IMMUNE : CONTROL REALLOCATION AFTER SURFACE FAILURES USING MODEL PREDICTIVE CONTROL

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Abstract

This paper describes how to exploit the linear model predictive control with quadratic programming to the well-known control surface reallocation problem once an actuator failure has been detected and isolated. Theoretical developments assume here an over-actuated aircraft model, only submitted to input constraints. Implementation improvements are added to allow real on-board applications. Simulations prove the efficiency of the algorithm for various failure cases.

1 Introduction

This paper deals more particularly with the so-called control surface reallocation problem once a surface failure has been detected and isolated. It is indeed possible to exploit the control surface redundancy or the ability of asymmetric control surface action. The aircraft A/C nominal behavior can be recovered on-line in spite of a failure by splitting the commanded control inputs into new physical actuator deflections. This is typically the case of the lateral motion : several spoilers, inner and outer ailerons on the left and the right wings can be used to obtain the same turn rate, even if one or more fail.

During these last decades significant research has been focused on the control allocation problem for over-actuated aircraft under control input constraints. All solutions use basically either weighting pseudo-inverse approach or linear/quadratic optimizations [5, 2]. Improvements can be reached through multiple allocation step schemes like Daisy Chaining [3] or through combinations with other techniques (sliding modes control...) [1]. In [6] Model Predictive Control addresses a specific case of the control allocation problem under input and output constraints : the goal is to limit a structural load output to an admissible level while keeping the flight behavior as close as possible to the initial one. This idea is re-used here in the context of actuator failures.

The paper is organized as follows. Section 2 gives an overview of the A/C closed-loop architecture and introduces the main notations. Section 3 presents the theoretical developments. And finally numerical simulation results illustrate the validity of this approach in Section 4.

2 Problem statement

Fig. 1 shows the closed loop system of the considered aircraft A/C. It was developed during the common ONERA/DLR IMMUNE project. IMMUNE stands for Intelligent Monitoring and Managing of UNexpected Events. For more de-

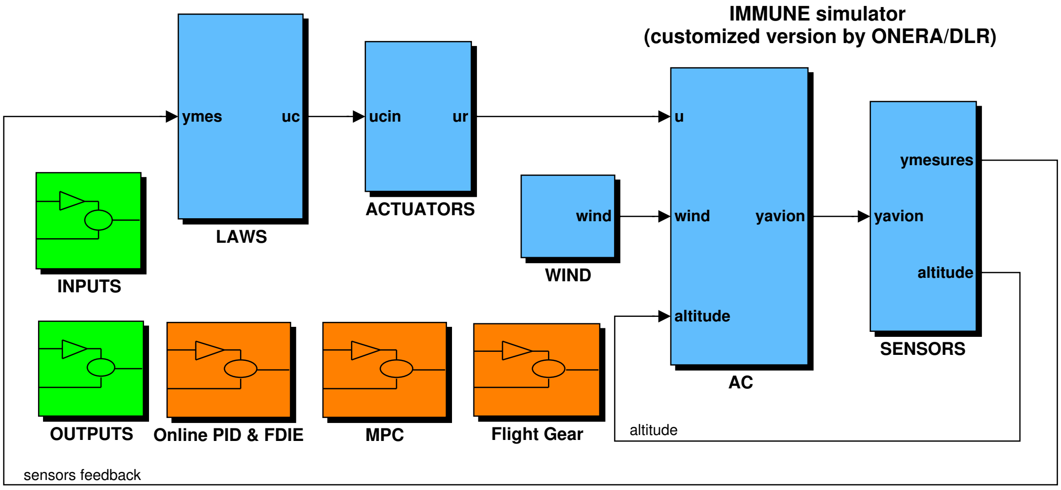

Fig. 1 Overall IMMUNE aircraft model in closed loop with on-line FDIE and MPC

tails on the IMMUNE project and the A/C model see [4]. Within the IMMUNE project several fault detection and isolation FDI and fault tolerant control FTC methods have been developed and validated, see for example [7, 8, 9].

The 11 A/C states are controlled by 20 first order control actuators. The control inputs u are the deflections δ of the inner (I) and outer (O) right (R) and left (L) ailerons (A), of the 12 spoilers (SP) and the rudder (R) for the lateral control as well as of the left and right elevators (EL) for the longitudinal control and the stabilizer (STAB) for trim. They are all expressed in ^{o}. The A/C states represent the main flight dynamics characteristics and are measured by sensors. These outputs y represent the angle of attack α, the sideslip angle β, the pitch angle θ and the roll angle φ, all given in ^{o}, the roll rate p, the pitch rate q and the yaw rate r, all given in $^o/s$, the altitude Z_{CG} given in m, the lateral acceleration $N_{y,CG}$ and the vertical acceleration $N_{z,CG}$ given in g as well as the vertical speed V_z given in m/s :

$$y = \begin{bmatrix} \alpha \\ \beta \\ p \\ q \\ r \\ \theta \\ \varphi \\ Z_{CG} \\ Ny_{CG} \\ Nz_{CG} \\ Vz \end{bmatrix} \qquad u = \begin{bmatrix} \delta AOR \\ \delta AIR \\ \delta SP1 \\ \delta SP2 \\ \vdots \\ \delta SP12 \\ \delta AIL \\ \delta AOL \\ \delta ELR \\ \delta STAB \\ \delta ELL \\ \delta RR \end{bmatrix}$$

The control deflections u are computed by the nominal control law in terms of equivalent control deflections in roll, pitch and yaw deflections. These equivalent control deflections are then allocated to the physical control actuators within the actuator split block, see Fig. 2.

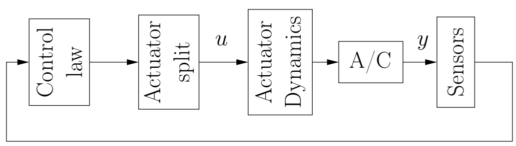

Fig. 2 Actuator split in the closed loop.

When no actuator failure occurs, the control law and the actuator split block are both tuned a priori to perform excellent reference flight qualities. On the other hand, the control law and the actuator split block are not designed to counter an actuator failure. In this case, the closed loop will deliver degraded outputs.

It is proposed here to modify the split block after one or several actuator failures in order to recover the nominal A/C behavior. Linear Model Predictive Control with quadratic programming will be used as a control reallocation algorithm for the on-line change of the split block. For illustration, the proposed approach is applied to ensure the nominal turn behavior even after several failures.

In normal flight, the ailerons are sufficient to provide the roll moment (the rudder is used for yaw). For such a manoeuvre, the lateral and longitudinal motions are coupled, the deflections of the elevators and the stabilizer are hence justified. In the case of an aileron failure, asymmetric use of spoilers can replace the failed actuator(s) to obtain the same roll moment. This is the redundancy which is exploited here.

Consider a complete linear aircraft model

$$(A_{sys}, B_{sys}, C_{sys}, 0)$$

which includes the A/C states at a given flight condition as well as the actuator and sensor dynamics. For on-board computation, the linearization is realized every $10s$. This model is then discretized with a sampling time T_{ech} using the well known TUSTIN equations :

$$\begin{array}{rcl} A & = & e^{A_{sys}T_{ech}} \\ B & = & A_{sys}^{-1}\left(e^{A_{sys}T_{ech}} - I\right)B_{sys} \\ C & = & C_{sys} \end{array}$$

leading to the following discrete time state space representation or recursive expression :

$$sys : \left\{ \begin{array}{rcl} x_{k+1} & = & Ax_k + Bu_k \\ y_k & = & Cx_k \end{array} \right. \qquad (1)$$

The sampling frequency $f_{ech} = \frac{1}{T_{ech}}$ is here $16Hz$.

Afterwards we will denote u_k (resp. y_k) the vector of the 20 deflections (resp. of the 11 outputs) values reached at the time step $k.T_{ech}$.

The deflections u_k are physically limited both in amplitude and rate. These limitations are taken into account in the following way :

$$u_{min} \leq u_k \leq u_{max} \text{ and } |\Delta u_k| \leq T_{ech}.\dot{u}_{max}, \ \forall k \geq 0 \qquad (2)$$

3 Proposed approach

Linear Model Predictive Control with quadratic programming is used after one or several actuator failures have been detected and isolated (see for example [7])

- at each time step
- by optimizing only nu_{MPC} control surface deflections
- in order to recover the nominal performances within a prediction horizon $N_p.T_{ech}$.

Using the recursive expression (1), the following matrix expression allows us to compute the outputs y_k from the present time step (denoted by the index 1) to $N_p.T_{ech}$ seconds later (index N_p) in function of the present value x_1 of state vector and the control inputs to be applied $(u_1, \ldots, u_{N_p})$:

$$\underbrace{\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ \vdots \\ y_{N_p-1} \\ y_{N_p} \end{bmatrix}}_{\hat{Y}} = \underbrace{\begin{bmatrix} C \\ CA \\ \vdots \\ \vdots \\ CA^{N_p-1} \\ CA^{N_p} \end{bmatrix}}_{\mathcal{V}} x_1 + \underbrace{\begin{bmatrix} 0 & \dots & \dots & \dots & 0 \\ CB & 0 & \dots & \dots & 0 \\ CAB & CB & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ CA^{N_p-2}B & CA^{N_p-3}B & \dots & CB & 0 \\ CA^{N_p-1}B & CA^{N_p-2}B & \dots & CAB & CB \end{bmatrix}}_{\mathcal{M}} \underbrace{\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ \vdots \\ u_{N_p-1} \\ u_{N_p} \end{bmatrix}}_{\hat{U}} \qquad (3)$$

The objective is now to compute the inputs $\hat{U}$ in order to minimize the difference between the obtained outputs $\hat{Y}$ and the reference outputs Y_{REF}, for instance the turn behavior of the nominal aircraft without failure :

$$\min_U J = \|\hat{Y} - Y_{REF}\|_2 = \|\left(\mathcal{M}\hat{U} + \mathcal{V}x_1\right) - Y_{REF}\|_2$$

which reduces to a quadratic programming problem for the inputs $\hat{U}$ as follows :

$$\min J = \hat{U}^T \mathcal{M}^T \mathcal{M} \hat{U} + a^T \hat{U} + d$$
$$\text{with} \quad \begin{cases} a^T &= 2x_1^T \mathcal{V}^T \mathcal{M} - 2Y_{REF}^T \mathcal{M} \\ d &= (\mathcal{V}x_1 - Y_{REF})^T (\mathcal{V}x_1 - Y_{REF}) \end{cases}$$

The limitations of u_k in both positions and rates given in the inequalities (2) are transformed into the following expressions :

$$\begin{cases} U & \leq \mathrm{U}_{max} \\ U & \geq \mathrm{U}_{min} \\ \mathcal{MVL}.U & \leq \underbrace{\begin{bmatrix} \Delta\mathrm{U}_{max} + \mathcal{U}_0 \\ \Delta\mathrm{U}_{max} - \mathcal{U}_0 \end{bmatrix}}_{\mathcal{VVL}} \end{cases} \quad (4)$$

where U_{max}, U_{min}, $\mathcal{U}_0$ and $\Delta\mathrm{U}_{max}$ are vectors of dimensions $(nu_{MPC}N_p \times 1)$:

$$\mathrm{U}_{max} = \begin{bmatrix} u_{max} \\ u_{max} \\ \vdots \\ u_{max} \end{bmatrix} \quad \mathrm{U}_{min} = \begin{bmatrix} u_{min} \\ u_{min} \\ \vdots \\ u_{min} \end{bmatrix} \quad \mathcal{U}_0 = \begin{bmatrix} u_0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\Delta\mathrm{U}_{max} = T_{ech} \,. \begin{bmatrix} \dot{u}_{max} \\ \dot{u}_{max} \\ \vdots \\ \dot{u}_{max} \end{bmatrix}$$

It can be easily proven that the square matrix $\mathcal{MVL}$ of dimension $(2nu_{MPC}N_p \times 2nu_{MPC}N_p)$ is equal to :

$$\mathcal{MVL} = \begin{bmatrix} \mathcal{VL} \\ -\mathcal{VL} \end{bmatrix}$$
with
$$\mathcal{VL} = \begin{bmatrix} I & 0 & \dots & \dots & 0 \\ -I & I & \ddots & \dots & \vdots \\ 0 & -I & I & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & -I & I \end{bmatrix}$$

Structural limitations for this aircraft are not known and were therefore not included into the cost function, but this approach accepts their potential inclusion as additional output constraints [6].

Once $\hat{U}$ is obtained, just the optimized control surface deflections u_1 at $k = 1$ is then applied to the aircraft.

For real time on-board application, the time-consuming optimization is not realized at each time step, but only at $m = (N_p - 1)/(N_i - 1)$ time steps (N_i is the interpolation horizon.). More precisely, instead of computing $\hat{U}$ directly, we search for

$$\hat{U}_{\text{downsampled}} = \begin{bmatrix} u_1 \\ u_{m+1} \\ \vdots \\ u_{Np} \end{bmatrix}$$

Between these points, the remaining values are linearly interpolated (with $\kappa = \frac{1}{m}$) :

$$\hat{U} = \underbrace{\begin{bmatrix} \mathrm{I} & 0 & \dots & \dots & 0 \\ (1-\kappa)\mathrm{I} & \kappa\mathrm{I} & 0 & \dots & 0 \\ (1-2\kappa)\mathrm{I} & 2\kappa\mathrm{I} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \mathrm{I} & 0 & \dots & 0 \\ \vdots & (1-\kappa)\mathrm{I} & \kappa\mathrm{I} & 0 & 0 \\ \vdots & (1-2\kappa)\mathrm{I} & 2\kappa\mathrm{I} & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & 0 & \mathrm{I} & 0 & 0 \\ \vdots & \vdots & 0 & \vdots & I \end{bmatrix}}_{C} \hat{U}_{\text{downsampled}}$$

then the whole procedure starts again. The interpolation also smooths the optimized control inputs.

There is a compromise between the accuracy in following the reference signal Y_{REF} and the computation time.

4 Application

The proposed algorithm is applied to several failure cases at various flight conditions (speed, mass and altitude) in the presence of uncertainties and external perturbations like the wind for the IMMUNE model, see [4]. The failures have been detected, isolated and estimated on-line using the algorithms proposed by [7]. It is first applied to cases where the nominal flight control law works well in order to determine the best compromise between the prediction horizon N_p and the interpolation horizon N_i as well as the number of control surfaces nu_{MPC} which should be optimized and the number of outputs ny_{MPC} which should be followed as references. These cases are :

- reconfiguration without failure using the spoilers in addition to the ailerons
- reconfiguration after an aileron jammed at 0^o using the remaining ailerons and spoilers
- reconfiguration after an aileron jammed at 40^o using the remaining ailerons and spoilers
- reconfiguration after loss of efficiency of an aileron

In Tab. 1, the computation time for the MPC is given in function of N_p, N_i, nu_{MPC} and ny_{MPC}.

Case	N_p	N_i	nu_{MPC}	ny_{MPC}	Time $[s]$	Ranking $[-]$
1	30	5	17	5	33,34	6
2	50	5	17	5	61,63	3
3	70	5	17	5	106,19	2
4	50	3	17	5	143,52	4
5	50	10	17	5	29,74	5
6	50	5	17	11	131,52	1

Table 1 Computation time and reference following quality for MPC

The flown manoeuvre takes $70s$. For accuracy reasons, all 17 lateral control inputs are reallocated :

- the rudder,
- the 4 inner and outer left and right ailerons,
- and the 12 spoilers

The best compromise for the A/C real time application is a prediction horizon N_p of $50.T_{ech} = 3.125s$ with an optimization every $N_i = 5.T_{ech} = 0.3125s$. It's case 2. The 5 reference signals Y_{ref} are here the nominal lateral behavior without failure:

$$Y_{ref} = \begin{bmatrix} \beta_{ref} \\ p_{ref} \\ r_{ref} \\ \varphi_{ref} \\ N_{y,ref} \end{bmatrix}$$

The reference signals are well followed and the control sequence is smooth. The best result would be obtained in case 6 where all longitudinal and lateral outputs y would be followed as Y_{REF}, but the computation time is twice too long for the real time application. In case 3 with a longer prediction horizon $N_p = 70.T_{ech}$, the reference signals are slightly better followed than in case 2, but the computation time is still 1.5 times too high. In case 4 with a smaller interpolation horizon $N_i = 3.T_{ech}$, the control sequence is a little bit more noisy and the computation time is twice longer than in case 2. In case 5 with a longer interpolation horizon $N_i = 10.T_{ech}$, the computation time is twice shorter and the control sequence very smooth, but the following of the reference signals is slightly worse than in case 2. In case 1, the computation time is also twice shorter than in case 2, but the control sequence is more noisy.

The reallocation by MPC is implemented in the actuator split block as shown in Fig. 3.

The control deflections computed by the nominal law and those ones optimized by MPC are combined except the rudder deflection. The MPC control inputs act hence as a feed-forward for the desired control deflections. The nominal control law ensures robustness to internal uncertainties like model errors and external perturbations like turbulences.

Once, the MPC control scheme is tuned, a very hard, but also very unlikely failure case is simulated in order to illustrate the interest of such a control reallocation approach. The reconfiguration is initialized after the simultaneous jam of the outer and inner left aileron at -40^o and -30^o respectively for a turn to the right with a heading change of $\Delta\Psi = +90^o$ at low altitude (about $700\,ft$) and at low speed (about $150\,kts$). The nominal law was not at all designed for such a case as the failure occurrence is less than the demanded one $\Phi_{coupled\ failure} < 10^9$).

In the following plots, the signals in the nominal case without failure are plotted in red, the signals resulting from the reallocation are plotted in blue lines.

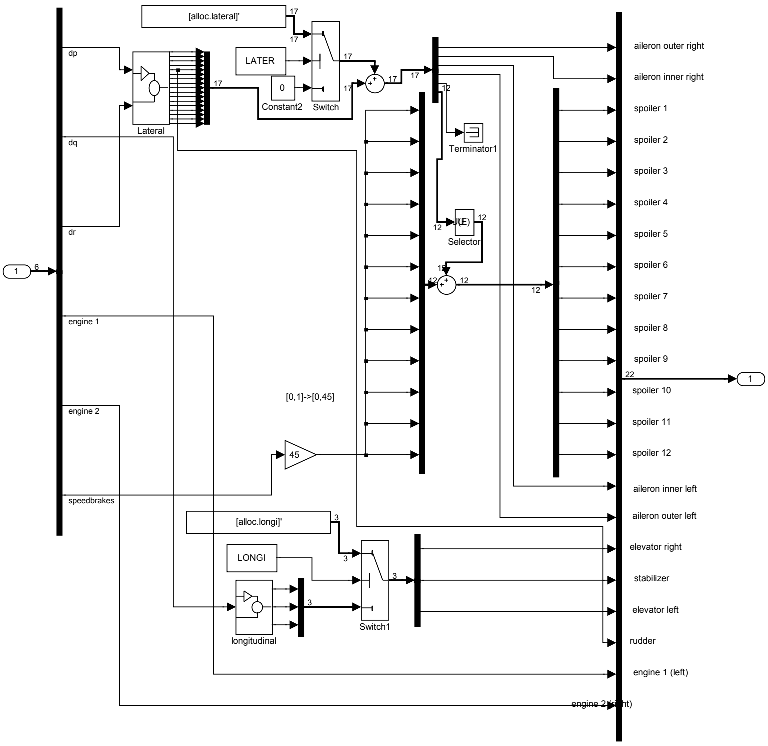

Fig. 3 Implementation of the MPC in the nonlinear simulation environment

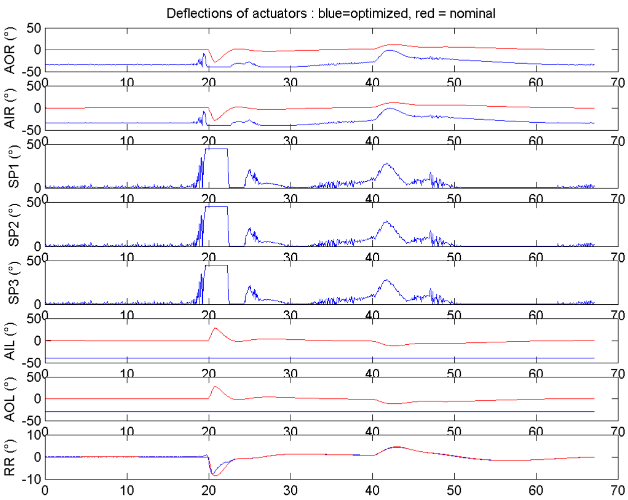

Fig. 4 Control inputs after optimization at each time step

In Fig. 4, the MPC reallocated control inputs without interpolation ($N_i = 1$, blue) after the simultaneous jam of the outer and inner left aileron are plotted. The jammed ailerons are deflected upwards. This would mean a negative lift on the left wing and hence a turn to the left. The nominal law immediately counters the induced roll moment by a symmetric deflection on the right ailerons. They are now saturated, but the aircraft continues a straight flight. In order to induce the turn to the right, the MPC orders the deflection of the spoilers 1-3 on the right wing. After the desired heading change, the MPC reduces the deflection of the right ailerons in order to less counter the effect of the jammed left ailerons. The aircraft comes back to a straight flight. In order to reduce the induced yawing moment of the jammed ailerons, the MPC orders a slight deflection of the spoilers 1-3 on the right wing. The optimized control input signals are a little bit too noisy. This is due to the optimization at each time step. The rudder is used almost in the same way as by the nominal law.

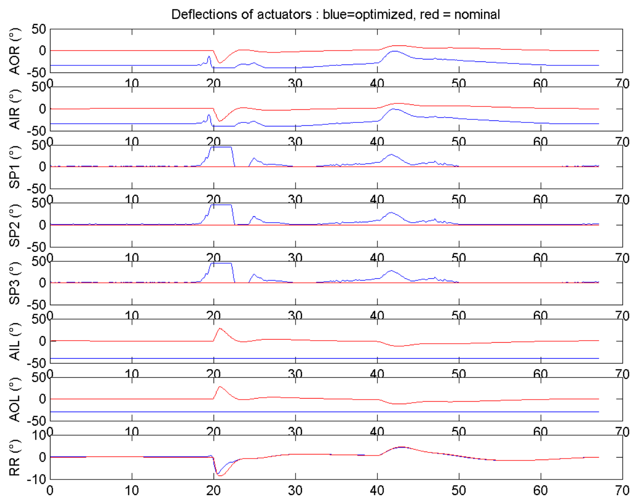

Fig. 5 Smoothed control inputs after optimization using interpolation

In Fig. 5, the MPC reallocated control inputs with interpolation ($N_i = 5$, blue) after the simultaneous jam of the outer and inner left aileron are plotted. Mainly, the system behavior is as before, but the control inputs are considerably smoothed. By the way, the computing time is divided by more than 2 during optimization.

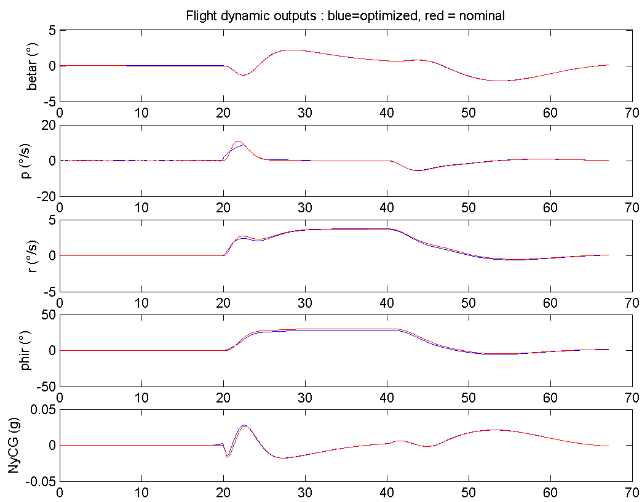

Fig. 6 The resulting lateral outputs compared to the reference signals

In Fig. 6, the resulting outputs $\hat{Y}$ (blue) due to optimization after the simultaneous jam are compared to the reference signals Y_ref without failure (red). Here a linear model is used. It can be seen that the A/C after failure behaves almost as the nominal A/C, especially in terms of sideslip angle β and roll angle φ. There is a slight difference in roll rate p during the initiation of the turn to the right. It can be stated that the minimization of the optimization criterion works well.

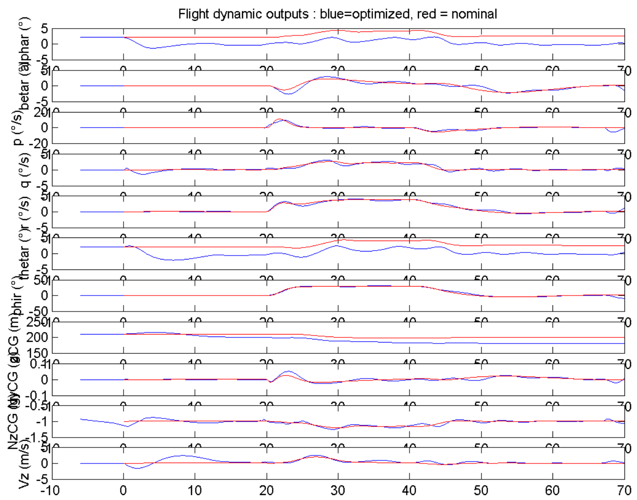

Fig. 7 Lateral and longitudinal outputs compared to the nominal case

In Fig. 7, the resulting lateral and longitudinal outputs y (blue) after the simultaneous jam are compared to the nominal signals without failure (red). Here the full nonlinear simulator is used. It can be seen that the A/C after failure behaves almost as the nominal A/C. The angle of attack α and the pitch angle θ differ from the nominal values which is due to the fact that the A/C has to compensate the additional drag due to the deflection of the jammed ailerons. Otherwise, the behavior is more or less the same than in the linear case. The sideslip angle β and the yaw rate r are slightly less damped than for the linearized model. This is due to small differences between the linear and the nonlinear model. But globally, the proposed linear MPC scheme works very well even with the nonlinear model. The combination of the MPC with the nominal control law ensures the robustness to internal uncertainties (model errors) and external perturbations.

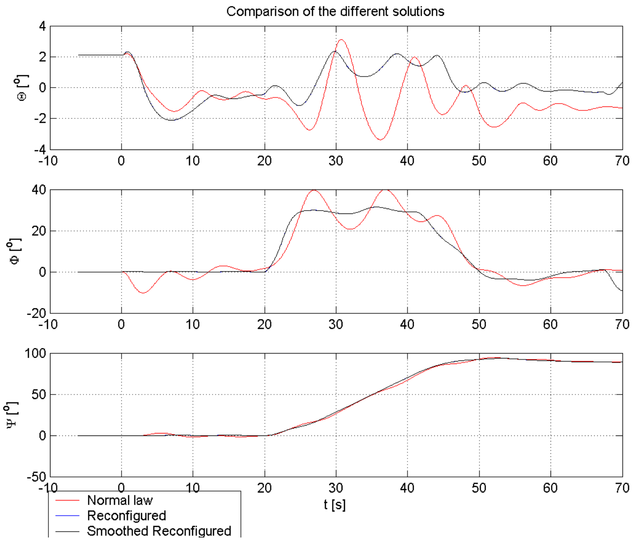

Fig. 8 θ, φ and ψ generated by the nominal law and the MPC reconfigured law after the simultaneous aileron jam

In Fig. 8, the pitch angle θ, the roll angle φ and the yaw angle ψ generated by MPC (blue) or smoothed MPC (black) are compared to those generated by the nominal law (red) in the case of the simultaneous jammed ailerons using the nonlinear simulator. It can clearly be seen that the nominal control law was not designed for this very hard and unlikely failure case. Especially, the roll angle oscillates by almost 10^o around its desired value of 30^o. The MPC reconfiguration with or without interpolation reassures an almost nominal behavior. The blue and the black lines are superposed. It improves significantly the A/C behavior with respect to the pure nominal law in this failure case.

5 Conclusions and perspectives

MPC works as a feed-forward, the nominal law in closed loop ensures robustness and rejects external perturbations. The proposed MPC algorithm permits to treat all failure cases in various flight conditions with uncertainties and external perturbations. The reconfiguration improves significantly the A/C behaviour in hard and very unlikely failure cases with respect to the pure nominal law.

However, the proposed MPC must still be extended to an integrated closed loop law in order to replace completely the nominal law after reconfiguration in order to ensure itself robustness and perturbation rejection. The computation time should still be reduced for implementation on onboard computers.

For the moment being, the reference signal was the nominal aircraft behavior without failure. It can be adapted to restricted achievable A/C performances after failure, especially after multiple failures.

6 Copyright Statement

References

[1] Alwi H and Edwards C. Fault tolerant control using sliding modes with on-line control allocation. *Automatica*, Vol. 44, pp 1859–1866, 2008.

[2] Beck R. *Application of Control Allocation Methods to Linear Systems with Four or More Objectives*. PhD thesis, Faculty of the Virginia Polytechnic Institute and State University, 2002.

[3] Buffington J and Enn D. Lyapunov stability analysis of Daisy Chain control allocation. *Journal of Guidance, Control and Dynamics*, Vol. 19, No 6, pp 1226–1230, 1996.

[4] Döll C, Varga A, and Kappenberger C. IMMUNE : Intelligent Monitoring and Managing of Unexpected Events. *Proc 27th ICAS Congress*, Nice, France, September 2010.

[5] Enns D. Control allocation approaches. *Proc AIAA Guidance, Navigation and Control Conference*, pp 98–108, 1998.

[6] Gaulocher S, Roos C, and Cumer C. Aircraft load alleviation during maneuvers using optimal control surface combinations. *Journal of Guidance, Control and Dynamics*, Vol. 30, No 2, 2007.

[7] Hardier G and Bucharles A. On-line parameter identification for in-flight aircraft monitoring. *Proc Proc. 27th ICAS Congress*, Nice, France, September 2010.

[8] Kappenberger C and Christmann J. Using simulation-based filtering approaches for in-flight monitoring of control surfaces. *Proc 27th ICAS Congress*, Nice, France, September 2010.

[9] Ossmann D, Hecker S, and Varga A. A versatile simulation environment of FTC architectures for large transport aircraft. *Proc 27th ICAS Congress*, Nice, France, September 2010.