APPLICATION OF COMPUTATIONAL AEROELASTICITY TO DESIGN OF HELICOPTER ROTOR BLADES

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Abstract

Design of efficient and stable rotors must account for the reciprocal influence of aerodynamic and structural forces. It is a challenging task, which relies on experience and on a few, validated tools, based on low fidelity models of structure and aerodynamics. Adopting high-fidelity modeling would provide a more accurate load prediction and more direct correlation between design parameters and rotor performances.

Numerical investigation of rotor flowfield based on Computational Fluid Dynamics (CFD) have been performed with increasing success in the latest ten years. The aerodynamic solver is normally a research code customised for rotor flow. However, rotor blades are normally modeled as beam elements. The potential for improvement might be represented by a more accurate, unsteady modeling of turbulence and by high-fidelity mdoeling of the structure.

In the latest few years - outside of the helicopter world - many software packages have been made available, which can couple Aerodynamics and Structure. They do not account for a helicopter control system and for helicopter trim in general. However, dynamic mesh, accurate modeling of turbulence and flexibility in boundary conditions might make them suitable also for rotor flow - provided they can be coupled to an external "trim" programme.

This paper presents the first incomplete results of a test, where the applicability of two powerful general purpose simulation tools is assessed on rotors.

1 Introduction and motivation

The flow around a rotor blade can be unsteady, three dimensional, transonic with shocks (on the advancing blade), reversed (on the retreating blade). It can contain dynamic stalls and a vortical wake. Its evolution depends not only on rigid motion of the blade - rotation around the flap, lag and pitch hinges - but also on the elastic deformations. The blade displacements - rigid and elastic - depend on airloads. The fully coupled aeroelastic problem must account for the mutual dependence between structure and aerodynamics. Besides, the problem is strongly nonlinear: a small change is one of the design parameters might provide a significant change in the dynamic response.

Traditionally, industry has relied on the so-called comprehensive codes, which are based on low-fidelity models: a "lifting-line" representation of the blade aerodynamics and "beam elements" for the structure.

As Bousman pointed out in 1999 [6] comprehensive codes are limited in their capability to mimic the aerodynamic phaenomena and their interaction with the rotor motion. This results in significant difference between predicted and measured airloads, especially on articulated rotors (i.e. where the motion is larger). As a consequence, the design process cannot rely on an analysis tool able to conveniently link changes in parameters to changes in loads.

Rotor design is therefore a relatively slow and inefficient process, as compared to the one of aerodynamic surfaces of fixed wing aircraft and of rotating elements in turbomachinery.

However, the last ten years of research have provided significant successes in the prediction of airloads on rotors. These prediction are based on a numerical coupled approach, where the flow is simulated with the aid of Computational Fluid Dynamics (CFD) tools using prescribed moving boundary conditions.

The computations normally include a comprehensive rotor code, coupled to Euler (refer for instance to [25] [1]) or Navier-Stokes (for instance [19] [20] [22] [21]) solvers.

The structure has been modeled in different ways but - as in comprehensive codes - mostly relies on a modified beam model or on monodimensional finite elements. In some cases ([12]) Navier-Stokes equations are coupled to a rigid rotor.

In virtually all cases, simulations use ”rotor” CFD codes, i.e. research codes written or modified especially to account for the characteristics of rotor flow. The same can be said for the structural modeling. If on one hand, customization has provided significant advantages in terms of efficiency, it has on the other hand limited the benefits of the latest developments in CFD and Computational Structure Dynamics (CSD) tools, such as more sophisticated modeling of turbulence and efficient adaptive grid refinement on the CFD side and transient solutions together with the flexibility to introduce a nonlinear material behavior on the CSD side. Moreover recent software packages have been developed also with the aim of having much simpler and more flexible interfaces with external programmes as well as a higher computational efficiency.

As open source software has greatly improved its reliability over the latest few years, it may also be considered. It has the advantages of including the “latest” algorithms and, by definition, offering an endless flexibility. Refer for instance to [13] [14] [18] for OpenFOAM, [9] [8] [7] for Overture, [29] for Nektar, [3] [2] [4] for PETs.

Open source CSD codes are also very powerful. For instance they offer explicit integration in time. Refer for instance to [23] for Code ASTER.

The broad availability of these and other ”advanced” features in simulation tools - which address a variety of industries - might let us wonder whether also non-rotor specific tools can be employed on the challenging problem of predicting rotor airloads and assess dynamic stability.

The aim of this work is to test the applicability of general purpose software packages to this aim. Two test case are presented where the commercial package Ansys Multiphysics and the open source package OpenFOAM have been used and tested on simple cases. The test still is in an early phase and no comparison with experimental data can be shown at this stage. Thsi paper focuses therefore on a single deliverable: the applicability of powerful general purpose tools.

This paper is organised as follows: The potential for improvement is described, in the form of a list of requirements for CFD, CSD and coupling. Two test cases are then presented. The first one - involving an industry-standard carbon and glass composite tail rotor blade - relies on the commercial package ANSYS which provides CFD, CSD and also the coupling between aerodynamics and structure computational domains. In the second test case, the open-source software package OpenFOAM in a first temporary stage is coupled to a helicopter ”trim” code, relying on a finite-elements description of the blade.

2 Potential for improvement in current CFD / CSD rotor analysis

2.1 Potential for improvement in CFD

The improvement of CFD over ”lifting-line” models is recognized. CFD provides a nonlinear, unsteady description of the flow surrounding the rotor. Nonlinearities include: high angle of attack, reverse flow, compressibility and wake. Unsteady phaenomena occur when the reduced frequency of the system is small enough. Load prediction at high speed (advance ratio $\mu \simeq 0.40$) is a typical ”difficult” test case, where CFD has provided results significantly more accurate than simple models. Refer for instance to [11], [22].

However, the results are yet not perfect: comparison with experiments still shows some phase differences in loads. The requirements for a successful CFD simulation of rotor flow could be summarizes as follows:

1. Correct representation of compressibility effects
2. Time accurate integration
3. Wake capturing and resolution
4. Correct representation of the flow on the blade, including boundary layer and detection of flow separations

Whereas requirements 1 and 2 can be ”simply” fullfilled with the use of unsteady, compressible solvers of the Navier-Stokes equations, requirements 3 and 4 require a slightly longer analysis.

2.1.1 Wake Capturing

An additional item in the wishlist - requirement 3 - is the capability to fully resolve the rotor wake. This means that the blade motion and loading must be accurately modeled in order to capture the starting points and strength respectively of the vortices. Moreover, the vortex wake must be properly convected through the domain. Although in some cases the wake can be modeled and passed over to the CFD solver as boundary conditions, being able to capture the wake in a general case, as part of the resolved flowfields, is one of the most interesting aspects of application of CFD to rotor flow. In forward flight solution of the flowfields around the whole rotor is necessary together with the capability to rotate the rotor inside the computational domain as the forward component of speed and wake do not rotate. As pointed out by [16], [10] and [26] the capability to fully resolve tip vortices for the duration of their life until they meet the following blade would require enormous computational resources. Note that an overset (chimera) grid approach is necessary in forward flight. The rotor wake play a critical role. The effects of the wake vortices on the blade flowfields con be dramatic in some flight conditions. It is well known that blade-vortex-interaction (BVI) in descent flow is accountable for significant noise generation. Note that noise reduction is one of the most important requirements in rotor design.

2.1.2 The Flow on the Blade

Flow separations and reattachments are correctly captured only if the turbulent phanomena are properly represented in a way that is physically meaningful. Whereas two-equation models are to be considered as reliable, the more appropriate way to represent turbulent structures in rotor flow would be Large Eddy Simulation (LES). LES consists in explicitly solving large turbulent anisotropic structures and modeling the smaller ones, which tend to be more homogeneous .

Rotor flow is characterised by large, unsteady turbulent structures: the rotor wake ”built” by the vortices shed by the blades, the vortices produced in dynamic stall. In the ”turbulent wake” working state of the rotor (see for instance [15]), the flow becomes a set of large, disorganized turbulent structures.

LES is being used for over twenty years in research ([17], [24])and, more recently, in industrial applications.

In general, LES is more demanding than RAS: it requires a finer mesh where turbulent strctures are expected and time-accurate integration. However, rotor flow requires in any case, time-accurate solution and sufficient resolution to capture the rotor wake. The additional cost of LES might not be that high.

Moreover, requirement 4, include the correct and accurate representation of boundary motion. Boundary motion is dictated by the rigid and elastic displacements of the blade, computed with CSD and a ”trim” code. Dynamic mesh motion is a feature which is normally included in CFD codes. It must be noted, however, that it many cases it provides additional fragility to the numerical solution.

Fulfillment of requirements 3 and 4 is demanding: many of the coupled CFD/CSD analysis published so far rely on simple algebraic turbulence model and prescribed wake.

2.2 Potential for improvement in CSD

Modal condensation and, in general, beam models reliably represent the behavior of a rotor blade in terms of displacements and rotations of the reference line. The interaction with aerodynamic forces can be suitably reproduced. However they do not allow a convenient and accurate reconstruction of stress and strain distribution in the whole blade structure. Root and tip regions are ”notoriously” tridimensional in both external shape and material layout. In a design process and especially in an weight saving exercise or in an optimization process (where the design parameters are systematically varied in given ranges) designers like to examine the stress and/or strain distribution in every composite layer of every structural element.

It is useless to remind how the human intelligence and creativity is still superior to numerical design techniques, as far as geometrically complex composite parts are concerned. It is therefore of the highest importance to provide senior designers and engineers with detailed information, as it is done in the design process of non-rotating parts.

A separate, completely different remark concerns the capability to reproduce the behavior of the material in special conditions, outside of the ”normal” elastic range, such as the state of stress due to ultimate loads or simulation of damaged conditions, where the material could enter the plastic range or the simulation could include damage propagation - analysis normally conducted on non-rotating parts. Simulation of the effects of ageing could also be a possibility. How many times discrepancies between flight test measurements and theory are justified with ”additional structural damping due to age of the rotor”?

2.3 Potential Improvement in CFD / CSD Coupling

The displacement of the rotor blade includes a rigid motion, dictated by the orientation of the swashplate, and elastic deformation due to the action of aerodynamic and inertial loads. As the orientation of the swashplate (i.e. the controls) depends on the force balance on the whole rotor or on the whole helicopter, depending on the scope of the analysis, the rigid motion cannot in general be resolved in a single-blade analysis.

The most pragmatic approach relies on a ”trim” code - or comprehensive code - which prescribes the controls and the position of the blade. On a rigid rotor, this would mean collective and cyclic pitch, and a sufficient number of harmonics of rigid flap and lag. The description of the motion of a flexible rotor requires the harmonics of a sufficient number of degrees of freedom, referring either to nodal displacements (if FEM) or modal or a combination.

A more ambitious option would consist in coupling CFD with a high-fidelity structural model, typically a Finite Element (FE) model. In this case, either the FE software can include rigid motion or a ”trim” code would still be necessary.

In all cases, coupling is a critical phase of the process. Coupling can be ”tight” (”strong”) or ”loose” (”weak”). In the tight coupling methodology, information between CFD and CSD is transferred at each time step. Subiterarion are often necessary, in order to enforce consistence of all flow and displacement fields.

In the loose coupling approach, loads and displacements (or velocities) are only periodically passed. In rotor analysis, typically once per revolution. Loose coupling has proven to be more convenient in a number of simulations. Details of one coupling framework can be found in the above references or in [27] [28].

Potential for improvement lies in the possibility of coupling high fidelity structural models with CFD, either directly or through a ”trim code” which would become a sort of ”motion manager”. Loose coupling normally assumes periodic flow and motion. This is a good assumption for load prediction, as transient are normally a fraction of a revolution.

However, the capability to handle fully unsteady behavior might help understanding specific flight conditions such as turbulent wake and autorotation flight states.

3 Evolution of the Requirements as a Function of Rotor Evolution

The requirements stated above might become more significant if we consider the evolution of rotors and helicopters. Tilt rotors and, possibly, compound helicopters, have ”rigid” rotors but fly at higher speed and experience potentially higher loads. The accurate prediction of compressible effects might become more important.

Accurate airloads prediction is critical to assess and reduce the vibratory and noise level - probably the highest priorities in design objectives.

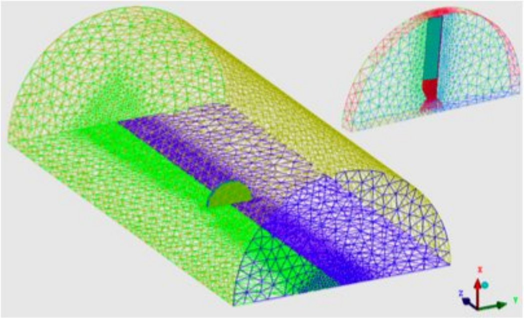

Fig. 1 Case study 1, the aerodynamic mesh

4 Case study 1: hover flight analysed with Ansys Multiphysics

In a first test case, the turbomachinery module of Ansys has been used. The steady state option has been used. Airloads are computed by CFX (the CFD solver part of the Ansys package) on an undeformed blade geometry and passed over to Ansys FEM which computes the structural deformations. The process is iterated until convergence is reached. Its application is straight forward; high-fidelity structural and aerodynamics models are generated and used at reasonable cost. The blade chosen is an industry-standard tail rotor blade, made out of carbon and glas fibers.

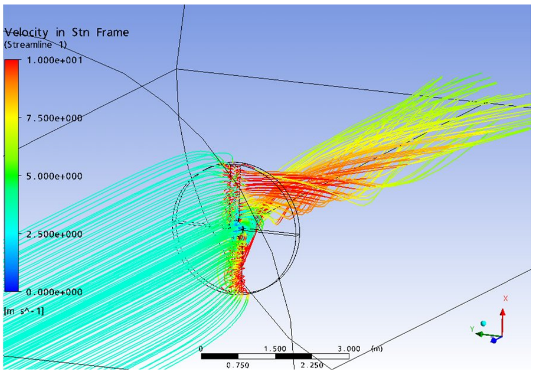

Fig. 2 Case study 1, flowlines computed in one of the flight conditions tested

The flowfield is plausible but no quantitative comparison of flow quantities has been conducted. Results impress for the abundance of information, as it is documented in figure 4. The high-fidelity structural (3) model provide a complete view on strain and stresses in every element of the structure. This analysis methodology could be used in practice also in industrial environment; it can be easily set up and provides all the information designers need.

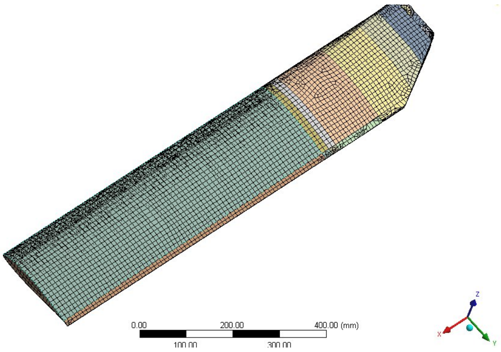

Fig. 3 Case study 1, a view of the FE mesh of the outer composite skin

However, its practical use is limited to hover flight, as we could not devise an easy way to properly account for rotor dynamics in forward flight. The aerodynamic mesh was not specially refined in order to capture the rotor wake.

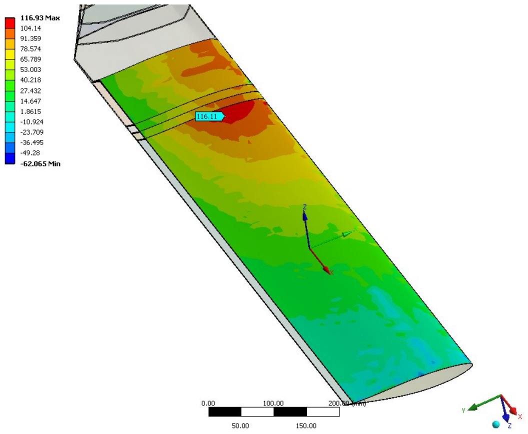

Fig. 4 Case study 1, dynamic stresses on one of the composite layers in one of the flight conditions tested

5 Case study 2: assessment of stability of a rotor in forward flight using OpenFOAM

Analysis of forward flight is significantly more demanding than hover flight. Loss of symmetry makes the single-blade approach insufficient. Besides, at hight speed compressibility effects arise on the advancing blade, pitch and flapping motion relative to higher harmonics causes unsteady behavior of the flowfields, the flow on a portion of the retreating blade reverses. Moreover, cyclic pitch excursion sets high requirements on the mesh deformation algorithm.

This case study aims at testing the prediction of airloads with a CFD package. At this stage, boundary motion has been prescribed (taken from the solution of a helicopter ”trim” programme) as well as the rotor wake (modeled externally). No high-fidelity structural model has been used in the ”trim” code. Objective of the test was the ease for the user to:

- Implement Dynamic boundary conditions, prescribed externally
- Implement the dynamic motion of the computational mesh
- Conduct a time-accurate simulation
- Provide a steady state converged solution as starting point of the unsteady simulation
- Assess the sensitivity to mesh refinement
- Assess the effects of compressibility
- Assess the difference between different turbulent models
- Assess the compatibility of the above mentioned parameters

The open source package OpenFOAM has been chosen for its versatility in choosing solver and turbulence models and its flexibility in accepting time varying, mapped boundary conditions.

A number of simulations have been conducted, testing various combinations of flow regimes and turbulent models, as well as mesh motion / deformation. Various solvers have been tested, they are listed in Tables 1 and 2. Note that the mesh has been generated analytically from airfoil shape and twist distribution. Dynamic mesh motion has been implemented with no particular difficulty despite geometrical complexity and a not-perfect-quality mesh. It must be said, though, that the mesh motion did not involve large displacements. Implementation of various two-equation turbulent models was also possible with no particular problem. The compressible solvers were also successfully tested but not with dynamic mesh motion. The overall user’s impression is very positive. The case was set up within a few weeks by one person with long experience on helicopters aeroealsticity but a very limited one on OpenFOAM.

The figures 5 to 17 show some of the results obtained from an unsteady incompressible simulation, conducted for about one degree in azimuth in a flight condition where the blade undergoes a negative flapping motion and a positive pitch rotation. Various mesh sizes have been used. The figures shown have been obtained on a relatively coarse mesh (about 1,4 million points). The computation has been carried out on a standard Core Duo processor and has required about 9000 sec.

The rotor of an average light twin helicopter has been taken as reference. The airfoil is the OA212 on the whole blade. The tip has been

Solver	Flow
`simpleFoam`	steady state, incompressible
`rhoSimpleFoam`	steady state, compressible
`pisoFoam`	transient, incompressible
`sonicFoam`	transient, compressible

Table 1 OpenFOAM fixed mesh solvers tested - in all cases were RANS and LES turbulence modeling available

Solver	Flow
`pimpleDyMFoam`	transient, incompressible

Table 2 OpenFOAM dynamic mesh solver tested - RANS and LES turbulence modeling are available

modeled with higher than average droop and sweep. The blade is therefore realistic but not real.

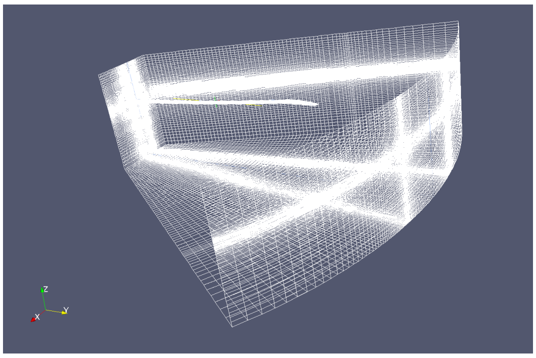

Fig. 5 Case study 2, The computational domain

6 Conclusions

Case studies 1 and 2 might contribute to provide the confidence that general purpose simulation packages such as OpenFOAM and Ansys Multiphysics can be employed to analyse the helicopter rotor in hover and forward flight. On the CFD front, the added value would be a more accurate modeling of turbulent structures and, possibly, more efficient computations. Concerning

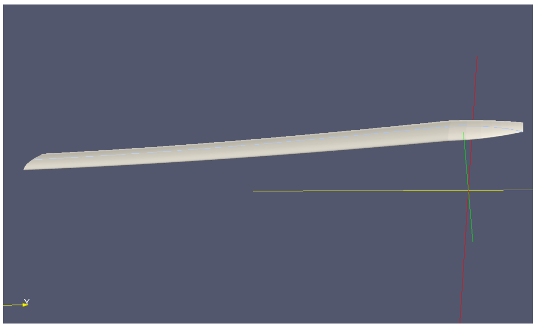

Fig. 6 Case study 2, Blade Surface

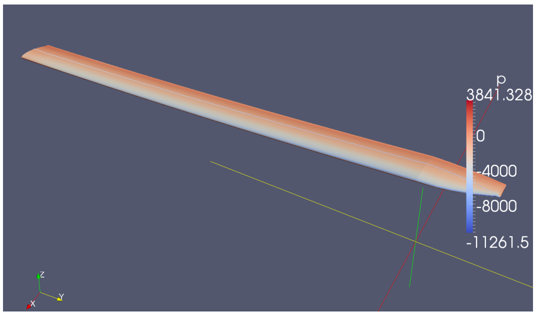

Fig. 7 Case study 2, Pressure distribution on the upper blade side

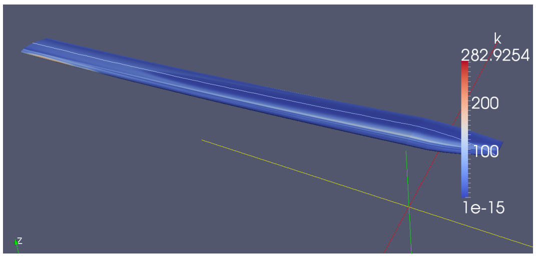

Fig. 8 Case study 2, Distribution of turbulent kinetic energy on the upper blade side

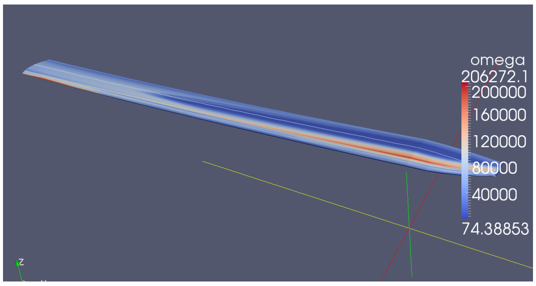

Fig. 9 Case study 2, Distribution of turbulent dissipation ω on the upper blade side

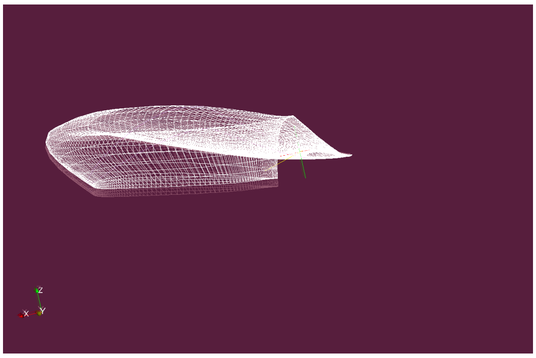

Fig. 10 Case study 2, Blade tip at start and final time

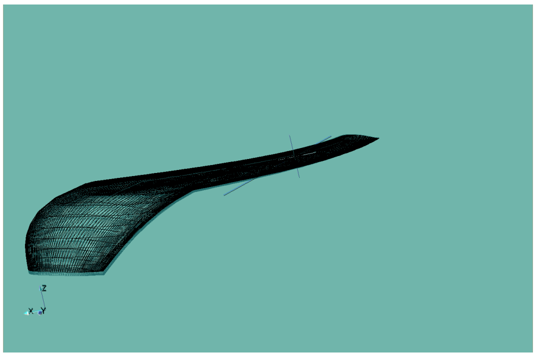

Fig. 11 Case study 2, Blade at start and final time

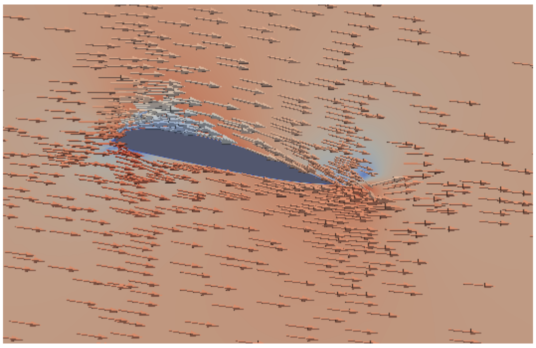

Fig. 12 Case study 2, Velocity flowfield at Y=40%

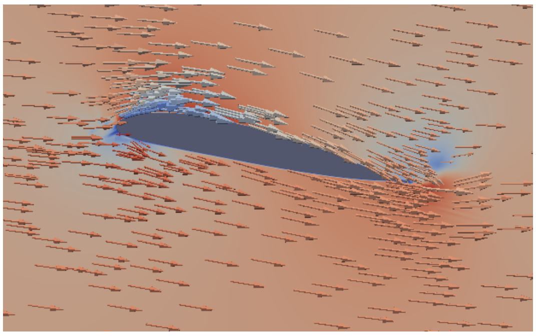

Fig. 13 Case study 2, Velocity flowfield at Y=60%

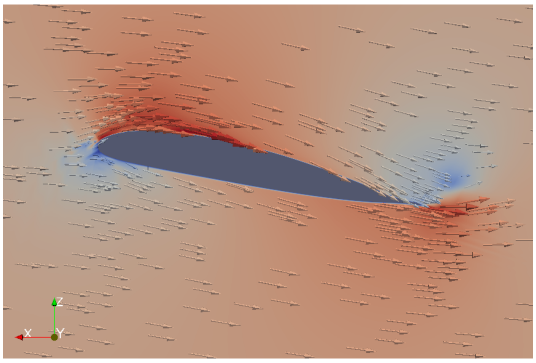

Fig. 14 Case study 2, Velocity flowfield at Y=75%

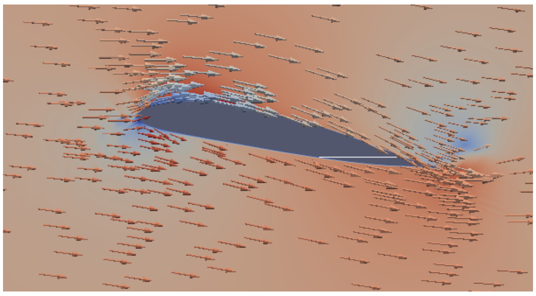

Fig. 15 Case study 2, Velocity flowfield at Y=95%

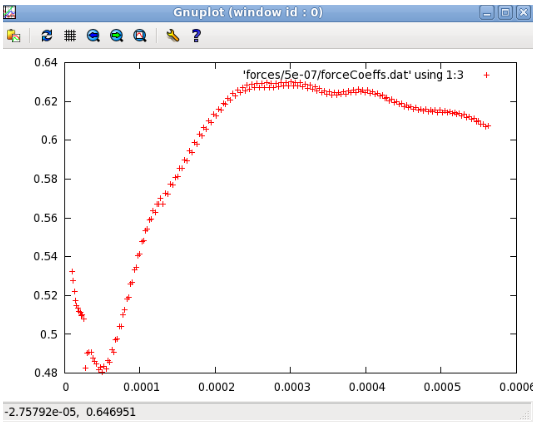

Fig. 16 Case study 2, Lift coefficient of the blade

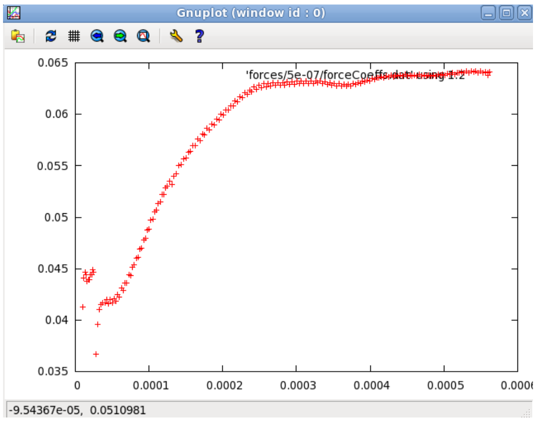

Fig. 17 Case study 2, Drag coefficient of the blade

CSD, high-fidelity models could provide the designers with a much better view on how design parameters influence rotor performances.

In the case of axial and hover flight, special "turbomachinery" modules, available for both software packages, provide the proper handling of centrifugal and Coriolis forces for both structure and airflow. Hovering rotor analysis can therefore be managed as a special case of a turbomachinery rotating element, with much fewer, more slender blades, hinged in the case of articulated hub.

In the case of forward flying rotor, the available off-the-shelf simulation codes must be coupled to a "trim" programme, able to determine the position of the swashplate (hence collective and cyclic pitch controls) corresponding to a given flight condition (forward speed, thrust). OpenFOAM can very well cope with the requirements for a single blade flow simulation, including compressibility, turbulence, dynamic flow variations, dynamic motion of the mesh. It can also work with "sliding" meshes and with overset meshes, but not out-of-the-box, it must be coupled to additional software ([5]). Simulation of the whole rotor would therefore require additional effort. However, it must be noted that OpenFOAM is not the only available tool and that the analysis in the present paper has been limited to standard available solvers.

A secondary motivation for this work, is that the applicability of general purpose analysis tools might lower the "entry fee" to rotor design. This could be beneficial to related industries such as wind turbine and rotorcraft UAVs, which might not have the same resources that helicopter manufacturers have.

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