DESIGN OF FLIGHT CONTROL SYSTEMS CONSIDERING THE MEASUREMENT AND CONTROL OF STRUCTURAL MODES

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Abstract

Traditionally, in the design of a flight control system, flight mechanics and aeroelasticity are treated separately: flight mechanics uses the rigid body approximation and an aeroelasticity phenomenon does not make use of real time simulations. This approach is valid once the aircraft presents sufficient stiffness, which leads to a wide separation among the structural and rigid-body modes. Conventional airplanes configurations usually do not present any significant influence between rigid and aeroelastic modes but, as the search for a more competitive aircrafts increases the use of low density materials and airfoils with lower thickness, this situation may not be true and conventional design technique can lead to error. This article compares the design of a flight control system whose aircraft model was obtained using rigid body approximation with the design using an elastic aircraft model. Current design techniques, such as robustness barriers and notch filter, will be compared with the design considering the measurement and control of structural parameters.

1 Introduction

Despite the aircraft is a flexible body, flight mechanics commonly makes use of the rigid body approximation (RBA) neglecting the structural dynamics, which is acceptable once the aircraft has sufficient stiffness and consequently a wide separation among the structural and rigid-body modes as depicted on figure1.

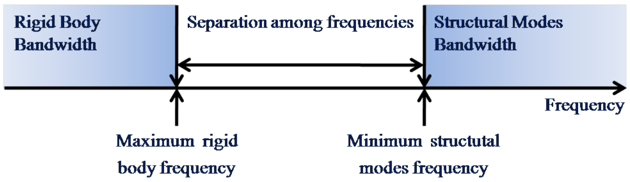

Fig. 1 Rigid Body and Structural modes separation

This hypothesis may not be valid for many aircrafts as the aircraft manufactures has increased the use of low-density materials (such as composites) and more tight criteria for structural design, bringing the safety margins to their limit. The search for more competitive aircrafts also brought the need for longer fuselages and airfoils with lower thickness. These new design techniques not only makes the aircraft lighter and decreases the drag but also reduces the structural modes natural frequencies, reducing the separation among the rigid body and structural modes.

The flight control system (FCS) must provide/enhance the aircraft's natural stability through the feedback of rates and acceleration. Modern Fly-By-Wire systems also have some protection features to avoid that the pilot exceeds the aircrafts limits in some maneuver.

The sensors used to feedback signals to the FCS have, not only the rigid body rates and accelerations but also high frequency content from the structural deformation, which may pass through the FCS and drive the controls at the frequency of the elastic modes leading to aeroservoelastic instability, i. e. flutter or limit cycle oscillations. To avoid this structural coupling, the design should not only consider the positioning of the sensors but also the the filtering of the FCS command and, as a side effect, there is a decrease in the phase margin in the Rigid Body Bandwidth, which leads to a decrease in the closed loop poles degrading the control law system performance.

This article compared the design considering the rigid body approximation (DRBA) where the filters are the only responsible to avoid the structural coupling with the FCS and the structural dynamics with the design considering the flexible aircraft model (DFAM), where structural modes are feedback increasing its damping and decreasing the need of filters leaving an open space to increase the control gains therefore improving the flight control system performance.

2 Development

2.1 Aircraft Model

The aircraft rigid body equations of motion can be obtained directly from Newton's second law using an inertial reference frame[5]. The work of Waszak et al[13] presents an integrated approach that unites the structural deformations performed by the aircraft structure and the aircraft dynamics while assuming that the structural deformation is sufficient small such as the linear elastic theory is valid. This formulation uses the Lagrangian Mechanics to deduce the equations and, adopting the mean axes, coupling between the rigid body degrees of freedom and the elastic degrees of freedom is avoided. In Silvestre[12] it is presented that the body and mean axis can be considered the same as the aircraft is performing common operations.

Here follows a brief description modal decomposition[1][8] and how the coupling between the structural and rigid body dynamics occurs according to the adopted formulation.

When submitted to external loads, the position of any point p in the aircraft structure can be written as linear combination of the position of n control points, therefore it is necessary $3n$ coordinates (considering displacements in the x, y and z axis of the chosen reference frame) to describe the structural displacement. This $3n$ generalized coordinates are given by the vector q as presented in equation 1, where the $N(x,y,z,t)$ is a matrix operator that interpolates the displacement in $q(t)$ to a generic displacement in $p_d(x,y,z,t)$.

$$\mathrm{p_d}\,(\mathrm{x,y,z,t}) = N(x,y,z,t)q(t) \tag{1}$$

Assuming that the structure has a linear behavior, its dynamics can be given by the equation 2 where M, B and K respectively mass, damping and stiffness matrices. Those matrices are dependant of several factors such as geometry and mass properties and can be obtained from commercial engineering softwares for example MSC NASTRAN. F is the external forces vector.

$$M\ddot{q}(t) + B\dot{q}(t) + Kq(t) = F(t) \tag{2}$$

The modal base can be derived from the free vibration problem of the equation 2, leading to equation 3, where η is the array of the $3n$ modal amplitudes and Σ is the matrix whose columns are the eigenvectors for each eigenvalue.

$$q(t) = \Sigma\eta(t) \tag{3}$$

Manipulating algebraically equations 2 and 3 leads to the new set of equations presented on equation 4, where ζ_i is the generalized damping, $\omega_{n,i}$ is the free vibration frequency of each mode given by $\omega_{n,i} = \sqrt{\lambda_i}$, μ_i is the generalized mass and $Q_{n,i}$ is the generalized force.

$$\ddot{\eta}_i + 2\varsigma_i\omega_{n,i}\dot{\eta}_i + \omega^2{}_{n,i}\eta_i = \frac{Q_{n,i}}{\mu_{ii}} \tag{4}$$

As already mentioned, the coupling between rigid and structural modes is not made through the rigid body or structural dynamics equations, instead it is modeled at the determination of the aerodynamic generalized loads. The total forces and moments presented in each axis at rigid body formulation[5] (X,Y,Z,L,M,N) are a function of the dynamic pressure, the engines position, geometry (reference area and mean aerodynamic chord) and dimensionless coefficients. The equation for the Pithing Moment (M) is depicted on equation 5 where it can be seen that there are coefficients related to the rigid body movement and also coefficients related to the structural deformation.

$$M = \frac{1}{2}\rho V_0^2 S\bar{c}\Big[CM_0 + CM_\alpha \alpha_v + CM_p p + \ldots + CM_q q + CM_\delta \delta + \sum_{i=1}^{\infty}\left(CM_{\eta_i}\eta_i + CM_{\dot{\eta}_i}\dot{\eta}_i\right)\Big] \tag{5}$$

In Waszak[13] the strip theory is applied to obtain the dimensionless coefficients, which is expanded later by Silvestre[12] for the complete aircraft dynamics. In Pogorzelski et al[7], the unsteady strip theory is applied to compute the aerodynamic loads. In Neto[6] the doublet Lattice method is applied to determine these loads. This work uses the model developed by Silva[10], which developed a conceptual aircraft for flight mechanics and flight controls studies. This model has three different configurations, each one with a different damping for the structural modes. The first configuration is used as it presents a behavior closer to the current aircrafts being developed by aeronautical industry considering the presence of four structural modes. This model has two pairs of elevators with are deflected symmetrically by the altitude hold autopilot. This model also presents two pairs of aileron which can be deflected symmetrically to control (damp) the structural mode.

2.2 Control Law Architecture

The altitude hold autopilot allows the aircraft to be held at a fixed altitude decreasing the pilot workload. This system is more concerned with the steady-state error and disturbance rejection than a fast response as the longitudinal control law of a Fly-By-Wire system. It has limited authority over the aircraft, usually limited at 0.15g over the longitudinal axis, allowing to keep the aircraft within $\pm 65\,ft$ from the selected altitude (RVSM requirement). It controls the pressure altitude after been filtered by a complementary filter. It is also used together with another control laws such as Mach, Speed and rate of climb hold to compose different autopilot modes such as Flight Level Change.

In the DRBA, the control law is composed by a PID controller added with and stability augmentation system (SAS), i. e. the aircraft short period states are feedback to improve the aircraft short term response characteristics[4]. Figure 2 presents the block diagram for this control law where it can be seb that the derivative term of the PID controller is made directly through the feedback of altitude rate ($\dot{H}$). It already includes the notch filters as it will be discussed later in section 2.3.

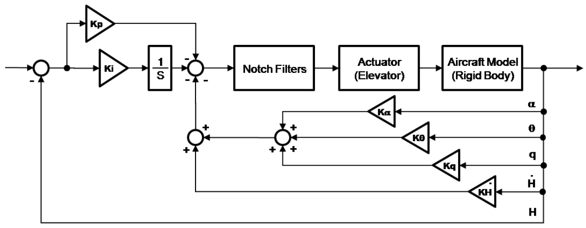

Fig. 2 Block Diagram for the design considering RBA

In the design considering the elastic body model, moreover the previous PID+SAS another control input is used, symmetrical deflection of Aileron (Elevon), to increase the damping of the structural modes through the feedback of the rate of the first two structural modes. A Low Pass Filter is included to attenuate effects of structural modes higher than two. Figure 3 presents its block diagram.

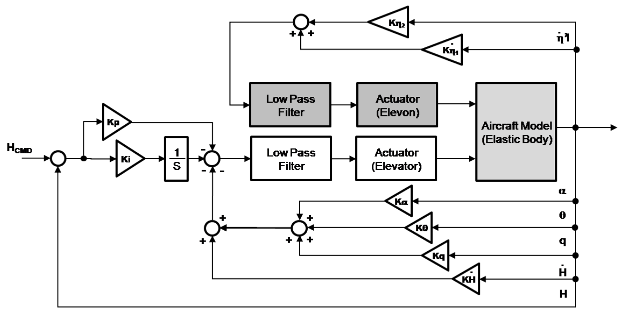

Fig. 3 Block Diagram for the design considering EAM

The altitude hold auto-pilot will be designed through frequency domain optimization, which consists in determinate the vector p (N-dimensional) of the decision parameters which minimizes a functional $J(\omega)$ in a desired frequency range. The vector p contains all the controller gains and as the optimization converges, the dynamic behavior of the system open loop transfer function L is close to the desired target loop transfer function G_d.

The use of optimization to design flight control systems has already been subject of study leaving to the designer the task of mapping all the requirements into a cost function[11]. The DRBA uses the functional presented at the equation 6, where ω is the frequency in rad/s, σ_{G_d} and σ_L are the singular values of G_d and L respectively. This functional computes the square error relative to the desired open loop response pondered by the frequency.

The way this functional was written, broken in two different integrals, gives more emphasis at the edges of the desired frequency range. The U factor is included to add a heavy penalty if the gains at vector p results unstable poles. Depending on the characteristics of the plant to be controlled and the desired open loop transfer function, it may be quite difficult to achieve a perfect fit between L and G_d , so the algorithm tries to at least maintain L close to G_d by a γ factor as presented on equation 6, which is accomplished by the f factor.

$$T_{DRBA} = \int_{\omega_0}^{1} f_1(\sigma_L(\omega)) \left(\frac{\sigma_{G_d}(\omega)-\sigma_L(\omega)}{\omega}\right)^2 d\omega + \ldots$$
$$+ \int_{1}^{\omega_1} f_1(\sigma_L(\omega))\left((\sigma_{G_d}(\omega)-\sigma_L(\omega))\omega\right)^2 d\omega + U \tag{6}$$

The DFAM uses the same functional including an extra term to bring the structural response bellow the threshold of $-9db$ between a pre defined range of frequencies as presented on equation 7,where $\sigma_{BL_{G_d}}$ and σ_{BL_L} are the singular values of the broken loop of G_d and L respectively. This threshold and the broken loop response will be more detailed in section 2.3.

The target open loop transfer function has a bandwidth of $0.15 rad/s$ as the auto pilot for the altitude hold has no need for fast responses. Any bandwidth higher than the aircraft's Phugoid should already give good response.

$$T_{DFAM} = \int_{\omega_0}^{1} f_1(\sigma_L(\omega)) \left(\frac{\sigma_{G_d}(\omega)-\sigma_L(\omega)}{\omega}\right)^2 d\omega + \ldots$$
$$+ \int_{1}^{\omega_1} f_1(\sigma_L(\omega))\left((\sigma_{G_d}(\omega)-\sigma_L(\omega))\omega\right)^2 d\omega + \ldots$$
$$\int_{\omega_2}^{\omega_3} f_2(\sigma_{BL_L}(\omega))\left(\sigma_{BL_{G_d}}(\omega)-\sigma_{BL_L}(\omega)\right)^2 d\omega \tag{7}$$

The result of the optimization for both designs are presented on figures 4 and 5, where it can be seen that L has a good fit on G_d, admitting a 15% error, up to $5 rad/s$, which is appropriate for this control law purposes. In the lower part of figure 5 its is presented the optimization regarding the bandwidth of the structural modes.

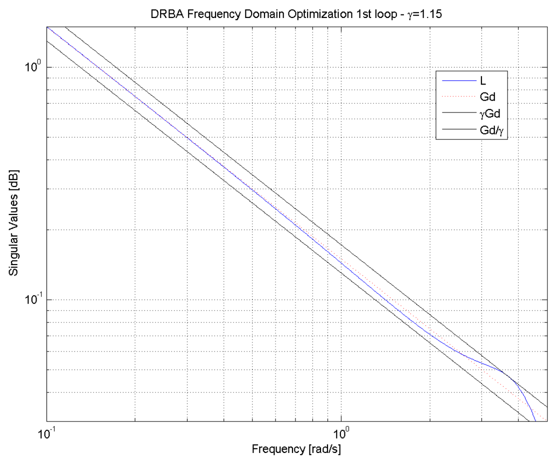

Fig. 4 Frequency Domain Optimization (DRBA)

2.3 Structural Modes Filtering

The sensors used to provide data to the flight control system detects not only the rigid body motion of the aircraft but also the high frequency structural dynamics. These dynamics may propagate through the flight control system and drive the control surfaces and, when interacting with the aerodynamic and inertial forces, may lead to instability. This issue has already been addressed by industry as it can be seem in Caldwell et al[2], Gangsaas et al[3] and Shin et al[9].

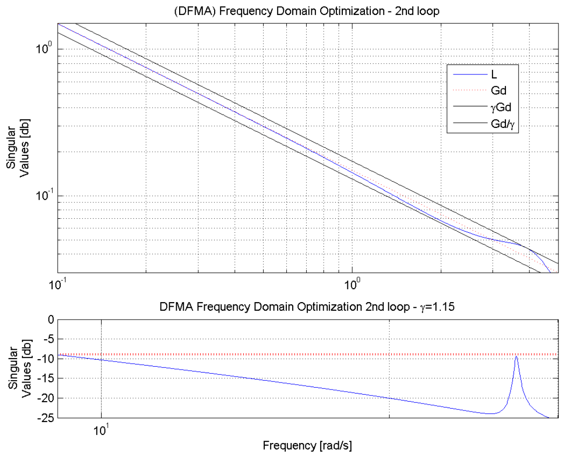

Fig. 5 Frequency Domain Optimization (DFAM)

The structural dynamics shall also be considered when installing the sensors at the aircraft because they could attenuate or amplify the high frequency structural content of the signal - e. g. if the sensor is located in a structural node, two or more structural modes may interact and attenuate each other. On the other hand, the sensor could be installed in the wing tip and detect different accelerations than the one performance by the aircraft center of gravity.

To avoid the structural dynamics being feedback to the flight control system, a set of filters are used to attenuate the signal content inside the range of the structural dynamics. This is usually done by a set of notch filter and/or low pass filters whose function is to preserve the high gain at low frequencies, keeping the flight control desired performance, and attenuate the high frequency content.

This analysis is done through the broken loop response from the control surface actuator to the flight control system output with the set of filters (if needed). The broken loop response shall lie bellow the FRF (Frequency Response Function).

In this article, the FRF used in the design considering the rigid body approximation is defined from the separation of the broken loop response of the rigid and elastic body model, considering the amount of attenuation needed to bring all the resonance peaks bellow to a certain threshold and the uncertainties in the frequency of these resonances. The Military Standard MIL-F-9490D recommends a gain margin of $8db$ in every loop, but it is an industry standard to adopt threshold $9db$ for this threshold[3]. The same threshold is used in the FRF defined for the design considering the elastic model.

The broken loop frequency response of both designs are depicted in the figures 6 and 7 together with both FRF. The notch filters acts like a band-stop filter with a narrow stop band centered at the frequency to be rejected. Its equation is presented on equation 8 where ω_0 is the desired frequency to be attenuated and the parameters d and c adjust the attenuation and how narrow the filter is.

$$H(S) = \frac{S^2 + 2\left({}^{d}/_{c}\right)\omega_0 S + \omega_0{}^2}{S^2 + 2\left({}^{1}/_{c}\right)\omega_0 S + \omega_0{}^2} \qquad (8)$$

It can be seen from the figures 6 and 7 that the requirements were satisfied: The broken loop response in the DRBA lies bellow the FRF and the broken loop response in the DFAM lies bellow the $-9db$ threshold.

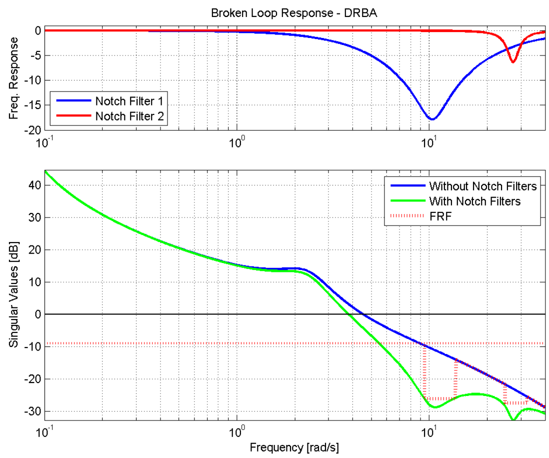

Fig. 6 Broken Loop Response (DRBA)

The filtering adds additional phase lag to the control loop as it can be seen in the figure 8 where its values at $1 rad/s$ and $5 rad/s$ is highlighted for comparison. The high phase delay introduced by the set of notch filters used in the DRBA adds

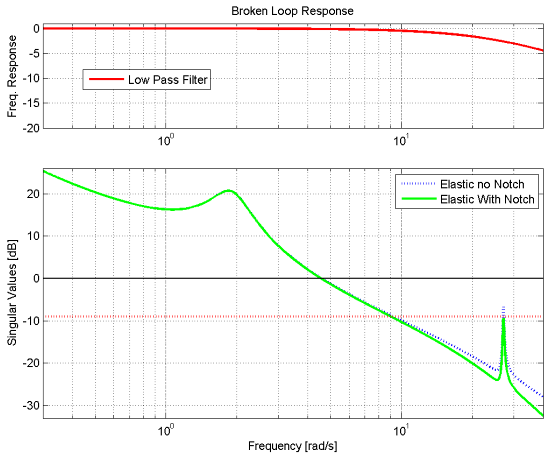

Fig. 7 Broken Loop Response (DFAM)

additional penalties in the project which may penalize the controlled aircraft performance.

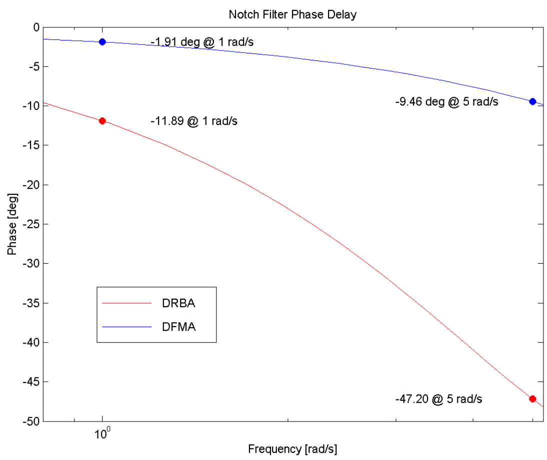

Fig. 8 Phase Delay Introduced by Filters

3 Results

Without the introduction of the notch filter in the control loop, the controlled aircraft in the DRBA has an acceptable damping but, as the notch filter were included in the control loop the aircraft became unstable. It was necessary to re-calculate the gains considering the filters and even though the damping felt dramatically as show in Figure 9. This occurs due to the high phase delay introduced by the filters in the control loop as already depicted in Figure 8.

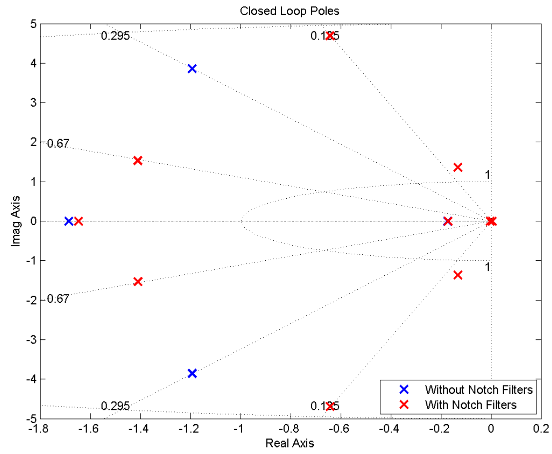

Fig. 9 Closed Loop Poles(DRBA)

The filter used in the DFAM did not introduce a phase delay as high as in the DRBA therefore there were no need for a second loop in the gains calculation. Of course the damping had a slight felt with the filters but it kept close to the damping of the aircraft in the DRBA without the filter as presented in Figure 10.

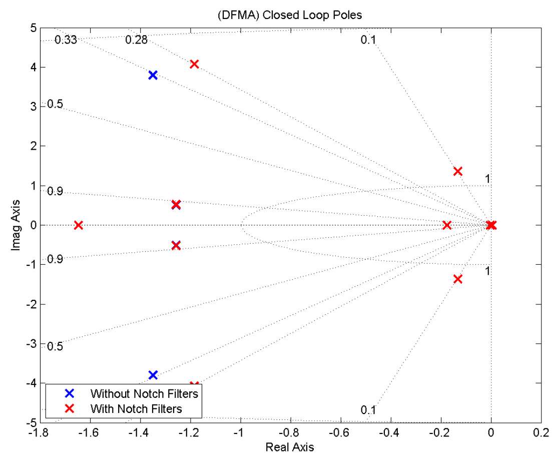

Fig. 10 Closed Loop Poles(DFAM)

The aircraft response to an altitude step is presented in the figures 11 and 12. Both designs provide the same altitude response but the damping in the pitch rate response is higher in the DFMA. The structural response in the DFMA is much lower than in the DRBA due to the feedback of the structural modes rate.

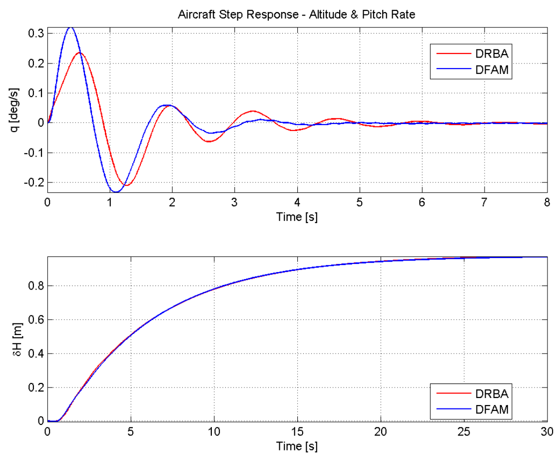

Fig. 11 Aircraft Step Response - Altitude & Pitch Rate

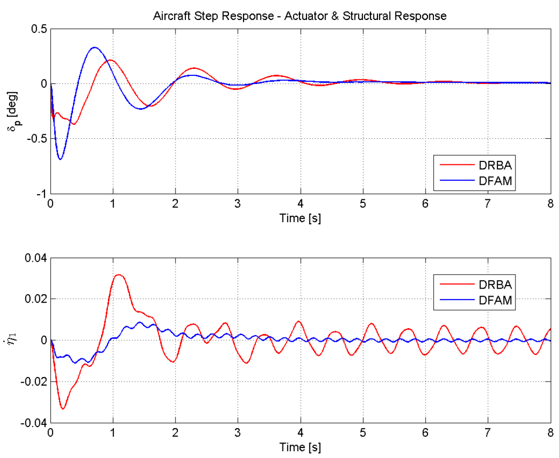

Fig. 12 Aircraft Step Response - Actuator & Structural Response

4 Comments and Conclusions

As the search for more competitive aircraft grows, the aircraft's structure becomes more flexible and it structural modes may interfere with the rigid body modes. The use of filter to avoid this coupling introduces large phase delay in the control loop that may compromise its performance or even the stability. Other approach to avoid the aeroservoelastic coupling is to control the structural modes through the feedback of its parameters. The control of the structural modes may not only avoid the extensively use of filters but also reduce the structural workload implying in a reduction of fatigue and the time between consecutive scheduled maintenance.

During development, the use of filtering implies also in multiple loops in the computation of the control gains and the filters parameters, i. e. after computing the gains and the filters parameter it may be necessary to re-evaluate the design and even change it.

Counterbalancing this advantages, the feedback of structural modes brings the need of the measurements of the structural modes, which involves not only the appropriate choice of the sensor location but also the hole development of the system which provides all the safety requirements pointed by the Fault Hazard Analysis (FHA). It shall be addressed also the safety and procedures involved in the case of the failure of such system. Other aspect that may brings difficulties to the project is the flexible aircraft model, whose parameter depends on a certain maturity of the project.

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