surface of the lifting body to be time-independent, i.e. expressible as $S(\mathbf{x},t)=S(x,y,z)=0$ The term $\partial S/\partial t$ is retained only in the boundary conditions. Then the boundary condition reduces further to

$$\frac{\partial\phi}{\partial n}=-\mathbf{U}_\infty\bullet\mathbf{n} \tag{5b}$$

on the surface of the lifting body.

To solve the problem it is convenient to transform it into an integral equation using Green function method. It can be shown that It can be shown that the integral equation for the pertrurbation potential ϕ can be written as[1]:

$$\phi(\mathbf{x})=\frac{1}{4\pi}\iint_{S'_{b+w}}\phi\frac{\partial}{\partial n}\left(\frac{1}{\|\mathbf{x}-\mathbf{x}'\|}\right)dS'_{b+w}-\frac{1}{4\pi}\iint_{S'_b}\frac{\partial\phi}{\partial n}\frac{1}{\|\mathbf{x}-\mathbf{x}'\|}dS'_b \quad (6)$$

In the subsequent development, it is convenient to utilize generalized Prandtl-Glauert transformation given by:

$$X=\frac{x}{L\beta};\ Y=\frac{y}{L\beta};\ Z=\frac{z}{L\beta}\quad,\ T=\frac{a_\infty\beta t}{L} \quad (7)$$

where $\beta=\sqrt{1-M_\infty{}^2}$ (8)

Since the present interest, consistent with earlier studies [1][2], is on harmonic oscillations (with frequency ω) about a fixed configuration, further simplification is obtained by introducing the complex potential

$$\phi(\mathbf{X},t)=\tilde{\phi}(\mathbf{X},t)e^{i\Omega(T+MX)} \quad (9)$$

And Eq. (6) can be written as:

$$\tilde{\phi}(\mathbf{X},t)=-\frac{1}{4\pi E}\iint_{\Sigma}\frac{\partial\tilde{\phi}}{\partial n}\frac{e^{i\Omega R}}{|\mathbf{X}_1-\mathbf{X}|}d\Sigma+\frac{1}{4\pi E}\iint_{\Sigma}\tilde{\phi}(\mathbf{X},t)\left(\frac{e^{-i\Omega R}}{|\mathbf{X}_1-\mathbf{X}|}\right)d\Sigma \quad (10)$$

where

$$R=\left[(X_1-X)^2+(Y_1-Y)^2+(Z_1-Z)^2\right]^{\frac{1}{2}}$$

$k\equiv\dfrac{\omega L}{U_\infty}$, reduced frequency and (11)

$\Omega\equiv\dfrac{\omega L}{\beta a_\infty}=\dfrac{kM}{\beta}$ compressible reduced frequency

Eq. (10) is a Fredholm integral equation of second kind, whose solution exists and is unique (for the steady state case, solution of the external Neumann's problem for the Laplace equation). Note, however, that the branch of the surface surrounding the wake, is not known.

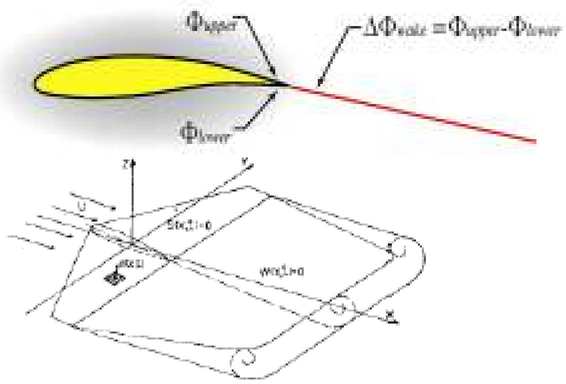

Fig 1 Lifting Body and Wake Configuration

In many Aerodynamic Lifting Surface Methods, the surface of the wake has been assumed to extend from the trailing edge to infinity, and the boundary condition at the wake is governed by the Kutta condition of the wake smooth flow off at the trailing edge and no pressure discountinuity across the wake surface.

2.2. The Evolution of the Wake Surface Trailing Vorticity and its Modeling for Oscillatory Flow

The Boundary Integral Equation (10) has been developed as the solution of the Fluid-Dynamic Equation (2) based on potential flow assumption, which is plausible since interest is focused on higher Reynolds number flows appearing in most practical aerodynamic applications. The presence of viscosity, which in the linearized potential flow situation is confined on very thin layer on the surface of the lifting body and at the trailing wake surface, is accounted for by the use of Kutta condition at the trailing edge, which allows unique solution for the governing equations.[2] For many other applications, in particular when there are strong interaction between the wake and the lifting surfaces (such as propeller and helicopter rotor), the influence of the wake has to be more carefully modeled, such as recently demonstrated by Winckelmans & Leonard [9], Opoku et al [10], Leroy et al [11], Willis [13][14], and Scott & Atassi [21], to mention just a few examples. Therefore, in the present

[1] The detail of such derivation can be obtained in Morino[17]

[2] Without losing generalities, this notion can be readily demonstrated in two-dimensional situation.

work, the influence of the wake vortices as these evolved due to the fluid motion will be accounted for in addition to the classical potential flow solution. For the present purpose, the approach of Willis [13]14] will be adopted and adapted.

In this conjunction, the velocity of the fluid in the domain external to the lifting body will be considered to be contributed by the irrotational (hence, conventional) velocity potential Φ and the rotational velocity potential Ψ. In the region of rotational flow, which encroaches the wake vortices and its dissipative propagation in the entire fluid domain, the solenoidal vector potential component $\mathbf{u}_\psi$ of velocity of the fluid can then be defined as

$$\mathbf{u}_\psi(\mathbf{x},t) = \nabla \times \boldsymbol{\Psi} \tag{12}$$

Then the fluid velocity at a given point in the domain can be expressed as the superposition of the scalar potential component, $\mathbf{U}_\phi$, and a solenoidal vector potential component, $\mathbf{U}_\psi$; hence:

$$\mathbf{U}(\mathbf{x},t) = \mathbf{u}_\Phi(\mathbf{x},t) + \mathbf{u}_\psi(\mathbf{x},t) = \nabla\Phi + \nabla \times \boldsymbol{\Psi} \tag{13}$$

The perturbation scalar velocity potential, which is typically the sought variable in Lifting Surface methods, can be obtained from Eq.(10). In general, the flow does not necessarily satisfy the irrotationality condition, and the assumption that it could be distinguished into irrotational and irrotational part, hence equation (13), should be valid everywhere in the domain.

The boundary condition (5a) will now reads:

$$\frac{\partial \phi}{\partial n} = -\left[\mathbf{U}_\infty + \mathbf{u}_\psi\right] \bullet \mathbf{n} \tag{5b}$$

on the surface of the lifting body. This boundary condition is acceptable on the surface of the body since the boundary layer on the surface of the body is assumed to be very thin and no flow separation is assumed commensurate with smooth flow off the trailing edge.

On the body fixed reference frame, the radiation condition for Eq. (2) is that the velocity in the far-field is the free-stream velocity, which implies that

$$\lim_{\mathbf{x}\leftrightarrow\infty} = \mathbf{U}_\infty(\mathbf{x},t) = \nabla\Phi\big|_{\mathbf{x}\leftrightarrow\infty} \tag{14a}$$

or

$$\mathbf{u} = \nabla\phi\big|_{\mathbf{x}\leftrightarrow\infty} = 0 \tag{14b}$$

2.3. Dynamical Condition Governing the Wake

From momentum consideration, the force **F** acting on the lifting configuration consisting of the lifting body and the wake is given by [15]:

$$\mathbf{F} = \frac{D}{Dt}\left[\iint_{S_B} \phi\mathbf{n}dS + \iint_{S_W} \Delta\phi_W \mathbf{n}dS\right] \tag{15}$$

Since the wake cannot sustain any pressure difference across it, it follows that

$$\frac{D}{Dt}\left[\iint_{S_W} \Delta\phi_W \mathbf{n}dS\right] = 0$$

which can be further reduced to

$$\frac{D}{Dt}\left[\Delta\phi_W\right] = 0 \tag{16}$$

This equation implies that once a fluid particle in the wake is imparted a doublet strength $\Delta\phi_W$, or equivalently a wake element has been imparted with shed vortex off the trailing edge with strength γ, this value remains constant as it is convected downstream.[3] This condition is equivalent to Kelvin's theorem.

2.3. Kelvin' Theorem and Kutta condition

By Kelvin's Theorem of conservation of vorticity, the vorticity generated on the lifting surface must be shed into the domain (fluid domain external to the solid boundary surface up to infinity). Since the flow is assumed to be inviscid external to the lifting body including the thin boundary layer surrounding it and the thin wake, Kutta condition applies at the trailing edge of the wing. The condition allows for a jump discontinuity in the surface potential across the geometric cusp representing the trailing edge in the infinitesimal dimension. Following Djojodihardjo & Widnall [15], and Morino & Kuo [16], there should be a disturbance velocity potential jump at the trailing edge, as represented in Fig. 1. Then the wake surface can be defined in a time-stepping procedure [15][13] in which the vortex strength

[3] However, it will be diffused following the vorticity diffusion law, which in the present scheme is regarded to be of higher order and will not be taken into consideration. The procedure for such diffusion procedure is elaborated in Appendix A.

at the wake elements is governed by Kelvin law and each wake element maintain its strength at the instant it was shed from the trailing edge. A simplified procedure used in [16][17] assumed the wake to be composed of vortex elements emanating from the trailing edge and parallel to the direction of the undisturbed flow. In all these cases, since the pressure is continuous across the wake, then the disturbance velocity potential jump across the wake is given by [13][15][16][17]

$$\Delta\phi_{wake} = \left(\phi_{upper} - \phi_{lower}\right)_{trailing-edge} \quad (15)$$

and is constant at each wake vortex element and equal to its value $\Delta\phi_{TE} \equiv \left(\phi_u - \phi_l\right)_{TE}$ when it was shed at the trailing edge. Numerically, the value of $\Delta\phi_{TE}$ is assumed to be equal to the value $\Delta\phi_W$ evaluated at the centroids of the elements in contact with the trailing edge : this assumption is reasonable in view of the Kutta condition that the pressure difference vanishes at the trailing edge [16]. For unsteady flow, time dependent component is also enforced:

$$\left[\frac{d\left(\Phi_{upper} - \Phi_{lower}\right)}{dt}\right]_{trailing-edge} = \left[\frac{d\left(\Delta\Phi\right)}{dt}\right]_{wake} \quad (16a)$$

or, translated into disturbance velocity potential:

$$\left[\frac{d\left(\phi_{upper} - \phi_{lower}\right)}{dt}\right]_{trailing-edge} = \left[\frac{d\left(\Delta\phi\right)}{dt}\right]_{wake} \quad (16b)$$

which is also equivalent to:

$$\left[\frac{d\Gamma_{span}}{dt}\right]_{wing} = -\left[\frac{d\Gamma_{span}}{dt}\right]_{wake} \quad (10)$$

To insure the Kelvin's vorticity conservation law. Here Γ_{span} represents the strength of the spanwise circulation (or vorticity) on the wing and at the wake at any section (or lifting line if no spanwise vorticity change occurs). Note that Γ is the integral of γ along designated streamwise direction, i.e. chord length for lifting body and from the trailing edge to infinity for the wake. This situation is depicted in Fig. 2.

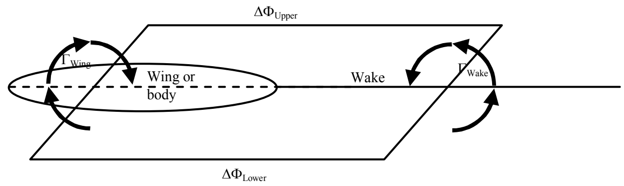

Fig. 2 Schematic description of Kelvin's theorem and relationship between $\Delta\Phi$ and Γ

Consequently, a method for representing the vorticity in the domain due to wake shear layer is in order, following the method described in Djojodihardjo & Widnall [15] and also Willis [13]. Close to the domain of interest vorticity is represented using a wake sheet dividing the continuous fluid-flow domain. The wake surface is represented by a sheet of prescribed velocity potential jump which account for the vorticity shed off the trailing edge in succession into the fluid domain. At each time step of the solution, a potential jump wake sheet is constructed to account for the wake shed vorticity. Such scheme represents the wake evolution process.

2.4. Representation of the Vorticity in the Domain

Vorticity ω is defined as the curl of the velocity:

$$\nabla \times \mathbf{U}_{\Psi} = \boldsymbol{\omega} \quad (17)$$

The velocity due to Vector potential Ψ is

$$\nabla \times \Psi = \mathbf{u}_{\Psi} \quad (18)$$

Substituting the vector potential relationship into thre definition of vorticity, and after some algebra, there is obtained

$$\nabla^2 \Psi = -\boldsymbol{\omega} \quad (19)$$

which is the Poisson equation relating the vector potential to the vorticity.

The vorticity evolution equation is derived starting from the incompressible Navier-Stokes Equation

$$\rho\frac{\partial \mathbf{U}}{\partial t} + \rho\mathbf{U}\bullet\nabla\mathbf{U} = -\nabla p + \mu\nabla^2\mathbf{U} \quad (20)$$

where ρ is fluid density, μ the fluid viscosity and p the static pressure. Taking the curl of (20) and yields:

$$\frac{\partial \boldsymbol{\omega}}{\partial t} + \left(\mathbf{U}\bullet\nabla\right)\boldsymbol{\omega} + \left(\boldsymbol{\omega}\bullet\nabla\right)\mathbf{U} = \nu\nabla^2\boldsymbol{\omega} \quad (21a)$$

where $\nu = \frac{\mu}{\rho}$ is the kinematic viscosity. Again, based on large Reynolds number assumption, the viscous term could be ignored at the present stage and Eq. (21) reduces to

$$\frac{\partial \boldsymbol{\omega}}{\partial t} + \mathbf{U} \bullet \nabla \boldsymbol{\omega} = -\boldsymbol{\omega} \bullet \nabla \mathbf{U} \qquad (21)$$

where the term $\boldsymbol{\omega} \bullet \nabla \mathbf{U}$ on the right-hand side represents the vorticity stretching as it is subject to the velocity gradient of the fluid. Following Willis [14], the vorticity in the domain is represented as the summation over all of the discrete vortex particles in the domain

$$\boldsymbol{\omega}(\mathbf{x},t) = \sum_p \boldsymbol{\omega}_p(t) vol_p \delta(\mathbf{x} - \mathbf{x}_p(t)) = \sum_p \boldsymbol{\alpha}_p(t) \delta(\mathbf{x} - \mathbf{x}_p(t)) \qquad (22)$$

where $\boldsymbol{\omega}_p(t) vol_p$ is represented as $\boldsymbol{\alpha}_p(t)$. Lagrangian reference frame for the evolution of the vorticity is used, such that the position $\mathbf{x}_p(t)$ of a discrete vortex particle at any given time is governed by:

$$\frac{d\mathbf{x}_p(t)}{dt} = \mathbf{U}_p(\mathbf{x}_p(t), t) \qquad (23)$$

The ***evolution of the vortex particle*** strength as it travels through the domain can be represented as

$$\frac{D\boldsymbol{\alpha}_p(t)}{Dt} = \boldsymbol{\alpha}_p(t) \bullet \nabla \mathbf{U}_p(\mathbf{x}_p(t), t) \qquad (24)$$

Each of the ***vortex particle (vortons)*** has an associated core in order to mimic the physical vortex core as well as to reduce the numerical instability of the vortex core interactions.

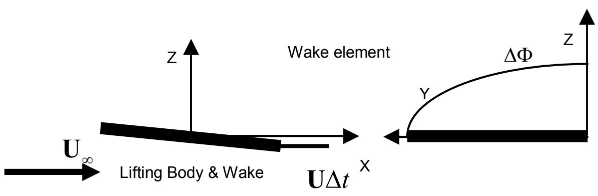

Fig.3 Wake Generation

2.5. Velocity Field in the Domain

The integral equation for computing the velocity in the domain, away from the body surface, due to the body, can be determined by taking the gradient of the integral equation for the velocity potential, Eq. (10).

Before proceeding, however, the following considerations should be taken into account:

a. The velocity $\mathbf{U}(\mathbf{x},t)$ at a given point in the domain, taking into account the contribution of vortex particles which give rise to velocity component due to vector velocity potential is given by Eq. (13).

b. Noting that the total potential and the perturbantion potential is related as $\Phi = \Phi_\infty + \phi$ where Φ_∞ and ϕ refers to the free-stream and perturbed local velocity potential, respectively, then the total velocity at any given point is given by

$$\mathbf{U} = \mathbf{U}_\infty + \mathbf{u}_\varphi + \mathbf{u}_\psi \qquad (25)$$

which is the key assumption in the present adopted and adapted approach.

c. The no-flux or flow tangency boundary condition on the surface of body implies

$$\frac{\partial \phi}{\partial n} = -[\mathbf{U}_\infty + \mathbf{u}_\Psi] \bullet \mathbf{n}_{body} \qquad (26)$$

From (13), (25) and (26)

$$\mathbf{U}(\mathbf{x},t) = \nabla \Phi_\infty + \nabla \phi + \nabla \times \nabla \Psi = \mathbf{U}_\infty + \mathbf{u}_\varphi + \mathbf{u}_\psi \qquad (27)$$

Then the integral equation for computing the velocity in the domain, away from the body surface, due to the body becomes

$$\mathbf{U}(\mathbf{x},t) \equiv \nabla \Phi = \frac{1}{4\pi} \nabla \iint_{S'_{body}} [\mathbf{U}_\infty + \mathbf{u}_\Psi] \bullet \mathbf{n}_{body} \frac{1}{\|\mathbf{x} - \mathbf{x}'\|} dS'_{body} + \frac{1}{4\pi} \nabla \iint_{S'_{body+wake}} \phi \frac{\partial}{\partial n} \left(\frac{1}{\|\mathbf{x} - \mathbf{x}'\|} \right) dS'_{body+wake} + \mathbf{U}_\infty + \mathbf{u}_\psi \qquad (28)$$

2.6. Integral equation for Pressure-Downwash Velocity relationship.

The Bernoulli equation can be used to determine the forces and pressures on the body,

$$\frac{D\Phi}{Dt} + \frac{\rho}{2} U^2 + p = Const \qquad (29)$$

where Φ is the velocity potential and p refers to the local static pressure. Utilizing disturbance velocity potential ϕ and harmonic motion approach, where $p = \overline{p} e^{i\omega t}$, or $p = \overline{p} e^{i\Omega(T+MX)}$, as appropriate, the mean pressure jump acros the lifting surface can be written as

$$\Delta \overline{p} = -\left[i\omega \overline{\phi} + U_\infty \frac{\partial \overline{\phi}}{\partial t} \right] \qquad (30)$$

where accordingly,

$$\Delta p = \Delta \bar{p} e^{i\omega t} \tag{31}$$

This pressure-velocity potential relationship allows the computation of the forces and moments on the body upon knowing the velocity field in the domain.

Alternatively, using the method of variation of parameter [22], the solution of the Bernoulli equation (22) can be obtained as

$$\bar{\phi} = \int_{-\infty}^{x} \frac{-\Delta \bar{p}}{\rho U_\infty} \left(e^{\frac{i\omega}{U_\infty}(\lambda - x)} \right) d\lambda \tag{32}$$

The boundary condition of no-flow through the surface of the body (and wing) defines the normal velocity or downwash w at the surface, and is given by

$$\frac{\partial \bar{\phi}}{\partial n} \cong \frac{\partial \bar{\phi}}{\partial z} = w \tag{33}$$

With appropriate algebraic manipulation [23], the following relationship is obtained:

$$\frac{w}{U_\infty} = -\frac{1}{8\pi} \iint \frac{\Delta \bar{P}_1 (x_1, y_1)}{\frac{1}{2} \rho U_\infty^{\ 2}} K(x - x_1, y - y_1) dS \tag{34}$$

where

$$K(x, y) = \lim_{z \to 0} \exp\left[-\frac{i\omega}{U_\infty} x \right] \left[\left[\int_{-\infty}^{x - x_1} \exp\left[\frac{1}{(1 - M^2)} \frac{i\omega}{U_\infty} \xi \right] \right] \frac{\partial^2}{\partial z^2} \left(\frac{e^{-iKr}}{r} \right) d\xi \right] \tag{35}$$

which is the well known standard Kernel function representation in lifting surface or panel method [19], and leads to further utilization of DPM or DLM in the unsteady aerodynamic problem as solution of (2) and (10) with appropriate boundary conditions.

3 Numerical Procedure

The present interest is to carry out computational procedure for harmonic motion in subsonic flow. Other types of flow will be addressed at a later stage using similar approach as an extension of this work.

In the present method, two assumptions are adopted:

a. The oscillatory motion has prevailed for sufficiently long time, so that a steady state situation is achieved.
b. In consequence to the continuous vortex-shedding process, the vortices give rise to velocity component due to vector potential component, as stipulated in Willis approach. The procedure for calculating the strength and evolution of the vortex shedding follows closely the procedure stipulated there.

With such principle in mind, one may proceed treating the oscillatory flow following the conventional practice in harmonic motion adopted in unsteady aerodynamics Lifting Surface Methods.

3.1. Computation of the disturbance velocity potential field in the domain

Following the principal assumptions above, for harmonic motion, the perturbation potential can be expressed as:

$$\phi(\mathbf{X}, t) = \bar{\phi}(\mathbf{X}, t) e^{i\Omega(T + MX)} \tag{36}$$

where $\bar{\phi}$ is the mean perturbation potential and

$$\beta = \sqrt{1 - M^2}\ ;\ M = \frac{u}{a_\infty} \tag{37a-i}$$

$$X = \frac{x}{L\sqrt{1 - M^2}} = \frac{x}{L\beta}\ ;\ Y = \frac{y}{L\beta};\ Z = \frac{z}{L\beta};$$

$$T = \frac{a_\infty \beta t}{L};\qquad \Theta = \frac{a_\infty \beta \theta}{L} = M(X_1 - X) + R \qquad \text{and}$$

$$R \equiv R(\mathbf{X}_1, \mathbf{X}) = \left[(X_1 - X)^2 + (Y_1 - Y)^2 + (Z_1 - Z)^2 \right]^{\frac{1}{2}}$$

$$\Omega = \frac{\omega L}{\beta a_\infty} = \frac{\omega L}{U} \frac{U}{a_\infty} \frac{1}{\beta} = \frac{kM}{\beta} \equiv \frac{kM}{\sqrt{1 - M^2}} = \frac{\omega L}{a_\infty} \frac{1}{\sqrt{1 - M^2}}$$

The general Boundary Integral Equation (10) now, as elaborated and adapted from Morino & Kuo [16] becomes

$$\begin{aligned}
\bar{\phi}(\mathbf{x}) = & -\frac{1}{4\pi} \iint_{S'_{body}} \left(-[\mathbf{U}_\infty + \bar{\mathbf{u}}_\Psi] \bullet \mathbf{n}_{body} \right) \frac{e^{i\Omega(T + MX)}}{|\mathbf{X}_1 - \mathbf{X}|} dS'_{body} \\
& + \frac{1}{2\pi} \iint_{S'_{body}} \bar{\phi} \frac{\partial}{\partial n} \left(\frac{e^{i\Omega(T + MX)}}{|\mathbf{X}_1 - \mathbf{X}|} \right) dS'_{body} \\
& + \frac{1}{2\pi} \int_{TE} \Delta \bar{\phi}_{TE} \left[(Y_1 - Y) \frac{\partial Z_{TE}}{\partial Y_1} - (Z_{TE} - Z) \right] dY_1 \int_{X_{TE}(S'_{wake})}^{\infty} \left(\frac{1}{|\mathbf{X}_1 - \mathbf{X}|} \right)^3 dX_{1wake}
\end{aligned} \tag{38}$$

If the surface of the Lifting body and the wake is represented by a set of surface (boundary) element of convenience, then the Boundary Integral Equation can be transformed into Boundary Element representation, which in matrix form assumes the form:

$$\phi_k - \sum_{i=1}^{N} c_{ki}\phi_i - \sum_{i=1}^{N} w_{ki}\phi_i = b_k \qquad \text{k=1,2,...n} \quad (39)$$

or

$$\sum_{i=1}^{N_{B+W}} A_{ki}\phi_i = b_k \equiv \sum_{i=1}^{N_{B+W}} B_{ki}\frac{\partial \phi_i}{\partial n} \quad (40a)$$

or $[\mathbf{A}]\bullet\{\phi\} = [\mathrm{B}]\bullet\left\{\dfrac{\partial\phi}{\partial \mathrm{n}}\right\}$ (40b)

Here

$$b_k = -\iint_{\Sigma_B} \phi^{(n)} \frac{e^{i\left(\frac{\Omega}{M}\right)R_k}}{2\pi R_k} d\Sigma \cong -\sum_{i=1}^{N}\left[\phi^{(n)}e^{-i\Omega R_k}\right]_{P=P^{(i)}} \iint_{\Sigma_B} \frac{1}{2\pi R_k} d\Sigma_i = -\sum_{i=1}^{N}\left[\left[-\left[\mathbf{U}_\infty + \bar{\mathbf{u}}_\Psi\right]\bullet \mathbf{n}_{body}\right]e^{-i\Omega R_k}\right]_{P=P^{(i)}} \iint_{\Sigma_B} \frac{1}{2\pi R_k} d\Sigma_i \quad (41)$$

representing the boundary condition on the surface of the Lifting body, and

$$c_{ki} = -\iint_{\Sigma_i} \frac{\partial}{\partial N_1}\left(\frac{e^{-i\Omega R_k}}{2\pi R_k}\right) d\Sigma_i \cong -\left[e^{-i\Omega R_k}\left(1+i\Omega R_k\right)\right]_{P=P^{(i)}} \iint_{\Sigma_i} \frac{\partial}{\partial N_1}\left(\frac{1}{2\pi R_k}\right) d\Sigma_i \quad (42)$$

which is the influence coefficient of a surface element on the surface of the Lifting body , representing the induced velocity at field point k due to uniform distribution of disturbance velocity potential at surface element i, and

$$w_{ki} = \pm\frac{1}{2\pi}\int_{\Delta Y_i}\left[\left(Y_1 - Y\right)\frac{\partial Z_{TE}}{\partial Y_1} - \left(Z_{TE} - Z\right)\right] I_W dY_1 \quad (43)$$

the influence coefficient of a surface element on the wake surface, representing the induced velocity at field point k due to uniform distribution of wake vortices given in terms of disturbance velocity potential jump across the wake surface. Here I_W has been evaluated by Morino & Kuo [16] and is given by

$$I_w = \left(Z - Z_1\right)e^{i\left(\frac{\Omega}{M}\right)\left(X_{TE}-X\right)}\left\{\begin{array}{l}\dfrac{\Omega\beta}{MR_0}\left[K_1(\kappa) + \dfrac{\pi i}{2}I_1(\kappa)\right] \\ \quad + \dfrac{X - X_1}{R_0^2}e^{i\Omega\left[\left(\frac{X-X_1}{M}\right)-R\right]} \\ \quad + \dfrac{1}{R_0^2}F\left(\dfrac{MR + X_1 - X}{\beta R_0}\right)\end{array}\right\}_{X_1 = X_{TE}} \quad (44)$$

where

$$R_0^{\,2} = \left(Y_1 - Y\right)^2 + \left(Z_1 - Z\right)^2 \qquad \kappa = \frac{\Omega\beta R_0}{M} \quad (45a\text{-}b)$$

and I_1 and K_1 are the modified Bessel function of the first order of first and second kind, respectively.

The function F is given by

$$F(u) = \sum_{1}^{\infty} F_n(u) \quad (46)$$

where

$$F_n(u) = \frac{1}{n!}(-1)^n \kappa^n \left(1+u^2\right)^{\frac{1}{2}} u^{n-1} + \frac{\kappa^2}{n(n-2)}F_{n-2}(u)$$

with

$$F_1(u) = -i\kappa\left(1+u^2\right)^{\frac{1}{2}} \qquad F_2(u) = \frac{\kappa^2}{2}\left\{u\left(1+u^2\right)^{\frac{1}{2}} - \ln\left[u + \left(1+u^2\right)^{\frac{1}{2}}\right]\right\} \quad (47a\text{-}c)$$

The surface of the lifting body is represented by quadrilateral element. Wake elements will also be represented by rectangular elements, at least initially. For certain values of Z, Morino [17] has shown that the wake integral I_W yields a closed form representation of the classical Kernel function of Ref. 24.

The values of $\Delta\phi_{wake}$ at the wake, following the relationship:

$$[\Delta\phi]_{wake} = [\Delta\phi]_{trailing-edge} = \left[\phi_{upper} - \phi_{lower}\right]_{trailing-edge} \quad (48)$$

can readily be evaluated upon solving Eq.(32). Following the computational approach adopted here for solving harmonic motion as stated at the beginning of this section, the following steps are followed. Initially, $\bar{\mathbf{u}}_\Psi$ in Eq. (32) will be assumed to be zero. This implies that initially, the wake sheet is assumed to extend from the

trailing edge infinity, and the influence of $\bar{\mathbf{u}}_\Psi$ is not taken into account yet.
Then for the subsequent steps, procedure adapted from Willis [13] will be followed to calculate the velocity in the field due to vortices shed in the wake in its evolution. However, this step should not be regarded as a time-stepping method, but rather an iterative procedure to establish the 'fully-developed" harmonic motion situation.

3.2. Computation of the evolutive wake vortex sheet and the strength of the vortices and the vector potential.

A novel approach will be introduced here to account for the influence of the evolutive wake vortex sheet. An extensive host of literature devoted to the vortex particle technique has been developed and indicates its potential in modeling and solving practical problems including aerodynamics, as an alternative to CFD approach, which is based on the solution of Navier-Stokes or Euler Equations, and Lifting Surface / Boundary Element Methods, which offer solution to the Fluid-Dynamic Equation (2) accompanied by Kutta-condition to account for the viscosity effects which materialized in smooth flow of the trailing edge of the lifting body as the ideal situation. For this purpose, the technique introduced by Willis [13][14] gives some guidance on the success of introducing vortex particle technique partially. However, since interest is focused on oscillatory flow that has developed into a "steady-state", a new approach will be synthesized. The approach can be summarized to capitalize on the following principle or assumptions:
a. Following the rationale utilized in [15], the strength of new vorticity released into the fluid domain should be equal to the value of the determined by utilizing Eq. (15) , which implies that the vorticity imparted on the wake element shed off the trailing edge, following Kelvin's theorem, is given by

$$\boldsymbol{\alpha}_{pW}(\mathbf{x},t)=\left[\left(\Phi_{upper}(t)-\Phi_{lower}(t)\right)\right]_{trailing-edge\ trailing-edge}$$
$$\equiv\left[\Delta\Phi(t)\right]_{trailing-edge}\equiv\Delta\Gamma_{pW}(\mathbf{x},t) \tag{49}$$

where, following the definition given by Willis in Eq.(22):

$$\boldsymbol{\alpha}_p(t)\equiv\iiint_p \boldsymbol{\omega}_p(t)\,dvol_p \tag{50}$$

represented subscript p refers to the vortex "blob" of fixed identity, as also implied by Ploumhans, Winckelmans & Salmon [25] and Ploumhans, Daeninck & Winckelmans[26].
b. For oscillatory case, commensurate with harmonic motion assumption, the vortex ($\boldsymbol{\Psi}_p$, $\boldsymbol{\omega}_p$ or $\boldsymbol{\alpha}_p$) are oscillatory in nature. Hence:

$$\boldsymbol{\Psi}_p=\bar{\boldsymbol{\Psi}}_p e^{i\Omega(T+MX)},$$
$$\boldsymbol{\omega}_p=\bar{\boldsymbol{\omega}}_p e^{i\Omega(T+MX)}$$

or

$$\boldsymbol{\alpha}_p=\bar{\boldsymbol{\alpha}}_p e^{i\Omega(T+MX)} \tag{51a-c}$$

c. Following Kelvin's Theorem, at any spanwise section of the lifting body:

$$\left(\int_{-T/2}^{T/2}\bar{\boldsymbol{\Gamma}}_p e^{i\Omega(T+MX)}d\tau\right)_{Wing\&Body}=-\left(\int_{-T/2}^{T/2}\bar{\boldsymbol{\Psi}}_p e^{i\Omega(T+MX)}d\tau\right)_{wake\,system} \tag{52}$$

If there are discontinuities, care will be taken to insure that the vortex system is governed by Kelvin's conservation theorem.
d. Correspondingly, the total sum of the wake vortices should be related to the total lift (following Kutta-Joukowski law, for simplification) commensurate with the wake geometry:

$$\bar{\boldsymbol{\Gamma}}_{wake\,system}=\frac{\bar{\mathbf{L}}_{Section}}{\rho U_\infty}\cong\bar{\boldsymbol{\Psi}}_{wake\,system} \tag{53}$$

Solution of the Poisson Differential Equation, Eq.(19) : $\nabla^2\boldsymbol{\Psi}=-\nabla\times(\nabla\times\boldsymbol{\Psi})=-\boldsymbol{\omega}$
can be written as an Integral Equation given by

$$\boldsymbol{\Psi}(\mathbf{x},t)=\frac{1}{4\pi}\iiint_W\frac{-\boldsymbol{\omega}}{\|\mathbf{x}-\mathbf{x}'\|}dV' \tag{54}$$

Using similar Boundary Element approach, Eq. (54) can be cast into the discretized form

$$\bar{\boldsymbol{\Psi}}(\mathbf{x})=\frac{1}{4\pi}\sum_p\bar{\boldsymbol{\alpha}}(\mathbf{x},t)\frac{1}{\|\mathbf{x}-\mathbf{x}_p(t)\|} \tag{55}$$

or, in matrix form:

$$[C_\omega]\bullet[\bar{\alpha}]=\bar{\boldsymbol{\Psi}}_p \tag{56}$$

where C_ω is a Matrix related to vorticity induced vector potential, where the vorticity is known and a single matrix vector product results in the vector potential, and equations (55) and (56) are already written for the mean values of $\bar{\boldsymbol{\Psi}}$ or $\bar{\boldsymbol{\Psi}}_p$. The velocity and the gradient of velocity can be determined by taking the curl and gradient of the curl of $\boldsymbol{\Psi}$ since

$$\mathbf{u}_{\Psi} = \nabla \times \boldsymbol{\Psi} \quad (57)$$

Therefore

$$\mathbf{u}_{\Psi} = \nabla \times \boldsymbol{\Psi}(\mathbf{x},t) = \frac{1}{4\pi}\sum_{p} \nabla \frac{1}{\left\| \mathbf{x} - \mathbf{x}_p(t) \right\|} \times \alpha(\mathbf{x},t) \quad (58)$$

The evolution of vorticity is computed by discretizing the governing vorticity evolution ODEs and computing the vortex particle position at time t+Δt. Willis current solver structure uses a simple approach following Forward Euler Finite Difference Formula

$$\mathbf{x}(t+\Delta t) = \mathbf{x}(t) + \mathbf{U}_p(\mathbf{x}(t),t).\Delta t$$

which allows progressive updating the vortex strength.

This scheme completes the first cycle of the computational procedure for the determination of the vortex strength at the wake.

The value of $\bar{\phi}$ should be updated with the additional influence of the vector velocity potential contribution to the no-flow boundary condition on the surface of the lifting body. Hence the new mean velocity potential $\bar{\phi}(\mathbf{x})$ is calculated using the relationship

$$\hat{\phi}(\mathbf{x}) = -\frac{1}{4\pi}\iint_{S'_{body}} \left(-[\mathbf{U}_\infty + \bar{\mathbf{u}}_\Psi]\bullet \mathbf{n}_{body}\right)\frac{e^{i\Omega(T+MX)}}{|\mathbf{X}_1 - \mathbf{X}|} dS'_{body}$$
$$+\frac{1}{2\pi}\oiint_{S'_{body}} \hat{\phi}\frac{\partial}{\partial n}\left(\frac{e^{i\Omega(T+MX)}}{|\mathbf{X}_1 - \mathbf{X}|}\right) dS'_{body}$$
$$+\frac{1}{2\pi}\int_{TE} \Delta\hat{\phi}_{TE}\left[(Y_1 - Y)\frac{\partial Z_{TE}}{\partial Y_1} - (Z_{TE} - Z)\right] dY_1 \int_{X_{TE}(S'_{wake})}^{\infty} \left(\frac{1}{|\mathbf{X}_1 - \mathbf{X}|}\right)^3 dX_{1wake} \quad (59)$$

which then complete the computational cycle for the determination of $\bar{\phi}$. The rationale of vortex diffusion process is given in the appendix, as adaptation of procedure established by Ploumhans et al [25][25] as well as Willis simplified approach [13][14].

5 Concluding Remarks and Work in Progress

A novel approach to account for the wake vortices in the unsteady aerodynamics of lifting body has been synthesized based on the classical unsteady lifting surface approach and the vortex particle approach. The latter has been introduced to allow further refinement of the classical approach, which has been proposed by many investigators. However, the approach taken to tackle periodic motion for further utilization in aeroelastic problems introduces some simplification as elaborated in the main text. The numerical computation for sample problems to verify the effectiveness of the work is presently in progress. However, for the first phase of the procedure, an alternative procedure has been developed and elaborated as described in a companion paper [23].

To calculate the unsteady pressure on the lifting body, a direct approach can be followed using the Bernoulli equation [15][16]. Integral equation for Pressure-Downwash relationship can alternatively be set up using the relationship between the velocity potential ϕ to the harmonically varying pressure difference ΔP by the Bernoulli equation:

$$\Delta\bar{p} = -\left[i\omega\bar{\phi} + U_\infty\frac{\partial\bar{\phi}}{\partial t}\right] \quad (30)$$

where

$$\Delta P = \Delta\bar{P}e^{i\omega t} \quad (31)$$

Using the method of variation of parameter [22], the solution of the Bernoulli equation (30) can be obtained, which has been utilized in the companion paper mentioned [23]. Further results addressing sample problems for benchmarking will be elaborated during the presentation.

Acknowledgement

The work reported here has been carried out within the current Universiti Putra Malaysia Research Univesity Grant Scheme (RUGS) initiated in April 2010 . The author would like to thank Universitas Al-Azhar Indonesia for the opportunity to carry out the work at Universiti Putra Malaysia.

References

[1]. Djojodihardjo, Harijono. BE-FE Coupling Computational Scheme For Acoustic Effects On

Aeroelastic Structures, paper IF-086, Proceedings Of The International Forum on Aeroelasticity and Structural Dynamics, held at the Royal Institute of Technology (KTH), Stockholm, Swede, 18-20 June 2007.

[2]. Djojodihardjo, Harijono. Unified BE-FE Aerodynamic-Acoustic-Structure Coupling Scheme For Acoustic Effects On Aeroelastic Structures, Paper I-586, 26th International Congress Of The Aeronautical Sciences, 14-19 September 2008, Anchorage, Alaska, USA.

[3]. Dowling, A.P. and Ffowcs Williams, J.E. Sound and Sources of Sound, Ellis Horwood Limited, Chichester – John Wiley & Sons, New York Brisbane Chichester Toronto, 1983, ©1983, A.P.Dowling and J.E. Ffowcs Williams.

[4]. Wröbel, L.C. The Boundary Element Methods, Vol.1, Applications in Thermo-Fluids and Acoustics, John Wiley & Sons, Chichester, West Sussex PO19 IUD, England, 2002

[5]. Norton, M.P. *Fundamentals of Noise and Vibration Analysis for Engineers*, Cambridge University Press, Cambridge, New York, Melbourne, Sydney,1989

[6]. Huang, X.Y. Active Control of Aerofoil Flutter, AIAA Journal, Vol.25, No.8, 1987.

[7]. Lu, P.J. and Huang, L.J. Flutter Suppression of Thin Airfoils Using Active Acoustic Excitations, AIAA Journal, Vol.30, No.12, December 1992

[8]. Nagai, Kenichiro; Toshimitsu, Kazuhiko and Namba, Masanobu. "Active Control of Cascade Flutter by Means of Sound Waves from Duct Wall Sources", JSME International Journal, Series B, vol. 39, No. 3, pp. 608-614, 1996.

[9]. Winckelmans, G.S. and Leonard, A. Contributions to Vortex Particle Methods for the Computation of Three-Dimensional Incompressible Unsteady Flows, J.Comp.Phys., 109, pp 247-273, 1993.

[10]. Opoku, D.G., Triatos, D.G., Nitzsche, F and Voutsinas, S.G. Rotorcraft Aerodynamic and Aeroacoustic Modeling Using Vortex Particle Methods, Proceedings, 22nd ICAS Congress, Toronto, Canada, 2002

[11]. Leroy, A., Buron, F. and Devinant, P. Unsteady Aerodynamic Model for Thin Wings with Evolutive Vortex Sheets, 2003, available at *www.dtic.mil/cgi-bin/.*

[12]. Eldredge,J.D., Colonius, T. and Leonard, A. A Vortex Particle Method for Two-Dimensional Compressible Flow, Journal of Computational Physics **179,** 371–399, 2002.

[13]. Willis,D., Peraire, J.P. and White,J. FastAero- A Precorrected FFT- Fast Multipole Tree Steady and Unsteady Potential Flow Solver, dspce.mit.edu/bitsream/handle/1721.1/7378

[14]. Willis, David Joe. Unsteady, Accelerated, High Order Panel Method with Vortex Particle Wakes-PhD-MIT-2006-74900794.pdf

[15]. Djojodihardjo, Harijono and Widnall, Sheila E. A Numerical Method for the Calculation of Nonlinear Unsteady Potential Flow Problems, AIAA J. 7 No.10, October 1969.

[16]. Morino, L and Kuo, C.C. Subsonic Potential Aerodynamics for Complex Configurations: A General Theory, AIAA JOURNAL VOL. 12, NO. 2, February 1974

[17]. Morino, L. A General Theory of Unsteady Compressible Potential Aerodynamics, NACA-CR-2464, 1974

[18]. Morino, L. “Unsteady Compressible Potential Flow Around Lifting Bodies Having Arbitrary Shapes and Motions,” TR-72-01, June 1972, Boston Univ., Boston, Mass.

[19]. Holström, F. Structure-Acoustic Analysis Using BEM/FEM; Implementation In MATLAB®, *Master's Thesis,* Copyright © 2001 by Structural Mechanics & Engineering Acoustics, LTH, Sweden, Printed by KFS i Lund AB, Lund, Sweden, May 2001, Homepage: http://www.akustik.lth

[20]. Ashley, H. and Landahl, M.T. Aerodynamic of Wings and Bodies , Addison Wesley, 1965

[21]. Scott, J.R. and Atassi, H.M. Numerical Solution of Periodic Vortical Flows about a Thin Airfoil, NASA Technical Memorandum 101998, 1989.

[22]. Kreyszig, Erwin. Advanced Engineering Mathematics, 9th Edition, John Wiley & Sons , 2006.

[23]. Djojodihardjo, H., Akbar, M., Gunawan, L. & Kusni, M. Unified Aerodynamic-Acoustic Formulation For Aero-Acoustic Structure Coupling, ICAS Paper ID-376, 27th International Congress Of The Aeronautical Sciences, 19-24 September 2010, Niece, France.

[24]. Watkins, C. E, Runyan, H. S., and Woolston, D. S. "On the Kernel Function of the Integral Equation Relating the Lift and Downwash Distributions of Oscillating Finite Wings in Subsonic Flow," Rept. 1234, 1955, NACA (supersedes NACA TN 3131).

[25]. Ploumhans, P., Winckelmans, G. and Salmon, J.K. Vortex Particles and tree codes: I. flows with arbitrary crossing between solid boundaries and particle redistribution lattice; II. Vortex ring encountering a plane at an angle, ESAIM: Proceedings, Vol.7, 1999, pp 335-348.

[26]. Ploumhans, P., Daeninck, G. and Winckelmans, G. Simulation of Three-Dimensional Bluff-Body Flows Using the Vortex Particle and Boundary Element Methods, Flow, Turbulence and Combustion 73, pp 117-131, 2004.

8 Contact Author Email Address

Harijono Djojodihardjo
harijono@djojodihardjo.com

Copyright Statement