# LONGITUDINAL STABILITY AUGMENTATION DESIGN WITH TWO DEGREE OF FREEDOM CONTROL STRUCTURE AND HANDLING QUALITIES REQUIREMENTS

**Francisco J. Triveno Vargas***, **Fernando J. O. Moreira*** , **Pedro Paglione****
***EMBRAER, **Technological Institute of Aeronautics ITA**

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## Abstract

*This paper presents the design of longitudinal stability augmentation system (SAS) using a two degree of freedom control structure based on the LQR*[1] *technique in the frequency domain, which is robust to plant uncertainties, sensor noise and external disturbances. The design incorporates various handling quality requirements involving modal and bandwidth domain criteria. The design is done looking its use in the possible alternative mode in fly-by-wire aircraft with relaxed stability. The approach is applied throughout the flight envelope of a commercial aircraft and the first results obtained are presented.*

## 1 Introduction

An aircraft model, characterized by its weight, center of gravity ($x_{cg}$) position, airspeed, altitude, flight path angle, flaps configuration, and landing gear position among others, is subject to a wide range of parameters variation. These characteristics change its dynamics and for this reason a dynamic mode that is stable and adequately damped in one flight condition may become instable or at least inadequately damped in another flight condition. In commercial aircraft a lightly damped oscillatory mode may cause a great deal of discomfort for passengers or make it difficult for the pilot to control the aircraft. For a combat aircraft this condition may lead to a more critical situation because the aircraft is already inherently unstable due to the maneuverability requirements and capability of attack.

These problems are overcome by using feedback control to modify the aircraft dynamics. Also aircraft's manufactures are currently developing improvements in terms of weight diminution, aerodynamic efficiency and optimization of fuel consumption. These changes are naturally leading to design of new airplanes with relaxed stability, increasing the use of feedback control laws [1]. Either a Stability Augmentation System (*SAS*) to increase and to change the natural frequencies of aircraft modes or a Control Augmentation System (*CAS*) to control the modes and to provide the pilot with a particular type of response are used as control laws. In this sense a Two Degree of Freedom ($TDOF$) controller [2] is normally used to project a Stability Augmentation System.

The $TDOF$ is a robust control strategy that guarantees a better performance against uncertainties, sensor noise and external disturbances when compared with some classical techniques [3]. The structure was successfully implemented for control position problem in a *SCARA* manipulator robot [4] and at the present moment, a technique extension, applicable to commercial and military aircrafts, is being developed.

The project focuses on finding optimal gains to shown robustness at different flight conditions using fixed gains instead of traditional gain scheduling. The gains are calculate using a *LQR*

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[1] Linear Quadratic Regulator

approach in the frequency domain via spectral factorization. The closed loop system is verified using traditional time response and stability analysis, and also some handling qualities criteria. HQ criteria used in this work can be divided in two categories: modal and bandwidth criteria.

In section 2 the mathematical model is described. Handling qualities criterions are presented in section 3. Main characteristics of the *TDOF* controller are presented in section 4. The project of controller and linear simulations are presented in section 5. The handling qualities analysis and results are presented in section 6. The digital implementation and some non linear simulations are presented in section 7. Finally, some conclusions are presented.

## 2 Mathematical Modelling

The aircraft dynamical behavior is modelled by a set of non-linear differential equations. A complete set of differential equations were derived considering only the aircraft rigid body motion, subjected to all usual external forces (aerodynamic lift and drag, thrust, and gravity). Stability derivatives calculation were performed based on historical data and geometric characteristics of the aircraft [5]. For the design phase and dynamical analysis, the non-linear model is linearized for a given flight condition of interest.

The linear models obtained in the trimming points used to design and analyze the controller are composed by following traditional set of linear equations:

$$\begin{bmatrix} \dot{V} \\ \dot{\alpha} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = A \begin{bmatrix} V \\ \alpha \\ q \\ \theta \end{bmatrix} + B\delta_e \tag{1}$$

where: $V$ is de airspeed, $\alpha$ is de angle of attack, $q$ is the pitch rate, $\theta$ is the attitude angle and $\delta_e$ is the elevator command.

### 2.1 Actuator and sensors models

The actuator model used for project and analysis corresponds a second order filter. The sensors dynamics were modelled as a 4th order Padé approximation.

### 2.2 Mass, velocity and center of gravity variations

The robustness specifications are based on the variation of the aircraft mass $m$, its velocity $V$ and its center of gravity along the body *x-axis* ($x_{cg}$). Nine models with mass varying between 10000 and 14000 kg with different velocities and center of gravity are considered. These models are presented in Table 1. A unique fixed parameter controller will be provided for all $V - x_{cg}$ flight condition.

| $x_{cg}$ <br> $V$ $[Kts]$ | 22 | 29 | 32 | 40 | 47 |
|---|---|---|---|---|---|
| 93 | | x | | x | x |
| 96 | x | x | x | | |
| 106 | | | x | x | x |

**Table 1** Linear models for $V$ and $x_{cg}$

## 3 HANDLING QUALITIES

Two handling quality criteria are used in the design. As suggested in references [6] and [7], use of the HQ criteria is made in the design verification phase to evaluate good response to a pitot input. From [8] the good response for the pilot are evaluated according to the following criteria.

- Satisfactory: Flying qualities clearly adequate for the mission Flight Phase. Desired performance is achievable with minimal pilot compensation.
- Acceptable: Flying qualities adequate to accomplish the mission Flight Phase, but some increase in pilot workload or degradation in mission effectiveness, or both.
- Controllable: Flying qualities such that the aircraft can be controlled in the context of the mission Flight Phase, even though pilot workload is excessive or mission effectiveness is inadequate, or both.

Level 1 is Satisfactory, Level 2 is Acceptable, and Level 3 is Controllable.

### 3.1 Modal Criteria

This criterion is related essentially with the damping ratios of aircraft modes: the short period and the phugoid modes [5].

The slow oscillating mode (phugoid) is generally underdamped. It mainly affects the pitch attitude and the true velocity. To reach the flying qualities required for level I, the phugoid damping ratio shall be greater than 0.04.

A rapid oscillating mode, short period, mostly affects the transient responses in angle of attack, pitch rate and load factor. To reach the flying qualities required for level I, the short period damping ratio shall be greater than 0.4.

### 3.2 Bandwidth Criterion

The criterion establishes that a measure of the handling qualities of an aircraft is based on its stability margin when operated in a closed pilot in the loop compensatory pitch attitude tracking task. The maximum frequency at witch such closed loop tracking can take place without spoiling stability is referred to as bandwidth [9].

The control bandwidth is further complicated by the fact that it varies with the inputs-outputs variables involved. Control and handling difficulties may arise when the bandwidth of an input-output relation is lower than it should be. Thus, all inputs-outputs bandwidth properties should be consistent to have good handling and adequate stability margins. Two measures for that are the pitch attitude and the flight path angle bandwidth [10].

From the frequency response of the considered output to stick input, the bandwidth frequency is the lower frequency for which the phase angle is -135, $\omega_{BW_{phase}}$, and gain margin of 6 $dB$ $\omega_{BW_{gain}}$ [11].

The phase delay requirement based on attitude response complete the bandwidth criteria. It is defined as:

$$\tau_p = \frac{\Delta\phi_{2\omega_{180}}}{2\omega_{180}}\frac{\pi}{180} \tag{2}$$

where $2\omega_{180}$ is twice the neutral stability frequency i.e., the frequency at -180 degrees phase, and $\Delta\phi_{2\omega_{180}}$ is the phase at twice the neutral stability frequency, i.e., is the phase for the frequency with a value twice the frequency for the phase equal to -180 degrees.

Table 2 summarizes the handling quality boundaries being considered in the design procedure:

## 4 Two Degree of Freedom Controller

The structure of the *TDOF* controller is presented in the Figure 1.

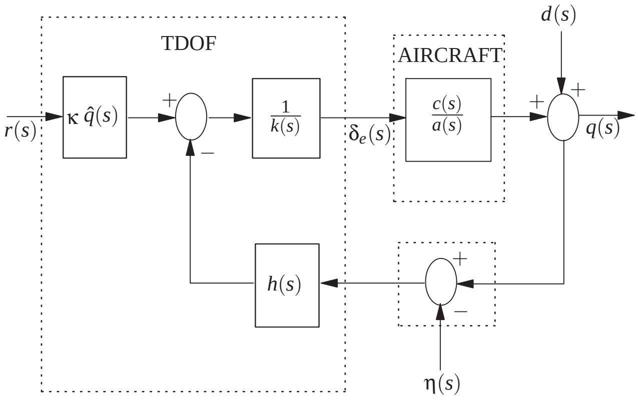

**Fig. 1** Controller structure

The system to be controlled is represented by the minimal, strictly proper and rational transfer function given by:

$$\frac{q(s)}{\delta_e(s)} = G(s) = \frac{c(s)}{a(s)} \tag{3}$$

where polynomials $a(s)$ and $c(s)$ are co-prime, with degrees $n$ and $m$ $(n > m)$. At the same time the output response, independently of disturbance $d(t)$ and the sensor noise $\eta(t)$ is:

$$\frac{q(s)}{r(s)} = \frac{\kappa c(s)\hat{q}}{a(s)k(s)+c(s)h(s)} = \frac{\kappa c(s)\hat{q}(s)}{\delta(s)} \tag{4}$$

where $\kappa$ corresponds to an arbitrary scalar and $\delta(s)$ represents the closed loop poles.

The controller guarantees a good output regulation, positioning the roots of $\delta(s)$ far enough

| Handling Qualities | Level I | Good Level I |
|---|---|---|
| Short Period Damping $\zeta_{sp}$ | $0.4 < \zeta_{sp} < 1.35$ | $0.7 < \zeta_{sp} < 1.35$ |
| $\theta$-Bandwidth | $> 1.5[rad/sec]$ | $> 1.75[rad/sec]$ |
| $\gamma$-Bandwidth | $> 0.6[rad/sec]$ | |
| Phase delay $\tau_p$ | $< 0.1[sec]$ | $< 0.09[sec]$ |
| Gain Margin | $> 6[dB]$ | $> 10[dB]$ |
| Phase Margin | $> 45[^\circ]$ | |

**Table 2** Handling qualities requirements

into the left half s-plane. However, the design could increase the bandwidth of system sufficiently to produce a control effort $\delta_e(t)$ saturation; in our case the elevator commands can not exceed the physical limits. One way of obtaining a desirable output regulation, without requiring an excessive control effort signal is design the controller minimizing the *LQR* performance index, which is:

$$J_{LQR} = \int_0^\infty \{\rho q(t)^2 + \delta_e(t)\} dt \qquad (5)$$

Minimization of equation (5) implies a desire to minimize both excessive output excursions and the control effort required to prevent such excursions. The state space solution of (5) is given by the *Ricatti* equation matrix, and the frequency solution is known as *Spectral factorization* [2]. In this sense considering that the polynomials $c(s)$ and $a(s)$ have real coefficients, requires a frequency $\omega$ with real values, that is it.

$$\begin{aligned} a(j\omega)a(-j\omega) &= |a(j\omega)|^2 \geq 0 \\ c(j\omega)c(-j\omega) &= |c(j\omega)|^2 \geq 0 \qquad (6) \end{aligned}$$

Therefore equation (6) with a weighing factor $\rho$ allows to obtain:

$$\Delta(s) = a(s)a(-s) + \rho c(s)c(-s) \qquad (7)$$

where the $2n$ roots of $\Delta(s)$ are obtained with $\rho$ varying from 0 to $\infty$, represents a special root locus which is termed a **root-square locus**. Then spectral factorization is given by:

$$\Delta(s) = [\Delta(s)]^+[\Delta(s)]^- = \delta^{F^*}(s)\delta^{F^*}(-s) \qquad (8)$$

where the optimal poles of system (8), are given by $n$ roots of $\delta^{F^*}(s)$. By duality it is possible to obtain the $n$ stable roots from $\delta^{H^*}(s)$ that are defined as:

$$\bar{\Delta}(s) = a(s)a(-s) + \sigma c(s)c(-s) = \delta^{H^*}(s)\delta^{H^*}(-s) \qquad (9)$$

In both cases, the variation of a single parameter $\rho$ and $\sigma$ allows to obtain the optimal poles locations. In this form, the gains $k(s)$ and $h(s)$ of degree *n-1* of the controller are given by:

$$k(s) = s^{n-1} + k_{n-2}s^{n-2} + \ldots + k_1 s + k_0 \qquad (10)$$

$$h(s) = h_{n-1}s^{n-1} + \ldots + h_1 s + h_0 \qquad (11)$$

Obtained solving the Diophantine equation:

$$a(s)k(s) + c(s)h(s) = \delta^{F^*}(s)\hat{q}(s) \qquad (12)$$

where the polynomial $\delta^{F^*}(s)$ is determined using $\rho$ and the polynomial $\hat{q}(s)$ of degree *n-1* is the result of $\delta^{H^*}(s)$ obtained using $\sigma$. The polynomials $k(s)$, $h(s)$ and $\hat{q}(s)$ represents the optimal gains of the $TDOF$ controller.

## 5 Project of Controller

The matrices $A$ and $B$ of the linear model used to design the controller corresponds to the cruise condition at (altitude= 2000 ft, V= 93 Knots, $x_{cg} = 0.47$ and mass = 11000 kg). The state-space equation of the longitudinal model obtained is given by:

$$A = \begin{bmatrix} -0.05332 & 5.331 & -0.07697 & -9.806 \\ -0.00825 & -0.64 & 0.9736 & -1.559e-8 \\ 0.00325 & -0.42 & -0.5484 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad (13)$$

$$B = \begin{bmatrix} 0.01932 \\ -0.055449 \\ -1.206 \\ 0 \end{bmatrix} \tag{14}$$

Zeros, phugoid mode and short period mode are identified on the zeros pole map on Figure 2:

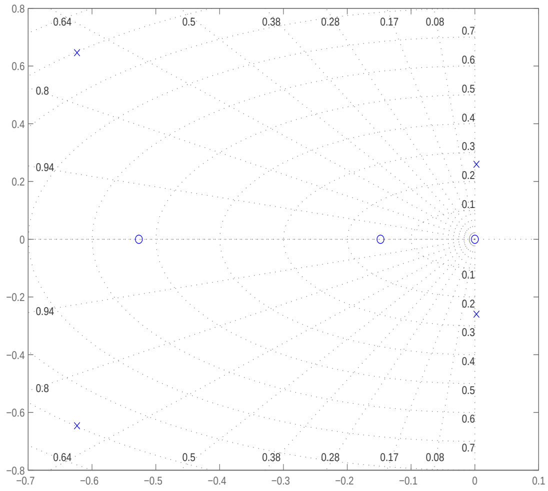

**Fig. 2** Pole-zero map of $\frac{q(s)}{\delta_e(s)}$

In order to verify the accuracy of the linear model, the response of aircraft to stick input was compared between linear and non linear models. Figure 3 shows the open loop responses of both models. Simulation results indicate that the linear model obtained is representative of aircraft dynamics at the equilibrium point and therefore adequate to the design purpose.

**Table 3** Weighting factor of controller

| ρ | σ |
|---|---|
| 5 | 2 |

The weighting factors used to design the controller are presented in Table 3 and the gains obtained in Table 4

The zeros and poles of linear augmented system are identified on the pole zero map illustrated in Figure 4:

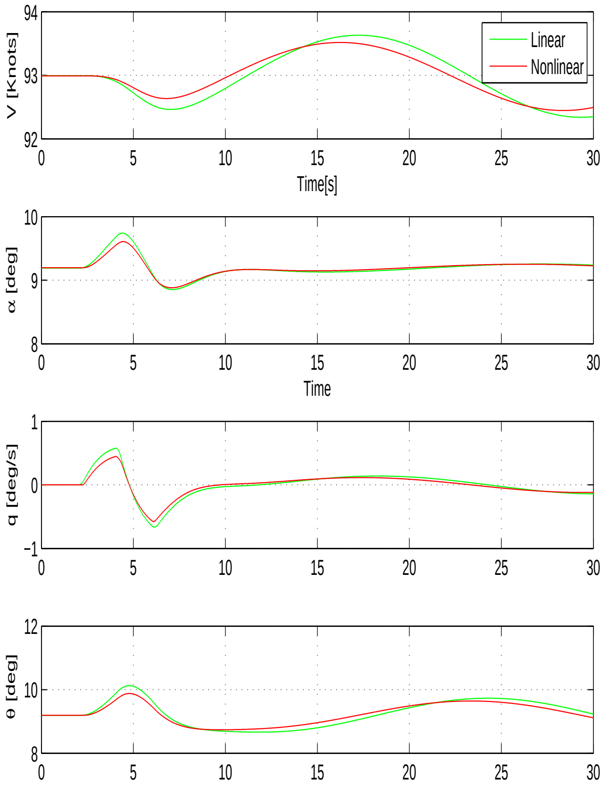

**Fig. 3** Linear and non linear simulation results

**Table 4** *TDOF* Controller gains

| $\kappa$ | $\hat{q}(s)$ | $h(s)$ | $k(s)$ |
|---|---|---|---|
| $-2.99$ | $1.0000s^3$ | $-1.8254s^3$ | $1.0000s^3$ |
| | $2.4088s^2$ | $-4.6310s^2$ | $2.4260s^2$ |
| | $1.5153s$ | $-3.1631s$ | $1.3493s$ |
| | $0.1270$ | $-0.6271$ | $0.1270$ |

## 6 Handling Qualities Analysis

Figure 5 it can be verified that the short period damping fulfills the requested requirements.

Figure 6 shows the results for $\theta - bandwidth$ versus $\gamma - bandwidth$. The limits presented on this figure were obtained from [6], [9] and are valid for a comercial aircraft. The results accomplish the requirements for bandwidth criteria presented in Table 2.

The figure 7 shows the results for bandwidth versus phase delay criteria. The results accomplish the requirements for the criteria presented on Table 2.

The Nichols plot for all linear models analyzed are shown in figure 8. The requirements for

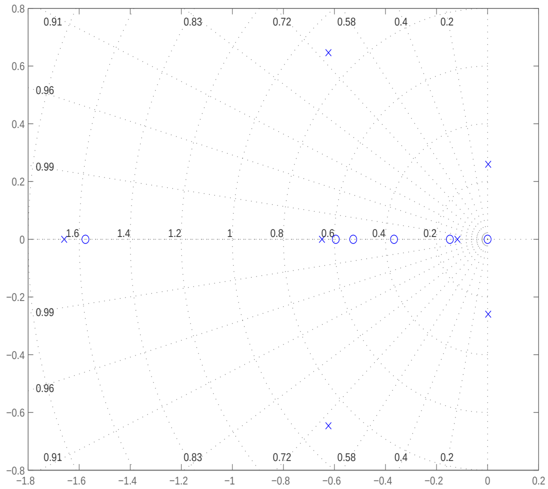

**Fig. 4** Pole-zero map of augmentation system

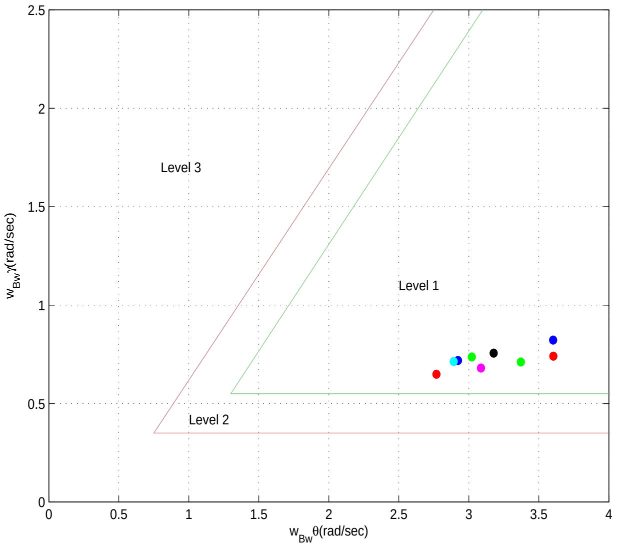

**Fig. 6** Bandwidth criterion

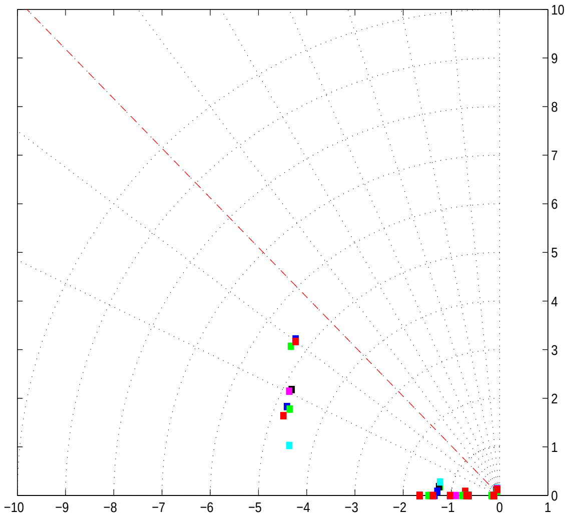

**Fig. 5** Modal criterion

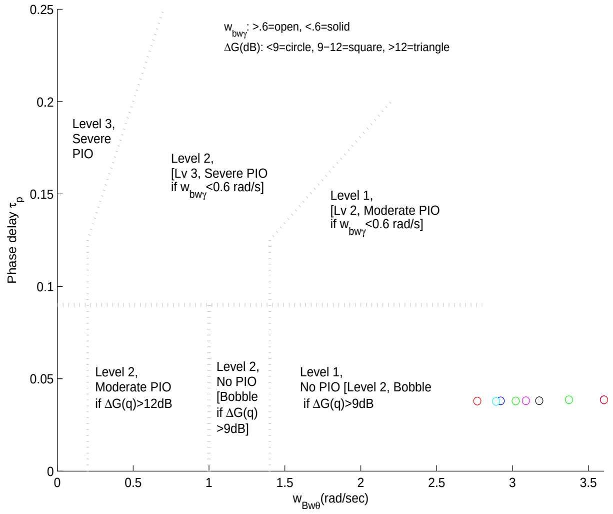

**Fig. 7** Bandwidth criterion

gain margin and phase margin are satisfied too.

## 7 Digital Implementation and Non-Linear Simulation

The digital implementation of controller was done using the *Tustin's approximation* where continuous poles are mapped to discrete poles according $z = e^{\frac{s}{f}}$ [5]. The sampling frequency $f$ corresponds to 100 [Hz]. The digital gains are presented in Table 5.

**Table 5** *TDOF* Digital gains

| $\kappa\,\hat{q}(z)$ | $h(z)$ | $k(z)$ |
|---|---|---|
| $-2.9898$ | $-1.8263$ | $1$ |
| $(z-0.9864)$ | $(z-0.9844)$ | $(z-0.9835)$ |
| $(z-0.9907)$ | $(z-0.9941)$ | $(z-0.9935)$ |
| $(z-0.9990)$ | $(z-0.9963)$ | $(z-0.9988)$ |

At this point, it is important to remind that one of the main advantages of the two degrees of freedom structure approach adopted, as compared with others such as, the $H_\infty$ analysis, is related with the order of the controller.

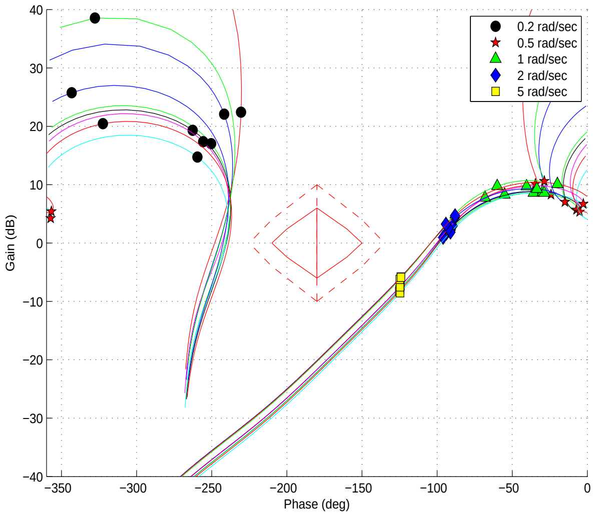

**Fig. 8** Nichols plot for stability analysis

Figure 9 shown the response to stick input considering the linear and non-linear aircrafts models for implemented digital controller. It can be observed that the aircraft has a good performance to the stick input, and that the linear and the non-linear models furnish results which differs within small and acceptable values.

## 8 Conclusions

This paper presents the design of a stability augmentation system using a two degree of freedom controller in the frequency domain considering some handling qualities criteria. For a given flight condition, a controller was obtained which a robustness to different flight conditions considering the mass, center of gravity and velocity variations. The linear model used in the design was validated comparing the response with non-linear model. The stability an handling qualities analysis were performed to fulfillment all established requirements. After the controller digital implementation, validation was performed using the non-linear model. Finally, it can be concluded that the control structure presented here can be used in the industrial environment.

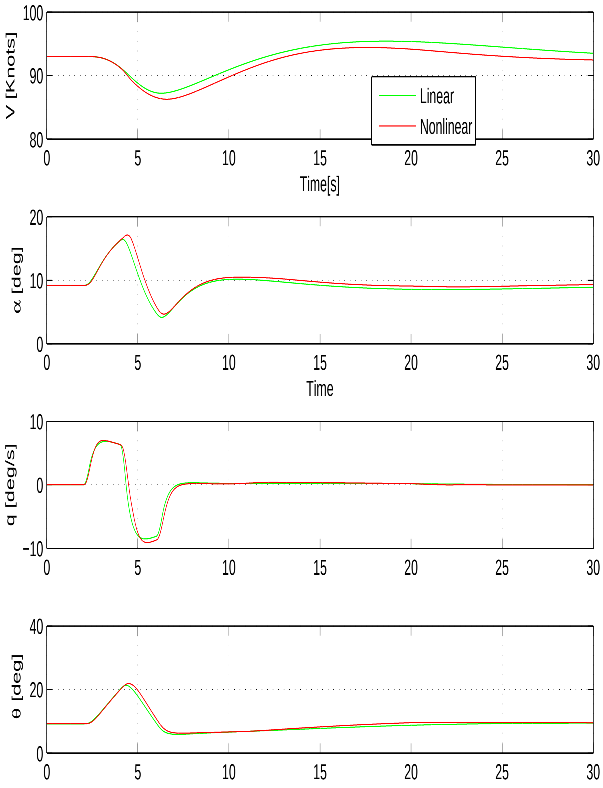

**Fig. 9** Results of simulation with controller implemented

## 9 Acknowledgments

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