OPTIMIZATION OF AIR RACE TRAJECTORIES

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Abstract

A novel approach for the optimization of air race trajectories that takes into account the highly non-linear nature of the dynamics of the participating aircraft is presented. For the optimization an enhanced, scalable multi-fidelity simulation model is utilized which is a sequential model extending the translation and position dynamics by different representations for the attitude and the rotational dynamics in the inner loop of the flight system. The scalable inner loop can either contain linear transfer functions for the load factors and the roll rate, linear state-space models for the longitudinal and the lateral motion of the aircraft or the fully non-linear rotational and attitude dynamics. With the sequential structure of the model, the complexity level of the inner loop and thus the optimization time and quality can easily be adapted to the required level. Furthermore, inversion controllers for the different loops are incorporated in the simulation model that enable the development of a procedure for the generation of robust and suitable initial guesses for the optimization with full non-linear 6-Degree-of-Freedom simulation models. The novel approach allows for the solution of highly complex trajectory optimization problems where classical methods failed mainly due to stiffness problems. The full dynamic order of the flight system considered is taken into account such that the resulting optimal race trajectory is truly achievable.

1 Introduction

Over the past years, air race events have become increasingly popular and even a global air race world series has been established with great success. These air races are attracting a large audience, thus enhancing the popularity and the fascination for aerospace in the public. The basic procedure of the regarded air races is as follows: after passing a starting point, which can be defined by a significant landmark, the aircraft consecutively have to fly a course defined by inflatable pylons at minimum time. The pylons form gates which either are to be passed wings level (level gate), or at 90° bank angle (knife edge gate). Other features are the "quad", consisting of two pylon pairs to be passed from perpendicular directions or the slalom which is a chicane built of a sequence of single pylons requiring rapid changes in turn direction. Furthermore, re-alignment and aerobatic maneuvers like vertical rolls or Half Cuban Eights are included to re-position the aircraft with respect to the track. The race ends by passing a finish gate which in many cases is equal to the start gate or again a significant landmark.

In order to win such a competition, the pilot has to find the fastest possible flight course through the gates, i.e. he tries to finish the race course in the minimum possible flight time. As the best trajectories only become apparent during training and at the race itself, shaped by the pilots' experience, it would be beneficial to know the optimum (i.e. minimum time) trajectory for a given aircraft right before the first aircraft enters the course and flight path optimization is the only efficient means to compute the optimal race track. This does not only help the pilots to find their best strategy but also the organizer and the local authorities to assess the track with respect to the single most important criterion that is safety. Moreover, so far the trajectory flown could only be judged by the final race time but no clear insights could be gained which parts of the race course flown deviate the most from the optimal trajectory and thus would offer room for improvement. Therefore, knowing the optimal race trajectory would be very interesting for the pilots, the planners, the evaluators as well as for the spectators.

The task of finding the necessary control inputs to produce the minimum race time for a specific air race track represents a typical multi-phase trajectory optimization problem that is to find the control inputs such that the resulting state histories become optimal with respect to a certain objective, here the final race time. Over the last decades, a vast number of trajectory optimization problems have been solved, mainly utilizing point-mass models (see e.g. [1], [7], [8], [9], [11], [13]) or only a single maneuver over a very short time span with a full 6-Degree-of-Freedom simulation model has been optimized ([5], [12], [14]). But for air race trajectories with their inherent, highly non-linear nature and flight at the limits of the envelopes and with frequently saturated controls, point-mass models can no longer adequately represent the dynamic order of the considered flight system. For the simulation and the optimization of air race trajectories it is mandatory to take into account the true attitude and the rotational dynamics of the flight system to achieve a realistic representation of the real aircraft. With the utilization of full, non-linear 6-Degree-of-Freedom simulation models for the optimization of trajectories over a long time span as it is the case for air races, optimization tasks suffer from many severe problems ranging from poor convergence properties to the difficulty of finding initial guesses for the optimal solutions. Thus, a novel approach has to be chosen for the optimization of air race trajectories.

This approach requires a simulation model of special structure. The outer loop contains the position and translation equations of motion followed by a scalable inner loop that represents the rotation and attitude equations of motion of the flight system. In the simulation model, three alternatives for the inner loop are implemented: first, linear transfer functions for the load factors and the roll rate, second, linear state-space models for the lateral and longitudinal motion and third, the full non-linear attitude and rotational dynamics. Additionally, inversion controllers for each inner loop are incorporated into the simulation model. With this scalable, multi-fidelity simulation model the complexity of the inner loop can easily be changed so that the fidelity of the entire simulation model and the computational effort for solving an optimization task can be adapted to the appropriate level while it is ensured that the full dynamic order of the flight system is taken into account for the optimization task. Thus, the resulting optimal trajectory is highly realistic. Furthermore, with the utilization of the mentioned simulation model for optimization tasks, a procedure for the generation of very good initial guesses for the solution of optimization problems based on the full non-linear simulation model can be established. Therefore, the optimal solution found for one complexity level is used as the starting solution for the higher level of complexity, thus improving the convergence time and stability for this optimization task since the applied starting solution presumably comes close to the final optimal solution.

In order to get as close as possible to reality, the simulation model also accounts for atmospheric effects like offset in the ambient temperature and pressure as well as for static and convective wind fields since the atmospheric conditions have a great influence on the optimal trajectory.

In the paper at hand, first the simulation model with its different alternatives for modeling the linear inner loop is described. Then, the trajectory optimization problem for air races utilizing this simulation model is formulated. Next, a procedure for generating good initial guesses for the various optimization problems is introduced. Finally, results of solved air race trajectory optimization problems are shown demonstrating the capability of the proposed approach.

2 Simulation Model

In this chapter, the scalable multi-fidelity simulation model that constitutes the basis for the optimization procedure to be explained in chapter 4 is described. Basically, the simulation model is divided into two parts that are an outer loop containing the position and translation equations of motion and an inner loop representing the attitude and rotation dynamics of the flight system. For the inner loop, three different alternatives are implemented and later on used for the optimization tasks: first, a linear inner loop that consists of linear transfer functions for the load factors and the roll rate, second, an inner loop with linear state-space models for the longitudinal and lateral motion of the aircraft and third a non-linear inner loop modeling the full attitude and rotational dynamics. The structure of the simulation model with its scalable inner loop is depicted in Fig. 1.

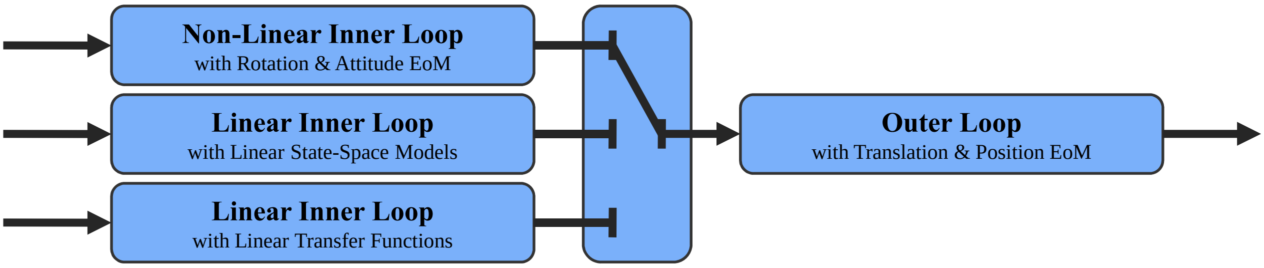

Fig. 1. Simulation Model Structure with Scalable Inner Loop

For conventional non-linear 6-Degree-of-Freedom simulation models, both the position and translation equations of motion and the attitude and rotation equations of motion are given with respect to the *NED*-Frame, i.e. that the inner loop and the outer loop are modeled "in parallel". In contrast to these standard simulation models, the herein presented simulation model features a sequential structure, where the inner loop is modeled in series to the outer loop, providing the input to the outer loop: the load factors in the Intermediate Kinematic Flight-Path Frame $\bar{K}$. This means that the attitude and rotation dynamics are not given with respect to the *NED*-Frame as usual but with respect to the Kinematic Flight-Path Frame K. The innovative structure of the simulation model is illustrated in Fig. 2. Since the inputs to the outer loop always remain the same irrespective of the modeling of the inner loop, the sort of modeling for the inner loop can easily be altered without affecting the outer loop. This specific structure allows for an easy adjustment of the simulation model to the required level of accuracy and complexity, to the desired computation time or computational robustness or to the aircraft data available for a specific simulation or optimization task. At the same time it is ensured that in principal the full dynamic order of the respective flight system is

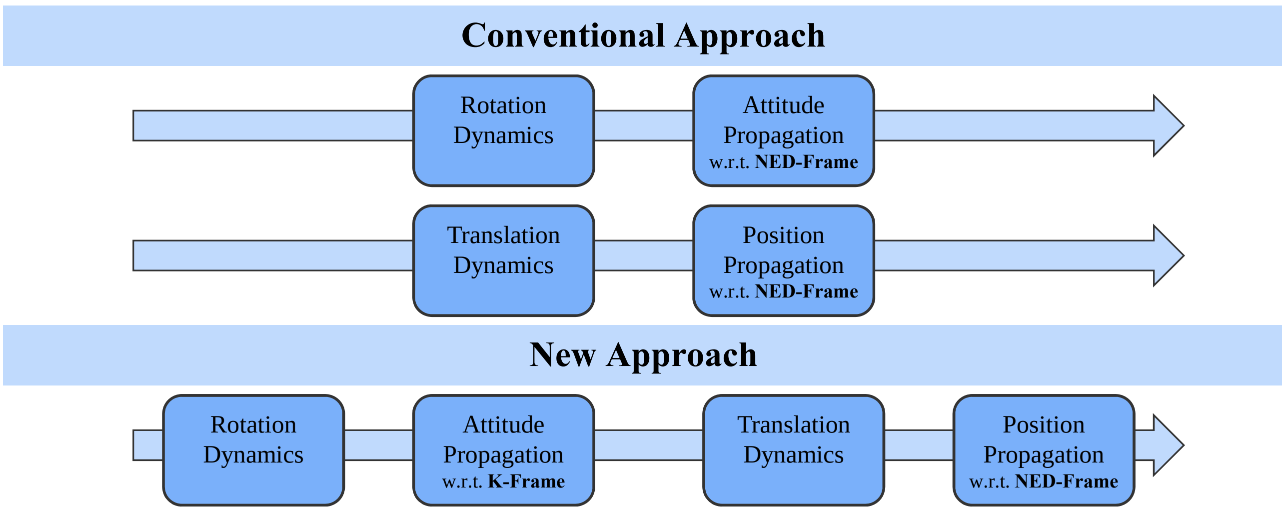

Fig. 2. Sequential Structure of the Simulation Model

taken into account for the optimization task so that the resulting optimal trajectory is highly realistic.

Furthermore, environmental influences like deviations from the standard atmosphere or static and convective wind fields are accounted for in the simulation respectively the optimization model in order to achieve flight paths that are as realistic as possible. Especially the influence of wind fields on the optimal trajectories is very significant and therefore cannot be omitted.

In addition to the outer loop with the translation and position equations of motion and the inner loop with the attitude and rotation dynamics, the simulation model is augmented by inversion controllers for the respective loops that are essential for the optimization procedure given below. The inversion controllers are based on a dynamic inversion of the principal physical causal chains of dynamic flight systems. At this, a causal chain describes the relation between a control surface deflection and a change in the position of the aircraft, e.g. the relation between an elevator deflection and the resulting change in the aircraft's altitude. With the dynamic inversion controller, the corresponding control surface deflections for given state time histories can be computed so that the flight system follows the prescribed state profiles. In the simulation model, inversion controllers for the outer loop as well as inversion controllers for the inner loop with its different depths of modeling are implemented.

A detailed description of the simulation model including the incorporated inversion controllers can be found in Ref. [2].

3 Statement of the Optimization Problem

In general, an optimal control problem can be stated as follows: Determine the optimal control history

$$\mathbf{u}_{opt}(t) \in \mathbb{R}^m \qquad (1)$$

and the corresponding optimal state trajectory

$$\mathbf{x}_{opt}(t) \in \mathbb{R}^n \qquad (2)$$

that minimize the Bolza cost functional

$$J = e\left(\mathbf{x}(t_f), t_f\right) + \int_{t_0}^{t_f} L\left(\mathbf{x}(t), \mathbf{u}(t), t\right) dt \qquad (3)$$

subject to the state dynamics

$$\dot{\mathbf{x}}(t) = f\left(\mathbf{x}(t), \mathbf{u}(t), t\right) \qquad (4)$$

the initial and final boundary conditions

$$\boldsymbol{\psi}_0\left(\mathbf{x}(t_0), \mathbf{u}(t_0), t_0\right) = \mathbf{0} \qquad \boldsymbol{\psi}_0 \in \mathbb{R}^q \qquad (5)$$

$$\boldsymbol{\psi}_f\left(\mathbf{x}(t_f), \mathbf{u}(t_f), t_f\right) = \mathbf{0} \qquad \boldsymbol{\psi}_f \in \mathbb{R}^p \qquad (6)$$

the interior point conditions

$$\mathbf{r}\left(\mathbf{x}(t_i), \mathbf{u}(t_i), t_i\right) = \mathbf{0} \qquad \mathbf{r} \in \mathbb{R}^k \qquad (7)$$

and the equality and inequality conditions

$$\mathbf{C}_{eq}\left(\mathbf{x}(t), \mathbf{u}(t), t\right) = \mathbf{0} \qquad \mathbf{C}_{eq} \in \mathbb{R}^{\mathrm{r}} \qquad (8)$$

$$\mathbf{C}_{ineq}\left(\mathbf{x}(t), \mathbf{u}(t), t\right) \leq \mathbf{0} \qquad \mathbf{C}_{ineq} \in \mathbb{R}^{\mathrm{s}} \qquad (9)$$

For the air race trajectory optimization problem, the Bolza cost functional reduces to a Mayer functional since the only objective is to minimize the final time:

$$J = t_f \qquad (10)$$

Finally, the state vector and control vector for the inner loop with the full, non-linear attitude and rotational dynamics are:

$$\mathbf{x} = \left[x, y, z, V_K, q_0, q_1, q_2, q_3, \alpha_K, \beta_K, p_K, q_K, r_K, \delta_T\right]^T \qquad (11)$$

$$\mathbf{u} = \left[\eta, \xi, \zeta, \delta_{T,CMD}\right]^T \qquad (12)$$

The initial boundary conditions for the optimization problem are given by the position of the start gate, whereas the final boundary conditions are determined by the location of the finishing gate and the direction the finishing gate has to be passed by the aircraft:

$$\begin{aligned} \vec{\mathbf{r}}(t_0) - \vec{\mathbf{r}}_{StartGate} &= \mathbf{0} \\ \vec{\mathbf{r}}(t_f) - \vec{\mathbf{r}}_{FinalGate} &= \mathbf{0} \end{aligned} \qquad (13)$$

The requirement that the pilot has to fly through certain gates in a certain direction and at given bank angles imposes interior point conditions to the trajectory optimization problem. Basically, there are two different types of gates, level gates and knife edge gates. Level gates have to be passed wings level, i.e. with the kinematic bank angle μ_K equal to zero whereas knife edge gates have to be flown through with a bank angle $\mu_K = 90°$. The resulting conditions read:

$$\begin{aligned} \mu_K\left(\bar{\mathbf{r}}_{KnifeEdgeGate}\right) - 90° = 0° \\ \mu_K\left(\bar{\mathbf{r}}_{LevelGate}\right) = 0° \end{aligned} \qquad (14)$$

By separating the entire race trajectory at these gates into multiple phases, the interior point conditions are transformed into final boundary conditions for each phase. The phases then have to be connected to the preceding phases to guarantee the continuity of the state and the control time histories:

$$\begin{aligned} \mathbf{x}_{i-1}\left(t_{f,i-1}\right) - \mathbf{x}_i\left(t_{0,i}\right) = 0 \qquad i = 2,\dots,n \\ \mathbf{u}_{i-1}\left(t_{f,i-1}\right) - \mathbf{u}_i\left(t_{0,i}\right) = 0 \qquad i = 2,\dots,n \end{aligned} \qquad (15)$$

where n denotes the number of phases, $t_{f,i}$ the final time of the i-th phase and $t_{0,i}$ the initial time of the i-th phase. Additionally, equality and inequality conditions have to be fulfilled along the flight path for an air race. Among others, the most important inequality path conditions result from safety regulations and require that a certain load factor limit must not be exceeded and that a minimum distance to the crowd has to be maintained. Further, inequality conditions affect the performance limits of the aircraft, i.e. the maximum roll rate or the maximum angle of attack. These conditions are cast in the following form:

$$\begin{pmatrix} -\left(n_{Z,\min} - n_Z(t)\right)\cdot\left(n_Z(t) - n_{Z,\max}\right) \\ -\left(d_{\min} - \left\|\bar{\mathbf{r}}(t) - \bar{\mathbf{r}}_{Crowd}(t)\right\|_2\right) \\ -\left(\left|p_K(t)\right| - \left|p_{K,\max}\right|\right) \\ -\left(\alpha_{A,\min} - \alpha_A(t)\right)\cdot\left(\alpha_A(t) - \alpha_{A,\max}\right) \end{pmatrix} \geq \mathbf{0} \qquad (16)$$

4 Description of the Optimization Procedure

A challenging task for the solution of every optimization problem is the creation of an initial guess that comes as close as possible to the optimal solution in order to guarantee stability and robustness of the optimization process as well as good convergence properties. The above

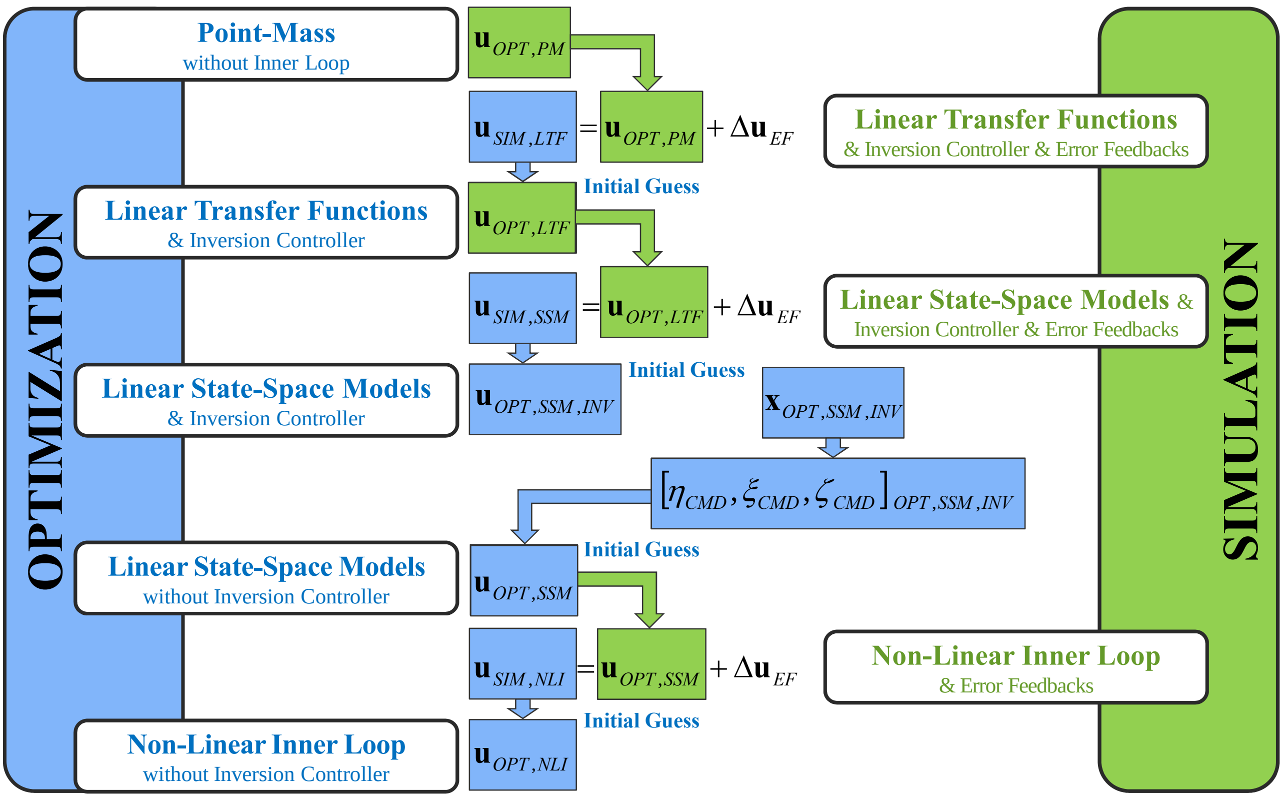

Fig. 3. Optimization Procedure

described simulation model allows for the setting up of an optimization procedure that is well suited for the generation of a good initial guess for the optimization of air race trajectories with full non-linear 6-Degree-of-Freedom simulation models. This novel procedure is described in the following, an overview of the various steps performed during this procedure is depicted in Fig. 3.

First, an optimal solution for the race course utilizing the point-mass model without an inner loop has to be found. The race course is divided into several phases, where each phase is defined as the flight path segment between two succeeding gates. An optimal solution for the first phase is computed using a homotopy procedure: at first, an optimization problem is solved where the starting gate and the next gate are positioned in line such that the aircraft can pass the two gates straight and level. Then, the position of the second gate and the attitude of the aircraft when flying through this gate are gradually changed until they meet the final boundary conditions given by the real position and type of the second gate. The final state and control values of the first phase are then utilized as the initial values for the second phase and an optimal solution for the second phase is computed just in the same manner as it has been computed for the first phase. This procedure is repeated until the last phase of the air race track is reached. Then, the computed states and controls of all the phases are put together, giving a quite good initial guess for the optimization of the whole air race trajectory based on the point-mass model

When the optimal air race trajectory for the point-mass model has been computed, the modeling complexity of the simulation model respectively its inner loop is increased step by step. First, the linear inner loop with transfer functions for the load factors and the roll rate is incorporated into the simulation model. The load factors and the roll rate that result from the optimization based on the point-mass model are set equal to the load factors and roll rate command inputs that are now the input signals to the inner loop with linear transfer functions. By simulating the air race trajectory with the "optimal" command inputs obtained from the optimization with the point-mass model without any inner loop, the resultant simulated trajectory deviates from the optimal trajectory found for the point-mass model since now the simulation model features an inner loop with linear transfer functions. Hence, for the simulation task error feedbacks are implemented in the simulation model to force the simulated trajectory for the model with linear transfer functions onto the optimal trajectory found for the model without inner loop. Of course, the control history resulting from the simulation is unlikely to be optimal since the cost function is not yet minimized by optimization, but an initial guess for the optimization task utilizing a model with linear transfer functions has been generated that fulfills all boundary conditions and might already come close to the final optimal solution.

Once the optimal solution for the air race trajectory utilizing a simulation model with the attitude and rotational dynamics represented by linear transfer functions has been computed, the obtained time histories for the controls can then be used as command inputs for the simulation of the air race trajectory taking into account an simulation model that features an inner loop with linear state-space models augmented by the appropriate inversion controller. Again, the simulated trajectory is likely to deviate from the optimal trajectory found for the model with linear transfer functions since now linear state-space models are incorporated into the simulation model. Thus, the modeling fidelity is increased and in contrast to the optimization based on linear transfer functions for the inner loop, the coupling between the states in the longitudinal motion respectively the coupling between the states in the lateral motion is now incorporated in the simulation model. Due to this reason, error feedbacks are used to force the aircraft model with state-space models onto the optimal trajectory based on a model with transfer functions.

In the next step of the optimization procedure, the time histories found by simulating the trajectory for the model with linear state-space models, inversion controller and error feedbacks in turn can be used as command inputs for the optimization of the air race track utilizing the simulation model with linear state-space models supplemented by the appropriate inversion controller as inner loop.

Computing the optimal race trajectory for this level of the simulation model in turn yields time histories for the control surface deflections. Now, these time histories serve as initial guess for the optimization of the air race trajectory utilizing the simulation model with linear state-space models in the inner loop but without the corresponding inversion controller.

Stepping forward in the optimization procedure, the control surface commands that are optimal for the simulation model with the state-space models (but without inner-loop inversion controller) are then used as command inputs for a simulation based on the model with the non-linear inner loop. As before, the resulting deviations from the optimal trajectory for the simulation model with state-space models are corrected by suitable error feedbacks, giving a time history for the controls that is not optimal for the increased modeling fidelity but that results in a sub-optimal trajectory that obeys all boundary conditions Thus, the resulting control surface deflection time histories can be utilized as quite good initial guesses for the optimization of an air race trajectory based on a full 6-Degree-of-Freedom simulation model with non-linear attitude and rotational dynamics without any inversion controller where the control surface deflections are the directly commanded inputs.

By the approach described, both the stability and the robustness of the optimization process and the convergence properties of the various optimization tasks can be increased significantly since for each step in the entire optimization procedure very good initial guesses for the optimal solution can be derived from the preceding step, while the optimization task itself starts with a comparably simple optimization problem.

5 Results

As mentioned above, wind has a great influence on the optimal race trajectory. This can be shown very illustrative for an aircraft flying the Half Cuban Eight. Without any wind,

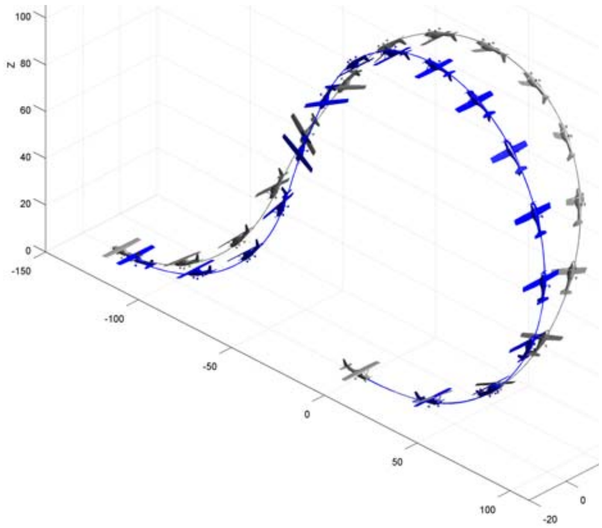

Fig. 4. Half Cuban Eight without wind (blue) and with wind (grey)

the aircraft accomplishes the Half Cuban Eight exactly in the vertical plane, while with crosswind the aircraft pulls up against the wind and flies down with the wind. This means that the plane in which the aircraft flies the Half Cuban Eight is inclined towards the direction

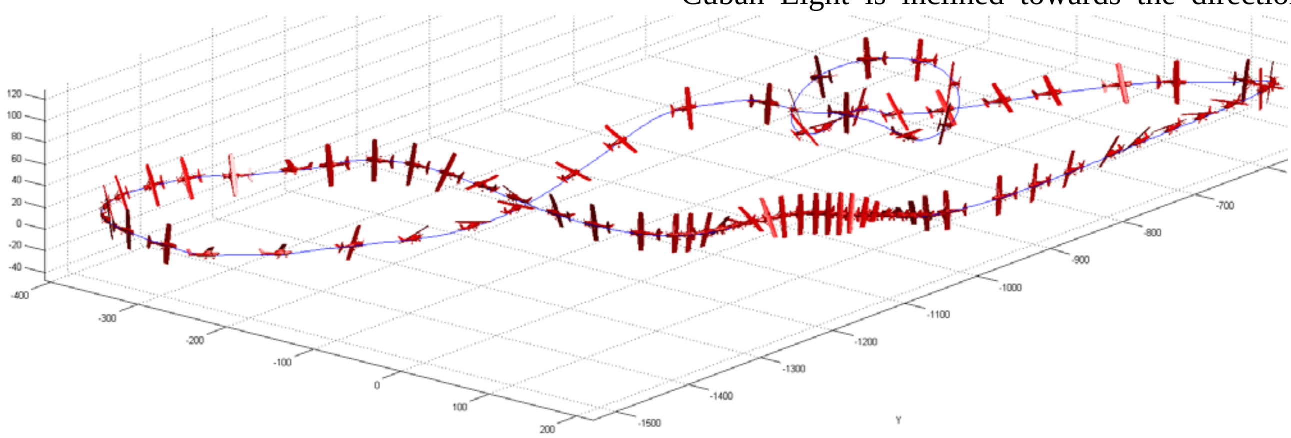

Fig. 5. Optimized Race Track

where the wind comes from. The described effect can be seen in Fig. 4. Here, an optimal trajectory for a Half Cuban Eight that is not influenced by any wind is depicted in comparison to the optimal trajectory for a Half Cuban Eight with wind where the wind blows from the west with the aircraft initially heading northward. Furthermore, the result shows the capability of the developed approach to optimize aerobatic maneuvers.

The above stated trajectory optimization problem has been solved using a direct multiple shooting method. As mentioned above, the complete air race track has been divided into sixteen phases, where the single phases have been defined as the flight path segments between the various gates. The optimized air race trajectory is shown in Fig. 5.

Although the race track is two-dimensional, i.e. the race gates are all on the same level, the resulting optimal trajectory is three-dimensional. This is especially true for the 270°-turn that is necessary for flying through the "quad": here, the aircraft pulls up in order to shorten the flight time for this maneuver. A complete air race track consists of two rounds and thus the optimization is also done for two rounds, but in Fig. 5 the second round is omitted for illustration purposes.

6 Conclusion

With the newly developed approach that is the utilization of a point-mass simulation model supplemented by rotation dynamics with scalable fidelity, the dynamic order of the flight system used for the simulation and optimization of air race trajectories can easily be adjusted to the required level of accuracy. Furthermore, an optimization procedure is presented where the complexity of the simulation model used for the optimization is increased step-by-step while the optimal solution that has been found for a specific level of complexity is utilized as initial guess for the subsequent level of complexity. Since the initial guesses are quite close to the particular optimal solution of the optimization task, the robustness and the stability of the optimization procedure is guaranteed.

Especially for the optimization of trajectories based on a full non-linear 6-Degree-of-Freedom simulation model, difficulties that would result from the instantaneous usage of such a simulation model for an air race optimization task can be avoided. The capability of the novel approach has been demonstrated by the optimization of an aerobatic maneuver, the Half Cuban Eight, and the optimization of an entire air race course.

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