A NEW HYBRID DYNAMIC PROGRAMMING APPROACH IN PERFORMANCE OPTIMIZATION OF AN AEROSPACE PLANE

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Abstract

A parallel hybrid dynamic programming approach for trajectory optimization is proposed. The application of this approach is exemplified by ascent trajectory optimization for an aerospace plane. The proposed algorithm is based on the combined utilization with the forward dynamic programming and the branch-and-bound method, and makes possible to obtain routinely and robustly numerical solutions to achieve the minimum-fuel ascent to Low-Earth-Orbit under complicated constraints. The lower bound of cost function required in this algorithm implementation is calculated by the energy-state approximation which is very effective in producing considerable reductions in computation requirements of the hybrid algorithm as well as storage requirements. An application to the preliminary design of aerospace planes are included.

1 INTRODUCTION

These days, many concepts concerning future space planes have been proposed: fully reusable concepts of the National Aerospace Plane(NASP) proposed by USA, of HOTOL by UK, and of Sanger by West Germany, and partially reusable concepts of HERMES by ESA, and of HOPE by JAPAN. A single-stage hypersonic aerospace plane using air-breathing engines is expected for more economical delivery of payloads into Low-Earth-Orbit. The advantages of such a vehicle are the operational flexibility of horizontal takeoff, the operational simplicity of a single stage, and the propellant mass reduction by using air-breathing engines. In these situations, suitable methods to compute routinely and robustly optimal atmospheric trajectories of vehicles containing from the subsonic to hypersonic flights for the purposes of preliminary design and performance estimation of the aerospace plane are greatly desired. The amount of difficulty and expense experienced in calculating optimal flight trajectories depends primarily upon the complexity of the dynamic model used to describe the aerospace plane, the existence of tabular functions on aerodynamic characteristics and propulsion models.

Various techniques have been developed to overcome these difficulties(Ardema, 1976 [2]; Bryson *et al.*, 1962 [4]; Kelley, 1973 [10]. The solution obtained by singular perturbation methods is in a good agreement with that obtained by established methods, but this method has a serious limitation which may be that the general characteristics of system behavior must be known a priori (Ardema [2]). An approach using the minimum principle has a computational difficulties such that guessing the missing boundary conditions at the initial points on the two-point boundary-value problems. On the other hand, dynamic programming(Bellman [3]) is a very powerful method for problems having various nonlinearities and nondifferentiability containing tabular functions in the system equations, but its direct applications are confronted with the difficulty of well known Bellman's the "curse of dimensionality problems". Alekseev and Vodos (1976 [1]), and Morin and Marsten (1976 [11]) used the branch-and-bound technique to eliminate unpromising state in discrete dynamic programming. Hanaoka and Tanabe (1982 [6]) applied the branch-and-bound combined with forward dynamic programming to the reduction of computational requirements in optimal control problems.

This paper considers an alternative dynamic programming approach for applications to aerospace planes. Our idea to overcome this difficulty is to make dynamic programming algorithm with a procedure to reject the state space which can not be contained in a part of the optimal trajectory by a combined use with a more simple method than dynamic programming. To doing this, the method to utilize the simple solution of the energy-state approximation (Bryson *et al.*, 1969 [5]; Rutowski, 1954 [12]) is indicated in this paper.

The proposed extended Hybrid Dynamic Programming(for a short, HDP) algorithm consists of three sub-algorithms which are the basic sub-algorithm(Hanaoka, 1994 [8]), the iterative sub-algorithm(Hanaoka, 1989 [7]) and a newly introduced parallel sub-algorithm. The basic sub-algorithm basically consists of interactions similar to Darwinian evolution through the cycles of selection, multiplication and mutation in the quantized state space, and generates a rough global optimal solution. Here, the selection process corresponds to the "preference order calculation with blocks" in this algorithm, and the multiplication process containing the mutation process corresponds to the "repetitive generation of trajectory groups by using regularly and randomly quantized controls". The iterative sub-algorithm which contains the most important idea successively improves not only optimal solutions but also accuracy of lower bound of cost function which gives a great influence upon the number of the computations as well as the computation regions in the hybrid algorithm. The parallel sub-algorithm is used for a guarantee to obtain the global optimal solution. In those algorithms, the "block" plays a very important role. A theoretical evidence of the hybrid algorithm is based on the forward dynamic programming(Hanaoka, 1982 [6]), The hybrid algorithm requires the lower bound used in the first computation of the basic sub-algorithm. Our idea for this requirement is to utilize a simple solution of the energy-state approximation performance optimization approach(Bryson *et al.*,1969 [5]; Rutowski, 1954 [12]) which often leads to unrealistic discontinuities in velocity and altitude and a poor solution, but can be used sufficiently as the lower bound of our hybrid algorithm. In Hanaoka(1994 [8]), the basic hybrid algorithm with the use of the energy-state approximation was applied effectively to performance optimization of a supersonic aircraft. In this paper, the extended hybrid algorithm with the energy-state approximation is applied to the minimum-fuel ascent trajectory generation of an aerospace plane.

2 DYNAMICS OF AEROSPACE PLANE

2.1 Equations of Motion

Assuming a spherical, nonrotating Earth and gravitational field with $g = \mu/r^2$, the two-dimensional point mass equations of motion for aerospace plane is described as

$$\dot{v} = \frac{T(h,v,\phi)\cos\alpha - D(h,v,\alpha)}{m} - \frac{\mu\sin\gamma}{r^2} \tag{1}$$

$$\dot{\gamma} = \frac{T(h,v,\phi)\sin\alpha + L(h,v,\alpha)}{mv} + \left(\frac{v}{r} - \frac{\mu}{vr^2}\right)\cos\gamma \tag{2}$$

$$\dot{r} = v\sin\gamma \tag{3}$$

$$\dot{\theta} = \frac{v\cos\gamma}{r} \tag{4}$$

$$\dot{m} = -\frac{T(h,v,\phi)}{g_0 I_{sp}} \tag{5}$$

where state variables are the radius from the center of the Earth r, velocity v, flight-path angle γ, polar angle θ and total mass m, and control variables are angle of attack α and engine fuel-equivalence ratio ϕ. $T(h,v,\phi)$ is thrust, I_{sp} is specific impulse of the propulsion system, $D(h,v,\alpha) = C_D(\alpha,M)\rho(h)v^2S/2$ is drag, $L(h,v,\alpha) = C_L(\alpha,M)\rho(h)v^2S/2$ is lift, $C_D(\alpha,M) = C_{D0}(M) + \eta(M)C_{L\alpha}(M)\alpha^2$ is drag coefficient and $C_L(\alpha,M) = C_{L\alpha}(M)\alpha$ is lift coefficient, $\rho(h)$ is air density, S is aerodynamic reference area, μ is gravitational constant, $M = v/a$ is Mach number, $a = a(h)$ is speed of sound, and where $T, C_{D0}, C_{L\alpha}, \eta$ and ρ are given as tabular functions. Fig.1 shows a nomenclature commonly used for this model.

2.2 Vehicle Modeling

The model of the aerospace plane used in this paper is based on (Shaughnessy, *et al.*, 1990 [13]) with some modifications by us. The gross takeoff weight is 136,080kg. It is a winged-cone configuration with a reference area of $S = 335\text{m}^2$ and an overall length of 61m. The thrust of the air-breathing propulsion system is given by

$$T(h,v,\phi) = C_T q \tag{6}$$

where q is dynamic pressure, and C_T is thrust coefficient. $C_T = C_T(M,q,\phi)$ and $I_{sp} = I_{sp}(M,q,\phi)$ are given by tabular functions. Engine throttling is controlled by ϕ.The model of the atmosphere given by tabular functions is based on Japanese Standard Atmosphere (Japanese Industrial Standards Committee,1990 [9]).

3 MINIMUM-FUEL ASCENT

3.1 Initial and Terminal Conditions

Assume the horizontal take-off condition and the terminal condition are specified as follows:

$$\begin{array}{ll} h(t_0) = 0 & h(t_f) = 80\text{km} \\ M(t_0) = \text{Mach}0.5 & M(t_f) = \text{Mach}24.0 \\ \gamma(t_0) = 0 & \gamma(t_f) = \text{free}. \\ \theta(t_0) = 0 & \\ m(t_0) = 13,876\frac{\text{kg.sec}^2}{\text{m}} & \end{array} \tag{7}$$

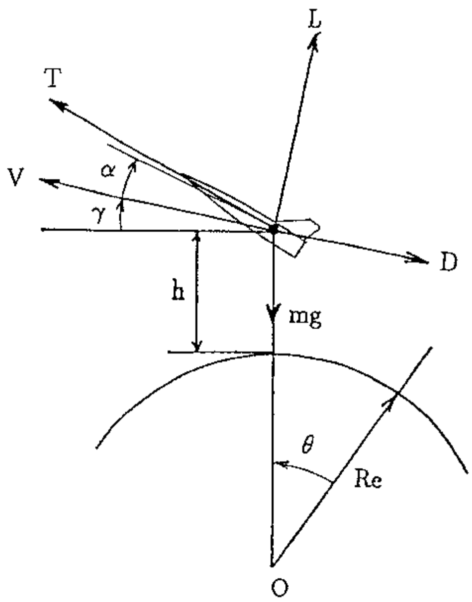

Fig. 1 Nomenclature used in aerospace planes.

3.2 State and Control Constraints

Two of the most important constraints which should be imposed on trajectories are

$$q = \frac{1}{2}\rho v^2 \le q_{max} \tag{8}$$

$$Q = K\sqrt{\rho}v^3 \le Q_{max} \tag{9}$$

where K is a constant. Eqs. (8) and (9) are constraints on dynamic pressure q and heating rate Q at a specified point of the vehicle, respectively.

3.3 Performance Index

Since this study is concerned with minimum-fuel ascent to orbit, the obvious performance index to be minimized is the fuel consumption.

$$J = \int_{t_0}^{t_f} -\dot{m}dt = \int_{t_0}^{t_f} |\dot{m}|dt \tag{10}$$

where t_0 and t_f are the starting and arriving times, respectively. Then the aerospace plane trajectory optimization problem(Original Problem) is to fined the sequence of the optimal control $\{\alpha^*(t)$ and $\phi^*(t), t = [t_0, t_f]\}$ such that J of (10) is minimized subject to Eqs.(1-9).

4 EXTENDED HYBRID APPROACH

For a simplification, vector form is used hereafter. Since most continuous-time systems can be treated as discrete-time systems under suitable assumptions, the present problem described as

Minimize

$$J = \sum_{i=0}^{N-1} L_i(x_k, u_k) \tag{11}$$

Subject to

$$\begin{aligned} x_{k+1} &= g_k(x_k, u_k) && (12) \\ x_0 &= c_0, \quad x_N \in \Omega_F && (13) \end{aligned}$$

where $x_k \in X_k \subset R^n$ and $u_k \in U_k \subset R^m$. X_k and U_k are the sets of admissible states and controls at stage $k(k = 0, \ldots, N)$, respectively. On the application to minimum-fuel ascent problems of an aerospace plane , $x_k = (v_k, \gamma_k, r_k, \theta_k, m_k)'$, $u_k = (\alpha_k, \phi_k)'$ and $L_k = \int_{t_k}^{t_{k+1}} |\dot{m}| dt$ are defined. L_k is the cost per single stage. g_k is the state transition function described by Eqs.(1)-(5).

4.1 Basic Sub-Algorithm

A procedure for quantizing a dynamic programming algorithm defined only for the state space generated by the algorithm is considered in order to utilize the concept that a deterministic problem is equivalent to finding a shortest path from the initial node of the graph to the destination node.

4.1.1 Forward Dynamic Programming and Quantization Procedures

The state space X_k of the problem having n-dimensional state vector x_k is quantized into reasonable subspaces $X_{k1}, X_{k2}, .., X_{kn_k}$ such that $X_k = \bigcup_{i=1}^{n_k} X_{ki}, (k = 1, .., N)$. Each subspace is called "block". The next, the set of admissible trajectories from the initial point x_0 such as $\{X_k^\circ\} = \bigcup_{i=0}^{k} X_i^\circ$ is generated by applying quantized control u_k repeatedly at each stage k. Here, X_k° is defined recursively such as $X_0^\circ = \{x_0\}, X_{k+1}^\circ = \{x_{k+1} \mid x_{k+1} = g_k(x_k, u_k), x_k \in X_k^\circ, u_k \in U(x_k)\}(k = 0, \ldots, N-1)$. This set $\{X_k^\circ\}$ has structures of a tree emanating from the initial point. For only block where the trajectory from the initial point exists, its own only representative point $x_{ki} \in X_{ki}$ is defined as follows:

$$\begin{aligned} f_k(x_{ki}) &= \min_{x_k} \{T(x_k) \mid x_k \in X_{ki}\} \\ & \qquad (i = 1, \ldots, n_k,\ k = 0, \ldots, N) \end{aligned} \tag{14}$$

where $T(x_k)$ is the cost from the initial point, which is given recurrently as

$$\begin{aligned} T(x_0) &= 0 (i = 1, \ldots, n_k, k = 1, \ldots, N) \\ T(x_k) &= f_{k-1}(x_{k-1i}) + L_{k-1}(x_{k-1i}, u_{k-1}) \\ x_k &= g_{k-1}(x_{k-1i}, u_{k-1}) \end{aligned} \tag{15}$$

Thus, the representative point x_{ki} for each block has the minimum cost from the initial point to the arrival point within the block. Then the forward dynamic programming algorithm defined on the finite representative points X_k° is constructed as

$$\begin{aligned} & f_0(x_{01}) = 0 \\ & f_{k+1}(x_{k+1i}) = \min_{x_{ki}, u_k} \{ f_k(x_{ki}) \\ & \quad + L_k(x_{ki}, u_k) \mid x_{ki} \in X_k^\circ, u_k \in U(x_{ki}) \} \\ & x_{k+1i} = g_k(x_{ki}, u_k) \quad (i = 1, \ldots n_k) \end{aligned}$$

where $f_k(x_k)$ is the forward minimum cost function. In this algorithm, the computation is proceed forward from the initial point x_0 using quantized controls u_k stagewise, step by step, and terminated at the time which one of trajectories generated is arrived first at the terminal point. Then the optimal solution is obtained with the optimal cost of $f_{0,N}^* = \min_{x_N}\{f_N(x_N) \mid x_N \in \Omega_F\}$.

The preference order calculation is to evaluate first the most optimal-like trajectory which has the smallest value of $T(x_k)$. Any trajectory arrives the second or the more later at the block can be rejected. This fact produces considerable reductions in computation requirements.

4.1.2 Bounding Operation and Clearance

To obtain more reductions in the computation requirements of the hybrid algorithm, the branch-and-bound technique is used [1, 11, 6]. As is well known, the state x_k can be eliminated if

$$f_k(x_k) + M_k(x_k) > I, \quad k = 0, .., N \tag{16}$$

where I is an upper bound of the final optimal cost $f^*_{0,N}$ and $M_k(x_k)$ is a lower bound of the backward minimum cost function $J_k(x_k)$. The conditions to be satisfied for those bounds are given by

$$\begin{aligned} M_k(x_k) &\leq \min_{u_k,u_{k+1},\ldots,u_{N-1}} \{\sum_{i=k}^{N-1} L_i(x_i,u_i)\} \\ &= J_k(x_k), \quad x_k \in X_k \\ I &\geq f^*_{0,N} \end{aligned} \tag{17}$$

So, the size of reductions of the computation requirements as well as the computation regions depends entirely on estimated accuracy of both I and $M_k(x_k)$. Defining Δa and Δb_k such as

$$\Delta a = I - f^*_{0,N}, \quad \Delta b_k = J_k(x_k) - M_k(x_k) \tag{18}$$

where Δa and Δb_k are considered as the strength of the upper and the lower bound. The size of the reduction depends on the value of the sum of Δa and Δb_k rather than separate Δa and Δb_k. Here, the sum $\Delta\varepsilon(\Delta\varepsilon = \Delta a + \Delta b_k)$ is called a clearance. Then the condition of the clearance to obtain the global optimal solution is given by

$$\Delta\varepsilon = \Delta a + \Delta b_k \geq 0 \tag{19}$$

Thus, the most desirable value of $\Delta\varepsilon$ is $\Delta\varepsilon = 0$.

4.2 Iterative Sub-Algorithm

To strengthen the bounding operation using Eq.(16) and to make more reductions of the number of the computations, the iterative sub-algorithm is considered. This algorithm improves successively not only lower and upper bounds but also a accuracy of the optimal solutions because state and control variables are quantized more finely, little by little in proportion to iteration progresses.

4.2.1 Calculation of Bounds

The upper bound I^i and the lower bound $M^i_k(x_k)(x_k \in X_k)$ at the ith iteration can be determined from values of the previous total cost $f*^{i-1}_{0,N}$ and the partial cost on the optimal trajectory as

$$I^i = f*^{i-1}_{0,N} \quad (i = 2,3,\ldots) \tag{20}$$

$$\begin{aligned} M^i_k(x_k) &= J^{i-1}_k(x^{*i-1}_k) \\ &= \sum_{l=k}^{N-1} L_l(x^{*i-1}_l, u^{*i-1}_l) \end{aligned} \tag{21}$$

where x^{*i-1}_l and α^{*i-1}_l are the state and the control on the previous optimal trajectory. I^i gives a good upper bound. M^i_k is a very tight lower bound in the neighboring states of the optimal trajectory. However the global optimal solution is not always obtained because for the states which are distant tolerably from that trajectory the lower bound condition(21) is not always satisfied. In the next section the method to obtain the global solution is considered.

4.3 Parallel Sub-Algorithm

The parallel sub-algorithm is used to obtain a global solution under the condition $\Delta\varepsilon \geq 0$. If $\Delta\varepsilon < 0$, then the lower bound $M^i_k(x_k)$ (21) is modified downward as

$$M^i_k(x_k) = J^{i-1}_k(x^{*i-1}_k) - |\Delta\,\tilde{\varepsilon}_{min}\,| \tag{22}$$

where $\Delta\,\tilde{\varepsilon}_{min}$ is the estimated minimum value of $\Delta\varepsilon(<0)$. If $\Delta\varepsilon \geq 0$, $\Delta\,\tilde{\varepsilon}_{min}= 0$ is used. However, for problems with multi-modal cost function, a big $|\Delta\,\tilde{\varepsilon}_{min}\,|$ may be required in order to satisfy $\Delta\varepsilon \geq 0$ over the domain of the problem considered, but it will cause an increase of the computation requirements. Parallel sub-algorithm is used to avoid this demerit. To apply this algorithm, the first the group of the tentative optimal trajectories is generated under a coarse quantization block using the basic sub-algorithm. This group will contain the global and the local optimal trajectories. The cost on those trajectories is used as the initial conditions to start the parallel sub-algorithm. That is, for individual local solutions the iterative sub-algorithms are applied separately in parallel with a smaller $|\Delta\,\tilde{\varepsilon}_{min}\,|$, and the most optimal trajectory among those improved local solutions is selected as the global optimal solution.

5 LOWER BOUND BY ENERGY-ASCENT

The energy-state approximation is used to obtain a solution of the relaxed problem to the original problem which can be used as the lower bound required in the hybrid algorithm. The relaxed problem is designed to be easier to solve than the original problem. In the energy-state approximation, the total energy per unit mass E of an aerospace plane is regarded as a state variable.

$$E = \frac{1}{2g}v^2 + h. \tag{23}$$

The time rate of change of energy E is obtained by differentiating (23) using (1),(3) and $h = r - R_e$ to eliminate $\dot{v}$ and $\dot{h}$

$$\begin{aligned} \dot{E} &= \frac{v(T(h,v,\phi)\cos\alpha - D(h,v,\alpha))}{m} \\ &\quad - \frac{2\mu h v \sin\gamma}{r^3} \end{aligned} \tag{24}$$

Dividing (24) into (5), and expressing h in terms of E and v using (23), then the equation

$$-dm = \frac{T(E,v,\phi)}{g_0 I_{sp} f_1} dE \tag{25}$$

is obtained where f_1 is given by

$$\begin{aligned} f_1 \equiv \dot{E} &= \frac{v(T(E,v,\phi)\cos\alpha - D(E,v,\alpha))}{m} \\ &\quad - \frac{2\mu h v \sin\gamma}{r^3} \end{aligned} \tag{26}$$

To minimize fuel-consumption to ascend to a given altitude and velocity using the energy-state approximation, it is clear that $I_{sp} g_0 f_1 / T(E,v,\phi)$ must be maximized with respect to v for a given E. To obtain the lower bound of (25), the second term in the right-hand side of (26) can be taken off if $\gamma \geq 0$. The assumption of $\gamma \geq 0$ is reasonable because in usual minimum-fuel ascent of aerospace planes with dynamic pressure constraints, the optimal trajectory is active on its constraint. This means $\gamma \geq 0$ with monotone increase of the ascent trajectories. Besides the drag D is modified as

$$\begin{aligned} &D(E,v,\alpha) \\ &\quad = \frac{1}{2}(C_{D0}(M) + \eta(M) C_{L\alpha}(M)\alpha^2)\rho(h) v^2 S \\ &\quad \geq \frac{1}{2} C_{D0}(E,v)\rho(E,v) v^2 S = D'(E,v) \end{aligned} \tag{27}$$

where $\alpha = 0$ is used and M is expressed in terms of E and v. Then, it is easy to show the relation

$$T\cos\alpha - D \leq T - D'. \tag{28}$$

Besides to obtain the lower bound strictly, the lower bound of mass m_b is used which is equal or less than the total mass of an aerospace plane excepting fuel and a fuel-equivalence ratio of ϕ'=unity is used which corresponds to maximum fuel efficiency. Thus the relaxed problem under those modifications is given as

$$\begin{aligned} \text{Minimize } J^M &= \int_{E_0}^{E_f} \frac{T(E,v,\phi')}{I_{sp} g_0 f_1} dE \\ &\leq \int_{t_0}^{t_f} -\dot{m} dt \end{aligned} \tag{29}$$

It is clear that the value of the cost function obtained by (29) is always less than that of the original problem(1)-(10). Hereafter, the method with those modifications is called the modified energy-state approximation. To minimize fuel-consumption to ascend to a given altitude and velocity using the modified energy-state approximation, it is clear that $I_{sp} g_0 f_1 / T(E,v,\phi')$ with respect to v for a given E should be maximized. This operation gives us the absolute minimum value of the fuel-consumption to ascend an energy section of two adjacent stages. Therefore, the lower bound $M_k(x_k)$ can be obtained as the sum of the individual absolute minimum value through the passage region on flights of the aerospace plane, and obtained by very reduced computational efforts.

6 OPTIMAL SOLUTIONS

First, the lower bound solution of the present minimum-fuel ascent problem was computed under the constraint of dynamic pressure $q = 9{,}766\text{kg}/\text{km}^2 (2{,}000\text{psf})$ by the modified energy-state approximation which was used in the first computation of the iterative sub-algorithm. The computation time is less than 1 second on a computer with speed of 100 MIPS. The trajectory of this solution, which goes along with the constraint curve of dynamic pressure $q = 9,766\mathrm{kg/km}^2$ completely. However it should be indicated that the lower bound solution under $q = 12,207\mathrm{kg/km}^2(2,500\mathrm{psf})$ in the same problem goes over slightly this constraint curve at the only neighborhood of the energy contour containing the target point. The next, to apply the hybrid algorithm more effectively to the current problem, the independent variable is changed from t to E. Each state variable of h (instead of r),v, γ and m is quantized into 64, 32, 32 and 8 levels per each admissible region, respectively. A stage variable E is divided into 20 levels and labeled them as stages k and E_k. A control variable α is quantized into 13 levels for its admissible range at the first computation and a fuel equivalence ratio ϕ of unity is used which is considered as almost optimal for the active constraint on q. This quantization requires cost function evaluations over 1×10^8 times if the conventional dynamic programming is used. The upper bound of the optimal solution required in the first computation of the hybrid algorithm can be estimated easily by the feasible solution obtained under the very rough quantization level with a use of the same lower bound solution. All the transitions of the states are computed by using numerical integration of the fourth-order Runge-Kutta method under single precision with a step size of $(E_N - E_0)/400$. An angle of attack history at the section $E_k \le E \le E_{k+1}$ is approximated by a straight line such as $\alpha_k(E) = \alpha(\alpha_k^*, \alpha_{k+1}, E)(\alpha_{k+1} \in U, E_k \le E \le E_{k+1})$ where α_k^* is the optimal control at representative points in the blocks at the stage k.

In Fig.2 is shown the optimal trajectory obtained after the first computation$(i = 1)$ by the Hybrid Dynamic Programming(HDP) algorithm. It can be seen that the trajectory undulates up and down over the constraint q. This means that a coarsely quantized control α_k cause a big undulation of trajectories but the hybrid algorithm is completely stable on computations. The region of crowded trajectories shown in Fig.3 designates the state space for which cost function is evaluated by the hybrid algorithm. This region is considerably reduced as compared with the whole definition region of problems considered. The convergence was effectively achieved after 8 iterations as shown in Fig 4. The computation time to obtain an optimal solution is about 300 seconds on the same computer. In Fig.5 is shown the converged optimal trajectory which goes along the constraint q and does not undulate almost. In Fig.6 are shown the histories of the state and the control variables on this fuel-optimal solution. The number of cost function evaluations required per iteration is less than 1/2,000 of that by the conventional dynamic programming. The size of the state space (that is, the number of blocks) for which cost function is evaluated is less than 1/1,000 of that by the conventional dynamic programming.

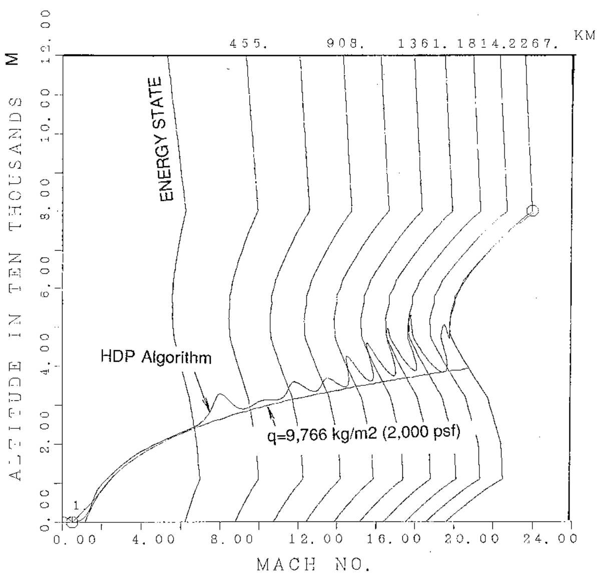

Fig. 2 Optimal ascent history by the basic sub-algorithm(i=1).

7 CONCLUSIONS

The hybrid algorithm based on the combined utilization of the forward dynamic programming and branch-and-bound method is properly interpreted, and is quite adequate for performance optimization of aerospace planes under the complexity of the dynamic model and complicated constraints. This hybrid algorithm gives us a unifying approach for obtaining routinely and

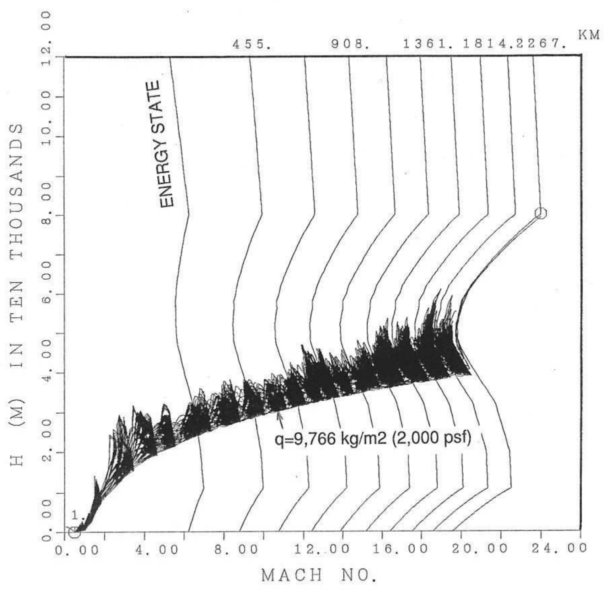

Fig. 3 State space for which cost function was evaluated by the hybrid algorithm.

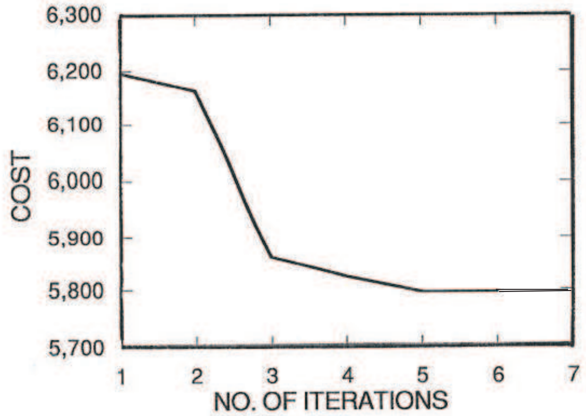

Fig. 4 Cost versus iteration for the minimum-fuel ascent problem.

robustly numerical solutions of the complicated high-dimensional optimization problems under various performance criteria such as minimum-fuel ascent and minimum time-to-climb. The lower bound required in this hybrid algorithm can be obtained easily by the energy-state approximation with some modifications by us, which reduces computational efforts considerably.

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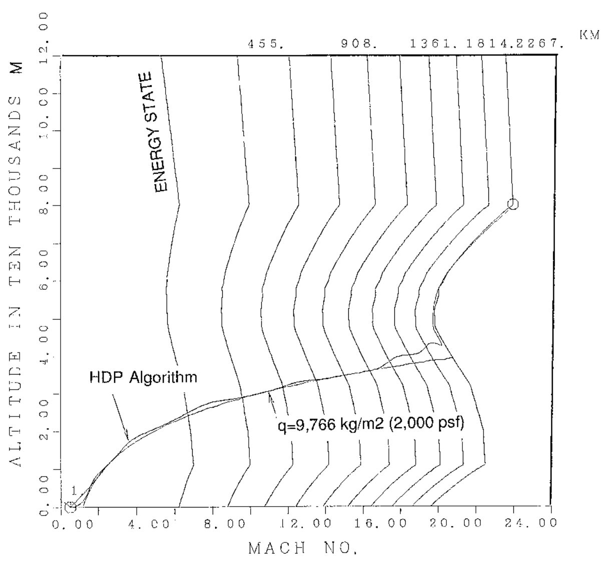

Fig. 5 Optimal ascent history by the iterative sub-algorithm(i=8).

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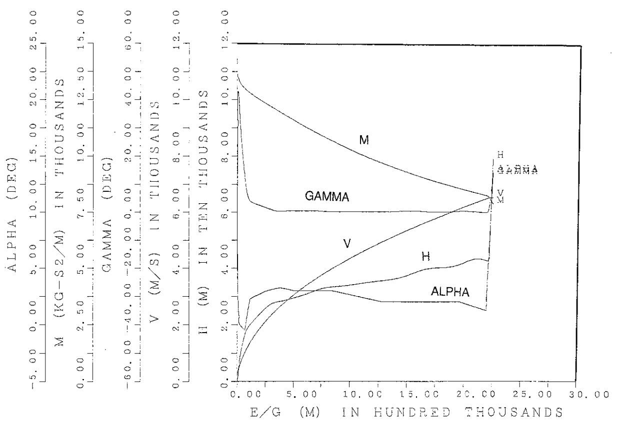

Fig. 6 Histories of state and control variables.