ATMOSPHERIC UNCERTAINTY ON CLEAN TAKE-OFF FLIGHT PATHS FOR CIVIL AIRCRAFT

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Abstract

The environmental impact of civil aviation at low altitudes concerns local air quality, noise nuisances in residential areas located in the vicinity of airports, and associated carbon dioxide emissions that may have an impact on climate. This research, integrated in the European JTI project Clean Sky, *addresses the mitigation of the environmental impact of a single aircraft by optimizing the departure procedure. In general it is not possible to minimize all the environmental agents at the same time, which means that there is not a unique optimal departure procedure. Therefore, in previous research this concept is formulated through a multi-objective optimization problem which permits to obtain the set of optimal departure procedures that represent the best trade-offs between the selected criteria.*

In an operational world, the optimal departure procedure would be established some time before take-off. Then, to assess the environmental efficiency of these innovating departure procedures, they must show a robust behavior face to a lack of accuracy on the atmospheric conditions predictions. In this paper we propose a methodology to measure the effect of uncertain inputs on a the solutions of multi-objective optimization problems. This methodology is applied to the set of environmentally friendly optimal departure procedures.

1 Introduction.

Today, one of the main driving problems on the evolution of commercial aviation is the implementation of sustainable infrastructures and technological progress to manage the increase of air traffic. This research, integrated in the European project *JTI Clean Sky*, addresses the study of aircraft departure procedures which reduce the environmental impact of aviation in the vicinity of airports in terms of air quality, perceived noise in populated zones and emissions of carbon dioxide, categorized as a greenhouse gas.

In previous work [1], this concept has been formulated through a multi-objective, non-linear, constrained optimization problem. Major findings show that significant environmental benefit can be obtained by using improved departure procedures. However, the efficiency of these procedures can only be assessed if they are robust face to variations of the atmospheric conditions.

In this paper a methodology is proposed to measure the influence of variations of input conditions on the solutions of a multi-objective optimization problem, and is applied to study the influence of uncertain atmospheric conditions on clean departure procedures.

This paper is organized as follows. Section 2 briefly introduces the mathematical formulation of the concept and main assumptions for the resolution of multi-objective optimal departure procedures. In section 3, a general methodology is proposed to measure the influence of uncertain inputs on the solutions of a general multi-objective optimization problem. This methodology is applied in section 4 to the concept of optimal, environmentally friendly atmospheric conditions striving to qualify the robustness of the obtained optimum procedures. Finally, in section 5 major results and concluding remarks are presented.

2 Optimum Departure Procedures.

In this section we recall main assumptions considered in the mathematical formulation proposed in [1] for the concept of multi-objective departure procedures.
As mentioned in the introduction, this research deals with new departure procedures that minimize the environmental impact of aircraft, which is defined in terms of local air quality, noise nuisances and carbon dioxides.
Improvement in air quality means, among other criteria, a reduction of the nitrogen oxides emissions at low altitudes. The air quality criterion considered in this paper consists of estimating the quantity of NO_x emissions up to a the mixing altitude, generally located at 3000ft [2].
The minimization of noise nuisances strongly depends on the departing scenario. For instance, a flight path minimizing noise on a certain zone around the airport will increase noise levels out of this zone. Then, the mitigation of noise nuisances relies on displacing the maximum levels of perceived noise on the ground to the non sensitive zones. Then, it is not possible to define a departure procedure minimizing noise for every aircraft, at every airport.
The third environmental factor considered are carbon dioxide emissions. So as CO_2 emissions are combustion products, the quantity of CO_2 emitted directly depends on fuel burn. In other words, minimizing CO_2 is equivalent to minimizing fuel burn. Remark that the impact of CO_2 emissions is similar along the whole aircraft mission, due to CO_2 resilience time in the atmosphere (estimated to 100 years) [3]. Therefore, the production of carbon dioxide must be minimized up to a common mission point (prescribed altitude, speed, aircraft configuration, and distance covered).
Finally, it worths noting that this flight phase is strongly constrained. Constraints include regulatory limitations, airport published procedures, aircraft systems capabilities or Air Traffic Management impositions. They can be modeled by non-linear functions, and they must be strictly satisfied.
The three environmental criteria considered cannot be minimized at the same time. Therefore, the concept of optimum procedures for minimum environmental impact can be formulated as a multi-objective, constrained, optimization problem which general formulation is shown in Eq. 1

$$\begin{aligned} \min \mathbf{F}(\mathbf{x},\mathbf{p}) \text{ with regard to } \mathbf{x} \\ \text{subject to } \mathbf{G}(\mathbf{x},\mathbf{p}) \leq 0 \end{aligned} \tag{1}$$

where:

- $\mathbf{F}(\mathbf{x},\mathbf{p}) \in \mathbb{R}^h$ represents the h objectives to minimize.
- $\mathbf{G}(\mathbf{x},\mathbf{p}) \in \mathbb{R}^l$,is the set of inequality constraints that guarantee the feasibility of the optimal procedures.
- $\mathbf{x} \in \mathbb{R}^n$ is a vector of n optimization variables.
- $\mathbf{p} \in \mathbb{R}^m$ corresponds to the m parameters that define the context of the departure. Although these parameters remain invariable in the optimization process, they may have an important influence on the optimal departure procedure.

In the concept of clean departure procedures, there are three optimization objectives ($h = 3$) and they represent the perceived noise on ground, NO_x emissions and CO_2 emissions. The number of constraints depends on the departing context (number of obstacles, take-off weight, aerodynamic configuration,etc.). As an order of magnitude, around 30 constraints must be satisfied. The optimization variables describe a segment-based departure procedure, which is based on a *Noise Abatement Procedure Departure* (NADP) profile [2]. The flight path model is detailed in subsection 2.1. Finally, there is a high number of parameters that define the departure procedure. They can be classified into three different groups: atmospheric conditions, airport characteristics, and aircraft take-off settings. Basic notions of multi-objective optimization problems and the resolution techniques used in this paper are presented in subsection 2.2.

2.1 Flight Path parametrization

In this study the take-off settings are assumed to be decided by other non environmental criteria such that all the regulation conditions for take-off are satisfied. This means that the aircraft speed, thrust and configuration at the beginning of the NADP procedure (35ft) are fixed. The NADP procedure finishes at the enroute configuration point, at which the climb phase starts.
In order to enforce the operational point of view of this research, the NADP procedure is modeled through a small number of segments that describe the aircraft acceleration and the thrust evolution during the departure procedure.
Aircraft speed schedule is controlled through 4 optimization variables (ΔV_2, Zp_a, V_{noise} and Zp_f). Then, the aircraft starts a first acceleration segment up to a speed defined as the addition of the regulatory variable V_2 and the optimization variable ΔV_2. Then, the aircraft flies at constant speed $V_2 + \Delta V_2$ up to the acceleration altitude Zp_a, where a second acceleration towards the target speed V_{noise} starts. Finally, the Zp_f altitude triggers the final acceleration that leads the aircraft to the enroute configuration.
As mentioned above, the take-off power is selected considering other non environmental criteria, such as engine life, in compliance with take-off regulations. Aircraft thrust evolution is controlled through three optimization variables (Zp_r, $N1_{noise}$ and Zp_f). Take-off rating is used up to reaching the cutback altitude Zp_r, where the reduced engine rating $N1_{noise}$ is settled. The segment at reduced thrust finishes at the Zp_f altitude, where the climb segment starts. It is also remarkable that climb engine rating is not optimized in this study.
To clarify these concepts, the variables are represented in an altitude-speed and altitude-thrust setting parameter on figure 1.

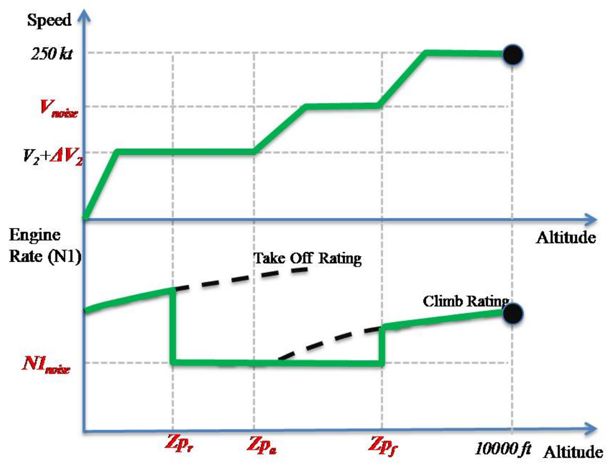

Fig. 1 Flight Paths Optimization Variables.

Finally, it worths noting that the flight path is strongly constrained during the departure phase. Regulation sets boundaries for the aircraft trajectory and airworthiness thus for the resulting aircraft parameters (e.g. take-off power, aerodynamic configuration, take-off speeds...). Therefore optimization of the subsequent trajectory segments does not jeopardize the compliance to regulatory limits. Operational limits are typically dictated by airspace prescriptions (published procedures) and commonly admitted aircraft parameters variation ranges. Bounds considered in this research are showed in table 1.

where $N1(\gamma_{min_{N-1}})$ refers to the maximum value of $N1$ such that a positive angle of climb is assured in case of engine failure, and $N1_{climb}$ represents the value of $N1$ while using the climb engine rating at the cutback altitude. However, an in-depth description of constraints can be found in [1].

Variable	Bounds
ΔV_2	$[10kt, 20kt]$
Zp_r	$[800ft, 3000ft]$
$N1_{noise}$	$[N1(\gamma_{min_{N-1}}), N1_{climb}]$
Zp_a	$[800ft, 3000ft]$
V_{noise}	$[V_2 + \Delta V_2, 250kt]$
Zp_f	$[800ft, 5000ft]$

Table 1 Bound Constraints.

2.2 Multi-Objective Optimization.

Multi-Objective Optimization (MOO) is the mathematical field that deals with the optimization of several objective functions. In MOO problems it is not possible to find out a solution which optimizes all the criteria at the same time, but a set of solution representing the best trade-off between objectives. According to the definition of Pareto optimality, a point $\mathbf{x}^* \in \mathbf{X}$ is Pareto optimal if there does not exist another point, $\mathbf{x} \in \mathbf{X}$, such that $\mathbf{F}(\mathbf{x}) \leq \mathbf{F}(\mathbf{x}^*)$, and $f_k(\mathbf{x}) < f_k(\mathbf{x}^*), k = 1, 2, ...h$ for at least one function. In other words, one solution is Pareto-optimal if any improvement in any of the objective functions necessarily degrades at least any other criteria [4]. The set of points verifying the Pareto optimality conditions (i.e. efficient or non dominated points) are called *Pareto front*.
A very common technique consists of transforming the tri-objective optimization problem into a bi-objective problem by prescribing a maximum level of one of the objectives. In this paper we propose the bi-objective, non-linear, constrained optimization problem is solved using BiMADS [5] and the NOMAD software [6]. This is a bi-objective optimization method that computes the Pareto front by a series of mono-objective optimizations, which are solved by the Mesh Adaptive Direct Search Method (MADS) [7]. Within this method optimal solutions are obtained without using any derivative information.

3 A Methodology for Measuring the Impact of Uncertain Inputs.

In this section we present a methodology for measuring the impact of uncertain input parameters on Pareto a front. Significant deformation of the optimal front can involve a lack of optimality of the solution, which may affect the selection of optimal procedures.
The uncertainty study can be divided into 3 different parts. Firstly, the uncertain variables are chosen and modelled. Then, uncertainty must be propagated along the system. Finally, the impact of uncertainty on the optimal solution is measured [8]. The methodology is formulated for a general MOO setting problem following the notation of Eq 1.

3.1 Uncertainty Modelling.

The first step of any uncertainty study is to determine the sources of uncertainty and model them. Uncertainties are defined through the random variable $\varepsilon_\mathbf{p} \in \mathbb{R}^{m_u}$, such that:

$$\tilde{\mathbf{p}} = \mathbf{p}_n + \varepsilon_\mathbf{p},\ \mathbf{p}_n,\ \varepsilon_\mathbf{p} \in \mathbb{R}^{m_u} \tag{2}$$

where $\mathbf{p}_n$ represents the nominal value of the $m_u \leq m$ conditions subject to uncertainty.
Uncertainty sources $\varepsilon_\mathbf{p}$ can be defined, for instance, through their probabilistic distribution.

3.2 Uncertainty Propagation.

In probabilistic-based uncertainty studies, the propagation of uncertainties throughout the system can be one of the most computational cost consuming steps. In order to improve this situation, effective methods to estimate the mean value and variance of the objective functions and constraints in presence of input uncertainties have appeared in the last years. The efficiency of these methods rely on the characteristics of the objective functions and variance (such as continuity, derivability) or on the use of surface-response approximations to describe the statistical estimators of the current evaluation [9].

However, Pareto-optimal departure procedures are obtained through a sequence of black box models with non linear behavior. Despite the important computational costs, the problem has been solved using the classic Monte Carlo method [10].
Remark that parallelization can be easily implemented in order to alleviate computational costs of Monte Carlo method.

3.3 Measuring Robustness.

As mentioned in the introduction of this section, the probabilistic distribution of the uncertain atmospheric condition is considered as an input data. Several propositions about robustness criteria for multi-objective optimization can be found in [11], [12], [13] and [14].
In this paper we three different indicators are proposed to measure the degree of robustness of a multi-objective departure procedure:

1. The probability P_F of obtaining a maximum prescribed deviation between all the uncertain objective functions and their deterministic values, $\Delta_F \in \mathbb{R}^h$:

$$P_F = P[\|\mathbf{F}(\mathbf{x}^*, \tilde{\mathbf{p}}) - \mathbf{F}(\mathbf{x}^*, \mathbf{p}_n)\| \leq \Delta_F] \quad (3)$$

2. The probability P_G of verifying constraints. Similarly, the feasible zone is extended using the control variable $\Delta_G \in \mathbb{R}^l$

$$P_G = P[\mathbf{G}(\mathbf{x}^*, \tilde{\mathbf{p}}) \leq \Delta_G] \quad (4)$$

3. A third, more restrictive criterion includes robustness in the objectives and constraints field at the same time. In this case, the robustness of a Pareto-optimal point can be measured through the given probability P_T, such that

$$\begin{aligned} P_T = & P[\|\mathbf{F}(\mathbf{x}^*, \tilde{\mathbf{p}}) - \mathbf{F}(\mathbf{x}^*, \mathbf{p}_n)\| \leq \Delta_F \\ & \cap \mathbf{G}(\mathbf{x}^*, \tilde{\mathbf{p}}) \leq \Delta_G] \end{aligned} \quad (5)$$

The third criterion is illustrated in figure 2 for a Pareto optimal solution $\mathbf{x}_i^*$ of a MOO problem of two objective functions and two active constraints. In this figure, Δ_{f1} and Δ_{f2} delimit the acceptance zone in the field of objectives, and Δ_{g1} and Δ_{g2} control the extended feasibility zone.

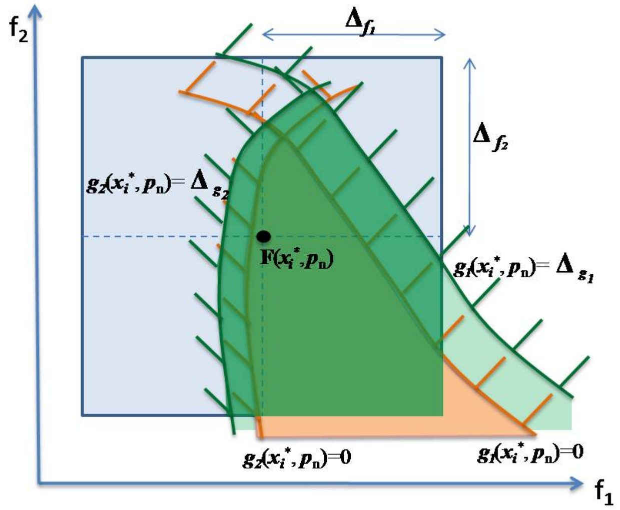

Fig. 2 Robustness criterion for uncertain inputs.

4 Numerical Application

In this section the methodology described above is applied to the concept of multi-objective optimal departure procedures. In this example, a single-aisle, medium range aircraft departs from an ideal airport (no obstacles, no ATM restrictions, no runway limitations). Departure procedures representing the best trade-off between the emission of NO_x and CO_2. In this reference airport there is a maximum noise level permitted placed at 6.5 km from the runway threshold. Moreover, in order to favor the robustness analysis instead of the analysis of optimum flight paths variables ΔV_2 and V_{noise} have been fixed to $10kt$ and $230kt$ respectively, so they have not been optimized.
Figure 3 represents the Pareto front resulting from the resolution of this problem, and the Pareto optimal points selected for the robustness study. A technique of non-linear regression has been applied in order to avoid oscillations due to

insensitive variables.

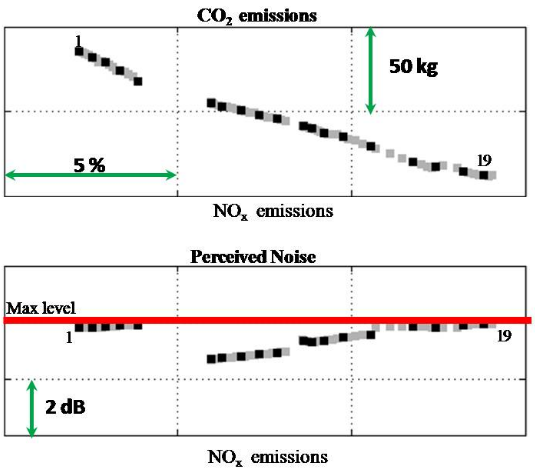

Fig. 3 Pareto front.

Figure 3 shows important gains in terms of CO_2 and NO_x emissions. However, it can be seen that the Pareto front does not show a continuous behavior. Regarding to Fig. 4, it can be seen that there are two different families of optimal flight paths. A first family of NADP 1 trajectories, in which thrust is cutback before accelerating, and a family of NADP 2 trajectories, where the aircraft accelerates firstly.

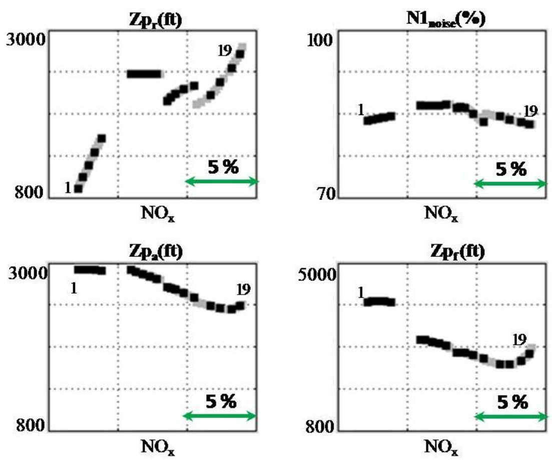

Fig. 4 Optimization variables along the Pareto front.

The minimum NO_x level is achieved through an NADP 1 flight path where thrust is cut back at its minimum value, and the aircraft accelerates at altitudes close to the threshold altitude, fixed at 3000ft. In this flight paths NO_x emissions are degraded to favor carbon dioxides emissions by increasing cutback altitude while the acceleration altitude remains constant, near the maximum acceptable value. The cutback altitude is upper bounded by a value in which the noise constraint is not satisfied. Then, the Pareto front is described by a second family of optimum flight paths in which the aircraft accelerates sooner and thrust is cut back later. In this family, the NADP procedure starts later, and both acceleration and cutback altitudes are also limited by the noise constraint. Decreasing even more the acceleration altitude would minimize CO_2 emissions, but the perceived noise level would be higher than the maximum prescribed level.

The robustness analysis strives to determine the sensibility of the optimal departure procedures face to a lack of accuracy on the predictions of the atmospheric conditions. Atmospheric conditions are represented by the air temperature and humidity, and the magnitude of axial wind. These parameters are assumed to be normally distributed, with a probability of 90% of achieving a maximum prescribed value: 5 °C of temperature variation, 20% of humidity variation, and 5kt of head/tail wind magnitude variation. The methodology described in section 3 considers a maximum level of degradation of the objective functions to determine the acceptance of the perturbed point. In this application, the maximum deviation allowed for each magnitude corresponds to 1.0% deviation for both objectives. Also, the feasible zone is extended through the variable Δ_G. In this application, each constraint can be unsatisfied on a 0.5% from their maximum value.

The robustness analysis is effectuated for the selected points in Fig. 3. Table 2 shows the probabilities P_F that represent the probability of acceptance of the objective function, P_G for the probability of acceptance in the constraint field, and P_T which gathers both the objective function and constraints.

Point	P_F	P_G	P_T
1	92.7%	95.5%	88.3%
2	92.5%	95.6%	88.2%
3	92.2%	95.3%	87.5%
4	91.9%	94.5%	86.5%
5	91.4%	90.0%	81.6%
6	90.5%	97.8%	88.5%
7	90.3%	97.9%	88.4%
8	89.9%	96.5%	86.8%
9	89.9%	97.7%	87.8%
10	89.7%	97.7%	87.6%
11	89.5%	98.3%	88.0%
12	89.6%	97.7%	87.5%
13	89.3%	97.9%	87.4%
14	89.1%	97.9%	87.3%
15	89.1%	98.2%	87.5%
16	88.8%	98.1%	87.2%
17	88.4%	97.6%	86.3%
18	87.6%	97.8%	85.7%
19	87.2%	97.5%	85.1%

Table 2 Robustness Analysis 5 °C, 20%, 5kt.

Table 2 shows that multi-objective optimum departure procedures have a probability of around 85 % of remaining optimal in presence of atmospheric uncertainty of the selected magnitude (3rd criterion). Moreover, this probability is higher for the NADP 1 (which favor low values of NO_x) rather than the NADP 2 trajectories that favor low values of CO_2. This behavoir can be justtified throughout the analysis of the robustness inidicators of the objectives and constraints separately (criterion 1 and 2).
Regarding at the results obtained, it can be said that operational constraints are robust with a probability higher than 95%, which induces that differences come from the impact on objectives. According to the robust criterion 1, the optimization objectives show a decreasing degree of robustness from the minimum NO_x solution to the minimum CO_2 solution. This phenomenon can be explained regarding at the evolution of the variables along the optimal fronts on figure 4. The argumentation is based on the fact that CO_2 emissions are computed up to a common mission point, while NO_x are harmful only at altitudes lower than 3000ft. Therefore it can be assumed that the impact of the atmopsheric uncertainty on the CO_2 criterion is similar in all the optimal trajectories. In contrast, NO_x is more sensitive to the atmospehric conditions when the aircraft performs an NADP 2 procedure than when NADP 1 departure is used.In figure 4 it can be seen that the NADP 1 flight paths are characterized by high values of the acceleration altitude, so in the NO_x exposure zone the only maneuvre performed is thrust cutback. In contrast, in the NADP 2 procedures the aircraft also accelerates in the NO_x under 3000ft. The addition of this maneuvres increases its sensitivity to the variation of atmospheric conditions, and so the probability of exceeding a maximum acceptable value of NO_x emissions is higher.

5 Conclusions

This paper stems from the necessity of characterizing the robustness characteristics of the multi-objective optimal departure procedures. To this effect, a new methodology has been developed to quantify the degree of robustness of the optimum flight paths. The measure of robustness proposed relies on the probability of obtaining acceptable levels of optimal environmental impact without penalizing the feasibility of the procedures. As any variation on the input conditions will affect the optimization objectives and constraints, the degree of robustness is referred to a maximum deviation allowed from the deterministic optimum objectives and a maximum level of non-compliance of constraints.
The methodology developed has been applied to a theoretical, but representative example of multi-objective optimum departures. The results obtained show that the degree of robustness of the optimum departure procedures are around 85%. Since these uncertain sources cannot be controlled, the degree of robustness can

only be improved by decreasing the maximum uncertainty allowed.
Future studies will deal with the implementation of robust optimization techniques to manage higher atmospheric uncertainty.

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