MATHEMATICAL STUDY OF LINEAR AND NONLINEAR ROTORCRAFT PILOT-INDUCED OSCILLATIONS

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Abstract

This paper deals with the mathematical study of linear and nonlinear rotorcraft-pilot couplings.

This sustained and uncontrollable phenomenon results from the inappropriate efforts of the pilot to control the aircraft. In order to avoid its appearance and to alleviate the pilot's workload, the modern design procedure of a command channel requires to take into account the pilot in the loop right from the beginning.

Several mathematical criteria do exist and are available so as to evaluate the pilot-induced oscillation proneness of a configuration and are here employed in the field of rigid body rotorcraft-pilot couplings.

The overall aim is finally to evaluate how useful and relevant the mathematical criteria are for the prediction of pilot-induced oscillation occurrences.

Introduction

Pilot-induced oscillations are sustained and uncontrollable oscillations, they result from the inappropriate efforts of the pilot to control the aircraft which may be inadequately designed. In order to avoid its appearance and to alleviate the pilot's workload, the modern design procedure of a command channel involves the analysis and prediction of possible rotorcraft-pilot couplings right from the beginning.

The first part of this research paper considers linear pilot-induced oscillations due to excessive overall phase delay by means of the bandwidth/phase delay and the Neal-Smith criteria.

In the second part, the contribution of nonlinear elements such as position or rate limiting of an actuator to the triggering of pilot-induced oscillations is examined. In order to assess the possible existence of PIO for a command channel, the describing function method and the open-loop onset point (OLOP) criterion are here employed. Some devices allow to alleviate flying qualities cliffs and their effects are also examined.

The study is concerned with a rotorcraft command channel containing rate-limited swashplates. Practical calculations are mainly made for the ADOCS helicopter prototype which is a Black Hawk UH-60 equipped for research purposes on advanced digital/optical flight control systems and for which data are widely available.

1 Linear pilot-induced oscillations

In this part devoted to linear PIO, the bandwidth/phase delay and Neal-Smith criteria are employed. They analyse the intrinsic properties of the aircraft so as to conclude on its controllability.

1.1 Bandwidth/Phase delay criterion

The bandwidth ω_{BW} is here the highest frequency below which the aircraft can follow all pilot commands (1) i.e. $6dB$ gain margin and 45 deg phase margin. The phase delay τ_{delay} is linked to the phase angle shape at frequencies above the bandwidth (2).

$$\omega_{BW} = \max\left\{\omega_{BW,dB(+6dB)}, \omega_{BW,\phi(+45\,\mathrm{deg})}\right\} \quad (1)$$
$$\tau_p = -\frac{180 - \phi_{2\omega_{180\,\mathrm{deg}}}}{2\omega_{180\,\mathrm{deg}}}\frac{\pi}{180} \quad (2)$$

The handling qualities properties of the ADOCS helicopter are assessed for several feedback gains and some results are summarised in the figure (1). It exposes the phase delay in function of the bandwidth and frontiers delimit the different types of predicted behaviours.

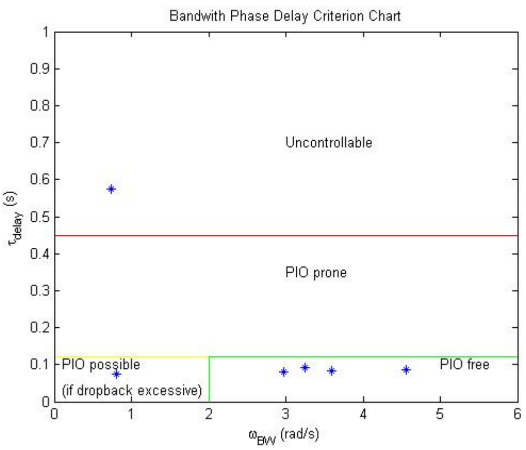

Fig. 1 Assessment of the handling qualities and the linear PIO proneness of the ADOCS helicopter for several feedback gains by means of the bandwidth/phase delay criterion

The feedback loop taken here is a proportional derivative one on the pitch angle whose gains are K_θ, K_q. According to the bandwidths ω_{BW} and to the phase delays τ_{delay} of the different configurations, it is visible that a badly designed feedback loop may lead to a PIO prone flight control system. Especially the sluggish handling qualities are met when there is only a feedback loop on the pitch rate and a possible PIO occurrence is diagnosed for the case the feedback loop concerns uniquely the pitch angle.

The Bandwidth/Phase delay criterion is the most widely used for handling qualities specifications [1]. As far as the longitudinal axis is concerned, the Neal Smith criterion deserves also much attention nevertheless.

1.2 Neal-Smith criterion

The Neal-Smith criterion evaluates the amount of compensation the pilot must furnish such that the pilot-aircraft system satisfies a required phase angle at the bandwidth frequency and maximum droop for the closed loop resonance. It focuses its attention on the longitudinal handling qualities and considers especially the aircraft mode with which the pilot can have an interaction (with a medium frequency range) that is to say the short period mode (neglecting the phugoid mode) and identifies the longitudinal dynamics with a low order model (3).

$$\frac{\theta(s)}{\delta(s)} = \frac{K_{\dot\theta}\left(s + 1/T_{\theta_2}\right)}{s\left(s^2 + 2\zeta_{sp}\omega_{sp}s + \omega_{sp}^2\right)} \quad (3)$$

Several feedback loop gains were tested (coming from available documentation) for the ADOCS helicopter. The Neal Smith criterion was then exploited directly so as to evaluate the PIO susceptibility.

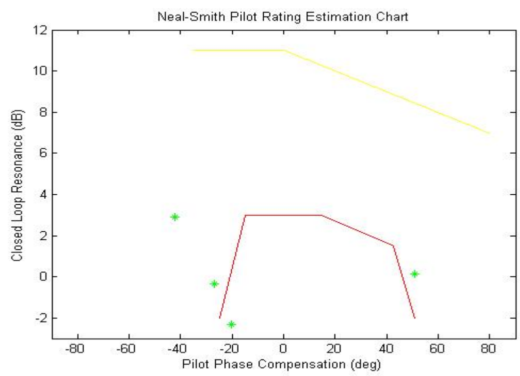

Fig. 2 Evaluation of PIO potential thanks to the Neal Smith criterion

A shown in figure (2), it seems to be difficult to reach a very safe configuration anyway uniquely thanks to an adequate feedback loop. The ADOCS command channel appears to be able to meet a PIO prone configuration whenever no special care is taken.

After having evaluated configurations where excessive phase delay may be responsible for PIO occurrences, the interest is now focused on cases where a nonlinear actuator may trigger such events.

2 Nonlinear pilot-induced oscillations

Several methodologies may be exploited in order to assess the PIO proneness of such nonlinear command channels as the one containing a rate limited actuator. The OLOP criterion and the describing function method are chosen amongst others to perform the analysis. A few elements on pilot modelling theory are furnished before the beginning of the analysis phase.

2.1 Pilot modelling

As far as the identification of the pilot model is concerned, two theories are mainly exploited and rest on different assumptions. The pilot control over the aircraft is most of the time said to be compensatory or precognitive synchronous [2],[3].

On the one hand, for compensatory control, the pilot corrects the observed error and tries to reduce it by adjusting the controls. In such a situation, the pilot-vehicle system tends to the so-called crossover model [2],[3]:
When the gain of the open-loop pilot-vehicle system is close to 1 (0 dB), the transfer function of the global system looks generally like $\frac{K}{j\omega}e^{-\tau j\omega}$ *which corresponds to an integrator plus a delay.*

On the other hand, for precognitive synchronous control, the way the pilot commands the aircraft results from experience and pre-learning. With a fully developed PIO, it can be effectively observed that the pilot reacts like a pure gain without time delay.

The link between the pilot gain and sensitivity is made through the crossover phase angle Φ_{cr}. A mathematical relation gives the value of the gain K_p required such that the overall gain of the pilot-vehicle transfer function $(K_p \cdot F)$ is unitary (i.e. equal to $0dB$) and such that the phase angle is fixed to Φ_{cr}:

$$K_p(\Phi_{cr}) \cdot |F(\omega_{cr})| = 1 \tag{4}$$

The gain spectrum corresponds to a crossover phase angle varying from $\Phi_{cr} = -110\,\text{deg}$ (low pilot gain) up to $\Phi_{cr} = -160\,\text{deg}$ (high pilot gain) for lateral manoeuvres and from $\Phi_{cr} = -90\,\text{deg}$ up to $\Phi_{cr} = -130\,\text{deg}$ for longitudinal operations.

2.2 OLOP criterion

The OLOP criterion [4] determines the handling qualities of the aircraft-pilot system when the rate limiter is activated. The implied drop may give PIO or not depending on the characteristics of the onset point. Principally the higher is the amplitude at which the OLOP is located, the more important the additional phase delay and closed-loop amplitude are, thus increasing the PIO susceptibility.

In the Nichols diagram (3), the points are linked to different rate limiter bounds varying from $R = 5$ $inches/s$ to $R = 25$ $inches/s$. Both examined configurations have got a pilot of the same nervousness i.e. with the same pilot gain. For the case examined on the top, the "command model" actuator (a second-order model involving and smoothing the pitch angle reference input) is suppressed of the command channel.

On the figure (3), the configuration on the left side is likely to meet PIO whereas the other one (on the right side) is not. This observation shows the importance of the command model (a second-order model) and its smoothing effect which contributes to make the PIO phenomenon disappear almost completely.

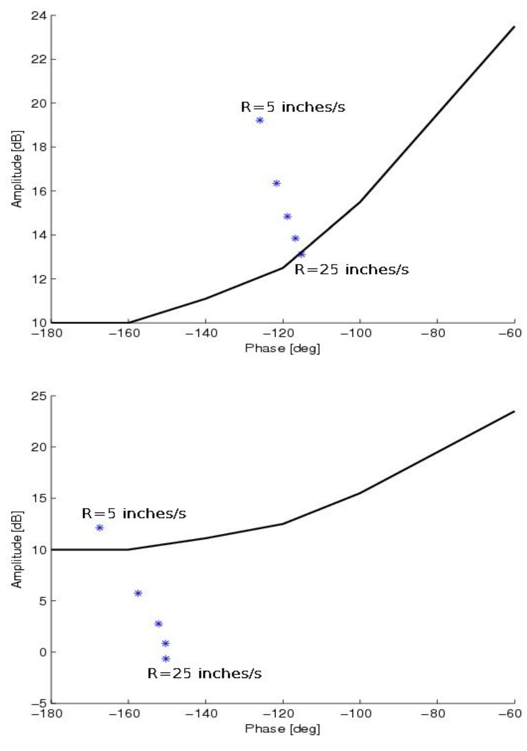

Fig. 3 ADOCS criterion applied for different configurations

2.3 Describing function method

In order to carry out successfully the describing function method [5], two conditions must hold. Fistly there must be a clearly identifiable nonlinear element which can be isolated from the linear part, secondly the linear part must behave like a low-pass filter.

The command channel considered is presented in the figure (4) and is the one of the ADOCS helicopter whose swashplate displacements are rate limited. The equation (5) expresses the link between the constituting actuators and involves amongst others the first-harmonic approximant of the nonlinearity $N(A,\omega)$, the amplitude θ_c or the pulsation ω of the sinusoidal input, the amplitude A and the phase delay ϕ of the entry signal of the rate limiter.

$$\begin{aligned}&1 + Rotor \cdot RigidBody \cdot N(A,\omega) \cdot \\ &(K_p \cdot Command + Feedback) \\ &= Actuator \cdot Command \cdot \theta_c / (A\exp(j\phi))\end{aligned} \tag{5}$$

The implicit equation (5) is solved by means of a continuation algorithm. This allows to determine the characteristics of the closed-loop system. The linear (blue) and nonlinear (green) behaviours of the equivalent open-loop systems are plotted in the Nichols chart (4).

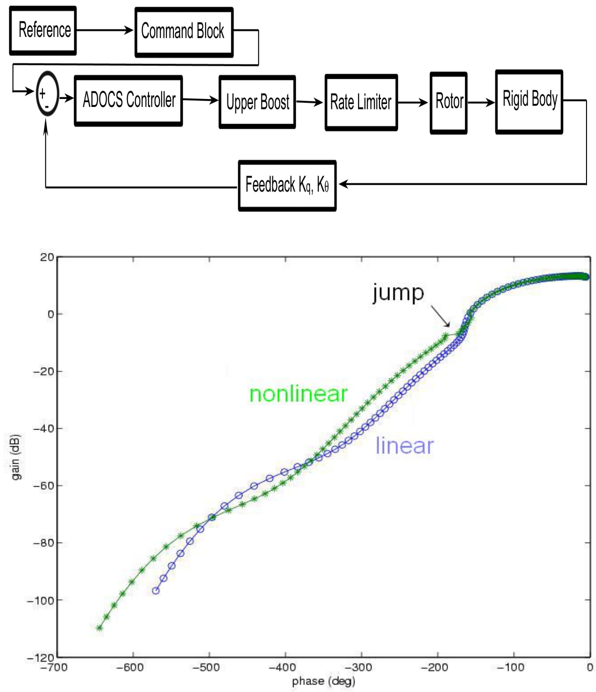

Fig. 4 Open-loop ADOCS command channel and Nichols charts for linear and nonlinear ADOCS command channels

When increasing the input pulsation, a so-called flying qualities cliff occurs suddenly at the critical pulsation. From the viewpoint of dynamical systems, it is called a resonance jump. The behavioural change of the rotorcraft dynamics may surprise the pilot.

Another case of interest is the one with a fixed target value to reach such as maintaining the pitch angle constant during a landing flare. A rate limiter is located in the forward path with the aim of restricting the command value transmitted to the rest of the FCS so as to smooth brusque stick inputs. But the disadvantage of such an element remains nevertheless that it can generate PIO.

The rate is bounded by a maximum of $15deg/s$. The transfer function relative to the longitudinal command channel expresses the pitch angle θ in function of the stick position δ_s besides:

$$\frac{\theta}{\delta_s} = \frac{5.26\,(0.2)\,e^{-0.244s}}{s\,[0.964, 2.35]} \tag{6}$$

This last one is composed of a second-order system plus a delay and uses the conventional notations (a) for $(s+a)$ and $[\xi,\omega]$ for $s^2+2\xi\omega s+\omega^2$.

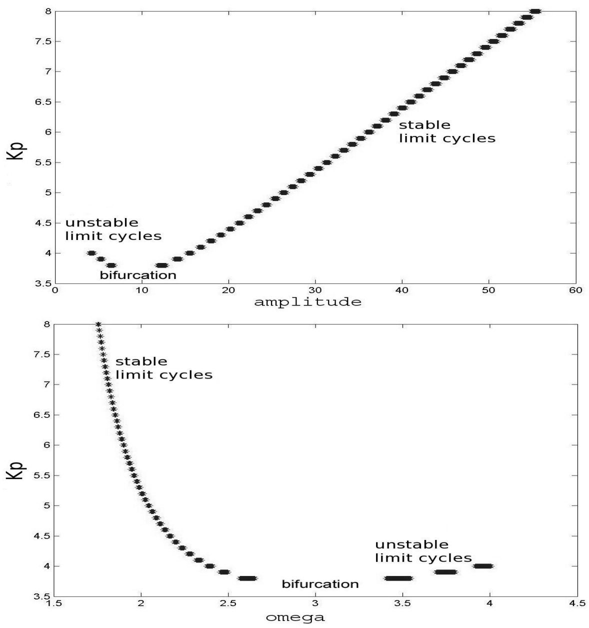

Fig. 5 Amplitude and pulsation of periodic orbits in function of pilot gain K_p

The figure (5) shows the configurations for which there are periodic orbits and estimate their pulsation and amplitude. Three ranges of pilot gain K_p can be distinguished:

- $K_p < 3.72$: no limit cycle ;
- $3.72 < K_p < 4.1$: two limit cycles, the one with the lowest amplitude and the highest pulsation is unstable whereas the one with the highest amplitude and the lowest pulsation is stable ;
- $K_p > 4.1$: one unique stable limit cycle.

At the critical pilot gain for which two periodic orbits appear (one stable and one unstable), the theory of dynamical systems stipulates that there is a saddle-node bifurcation of limit cycles [6]. It correspond also to the triggering condition of a PIO phenomenon.

The previous methodologies were interested more or less by the first harmonic evolution and properties. But there exist other approaches aiming at analysing nonlinear flight control system as the one investigated next based on optimisation and time simulation.

2.4 Time domain Neal-Smith criterion

The Time domain Neal-Smith criterion was developed amongst others by the Air Force Research Laboratory of Wright-Patterson [7] and Calspan [8]. The piloting objectives are translated in terms of constraints and performance requirements. The pilot must reach a target as quickly as possible and with a minimum of oscillation and overshoot. An acquisition time is defined such that after it, the pilot must be in a region close to the target value. The root mean square of the attitude error after the acquisition phase is minimised. Variations of the acquisition time correspond to changes of the aggressiveness of the task performance and of the swiftness of the closed-loop response.

The pilot model is identified such that the closed-loop pilot-vehicle system satisfies a required phase angle at the bandwidth frequency and a limited gain loss. The evolution of the root mean square error and its derivative in function of the acquisition delay is examined. According to these features, the flight control system is then estimated as PIO prone or not.

The acquisition time is reduced and the behaviour of the system described by equation (6) is observed. The real state must be close to the target after an acquisition time. An optimisation process is launched so as to achieve this goal and to minimise the root mean square error.

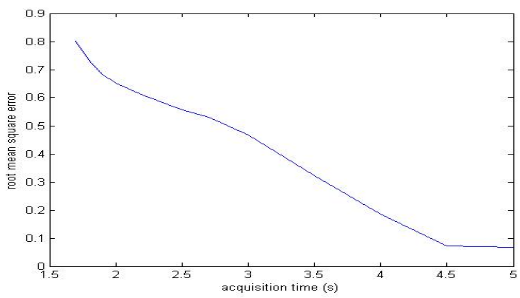

Fig. 6 Evolution of the root mean square error in function of the acquisition time

Even if the target value was quite high, in the figure (6), it is visible that the root mean square error is increasing too quickly when the acquisition delay is reduced. That is why according to this criterion, the inspected flight control system is clearly susceptible to PIO and needs some modification to alleviate the appearance of such phenomenon.

3 Anti-windup strategy

Once the actuator is saturated, the overall system can become unstable and divergent. In order to remedy to this situation, the level of saturation can be taken into account in the control value transmitted in order to increase the stability or to reduce the quadratic error between the linear and nonlinear configurations. The graph (7) exposes the results of an anti-windup strategy based on the optimisation of a basin of attraction and on a Lyapounov-like theorem [9].

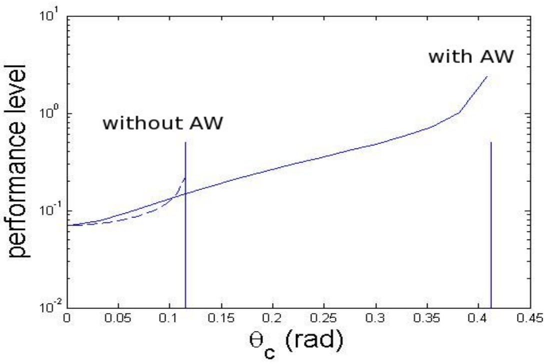

Fig. 7 Impact of an anti-windup device : Improvement of the performance level by means of an anti-windup strategy

With an anti-windup element, the acceptable pitch amplitudes for which the helicopter remains stable are higher. For low amplitudes, the quadratic performance level is besides better than without any anti-windup compensator. This improvement may also be employed so as to reduce PIO occurrences due to an excessive saturation level.

Conclusion

Several approaches were carried out in order to diagnose the level of susceptibility of several helicopter flight control systems. It appears to give some interesting insights concerning the design procedure that must be chosen for avoiding PIO occurrences. The judgement passed by diverse criteria are not always the same but nevertheless it proves to be often concordant or in the same appreciation field. Especially the influence of the modification of a feedback loop is apparently well estimated and a tendency may be observed. This paper reviews more or less the different mathematical criteria that were developed so as to estimate the PIO susceptibility. Even if some results are here valuable, it would still require further investigation and work on actual rotorcrafts to be complete.

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