ON THE COUPLING OF REANALYSIS TECHNIQUES WITH A SURROGATE-BASED DESIGN OPTIMIZATION METHOD

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Abstract

In this research, a Surrogate-Based Optimization (SBO) method is coupled with reanalysis techniques to improve the computational efficiency during optimization even further. The reanalysis techniques are used to speed up a reduction and a substructuring method, Craig-Bampton, which is utilized at the analysis step of the proposed SBO strategy. This strategy is suitable for solving problems where the modal and the harmonic responses of structures are required to be modified. An academic test problem is utilized for the demonstration.

1 Introduction

In engineering applications, designing and producing both economical and efficient products are necessary in order to be able to withstand global competition in the market. This is one of the main motivation of using optimization methods for many manufacturing companies. *Structural design optimization* problems require reliable analysis which is generally carried out by the Finite Element (FE) method. One of the main difficulties is that optimization of complex structures often requires numerous computationally demanding FE analyses.

Direct coupling of an FE model with numerical optimization algorithms is inevitable for problems which have many design variables, i.e. large scale optimization problems. If it is feasible to calculate the derivatives, the gradient-based algorithms are the most suitable because they require less function evaluations (FE analysis calls) than the derivative-free algorithms. On the other hand, efficient and accurate calculation of the derivatives are remaining issues in their application. Moreover, the analysis of a structure may fail when some of the design values are not feasible. For instance, direct coupling of a *Branch and Bound* type *Mixed-Integer-NonLinear-Programming* (MINLP) algorithm with an FE model might be problematic when non-integer values are assigned to some of the design variables.

For small-scale optimization problems (i.e. problems with small number of design variables), where the design domain wants to be explored globally, *Surrogate-Based Optimization* (SBO) can be a good alternative to algorithms based on direct coupling. The motivation of SBO is replacing expensive-to-evaluate FE models with their fast-to-evaluate approximations in optimization problems. These approximations are known as *meta-models*, *surrogate models* or *response surfaces* in the literature. When they are defined on the overall design domain, they are also called *global approximations*. Meta-models are built to predict the trends in the data collected from an FE model. The data consists of a set of values for the selected design variables and the response of the structure for these design values. Therefore, surrogate models can be considered as highly simplified versions of FE models.

Once a surrogate model is built, it is many orders of magnitude faster to evaluate than the FE model. Thus, it can be effectively employed in global optimization schemes. The number of

function evaluations in an optimization algorithm is not a big issue due to the simplicity of the surrogate models.

Analytical derivatives of the FE models are not required for building the surrogates. Additionally, analytical derivatives of the surrogates are not essential during optimization. Derivatives of these can accurately be calculated by the finite difference approximation.

When an optimization algorithm is directly coupled with an FE model, evaluation of the model is done sequentially during the search of an optimum. On the contrary, the FE model is only required for generating data for metamodeling in SBO. Hence, the data can be gathered all at once by parallel processing.

Data generation is still the challenging step of surrogate modeling. For obtaining certain accuracy, the total number of the data should be sufficient. On the other hand, with an increasing number of the design variables, the required number of data grows rapidly. Accordingly, the number of the FE analysis calls increases significantly. In order to reduce this computational burden, an SBO method is proposed in [2] for optimizing the dynamic behavior of structures where global approximations are utilized as surrogates. In the method, a reduction and a substructuring method, Craig-Bampton (CB), is used for offering solutions to one of the major difficulties in SBO, *the analysis time*. Using the CB method with SBO has the following additional advantages: **(1)** Reduction in the total d.o.f. leads to fast analysis of the complete structure. Meanwhile, the accuracy of the analyses are preserved within a low-frequency range. **(2)** Independent condensation of each substructure encourages parallel processing even further. **(3)** Preventing unnecessary calculations of the unmodified substructures, only the modified components can be analyzed and coupled with the already computed ones. **(4)** For structures having *repeated* components, modeling of one component is sufficient.

For reducing the analysis time even further, employing reanalysis methods can also be very useful. The objective of these methods is to evaluate the structural response due to the modifications in the design variables, using knowledge of the initial model. Therefore, solving a complete set of new equations is avoided. Integration of some reanalysis methods into the CB method is discussed in [1].

In this research, the SBO method proposed in [2] is coupled with the reanalysis techniques introduced in [1]. The structure of the paper is organized as follows: The CB method and the reanalysis methods are introduced briefly in Section 2 and Section 3. The new SBO method is introduced in Section 4. The final section includes the demonstration of the introduced concepts.

2 Craig-Bampton Method

The Craig-Bampton (CB) method [3] consists of breaking up a large structure into several substructures (components), obtaining reduced order system matrices of each component and then assembling these matrices to obtain the reduced order system matrices of the entire structure.

Assume that an FE model of a structure is constructed on a domain Ω and is divided into S non-overlapping substructures such that each component is defined on the sub-domain Ω^c. Thus, excepting the nodes on the interface boundaries, each node belongs to one and only one component. The linear dynamic behavior of an undamped component, labeled c, is governed by the equations,

$$\begin{bmatrix} \mathbf{M}^c_{ii} & \mathbf{M}^c_{ib} \\ \mathbf{M}^c_{bi} & \mathbf{M}^c_{bb} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{d}}^c_i \\ \ddot{\mathbf{d}}^c_b \end{Bmatrix} + \begin{bmatrix} \mathbf{K}^c_{ii} & \mathbf{K}^c_{ib} \\ \mathbf{K}^c_{bi} & \mathbf{K}^c_{bb} \end{bmatrix} \begin{Bmatrix} \mathbf{d}^c_i \\ \mathbf{d}^c_b \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}^c_i \\ \mathbf{f}^c_b \end{Bmatrix} + \begin{Bmatrix} \mathbf{0} \\ \mathbf{g}^c_b \end{Bmatrix} \tag{1}$$

where "i" and "b" refer to *interior* and *boundary*, respectively. In the formulation, $\mathbf{M}^c$, $\mathbf{K}^c$ and $\mathbf{d}^c$ are respectively the mass matrix, the stiffness matrix and the vector of the local d.o.f of the component. The vector $\mathbf{f}^c$ represents the external loads, and the vector $\mathbf{g}^c$ represents the interface loads between the component c and the neighboring components that ensure compatibility at the interfaces.

For reducing the size of the component matrices, $\mathbf{K}^c$ and $\mathbf{M}^c$, a subspace spanned by the columns of $\mathbf{T}^c$ is built in such a way that the solution of Equation (1) can be written in the form:

$$\mathbf{d}^c \approx \mathbf{T}^c \mathbf{q}^c \tag{2}$$

where $\mathbf{q}^c$ is a vector of generalized coordinates and $dim(\mathbf{q}^c) \ll dim(\mathbf{d}^c)$. $\mathbf{T}^c$ is referred to as a *reduction basis*, a *transformation matrix* or a *Ritz basis*.

The CB reduction basis is obtained utilizing the *fixed interface normal modes*, $[\mathbf{\Phi}_{\mathrm{i}}^c \;\; \mathbf{0}]^{\mathrm{T}}$, and the *constraint modes*, $[\mathbf{\Psi}_{\mathrm{ib}}^c \;\; \mathbf{I}_{\mathrm{bb}}]^{\mathrm{T}}$.

The fixed interface normal modes describe the internal dynamic behavior of a substructure. These modes are calculated by restraining all d.o.f. at the interface and solving an undamped free vibration problem

$$(\mathbf{K}_{\mathrm{ii}}^c - \omega_j^2 \mathbf{M}_{\mathrm{ii}}^c)\{\Phi_{\mathrm{i}}^c\}_j = 0 \quad j = 1, 2, \ldots, N_{\mathrm{T}} \tag{3}$$

where ω_j, $\{\Phi_{\mathrm{i}}^c\}_j$ are the jth natural frequency and the corresponding mode shape respectively, and, N_{T} is the truncated number of the normal modes which is usually a lot less than the actual number.

The motion on the substructure interfaces, the propagation of the forces between substructures and the necessary information about the rigid body motions are defined by the *constraint modes*. These modes are calculated by statically imposing a unit displacement to the interface d.o.f. one by one while keeping the displacement of the other interface d.o.f. zero and assuming that there are no internal reaction forces, i.e.,

$$\begin{bmatrix} \mathbf{K}_{\mathrm{ii}}^c & \mathbf{K}_{\mathrm{ib}}^c \\ \mathbf{K}_{\mathrm{bi}}^c & \mathbf{K}_{\mathrm{bb}}^c \end{bmatrix} \begin{bmatrix} \mathbf{\Psi}_{\mathrm{ib}}^c \\ \mathbf{I}_{\mathrm{bb}}^c \end{bmatrix} = \begin{bmatrix} \mathbf{0}_{\mathrm{ib}}^c \\ \mathbf{R}_{\mathrm{bb}}^c \end{bmatrix}. \tag{4}$$

In Equation (4), $\mathbf{R}_{\mathrm{bb}}^c$ is a matrix including the unknown reaction forces acting on the interface.

Therefore, the *Craig-Bampton transformation matrix* $\mathbf{T}_{\mathrm{CB}}^c$ for component c is defined as,

$$\mathbf{T}_{\mathrm{CB}}^c = \begin{bmatrix} \mathbf{\Phi}_{\mathrm{i}}^c & \mathbf{\Psi}_{\mathrm{ib}}^c \\ \mathbf{0} & \mathbf{I}_{\mathrm{bb}} \end{bmatrix}. \tag{5}$$

After defining the CB reduction basis $\mathbf{T}_{\mathrm{CB}}^c$, first, the right-hand side of Equation (2) is substituted into Equation (1) and then, Equation (1) is pre-multiplied by ${\mathbf{T}_{\mathrm{CB}}^c}^{\mathrm{T}}$. Hence, the reduced matrices of each component are defined by: $\bar{\mathbf{K}}^c = {\mathbf{T}_{\mathrm{CB}}^c}^{\mathrm{T}} \mathbf{K}^c \mathbf{T}_{\mathrm{CB}}^c$, $\bar{\mathbf{M}}^c = {\mathbf{T}_{\mathrm{CB}}^c}^{\mathrm{T}} \mathbf{M}^c \mathbf{T}_{\mathrm{CB}}^c$. The external loads and the interface loads are $\bar{\mathbf{f}}^c = {\mathbf{T}_{\mathrm{CB}}^c}^{\mathrm{T}} \mathbf{f}^c$ and $\bar{\mathbf{g}}^c = {\mathbf{T}_{\mathrm{CB}}^c}^{\mathrm{T}} \mathbf{g}^c$, respectively.

In the CB method, the assembly of the components is done using the compatibility of the interface d.o.f. [5]. This implies matching FE meshes at the interfaces.

3 Reanalysis Methods for Updating the CB Reduction Basis

Updating the fixed interface normal modes:

For updating the initial fixed interface normal mode set of a substructure, the Enriched CB method proposed by Masson et al. [4] is utilized. The idea behind the method is, first, calculating the residual forces $\mathbf{R}_{\mathrm{L}} = [\mathbf{f}_{\mathbf{\Delta}}(\omega_1), \ldots, \mathbf{f}_{\mathbf{\Delta}}(\omega_{N_{\mathrm{T}}})]$ acting on the initial substructure due to the design modifications where $\mathbf{f}_{\mathbf{\Delta}}(\omega_j) = -[\mathbf{\Delta K}_{\mathrm{ii}} - \omega_j^2 \mathbf{\Delta M}_{\mathrm{ii}}]\{\Phi_{\mathrm{i}}\}_j$, $\mathbf{\Delta K}_{\mathrm{ii}}$, $\mathbf{\Delta M}_{\mathrm{ii}}$ stand for the introduced modifications on $\mathbf{K}_{\mathrm{ii}}$ and $\mathbf{M}_{\mathrm{ii}}$ and, ω_j, $\{\Phi_{\mathrm{i}}\}_j$ are the jth natural frequency and the corresponding mode shape of a modified substructure, respectively. Afterwards, these residual forces are used to define a correction to the initial displacement field. However, their exact calculation is not possible by only knowledge of the initial substructure data. Therefore, they are approximated by, first, computing the residual forces $\hat{\mathbf{R}}_{\mathrm{L}} = [\hat{\mathbf{f}}_{\mathbf{\Delta}}(\omega_1), \ldots, \hat{\mathbf{f}}_{\mathbf{\Delta}}(\omega_{N_{\mathrm{T}}})]$ acting on the modified structure where $\hat{\mathbf{f}}_{\mathbf{\Delta}}(\omega_j) = -[\mathbf{\Delta K}_{\mathrm{ii}}\{\omega_j^0\}^2 \mathbf{\Delta M}_{\mathrm{ii}}]\{\Phi_{\mathrm{i}}^0\}_j$. The fixed interface normal modes and the corresponding eigenvalues of the initial model are represented by $\{\Phi_{\mathrm{i}}^0\}_j$ and $\{\omega_j^0\}^2$, respectively. Then, the approximate residual forces are defined as $\mathbf{f}_{\mathbf{\Delta}}(\omega_j) \approx y_1 \hat{\mathbf{f}}_{\mathbf{\Delta}}(\omega_1) + \ldots + y_{N_T} \hat{\mathbf{f}}_{\mathbf{\Delta}}(\omega_{N_T})$ where $\mathbf{y}^{\mathrm{T}} = \{y_1, y_2, \ldots, y_{N_T}\}$ is a vector of unknown coefficients. The residual forces $\hat{\mathbf{R}}_{\mathrm{L}}$ can also be utilized to replace $\mathbf{R}_{\mathrm{L}}$ and the corrections to the displacement field can be imposed using them. The essential idea of doing this is: if the subspace spanned by $\hat{\mathbf{R}}_{\mathrm{L}}$ does not contain the exact

residual force vectors with respect to a specific design modification, it may at least contain a reasonable representation of these vectors. The approximate correction matrix $\tilde{\mathbf{R}}_{\mathrm{D}}$ is then defined as $\tilde{\mathbf{R}}_{\mathrm{D}} = \mathbf{K}_{\mathrm{ii}}^{-1}\tilde{\mathbf{R}}_{\mathrm{L}}$ where $\tilde{\mathbf{R}}_{\mathrm{L}}$ is the reconditioned form of $\hat{\mathbf{R}}_{\mathrm{L}}$ by Singular Value Decomposition. Finally, the initial fixed interface normal mode set is enriched by $\tilde{\mathbf{R}}_{\mathrm{D}}$, that is,

$$\mathbf{\Phi} = \begin{bmatrix} \mathbf{\Phi}_{\mathrm{i}}^{0} & \tilde{\mathbf{R}}_{\mathrm{D}} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}.$$

This extended set of vectors is then used in the CB transformation matrix for the condensation of the modified component.

Updating the constraint modes:

For updating the initial constraint mode set of a substructure, a method based on the Combined Approximations (CA) approach is utilized [1]. The idea behind the method is approximating the residual constraint mode matrix, $\mathbf{\Delta\Psi}_{\mathrm{ib}}$, using the *conditioned binomial series expansion*. A brief description of the procedure is as follows:

The tth residual constraint mode $\{\mathbf{\Delta\Psi}_{\mathrm{ib}}\}_t$ is approximated in the space spanned by the vectors of the basis $\mathbf{H}_t = [\{\mathbf{\Delta r}_1\}_t, \ldots, \{\mathbf{\Delta r}_{N_b}\}_t], \;\; t = 1, 2, \ldots, N_{\mathrm{s}}$ where

$$\{\mathbf{\Delta r}_1\}_t = \mathbf{K}_{\mathrm{ii}}^{-1}\mathbf{R}_t, \; \{\mathbf{\Delta r}_k\}_t = -\mathbf{K}_{\mathrm{ii}}^{-1}\mathbf{\Delta K}_{\mathrm{ii}}\{\mathbf{\Delta r}_{k-1}\}_t.$$

In the formulation, $k = 2, 3, \ldots, N_{\mathrm{b}}$ indicates the number of the basis vector (binomial series term), N_{b} is the total number of the binomial series terms used in the approximation and $\mathbf{R}_t$ is the tth column of $\mathbf{R} = -\mathbf{\Delta K}_{\mathrm{ii}}\mathbf{\Psi}_{\mathrm{ib}}^{0} - \mathbf{\Delta K}_{\mathrm{ib}}$. The initial constraint mode matrix is represented by $\mathbf{\Psi}_{\mathrm{ib}}^{0}$.

Having defined the basis $\mathbf{H}_t$, $\{\mathbf{\Delta\Psi}_{\mathrm{ib}}\}_t$ can be approximated as

$$\begin{aligned} \{\mathbf{\Delta\Psi}_{\mathrm{ib}}\}_t &\approx \{\mathbf{\Delta r}_1\}_t y_{t,1} + \ldots + \{\mathbf{\Delta r}_{N_{\mathrm{b}}}\}_t y_{t,N_{\mathrm{b}}} \\ &= \mathbf{H}_t\mathbf{y}_t \end{aligned} \tag{6}$$

where $\mathbf{y}_t^{\mathrm{T}} = \{y_{t,1}, y_{t,2}, \ldots, y_{t,N_{\mathrm{b}}}\}$ is a vector of unknown coefficients. These coefficients can be obtained by solving a linear system of equations

$$\begin{aligned} &[\mathbf{H}_t^{\mathrm{T}}(\mathbf{K}_{\mathrm{ii}} + \mathbf{\Delta K}_{\mathrm{ii}})\mathbf{H}_t]\mathbf{y}_t = \\ &\qquad \mathbf{H}_t^{\mathrm{T}}(-\mathbf{\Delta K}_{\mathrm{ii}}\{\mathbf{\Psi}_{\mathrm{ib}}^{0}\}_t - \{\mathbf{\Delta K}_{\mathrm{ib}}\}_t) \end{aligned}$$

whose size is much smaller than that of the original one (the original system has the same size as Equation (4)). When this system is solved for $\mathbf{y}_t$ and the solution is inserted back into Equation (6), the tth residual constraint mode $\{\mathbf{\Delta\Psi}_{\mathrm{ib}}\}_t$ is computed approximately. Performing the above defined operations for each residual constraint mode, the CA approach of the residual constraint mode matrix $\mathbf{\Delta\Psi}_{\mathrm{ib}}$ is defined as

$$\mathbf{\Delta\Psi}_{\mathrm{ib}} = [\{\mathbf{\Delta\Psi}_{\mathrm{ib}}\}_1, \{\mathbf{\Delta\Psi}_{\mathrm{ib}}\}_2, \ldots, \{\mathbf{\Delta\Psi}_{\mathrm{ib}}\}_{N_s}].$$

Hence, the approximate constraint mode matrix is given by

$$\mathbf{\Psi} \approx \begin{bmatrix} \mathbf{\Psi}_{\mathrm{ib}}^{0} + \mathbf{\Delta\Psi}_{\mathrm{ib}} \\ \mathbf{I}_{\mathrm{bb}} \end{bmatrix}.$$

It is possible to automatize the calculation of the constraint modes. To do that,first, a value is assigned to the initial number of the basis vectors in the CA approach. Next, the number of FLoating-point OPerations (FLOPs) is counted [1]. This number is compared with the number of FLOPs of the exact analysis. The CA approach is used only when it requires less FLOPs than the exact analysis. If it is computationally efficient to be employed, the residual constraint mode matrix $\mathbf{\Delta\Psi}_{\mathrm{ib}}$ is calculated using CA. The accuracy of the approximation is verified [1]. If the accuracy is not satisfactory and the number of FLOPs of CA is still less than the exact analysis when a new vector is added to the basis $\mathbf{H}_t$, it is extended with this vector. The reanalysis is performed again. Otherwise, the constraint modes are computed with the exact analysis.

4 Surrogate-Based Optimization Method

The solution process of the SBO method is as illustrated in Figure 1. It starts with the problem analysis which involves, first, understanding the problem under consideration. Then, selection of the design variables and parameterization of the computational model are carried out. Finally the objective function and the constraints are defined.

The second step is to generate the surrogate model. Here, firstly a set of sample points is selected from the design space which is called Design of Experiments (DOE). In the method, Latin

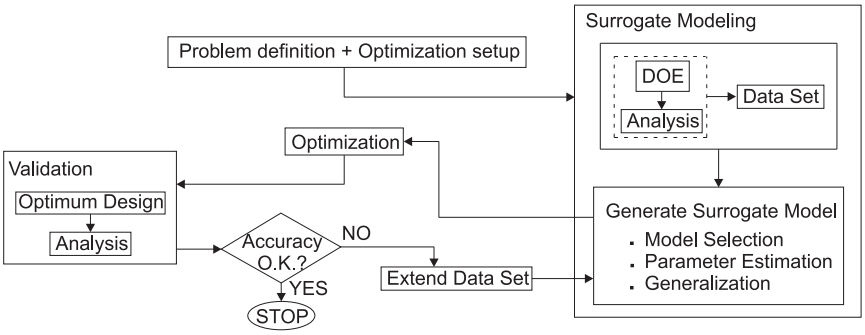

Fig. 1 Schematic illustration of the SBO method.

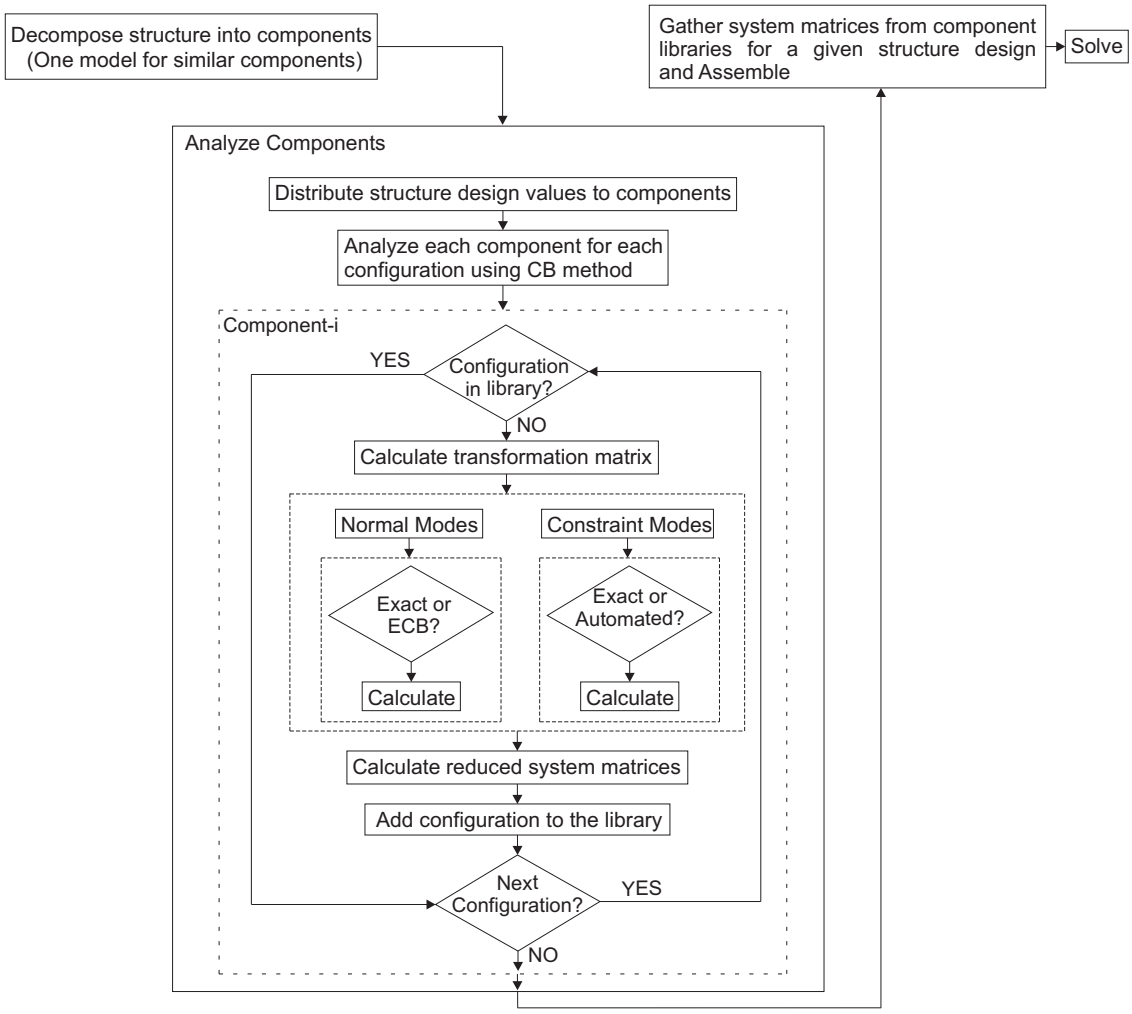

Fig. 2 Schematic illustration of the analysis step of the SBO method.

Hypercube Sampling (LHS) scheme is utilized to generate the DOE set. Afterwards, for each sample point, the FE model is run and data is gathered for training the surrogate. At the *analysis* step, the Craig-Bampton (CB) method is used as a CMS technique. Furthermore, reanalysis methods are considered for efficient calculation of the CB transformation matrices of the modified components. The followed steps at the analysis phase are shown schematically in Figure 2. For the dynamic analysis of a structure, first, the complete structure is divided into components. Then the

parameterized FE model of each component is built. If there are similar components, only one of them is modeled. Afterwards, the design values of the complete structure are distributed to components based on the design variables captured in the component models. An FE model standing for similar components may get multiple configurations for its design variables. The next step is the calculation of the reduced system matrices of each component for the assigned design values. In the proposed scheme, libraries are used to store the information about the already analyzed components. Hence, unnecessary analyses are prevented. Before generating the system matrices of a given component design, first, the corresponding library is checked. If the requested information is not there, it is computed and stored in the library. In the computation, first of all, the transformation matrix, consisting of the normal and the constraint modes, is calculated. The normal modes can be computed either using the exact analysis methods or using the Enriched Craig-Bampton (ECB) method. Unfortunately, there is no automated switch from ECB to the exact methods based on the accuracy and/or the computational efficiency of ECB. On the other hand, calculation of the constraint modes, either by the exact or the approximate methods, can be automated. The approximate constraint modes are calculated using the Combined Approximations (CA) approach. After the transformation matrix is determined, the reduced component matrices are computed. The given component design, its transformation and the reduced matrices are saved in the component library. This procedure is repeated for each component and the corresponding configurations. This ensures that all the necessary information to generate the reduced system matrices of the complete structure is readily available in the libraries for further use. Thereafter, the stored reduced matrices are gathered from the libraries for the given structure design and assembled to obtain the reduced matrices of the entire structure. Finally, the dynamic analysis of the structure is performed.

After defining the data set, a suitable meta-modeling approach is selected and the unknown parameters of the chosen meta-model are determined using the available data. In proposed method, Neural Networks are employed for this purpose.

Having generated the surrogate model, the next step is the optimization where the global optimum is sought using a Multi-Level Hybrid Optimization (MLHO) scheme. In MLHO, a stochastic derivative-free global optimization method, the Genetic Algorithm (GA), is employed to locate the global optimum. A gradient-based method, Sequential Quadratic Programming, is initialized with the solution of GA to find an exact optimum solution.

Since the calculated optimum is not directly related with the FE model but the surrogate model, the results need to be validated. In order to do that, the response of the FE model is obtained by the computed optimum design values. This is then compared with the response of the surrogate model for the same design values. If the accuracy is acceptable, the scheme is stopped. Otherwise, the data set is extended with the optimum design values and the corresponding response of the FE model. New parameters for the selected surrogate model are computed using the extended data set and the optimization step is repeated. This procedure is iterated until the validation results are acceptable.

5 Demonstration of the Concepts

For the demonstration of the introduced concepts, an idealized fuselage structure, shown in Figure 3, is utilized. The structure is composed of 8 identical components and it is free at the boundaries. A component consists of a cylinder skin including a floor panel, frames and stiffeners whose geometry is as illustrated in Figure 3.

The reduced system matrices of the entire structure are obtained by only modeling one component. The FE model of a component is generated in the commercial FE software ANSYS. Its system matrices are calculated for the defined design variables and then they are transferred to MATLAB. For obtaining the reduced system matrices of the components, first of all the trans-

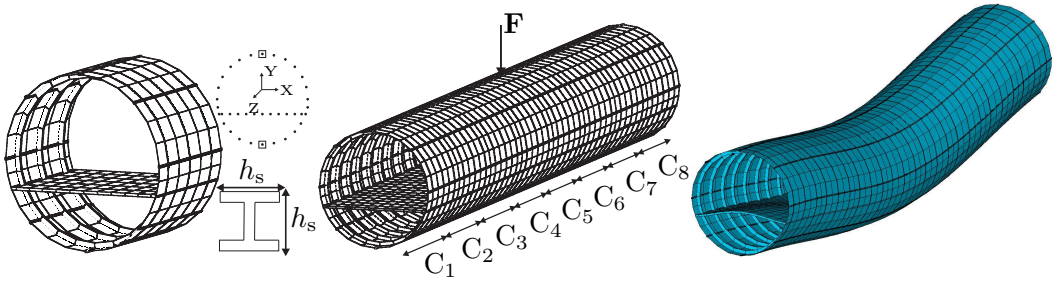

Fig. 3 Test problem. (Left) Component model, (Middle) Selected structure under applied force, (Right) First bending mode of the initial design.

formation matrices are computed and afterwards condensation is performed. In the transformation matrices 18 fixed interface normal modes are used. The number of the nodes on one interface of a component is 37. After the reduced matrices of all the components are obtained, these matrices are assembled and the reduced system matrices of the entire structure are gathered.

The skin, floor and frames are modeled using a 4-node shell element which has 6 d.o.f. at each node and is suitable to analyze thin to moderately thick shell structures. The stiffeners with *I cross-section* are modeled with a three dimensional beam element which has 6 d.o.f. at each node. It allows different cross sections and permits the end nodes to be offset from the centroidal axes of the beam. The cross section width and height of the stiffeners ($\mathrm{h_s}$) in the components (see Figure 3) are defined as the design variables and all the stiffeners of a component are assumed to have the same design values. Therefore, there exist 8 design variables in total in the overall structure. Each component has one design variable. For the initial design h_{s_i}, $i = 1, 2, \ldots, 8$ are set to 0.05m.

There is a harmonic force acting on the structure. The applied load has an amplitude of 100 kN and is in the y-direction. It is applied on the top interface node of the 4th component C_4 and the 5th component C_5 as shown in Figure 3. For the harmonic response analysis, structural damping with an energy dissipation of 3% is assumed which is imposed directly on the reduced stiffness matrix of the structure.

For the harmonic response analysis, focus is on the frequency range of $10 - 30$ Hz. This interval involves the first bending frequency of the initial design. Figure 3 shows the mode shape of this frequency. The objective is to reduce the amplitude of the displacement response in this frequency range, thereby decreasing the displacement response of the structure for the first bending mode. The nodes that lie on the top and the bottom interface of the components are selected to prescribe the objective function. Figure 3 illustrates the nodes corresponding to the interface of a component. The selected nodes are identified with squares around them. The displacement magnitudes in the y-direction are computed for these nodes in the frequency range of $10 - 30$ Hz and then summed up. The response curve that represents the "frequency-displacement magnitude" relationship of the initial design is plotted in Figure 4. The results displayed in the figure are obtained by the full FE analysis performed in ANSYS.

The objective function of the problem is defined as minimizing the total area, $\mathrm{A}(\mathbf{h})$, beneath the response curve. The area beneath the response curve is 0.95 m.Hz for the initial design.

The constraints of the problem are as follows:

- Keeping the first bending frequency around 22 Hz. This constraint is defined as $22 - \epsilon \leq f_7(\mathbf{h}) \leq 22 + \epsilon$ where $\epsilon = 0.02$.
- Keeping the total final mass of the stiffeners less than the total initial mass of the stiffeners. This is given as, $\sum_{i=1}^{8} [\rho\, \mathrm{V}_i(\mathrm{h}_{s_i})] \leq 23$ where $\mathrm{V}_i(\mathrm{h}_{s_i})$ is the total volume of the stiffeners in component C_i and ρ is the density of the stiffeners.

- Preserving the mode shape of the first bending frequency. This is assured by the MAC criterion. $\mathrm{MAC}_7(\mathbf{h}) \geq 0.9$.

- Having a symmetric final configuration. This constraint is prescribed by forcing the design variables of the component pairs; C_1-C_8, C_2-C_7, C_3-C_6, C_4-C_5 to have similar values. This is imposed by: $h_{s_j} - h_{s_{(9-j)}} \leq 10^{-4}$, $h_{s_{(9-j)}} - h_{s_j} \leq 10^{-4}$, $j = 1, 2, \ldots, 4$.

- The upper and the lower bounds for the design variables are selected as $0.01 \leq h_{s_i} \leq 0.1$, $i = 1, 2, \ldots, 8$.

In the optimization problem, 3 surrogate models are used. These surrogates stand for $\mathrm{A}(\mathbf{h})$, $f_7(\mathbf{h})$ and $\mathrm{MAC}_7(\mathbf{h})$.

The DOE set $\mathbf{D}_\mathrm{T}$ of the whole structure has 81 designs where each design defines a new structure configuration.

At the analysis step of the SBO method, the transformation matrix of each modified component is computed using one of the following methods:

Exact: The fixed interface normal modes and the constraint modes of the Craig-Bampton (CB) transformation matrix are computed by exact analysis methods all over again.

ECB+CA: The initial fixed interface normal mode set is extended using the Enriched Craig-Bampton (ECB) method. The constraint modes are approximated by the Combined Approximations (CA) approach. The minimum number of the basis vectors are set to 3. The accuracy of the modes are verified and if it is not satisfactory, the set of the CA basis vectors is extended with a new vector and the reanalysis step is repeated.

ECB+CA Automated: The initial fixed interface normal mode set is extended using the Enriched Craig-Bampton (ECB) method. For the calculation of the constraint modes, the automated update scheme defined in Section 3 is used. The minimum number of the basis vectors are set to 3 in the CA approach.

After the transformation matrix of a component is calculated using one of the above methods, condensation of the component matrices are performed.

The responses, $\mathrm{A}(\mathbf{h})$, $f_7(\mathbf{h})$ and $\mathrm{MAC}_7(\mathbf{h})$, of the structure for each configuration in $\mathbf{D}_\mathrm{T}$ are calculated using the assembled reduced component system matrices and the training data sets are gathered for meta-modeling.

3 separate libraries are used for storing the transformation, the reduced stiffness and the reduced mass matrices of each new component design. The first library is for component C_1, the second one is for components $C_2, C_3, \ldots, C_7$ and the third one is for C_8.

The following cases are considered.

Case 1: The optimization problem is solved twice. First, the **Exact** approach is used for the calculation of the transformation matrices during the analysis step of the SBO method. In the second solution, instead of the **Exact** approach, the **ECB+CA Automated** approach is used. The performance of the SBO method is evaluated regarding the accuracy of the results and the computation time. In short, in Case 1, the computational efficiency and the accuracy of the reanalysis methods are tested. It is important to emphasize that the number of FLOPs for the exact analysis of the constraint modes is smaller than that of the CA approach in the selected structure. Accordingly, in the **CA Automated** approach, the constraint modes are always computed by the exact analysis.

Case 2: As mentioned in Case 1, the constraint modes are always computed by the exact methods in the **CA Automated** approach. In order to examine the accuracy of the CA method, the **ECB+CA** approach is used in the analysis step of the SBO method. The final design configuration and the corresponding analysis results are compared with the solutions found in Case 1.

In all the test cases, the NN model is employed with 25 hidden layer neurons.

The search for optimum is repeated until the relative errors between the responses of the FE model and that of the surrogates are smaller than 0.005 for the computed optimum design values. The relative error is computed with respect to the FE analysis results.

Before calculating the reduced system matrices of the components for the optimum design values, first the libraries are checked for similar component designs. These designs are sought with a relative error tolerance of 10^{-3}. The relative error is calculated with respect to the investigated optimum design.

5.1 Results and Discussions

Case 1:
The results of Case 1 are summarized in Table 1. The "frequency-displacement magnitude" curves that correspond to the final configurations are shown in Figure 4. To validate the results, the response of the structure is calculated in ANSYS using the full FE analysis for the final design values. These solutions are also presented in Figure 4.

Both of the final configurations are feasible. These configurations have almost the same design values. The optimal configuration is stiffest in the middle while the stiffness decreases towards the free ends of the structure. The total area beneath the "frequency-displacement magnitude" curve is reduced by almost 14% in both **Exact** and **ECB+CA Automated**.

The total required time for the optimization process decreases around 30% when the **ECB+CA Automated** approach is utilized in the SBO method.

As observed from the results, the total number of the iterations required in the SBO method are very low.

The accuracy and the computational efficiency of the SBO method with **ECB+CA Automated** approach is very satisfactory for the selected problem.

Case 2:
The results of the SBO method with the **ECB+CA** approach are summarized in Table 2.

The "frequency-displacement magnitude" curve that corresponds to the final configuration is plotted in Figure 5.

The final design is very similar to those obtained in Case 1 and it fulfills all the constraints. The total area beneath the "frequency-displacement magnitude" curve is reduced by almost 14%.

As seen in Figure 5, the accuracy of **ECB+CA** is satisfactory compared to the full FE analysis results for the final design values.

Table 2 Summary of Case 2 results.

	ECB+CA
Final Design (m)	$[0.01, 0.01, 0.059, 0.1]$ $[0.1, 0.059, 0.01, 0.01]$
Final Area (m.Hz)	0.8215
Final Mass (kg)	21.9
Total # of iterations	6

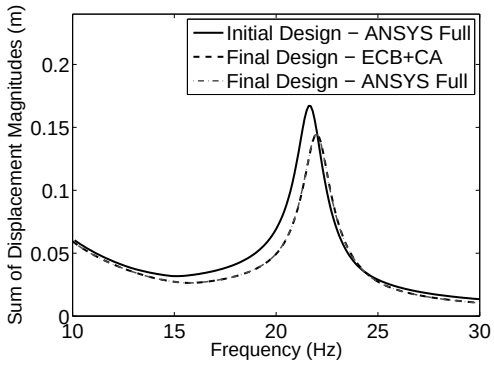

Fig. 5 Results of Case 2. The CB transformation matrices are computed by the **ECB+CA** approach in the SBO method.

6 Summary and Conclusions

The contribution of this research is proposing solutions to one of the major difficulties, *analysis time*, in structural optimization by taking the advantage of effective structural analysis and reanalysis techniques in an SBO scheme.

Integration of two reanalysis techniques into the Craig-Bampton (CB) method is introduced. This is then used at the analysis step of a Surrogate-Based Optimization strategy for improving the computational efficiency during optimization. The strategy is demonstrated by an

Table 1 Summary of Case 1 results.

	Exact	**ECB+CA Automated**
Final Design (m)	$[0.01, 0.01, 0.058, 0.1, 0.1, 0.058, 0.01, 0.01]$	$[0.01, 0.01, 0.063, 0.1, 0.1, 0.063, 0.01, 0.01]$
Final Area (m.Hz)	0.8238	0.8161
Final Mass (kg)	21.7	22.5
Total # of iterations	4	4
Computation time	5h06min	3h34min

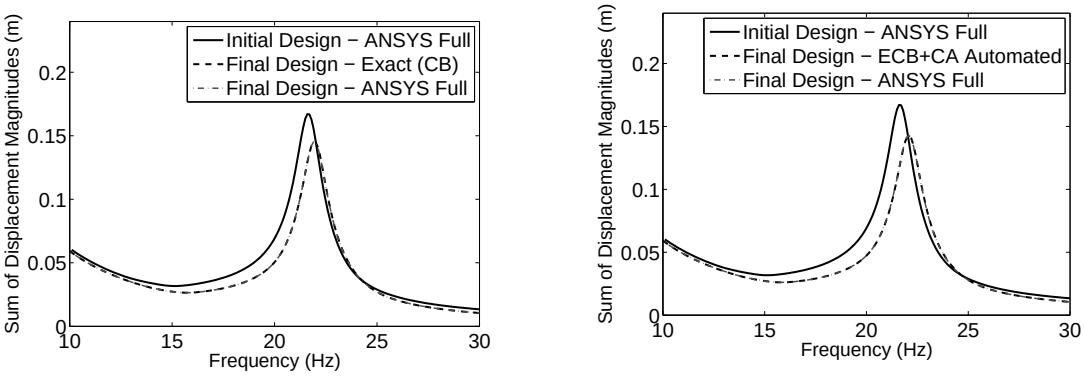

Fig. 4 Results of Case 1. (Left) The CB transformation matrices are computed by the **Exact** approach in the SBO method, (Right) The CB transformation matrices are computed by the **ECB+CA Automated** approach in the SBO method.

academic test problem.

The results of the test case are very promising for the application of the proposed strategy on small-scale optimization problems where the dynamic behavior of large complex structures wants to be modified. It is believed that the efficiency of the strategy will be more pronounced when tested on more complex problems.

6.1 Copyright Statement

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