APPLICATION OF HYBRID AND VMS-LES TURBULENT MODELS TO AERODYNAMIC SIMULATIONS

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Abstract

*We report on investigations for designing and validating VMS-LES and hybrid approaches between RANS and VMS-LES. Beside several more usual hybridization, We propose a new hybrid RANS/LES approach for the simulation of turbulent flows, which can be interpreted in the framework of the Non-Linear Disturbance Equation (*__NLDE__*) aproach. The solution of the Navier-Stokes equations is decomposed into a mean part (RANS), a perturbed/corrected part that takes into account the turbulent large-scale fluctuations and a third part made by the unresolved (SGS) fluctuations. Several flows past cylinder and spheres are computed for validating the considered approaches. The NLDE approach is used to simulate the flow around a circular cylinder at Reynolds number 140000; results are compared with a simplified version of the hybrid model (see for example [16]).*

1 Introduction

The numerical simulation of turbulent flows is one of the great challenges in Computational Fluid Dynamics (CFD). It is commonly accepted that the physics of the flow of a continuous fluid is well represented by the Navier-Stokes equations. The Direct Numerical Simulation (**DNS**) (for a review, see [10]) discretizes directly the three-dimensional Navier-Stokes equations. The basic requirement for such a simulation to succeed is the use of numerical schemes of high-order accuracy and meshes fine enough to capture the smallest scales of motion, to the order of the Kolmogorov scales. However, when the ratio of inertial forces to viscous ones, quantified by the Reynolds number, increases, the smallest scales become smaller, and the amount of information (handled and processed) necessary for a Navier-Stokes based prediction becomes enormous. In order to deal with the complex flows associated with higher Reynolds numbers and complex geometries, as those encountered in practical engineering applications, turbulence modeling was introduced.

One of the most widely used turbulence approach is Reynolds Averaged Navier-Stokes (**RANS**), in which only the averaged flow is solved. Due to nonlinearities, the pure mathematical averaging of the Navier-Stokes system introduces new terms, which must be modeled to close the problem. The closure of the new system needs thus to be obtained from phenomenological information provided by the study of simplified flows. Averaging and the need of extra information for closure are indeed two limitations of RANS. However, the RANS models made possible the prediction of high Reynolds number flows.

In the Large Eddy Simulation (**LES**) approach, the reduction of the simulation unknowns is obtained through the application of a spatial filter to the Navier-Stokes equations. In most cases, the filter size is strictly related to the typical size of the computational grid (grid scale). Only the set of scales larger than the grid size (grid scales) is computed explicitly, while the small scales (subgrid-scale, **SGS**) are modeled. For solving unsteady and massively separated flows as the flow around bluff-bodies, the Large-Eddy Simulation (LES) approach gives generally more accurate predictions than the computationally cheaper RANS models, and can also deliver an increased level of details. While RANS methods provide averaged results, LES is able to predict a part of instantaneous flow characteristics and to resolve important turbulent flow structures. However, a major practical difficulty for the LES simulation of complex flows is the fact that the computational costs of LES are generally much larger than those of RANS. Further, these costs rapidly increase as the flow Reynolds number increases. Indeed, the grid has to be fine enough to resolve a significant part of the turbulent scales, and this becomes particularly critical in the near-wall region at high Reynolds numbers.

Initiated by a few pioneering papers like [19], a new class of models has recently been proposed in the literature which combines RANS and LES approaches. The purpose is to obtain simulations as accurate as in the LES case but at reasonable computational costs. In the perspective of the simulation of massively separated unsteady flows in complex geometry, as occur in many cases of engineering or industrial interest, we are primarily interested in the so-called universal hybrid models [14], which should be able to automatically switch from RANS to LES throughout the computational domain. Among the universal hybrid models described in the literature, the Detached Eddy Simulation (**DES**) has received the largest attention. The DES approach [18] is generally based on the Spalart-Allmaras RANS model modified in such a way that far from solid walls and with refined grids, the simulation switches to the LES mode with a one-equation SGS closure.

A major difficulty in combining a RANS model with a LES one is due to the fact that the unknown of the RANS model is an averaged flow, which can in many cases be a steady one. In particular, RANS does not naturally allow for time-fluctuations, due to its tendency to damp them and to "perpetuate itself", as explained in [18]. On the other hand, LES needs a significant level of fluctuations in order to accurately model the turbulent flow. The abrupt passage from a RANS region to a LES one may produce the so-called "modeled stress depletion" [18]. We propose here a strategy for blending RANS and LES approaches in a hybrid model [16, 12], which is based on the NLDE aproach [9].

To this purpose, as in [9], the flow variables are decomposed in a RANS part (*i.e.* the averaged flow field), a correction part that takes into account the fluctuations that can be resolved in LES, and a third part made of the unresolved or SGS fluctuations. The basic idea is to solve the RANS equations in the whole domain and to correct the obtained averaged flow field by adding the resolved fluctuations in some regions of the domain, where the grid is adequately refined for LES. The NLDE technique was formulated and used as a zonal approach [9, 20], in which the RANS and LES regions are a-priori defined. Conversely, as previously stated, we search here for a hybridization strategy in which the RANS and LES models are built in the whole computational domain and blended in it, following a given criterion. To this aim, a blending function is introduced, θ, which smoothly varies between 0 and 1. The correction term which is added to the averaged flow field is damped by a factor $(1-\theta)$, obtaining a model which coincides with the RANS approach when $\theta = 1$ and recovers the LES approach for θ vanishing. For $0 < \theta < 1$, *i.e.* when the model switches from the RANS to the LES mode, a progressive addition of fluctuations is obtained and is aimed at avoiding, although in an empirical manner, the previously mentioned problems occuring for abrupt switches from RANS to LES. Following strictly these guidelines implies that the two fields, RANS and LES correction, need to be computed separately. In this case where the RANS is computed separately, the spurious influence of the LES fluctuations on the RANS mean flow is avoided and the physical meaning of the two sets of variables is clear. However, these calculations imply larger computational costs than hybrid simulations characterized by only one set of variables. Following this consideration, we developed and used in previous works [12, 16] a single-field simplified version of the hybridization strategy. It is then interesting to analyse the merits of the two options from the standpoint of predictivity. This paper is organized as follows : We first give some features and the description of our hybrid model followed by a few explanations on the near wall treatment. Then we give a description of the new hybrid model as a general case of a NLDE approach, and lastly some numerical results involving subcritical and supercritical flows past a sphere and a cylinder, and focalize to the first comparisons between single-field and two-field versions for the flow around a circular cylinder at Reynolds number 140,000.

2 Methodology

2.1 NLDE formulation

Following Labourasse and Sagaut [9], the following decomposition of the flow variables is adopted:

$$W = \underbrace{<W>}_{RANS} + \underbrace{W^c}_{correction} + W^{SGS}$$

where $<W>$ are the RANS flow variables, obtained by applying an averaging operator to the Navier-Stokes equations, W^c are the remaining resolved fluctuations (*i.e.* $<W> + W^c$ are the flow variables in LES) and W^{SGS} are the unresolved or SGS fluctuations. Writing the Navier-Stokes equations for the averaged flow $\langle W \rangle$ and applying a filtering operator, the LES equations are obtained and we get first a closure term given by a RANS turbulence model and then a SGS term. An equation for the resolved fluctuations W^c can thus be derived (see also [9]). The basic idea of the proposed hybrid model is to solve the equation for the averaged flow in the whole domain and to correct the obtained averaged flow by adding the remaining resolved fluctuations (computed through the equation of the resolved fluctuations), wherever the grid resolution is adequate for a LES. To identify the regions where the additional fluctuations must be computed, we introduce a *blending function*, θ, smoothly varying between 0 and 1. When $\theta = 1$, the RANS approach is recovered, wherever $\theta < 1$, additional resolved fluctuations are computed. Thus, the equations for the averaged flow and for the correction term in the proposed *NLDE or two-field hybrid model* become respectively:

$$\begin{aligned} &\frac{\partial \langle W \rangle}{\partial t} + \nabla \cdot F_c(\langle W \rangle) + \\ &\nabla \cdot F_v(\langle W \rangle) = -\tau^{RANS}(\langle W \rangle) \end{aligned} \tag{1}$$

$$\begin{aligned} &\frac{\partial W^c}{\partial t} + \nabla \cdot F_c(\langle W \rangle + W^c) - \\ &\nabla \cdot F_c(\langle W \rangle) + \nabla \cdot F_v(W^c) = \\ &(1-\theta)\left[\, \tau^{RANS}(\langle W \rangle) - \tau^{LES}(\langle W \rangle + W^c)\right] \end{aligned} \tag{2}$$

where $\tau^{RANS}(\langle W \rangle)$ is the closure term given by a RANS turbulence model, and $\tau^{LES}(W)$ is given by one of the SGS closures described in Sec. 2.4 and in[17] [21] [11]. To avoid the solution of two different systems of PDEs and the consequent increase of required computational resources, Eqs. (1) and (2) can be recast together in a more classical formulation, which we call the *single-field formulation* defined as follows:

$$\begin{aligned} &\frac{\partial W}{\partial t} + \nabla \cdot F_c(W) + \nabla \cdot F_v(W) = \\ &-\theta\, \tau^{RANS}(\langle W \rangle) - (1-\theta)\tau^{LES}(W) \end{aligned} \tag{3}$$

where W stands now for $\langle W \rangle + W^c$. Clearly, if only Eq. (3) is solved, $\langle W \rangle$ is not available at each time step. Two different options are possible: either to use an approximation of $\langle W \rangle$ obtained by averaging and smoothing of W, in the spirit of VMS-LES, or to simply use in Eq. (3) $\tau^{RANS}(W)$. This simplified formulation has been firstly tested by our team [12, 15].

The novelty we introduce in (1),(2) is to use two different systems connected with each other. The RANS system (1) is first solved, followed by the hybrid (2) one. This involves the solution of two different systems at each time step.

2.2 Hybridization

As discussed in the introduction, the LES fluctuation corrections are introduced following a blending strategy based on the blending function θ.

As a possible choice for θ, the following function is used in the present study:

$$\theta = F(\xi) = tanh(\xi^2) \quad (4)$$

where ξ is the *blending parameter*, which should indicate whether the grid resolution is fine enough to resolve a significant part of the turbulence fluctuations, *i.e.* to obtain a LES-like simulation. The choice of the *blending parameter* is clearly a key point for the definition of the present hybrid model. Different options are proposed and investigated, namely: using the ratio $\xi_{VR} = \mu_s/\mu_t$ between the eddy viscosities given by the LES and the RANS closures, which is also used as a blending parameter in LNS [3], and using the ratio $\xi_{LR} = \Delta/l_{RANS}$, l_{RANS} being a typical length in the RANS approach, *i.e.* $l_{RANS} = k^{3/2}\varepsilon^{-1}$ and, Δ measures the local mesh size. Note that none of these criteria involves explicitly the distance from the wall. This is interesting for complex geometries and complex grid topologies, as for unstructured grids (used herein), for which the computation of the distance from the wall yields practical difficulties.

2.3 Main numerical ingredients

The governing equations are discretized in space using a mixed finite-volume/finite-element method applied to unstructured tetrahedrizations. This scheme is a variational one relying on a finite-volume formulation for the convective terms, with a basis and test function χ_l, associated with the finite-volume cell centered on vertex l. A finite-element formulation is used for the diffusive terme, with a basis and test function ϕ_l continuous, linear by element, equal to 1 at vertex l and vanishing at other vertices. The Roe scheme [13] represents the basic upwind component for the numerical evaluation of the convective fluxes. A Turkel-type preconditioning term is introduced to avoid accuracy problems at low Mach numbers [7]. To obtain second-order accuracy in space, the Monotone Upwind Scheme for Conservation Laws reconstruction method (MUSCL) is used, in which the Roe flux is expressed as a function of reconstructed values of W at each side of the interface between two cells. The introduced numerical dissipation is made of sixth-order space derivatives [4] and, thus, concentrates on a narrow-band of the highest resolved frequencies. Moreover, it can be tuned to the minimum amount required for the simulation stability by means of an ad-hoc parameter γ_s. Finally, an implicit linearized time-marching algorithm using into a Restrictive Additive Schwarz formulation a local ILU solver can be efficiently used due to the smaller number of nodes in a vertex-centered formulation. The numerical accuracy is second-order accurate in space and time. More details can be found in [4].

2.4 VMS-LES and RANS modelling

For the LES mode, we consider the Variational Multi-Scale approach (**VMS-LES**), in which the flow variables are decomposed as $W = \overline{W} + W'$, where $\overline{W}$ are the large resolved scales (**LRS**) and W' are the small resolved scales (**SRS**). We follow here the VMS-LES approach proposed in [8] for the simulation of compressible turbulent flows through a finite volume/finite element discretization on unstructured tetrahedral grids. In order to obtain the VMS-LES flow decomposition, basis and test functions can be expressed as: $\chi_l = \overline{\chi}_l + \chi'_l$ and $\phi_l = \overline{\phi}_l + \phi'_l$, where the overbar denotes the basis functions spanning the finite dimensional spaces of the large resolved scales and the prime those spanning the SRS spaces. As in [8], the basis functions of the LRS space are defined through a projector operator in the LRS space, based on spatial average on macro cells, which are obtained by an agglomeration process. Finally, a key feature of the VMS-LES approach is that the SGS model is added only to the smallest resolved scales. Eddy-viscosity models are used here, and, hence, the SGS terms are discretized analogously to the viscous fluxes. In this context, the Galerkin projection for the computation of the LES approximation W^{VMS} is:

$$\begin{aligned}&\left(\frac{\partial W^{VMS}}{\partial t},\chi_l\right) + \left(\nabla\cdot F_c(W^{VMS}),\chi_l\right) + \\ &\left(\nabla\cdot F_v(W^{VMS}),\phi_l\right) = -\left(\tau^L(W'),\phi'_l\right) \\ &l = 1,N\end{aligned} \quad (5)$$

For defining more precisely the expression of τ^L in Eq.(5), three different eddy-viscosity models have been considered, namely those proposed by Smagorinsky [17] and Vreman [21] and the WALE model introduced in [11]. The eddy-viscosity introduced by these models, within the VMS approach, is computed as a function of the SRS flow variables, and the filter width Δ has been selected as the cubic root of the volume of each tetrahedron. Finally, the model constant has been set equal to 0.1 for the Smagorinsky and WALE models and to 0.158 for the Vreman one. In the present study, the RANS closure model is the low-Reynolds $k-\varepsilon$ one proposed in [5]. For flows with complex geometry, we found it preferable to build a formulation that does not need the distance to the wall. The low Reynolds $k-\varepsilon$ formulation of [6, 5] enjoys this property. Furthermore, this low Reynolds $k-\varepsilon$ model was designed to improve the prediction of the standard k-eps one for adverse pressure gradient flows, including separated flows. In order to get a robust formulation applicable to high Reynolds numbers, we combine it with Reichardt's wall law which gives a smooth matching between linear, buffer and logarithmic regions. Because the y^+ normalised distance is generally subject to large variations in complex geometries, we have found mandatory to combine both wall law and low Reynolds modelling with damping of the fully-turbulent model.

2.5 Global formulation

We denote by $\tau^{RANS}(\langle W \rangle$ the closure term given by the RANS turbulence model and by $\tau^{LES}(W')$ the term from the SGS closures. The Galerkin projections of the equations for the averaged flow and for the correction term in the proposed hybrid model become respectively:

$$\begin{aligned}&\left(\frac{\partial \langle W \rangle}{\partial t},\psi_l\right)+(\nabla\cdot F_c(\langle W \rangle),\psi_l)\\&+(\nabla\cdot F_v(\langle W \rangle),\phi_l)=-\left(\tau^{RANS}(\langle W \rangle),\phi_l\right)\\&l=1,N\end{aligned} \quad (6)$$

$$\begin{aligned}&\left(\frac{\partial W^c}{\partial t},\psi_l\right)+(\nabla\cdot F_c(\langle W \rangle+W^c),\psi_l)-\\&(\nabla\cdot F_c(\langle W \rangle),\psi_l)+(\nabla\cdot F_v(W^c),\phi_l)=\\&(1-\theta)\left[\left(\tau^{RANS}(\langle W \rangle),\phi_l\right)-\left(\tau^{LES}(W'),\phi_l'\right)\right]\\&l=1,N\,.\end{aligned} \quad (7)$$

3 Numerical Results

We discuss now the behavior of the models introduced for computing flows around simplified geometries for comparison with well known experiments. In order to prepare industrial applications, we try in some cases to use meshes that are as coarse as possible. Depending on the Reynolds number, the boundary layer can stay in a laminar regime (subcritical cases) or be turbulent (supercritical cases). Low Reynolds number subcritical flows are then considered without hybridization.

3.1 Flow around a circular cylinder (VMS-LES)

Risers are cylinders of rather small diameters but large lengths. For the lowest Reynolds numbers, let us choose 20,000 for fixing the ideas, a thin laminar boundary layer must be captured by the mesh. Since spanwise length is as large as thousands diameters, the challenge is to find a mesh as coarse as possible allowing the 3D vortex shedding mechanisms to be captured and, thus, important bulk quantitites to be predicted accurately enough. In this case, the performance of VMS-LES is a determining factor since a very small number of cells is devoted to the boundary layer.

After some investigations, with the help of the unstructured meshing, we have chosen a mesh made of planes with 2800 cells with a first layer allowing sufficient resolution of the laminar boundary layer before separation (Fig.1). The mesh stretching is more than 100 in spanwise direction. As a consequence, the VMS-LES produces a filtering of very small width in the spanwise direction. Thanks to this, 3D instabilities with sufficiently small spanwise wave lengths are obtained. Further, mean behaviors are obtained earlier thanks to the average in the homogeneous

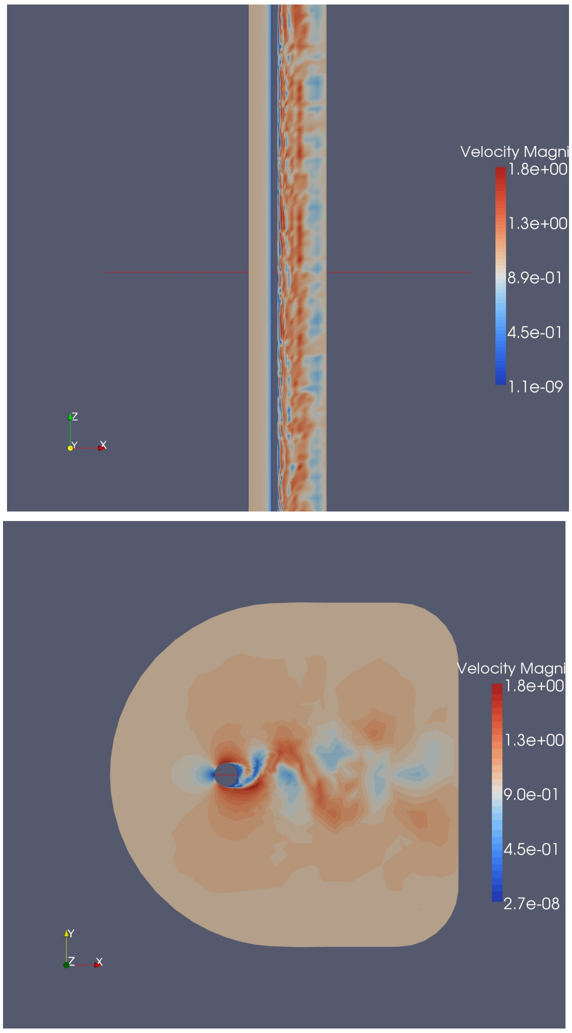

Fig. 1 *Top: Flow around a riser at Reynolds number* $20,000$. *Velocity module in a vertical (top) and in an horizontal (bottom) cut*

spanwise direction, taking into account a large collection of vortex configurations. We get a mean drag of 1.2 and a Strouhal of .16 while experiments give respectively 1.1 and .21.

3.2 Flow around a circular cylinder (Hybrid RANS/VMS-LES)

The single-field hybrid model has been applied to the simulation of the flow around a circular cylinder at $Re = 140,000$ (based on the far-field velocity and the cylinder diameter). The considered mesh is unstructured, tetrahedral, rather coarse with 458K vertices.

The numerical parameter γ_s, which controls the amount of numerical viscosity introduced in the simulation, has been set equal to 0.3.

Fig.2 shows the pressure distribution on the cylinder surface averaged in time and in the homogeneous z direction for the new proposed hybrid model, compared with the single-field hybrid version. These preliminary results show noticeable differences between the NLDE formulation and its single-field version. In particular, as also shown in Fig. 3, it seems that the vortical structures form closer to the cylinder in the NLDE simulation, this leading to slightly larger absolute value of the base pressure. Further analysis is needed, together with a systematic comparison with experimental data.

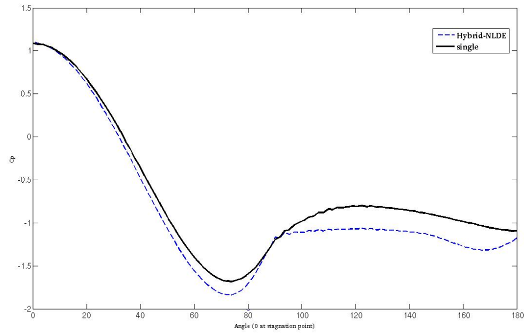

Fig. 2 Time-averaged and z-averaged pressure distribution on the surface of the cylinder, comparison of the NLDE hybrid model with the single-field one.

3.3 Flow past a sphere (VMS-LES and Hybrid RANS/VMS-LES):

As a contribution to the validation of the VMS-LES model alone, we computed several subcritical flows, with Reynolds numbers $10,000$ and $50,000$. The corresponding drag are indicated on Fig.5. In the case of Reynolds number $50,000$, vertical wake cuts of the horizontal velocity are compared in Fig.4 with the experiments of Bakic, [2]. We use the *single-field hybrid formulation* for computing the supercritical flow around a sphere at a Reynolds number of $400,000$ with a mesh of $500,000$ nodes.Fig.5 shows that the prediction of drag is in good agreement with experi-

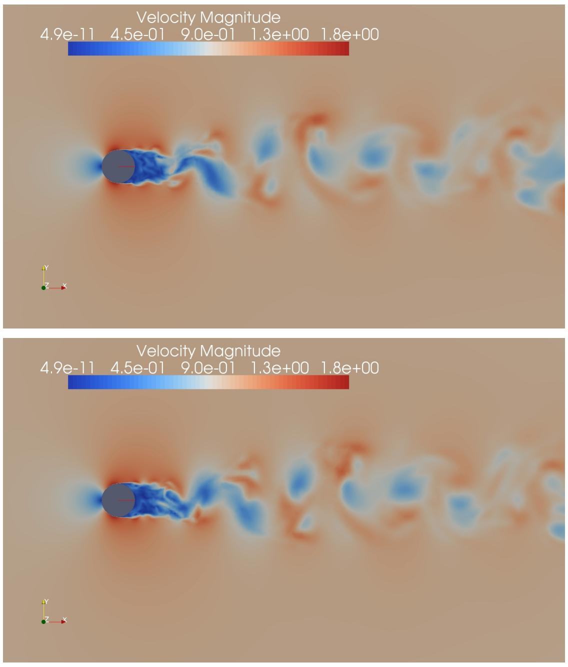

Fig. 3 Two-Field model: velocity fields for hybrid double field approach (top) and for the single-field hybrid model based on one equation (bottom)

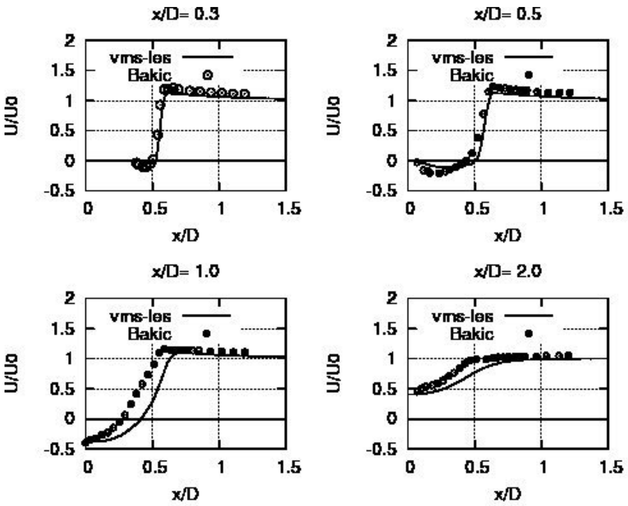

Fig. 4 Flow past a sphere at Reynolds 50,000 number: velocity wake and comparison with experiments by Bakic [2]

menatl data for all the considered cases.

4 Conclusions

We have described a software platform for studying RANS-LES hybridization.

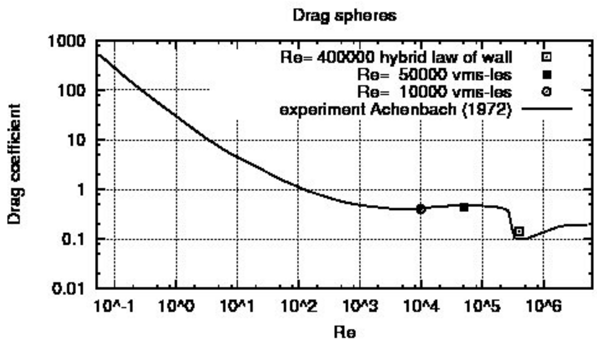

Fig. 5 Flow past a sphere: Study of computed drag as a function of Reynolds number. Experiments from Achenbach [1]

The numerical scheme is stabilised with sixth order derivatives, and therefore it allows a very low level of numerical dissipation. Although this platform is based on an aerodynamic numerical technology (compressible model with collocated nodes, in particular), accuracy is good even for the incompressible case. The riser example shows a rather good prediction with a mesh of only 4,800 nodes per span plane. This ability is also increased by the use of the VMS-LES formulation. This is shown for instance by the prediction of the drag acting on a sphere at different Reynolds numbers. Since RANS and LES use different variables, their combination onto a unique solution field rises many open questions. RANS production term can be defavourably influenced by the fluctuation allowed by the LES component. The local transfer of a RANS field into a turbulent LES one is also a challenge. The two formulations, single-field and NLDE two-field, which we have introduced allow to address a part of these questions. Preliminary results obtained with this approach have been shown for the flow around a circular cylinder.

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