A MAXIMUM PRINCIPLE APPLICATION FOR THE DECOMPOSITION OF THE AIRCRAFT MULTIDISCIPLINARY OPTIMIZATION PROBLEM

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Abstract

The problem of multidisciplinary optimization of aircraft parameters and trajectories is considered. The method of the problem decomposition into subtasks of each discipline, as flight dynamics and control, aerodynamics and structures, is described. The decomposition is based on the sensitivity analysis of the aircraft mission efficiency with respect to design variables. These necessary data are calculated using adjoint system solution within the trajectory optimization by the Pontryagin maximum principle.

Nomenclature

Symbols:

A	aerodynamic force vector
C_D	aerodynamic drag coefficient
C_{D0}	zero-lift drag coefficient
C_L	aerodynamic lift coefficient
C_L^α	$= \partial C_L / \partial\alpha$
C_m	pitching moment coefficient
C_m^α	$= \partial C_m / \partial\alpha$
F_0	reference cross section area
$\boldsymbol{g}$	gravity acceleration vector
M	Mach number
m	vehicle mass
n_z	transverse loading
q	dynamic pressure
r	radius-vector from the mass center
T	thrust value
T	thrust vector
t	time
V	velocity value
V	velocity vector
α	angle-of-attack
μ	mass flow rate
Ω	the acceleration vector due to coordinate system noninertiality

Subscripts:

$(\,)_f$	at the final point
$(\,)_i$	at the initial point
$(\,)_{max}$	maximal value
$(\,)_{opt}$	optimal value

1 Introduction

As the requirements for the aircraft performance grow and their structure becomes more complex, thorough theoretical and experimental studies in different aerospace disciplines are necessary.

There is a growing interest to use more complicated computational methods at the early design stages. This allows to make more competent decisions at the stage of conceptual studies and reduces the risk of dead-end base concepts that cannot be “corrected” by small modifications. A rapid progress in computational capabilities also promotes a broad application of more "advanced" numerical methods.

Previously, the empirical formula dependences and very simplified calculations were used at the stages of the conceptual design. However, more complex methods, which were frequent in the past on the last design stages, are currently being used.

Current trends of broad cooperation among specialized scientific institutions as well as geographical expansion of the aerospace corporations have forced development of new approaches to multidisciplinary optimization

(MDO). The new approaches must allow for combining diverse and possibly remote programs intended for detailed single-discipline investigations into a unified framework. The review of MDO approaches and methods can be found, for example, in [1]–[5].

Traditionally, at the detailed research stage each aerospace discipline uses its own optimization criteria, which convey intuitively the representation about «the best vehicle» and do not depend formally on the global target performance. For example, by optimizing the aircraft structure, the minimum structural mass is provided; by optimizing an aerodynamic layout, a minimum aerodynamic drag or maximum lift-to-drag ratio on the particular flight regimes is attained, etc. The results of the comparative analysis [5] showed that such approach becomes ineffective for problems in which a mutual interaction of aircraft parameters in separate aerospace disciplines is strong.

State-of-the-art MDO techniques (see [2],[4]) include the construction of objective functions for separate disciplines on a certain algorithm depending on the global objective functions. If direct optimization methods are used and the number of internal parameters is large, a weak effect of their variations on the objective functions may be observed that results in a numeral noise at calculation sensitivity derivatives [6].

To eliminate such a drawback in aerodynamic shape optimization problems, for example, special methods, including a solution of an additional set of equations for the adjoint variables are done. [7]. Use the adjoint set of equations allows to determine sensitivities automatically and to reduce algorithm complexity [8].

In this paper a decomposition of a multidisciplinary problem into single-discipline subtasks is offered using the local complex distributed criteria (LDC) [8]. The subtasks are formed on the basis of the adjoint system solution and the Lagrangian multipliers determination during aircraft trajectory optimization using the Pontryagin maximum principle [9].

2 Decomposition of the multidisciplinary optimization problem using the maximum principle

Let us consider the problem of the multidisciplinary optimization of an aircraft trajectory and parameters by the criterion

$$\Phi \Rightarrow \max. \tag{1}$$

The criterion Φ depends on the selection of the control law $\mathbf{u}(t) \in \mathcal{U} \subset \mathbf{R}^m$, $t \in [t_i, t_f]$ and a vector parameter $\mathbf{p} \in \mathcal{P} \subset \mathbf{R}^p$. The optimal solution is to find:

$$\{\mathbf{u}, \mathbf{p}\}_{\text{opt}} = \arg\max \Phi. \tag{2}$$

The optimization problem breaks up to the control optimization problem:

$$\{\mathbf{u}\}_{\text{opt}} = \arg\max \Phi|_{\mathbf{p}=\text{fix}},$$

and the non-linear programming problem:

$$\{\mathbf{p}\}_{\text{opt}} = \arg\max \Phi|_{\mathbf{u}=\mathbf{u}\text{opt}}.$$

The parameter $\mathbf{p}$ influences on constraints of the admissible control $\mathcal{U}(\mathbf{p})$, on the admissible state area $\mathcal{X}(\mathbf{p})$, including initial conditions, and on the right member of the motion equation:

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{p}, t), \tag{3}$$

where $\mathbf{x} \in \mathcal{X} \subset \mathbf{R}^n$ is the state vector.

Let us consider the aircraft trajectory optimization problem in more detail.

The state and control vectors are limited to a system of inequalities:

$$\mathbf{x} \in \mathcal{X} = \left\{\mathbf{x} \in \mathbf{R}^n : \mathbf{X}(\mathbf{x}, \mathbf{p}, t) \le 0, \mathbf{X} \in \mathbf{R}^N\right\}, \tag{4}$$

$$\mathbf{u} \in \mathcal{U} = \left\{\mathbf{u} \in \mathbf{R}^m : \mathbf{U}(\mathbf{x}, \mathbf{u}, \mathbf{p}, t) \le 0, \mathbf{U} \in \mathbf{R}^M\right\}. \tag{5}$$

The state inequality constraints (4) break up according to [10] into constraints of equality type on the state vector and the control

$$\mathbf{W}(\mathbf{x}, \mathbf{u}, \mathbf{p}, t) = 0, \mathbf{W} \in \mathbf{R}^N. \tag{6}$$

and constraints on the function of state vector at the isolated points of the trajectory $\mathbf{x}(t_j)$:

$$\mathrm{D}_j(\mathbf{x}(t_j), \mathbf{p}, t_j) = 0, \; j = 1, \ldots, q_1. \tag{7}$$

To solve the problem, the indirect optimization method, the Pontryagin maximum principle, is used. In accordance with the maximum principle the Hamiltonian is introduced as

$$\begin{aligned} \mathcal{H} = {} & \boldsymbol{\psi}^T \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{p}, t) + \boldsymbol{\lambda}^T \mathbf{U}(\mathbf{x}, \mathbf{u}, \mathbf{p}, t) + \\ & + \boldsymbol{\xi}^T \mathbf{W}(\mathbf{x}, \mathbf{u}, \mathbf{p}, t), \end{aligned} \tag{8}$$

where $\boldsymbol{\psi} \in \mathbf{R}^n$ is the adjoint vector, $\boldsymbol{\lambda} \in R^M$, $\boldsymbol{\xi} \in R^N$ are the vectors of Lagrangian multipliers.

The adjoint vector $\boldsymbol{\psi}$ is determined by the equation [9]:

$$\frac{d\boldsymbol{\psi}}{dt} = -\left[\frac{\partial \mathcal{H}}{\partial \mathbf{x}}\right]^{\mathrm{T}}, \tag{9}$$

and transversality conditions (see [8]). The optimal control is found from:

$$\mathbf{u}_{opt} = \arg\max_{\mathbf{u} \in \mathcal{U}} \mathcal{H}. \tag{10}$$

Lagrangian multipliers $\boldsymbol{\lambda}$, $\boldsymbol{\xi}$ are found by conditions:

$$\lambda_k = \begin{cases} < 0, \text{ if } U_k = 0; \\ = 0, \text{ if } U_k < 0; \end{cases}$$

$$\xi_k = \begin{cases} \neq 0, \text{ if } \mathrm{W}_k = 0, X_k = 0; \\ = 0, \text{ if } X_k < 0. \end{cases}$$

Thus, the initial optimization problem is reduced to a multipoint boundary value problem for Eqs. (3), (9).

The variation $\delta\Phi$ due to variation of the parameter vector $\mathbf{p}$ is determined as follows:

$$\begin{aligned} \delta\Phi &= \delta_p \Phi + \mathbf{v}^T \delta_p \mathbf{D} + \\ &+ \int_{t_i}^{t_f} \left(\boldsymbol{\psi}^T \delta_p \mathbf{f} + \boldsymbol{\lambda}^T \delta_p \mathbf{U} + \boldsymbol{\xi}^T \delta_p \mathbf{W}\right) dt = \\ &= \nabla_p \Phi\, \delta\mathbf{p} \Rightarrow \max \end{aligned} \tag{11}$$

where δ_p is an increment caused by $\delta\mathbf{p}$. The state and adjoint variables, control and Lagrangian multipliers $\mathbf{v}$, $\boldsymbol{\lambda}$, $\boldsymbol{\xi}$ in (11) correspond to the optimal solution at the nonperturbed (nominal) vector $\mathbf{p}$. The gradient:

$$\begin{aligned} \nabla_{\mathbf{p}}\Phi = \frac{\partial \Phi}{\partial \mathbf{p}} + \mathbf{v}^T \frac{\partial \mathbf{D}}{\partial \mathbf{p}} + \int_{t_i}^{t_f} \Bigg[\boldsymbol{\psi}^T \frac{\partial \mathbf{f}(\mathbf{x},\mathbf{u},\mathbf{p},t)}{\partial \mathbf{p}} + \\ + \boldsymbol{\lambda}^T \frac{\partial \mathbf{U}(\mathbf{x},\mathbf{u},\mathbf{p},t)}{\partial \mathbf{p}} + \boldsymbol{\xi}^T \frac{\partial \mathbf{W}(\mathbf{x},\mathbf{u},\mathbf{p},t)}{\partial \mathbf{p}} \Bigg] dt \end{aligned} \tag{12}$$

sets the direction of the improving variation in the $\mathbf{p}$-space.

The optimization of aircraft layout parameters $\mathbf{p}$ by the criterion (1) is reached as a result of iterations containing calculations of the characteristics in Eq. (12) within the framework of separate disciplines. Aircraft parameters improvement is possible so long as the gradient of the functional (1) has a positive projection to the cone of permissible variations in P:

$$\nabla_{\mathbf{p}} \Phi\, \delta\mathbf{p} \geq 0, \quad \mathbf{p} + \delta\mathbf{p} \in \mathscr{P}. \tag{13}$$

Under variation of $\mathbf{p}$ the vector of aircraft characteristics $\mathbf{C}(\mathbf{x}, \mathbf{p})$ that appears in the right member $\mathbf{f}$ of equations (1), boundary conditions and constraints (4)–(7) is changed. In turn, effect of local variations of characteristics $\mathbf{C}(\mathbf{x}, \mathbf{p})$ on functional Φ which can be determined by gradient $\partial\Phi/\partial\mathbf{C}$ varies along trajectory. One of the important problems of MDO is the taking objectively into account the indicated distributed effect of aircraft parameter changes.

The total variation of a functional (11) can be recorded as:

$$\begin{aligned} \delta\Phi = \frac{\partial \Phi}{\partial \mathbf{p}} \delta\mathbf{p} + \mathbf{v}^{\mathrm{T}} \frac{\partial \mathbf{D}}{\partial \mathbf{C}} \frac{\partial \mathbf{C}}{\partial \mathbf{p}} \delta\mathbf{p} + \\ + \int_{t_i}^{t_f} \Bigg(\boldsymbol{\psi}^{\mathrm{T}} \frac{\partial \mathbf{f}}{\partial \mathbf{C}} \frac{\partial \mathbf{C}}{\partial \mathbf{p}} \delta\mathbf{p} + \boldsymbol{\lambda}^{\mathrm{T}} \frac{\partial \mathbf{U}}{\partial \mathbf{C}} \frac{\partial \mathbf{C}}{\partial \mathbf{p}} \delta\mathbf{p} + \\ + \boldsymbol{\xi}^{\mathrm{T}} \frac{\partial \mathbf{W}}{\partial \mathbf{C}} \frac{\partial \mathbf{C}}{\partial \mathbf{p}} \delta\mathbf{p} \Bigg) dt. \end{aligned} \tag{14}$$

Multipliers $\mathbf{v}$, $\boldsymbol{\psi}$, $\boldsymbol{\lambda}$, $\boldsymbol{\xi}$ in (14) are determined from the solution of the trajectory optimization problem on the base of the maximum principle. The derivatives $\frac{\partial \Phi}{\partial \mathbf{p}}$, $\frac{\partial \mathbf{D}}{\partial \mathbf{C}}$, $\frac{\partial \mathbf{f}}{\partial \mathbf{C}}$, $\frac{\partial \mathbf{U}}{\partial \mathbf{C}}$, $\frac{\partial \mathbf{W}}{\partial \mathbf{C}}$ are known and expressed as formulae. Multiplier $\frac{\partial \mathbf{C}}{\partial \mathbf{p}}$ should be obtained during the research in related disciplines (aerodynamics, strength, etc.). Using (14), it is possible to conduct optimization directly by global target criterion (1).

The variation (11) contains sensitivity functions that have a meaning of local distributed criteria (LDC) for contiguous disciplines [8].

The main advantages of the method are:

- it takes into account the specific effect of each trajectory section on the functional;

- automatic modification of control structure at a variation of aircraft parameters;
- the objectivity of obtained solutions due to thoroughness of the used optimization method;
- the greatest possible achievable effect at the expense of natural subordination of all single-discipline variables and parameters to one general problem – optimization by the aircraft mission efficiency criterion;
- possibility of using advanced research methods inside each discipline, since simplification of single-discipline calculations is not required.

To use the method of decomposition it is necessary to provide a regular numerical procedure for the solution of a multipoint boundary value problem (MPBVP) for the state (3) and adjoint (9) differential equations.

Practically the regular procedure for the solution of such problems is realized, for example, in the program complex ASTER[11]. The implementation of the modified Newton method and the continuation method in combination with the principle of local extremal selection and the vast database of the solutions obtained previously makes it possible essentially to extend the convergence domain of the MPBVP solution technique.

For definition of aerodynamic characteristics of aircraft the complex ANTARES designed in TsAGI is used. The complex allows solving Navier-Stokes equations for viscous compressible perfect gas.

3 Example of multidisciplinary optimization of multiregime aircraft

Let us consider as an example the problem of multidisciplinary optimization of two-stage launcher using the payload mass injected into an Earth orbit as the criterion.

The given aircraft passes a broad range of velocities and altitudes and its path does not contain preferred steady flight regimes.

The input data are given in Appendix A. Two trapezoidal airfoil consoles can be mounted on I-st stage booster (Fig. 1).

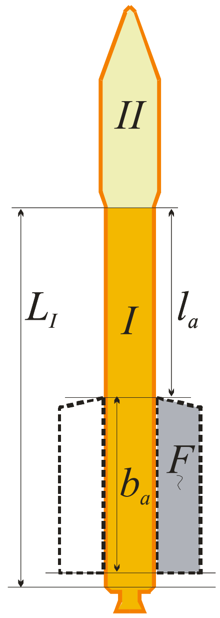

Fig 1. The scheme of the optimized aircraft.

Consider as the components of the optimized vector $\mathbf{p}=\{F, b_a, l_a\}^T$ the following parameters (see Fig.1):

1. $\bar{F}$ is ratio of airfoil console area to reference cross section area: $0\leq \bar{F} \leq 3$;
2. b_a is aerodynamic chord of the console: $\frac{1}{2}L_I \leq b_a \leq L_I$;
3. l_a is the distance from a plane of connection of I-st and II-nd stage boosters up to a leading edge of the console: $0 \leq l_a \leq \frac{1}{2}L_I$, $l_a \leq L_I - b_a$.

The variation of $\mathbf{p}$ results in change of aerodynamic characteristics of aircraft, of structural mass and of the optimal injection trajectory.

The payload is calculated as a difference of the final injected mass m_f and the structural mass of II-nd stage booster m_s that is considered to be fixed:

$$m(\mathbf{p}) = m_f(\mathbf{p}) - m_s. \qquad (15)$$

Thus, the optimum solution is determined by a condition:

$$\Phi = m_f \Rightarrow \max . \qquad (16)$$

The optimal aircraft trajectory should be built to define m_f. The aircraft mass centre motion is considered in the coordinate system fixed at the start point [11]:

$$\left.\begin{aligned} &\frac{d\mathbf{r}}{dt}=\mathbf{V}, \\ &\frac{d\mathbf{v}}{dt}=\frac{\mathbf{A}+\mathbf{T}}{m}+\mathbf{g}+\mathbf{\Omega}, \\ &\frac{dm}{dt}=-\mu, \end{aligned}\right\} \tag{17}$$

The vector of aerodynamic forces is written in the form [11],[13]:

$$\mathbf{A}=\frac{1}{2}\rho V^2 F_0\Big(C_L^\alpha \mathbf{e}_\tau-\big(D_0+ +\big(C_L^\alpha+D_\alpha\big)(\mathbf{e}_\tau,\mathbf{e}_\mathrm{v})\big)\mathbf{e}_\mathrm{v}\Big), \tag{18}$$

where $\mathbf{e}_\tau$ - is the unit vector directed along the vehicle longitudinal axis, $\mathbf{e}_\mathrm{v}$ - is the velocity unit vector.

The following form of aerodynamic coefficients is used[11],[13]:

$$C_L=C_L^\alpha\sin\alpha,\quad C_D=D_0+D_\alpha\cos\alpha,$$

that is in accordance with the square aerodynamic polar

$$C_L\cong C_L^\alpha\alpha,\quad C_D\cong C_{D0}+k\alpha^2,$$
$$D_0=C_{D0}+2k,\quad D_\alpha=-2k,$$

at a small angle of attack. Pitching moment coefficient C_m relative to vehicle nose is presented as

$$C_m=C_m^\alpha\sin\alpha.$$

Thus, aerodynamic forces and moments are determined by four characteristics C_L^α, C_{D0}, k and C_m^α, which are considered to be dependent on the Mach number only.

The thrust is constrained by minimum and maximum values:

$$T_{\min}\le T\le T_{\max}$$

The thrust vector $\mathbf{T}$ deflects from longitudinal axis at the angle δ that is determined by the condition of pitch trim:

$$\begin{aligned}\bar{l}_T\cdot T\sin\delta&=\frac{1}{2}\rho V^2F_0(C_m+\bar{l}_m C_L)=\\ &=\frac{1}{2}\rho V^2F_0(C_m^\alpha+\bar{l}_m C_L^\alpha)\sin\alpha\end{aligned}, \tag{19}$$

where $\bar{l}_T$ is distance between thrust application point and vehicle nose divided by reference length, $\bar{l}_m$ is distance between mass center and vehicle nose divided by reference length.

The control vector is

$$\mathbf{u}=\{\mathbf{e}_\tau, T\}^\mathrm{T}\in\mathcal{U}.$$

Optimum control is determined at the solution of a boundary value problem of maximum principle from the condition (10). The angle of attack α is calculated from:

$$\cos\alpha=(\mathbf{e}_\tau,\mathbf{e}_\mathrm{v}).$$

The constraint on transverse g-load is taken into account:

$$\mathrm{U}\triangleq\left|\frac{N}{m}\right|-n_{z\max}<0, \tag{20}$$

where N is the resultant force acting transversally to the vehicle longitudinal axis.

The variation of $\mathbf{p}$ results in change of the structural mass of I-st stage booster, relations of characteristics C_L^α, C_{D0}, k and C_m^α on a Mach number and consequently the right member of Eqs. (17) and constraint (20). The change in specific structural mass Δm_i of I-st stage booster due to installation of airfoils is approximately taken into account with the use of the technique which is based on outcomes of [14],[15]:

$$\Delta m_i=\sigma\cdot F\cdot n_{z\max}\cdot\Delta Y, \tag{21}$$

where σ is the constant equal to $9.7\cdot10^{-6}$; $n_{z\max}$ is the maximum transverse g-load; ΔY is the maximum ratio of lift force induced by airfoils to total lift force acting the vehicle at $n_z=n_{z\max}$.

In application to considered problem the gradient of the functional is given by:

$$\begin{aligned}\frac{\partial\Phi}{\partial\mathbf{p}}&=\mathbf{v}^T\frac{\partial\mathbf{D}}{\partial\mathbf{p}}+\int_{t_i}^{t_f}\left[\frac{\mathbf{S}^T}{m}\left(\frac{\partial\mathbf{A}}{\partial\mathbf{p}}\right)+\boldsymbol{\lambda}^T\frac{\partial\mathbf{U}}{\partial\mathbf{p}}\right]dt=\\ &=\mathbf{v}^T\frac{\partial\mathbf{D}}{\partial\mathbf{p}}+\int_{t_i}^{t_f}\left(-\frac{\mathbf{S}^T(\mathbf{A}+\mathbf{T})}{m^2}\frac{\partial m}{\partial\mathbf{p}}+f_k\frac{\partial k}{\partial\mathbf{p}}+\right.\\ &\left.+f_{C_{D0}}\frac{\partial C_{D0}}{\partial\mathbf{p}}+f_{C_L^\alpha}\frac{\partial C_L^\alpha}{\partial\mathbf{p}}+f_{C_m^\alpha}\frac{\partial C_m^\alpha}{\partial\mathbf{p}}\right)dt,\end{aligned} \tag{22}$$

where $\mathbf{S}$ is vector adjoint to velocity vector $\mathbf{v}$, functions $f_k, f_{C_{D0}}$, $f_{C_L^\alpha}$, $f_{C_m^\alpha}$ reveal the distribution of specific influence of k, $C_{D0}, C_L^\alpha, C_m^\alpha$ on functional (16) along the trajectory. According to (14):

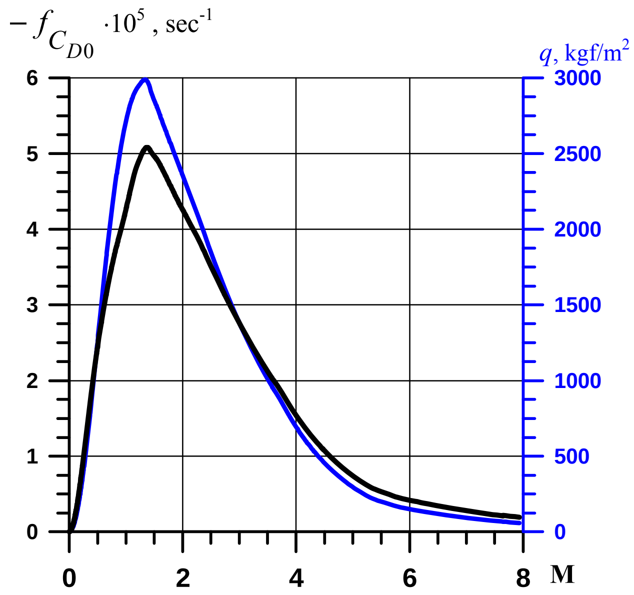

Fig 2. Relations of $f_{C_{D0}}$ and dynamic pressure q to Mach number at $\bar{F}$=0.

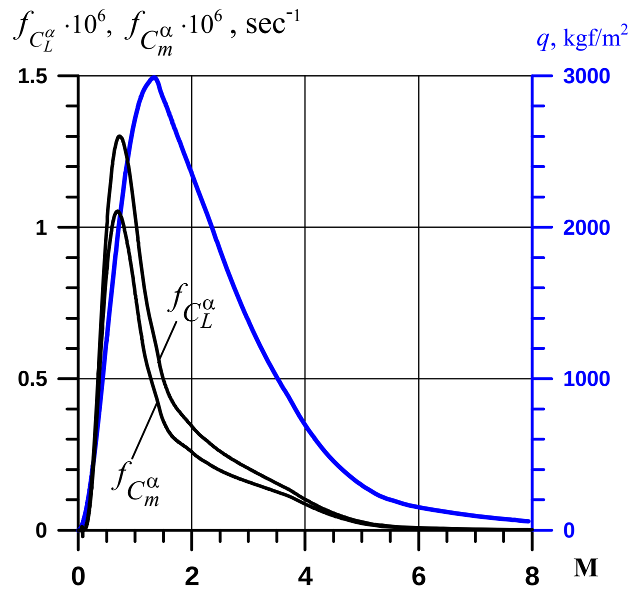

Fig 3. Relations of $f_{C_L^\alpha}$, $f_{C_m^\alpha}$ and dynamic pressure q to Mach number at $\bar{F}$=0.

$$f_k = \frac{\mathbf{S}^T}{m}\frac{\partial \mathbf{A}}{\partial k},$$

$$f_{C_{D0}} = \frac{\mathbf{S}^T}{m}\frac{\partial \mathbf{A}}{\partial C_{D0}},$$

$$f_{C_L^\alpha} = \frac{\mathbf{S}^T}{m}\frac{\partial \mathbf{A}}{\partial C_L^\alpha} + \boldsymbol{\lambda}^T \frac{\partial \mathbf{U}}{\partial C_L^\alpha},$$

$$f_{C_m^\alpha} = \frac{\mathbf{S}^T}{m}\frac{\partial \mathbf{A}}{\partial C_m^\alpha} + \boldsymbol{\lambda}^T \frac{\partial \mathbf{U}}{\partial C_m^\alpha},$$

are determined in the process of the solution of a problem of trajectory optimization on the basis of maximum principle.

For vehicle layout under study it is possible to assume $k = C_L^\alpha$ and consider

$$f_{C_L^\alpha} = \frac{\mathbf{S}^T}{m}\left(\frac{\partial \mathbf{A}}{\partial C_L^\alpha} + \frac{\partial \mathbf{A}}{\partial k}\right) + \boldsymbol{\lambda}^T \frac{\partial \mathbf{U}}{\partial C_L^\alpha} .$$

Figures 2 and 3 show dependence of $f_{C_{D0}}$, $f_{C_L^\alpha}$, $f_{C_m^\alpha}$ for the base layout ($\bar{F}$=0) on Mach number on the optimal trajectory. The function $f_{C_{D0}}$ is negative at any point on the trajectory. The maximum of $f_{C_{D0}}$ modulus is reached at the point corresponding to the maximum dynamic pressure. So the maximum payload increase can be provided by reducing C_{D0} at M~1.7.

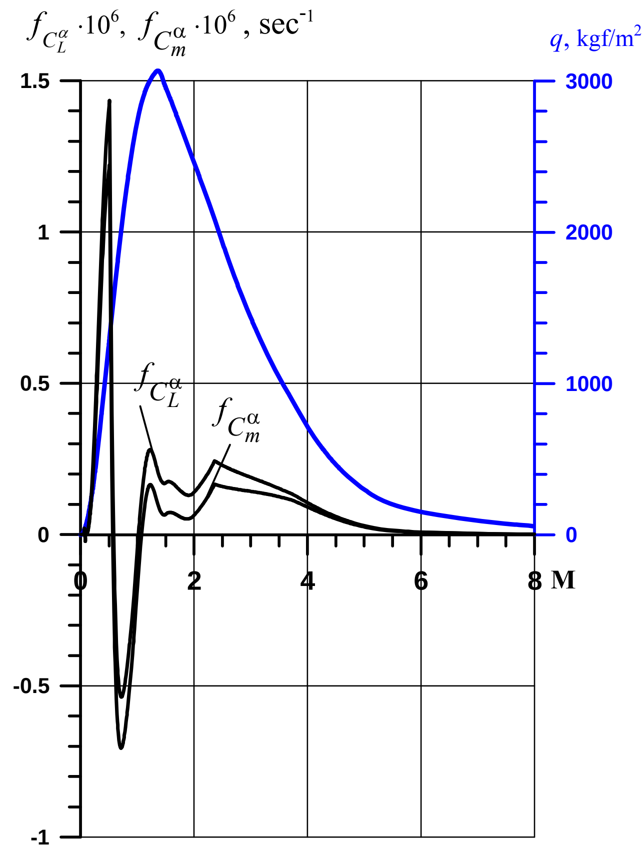

Fig 4. Relations of $f_{C_L^\alpha}$, $f_{C_m^\alpha}$ and dynamic pressure q to Mach number at $\bar{F}$=1.1, b_a= L_{I}, l_a=0.

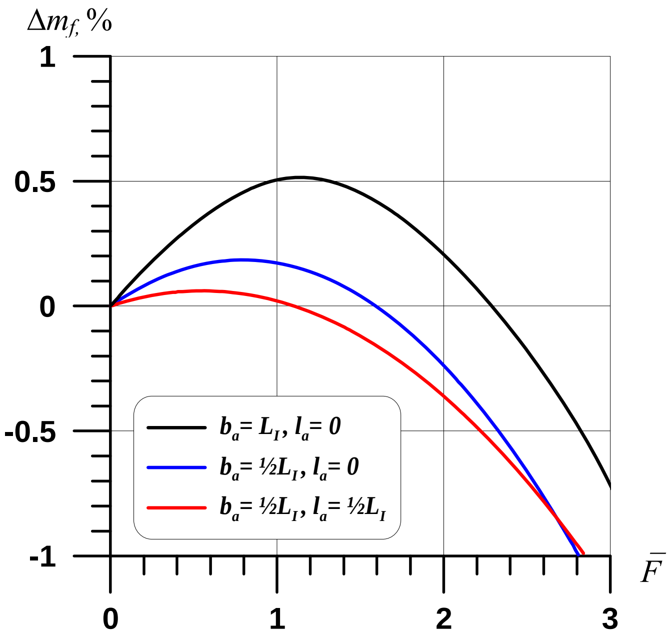

Fig 5. Relations of Δm_f to $\bar{F}$ at constant b_a and l_a.

Functions $f_{C_L^\alpha}$ and $f_{C_m^\alpha}$ are positive at any point on the trajectory, so an increase in C_L^α and C_m^α results in an increase of the payload. The maximums of $f_{C_L^\alpha}$ and $f_{C_m^\alpha}$ are observed at M≈0.7.

During the optimization of vector **p** the optimal trajectory changes. In particular, when $\bar{F}$ increases, the constraint (20) becomes active. It results in changes to $f_{C_L^\alpha}$ and $f_{C_m^\alpha}$ (see Fig. 4), they become negative at $0.5 < M < 1$. This effect can be explained as follows.

An increase in C_L^α and C_m^α results in an increase of the resultant force acting transversally to the vehicle longitudinal axis. Action of the constraint on transverse g-load demands to reduce the angle of attack when C_L^α of C_m^α increase. So the control deviates from the optimal one and payload decreases.

Thus the method can present detailed information about specific effect of each elementary trajectory section on the aircraft mission efficiency. It can be used for optimization within the framework of separate disciplines.

The dependencies of the relative increment of the payload

$$\Delta m_f = m_f(\bar{F}) / m_f(\bar{F}=0)$$

on $\bar{F}$ parameter at constant b_a and l_a are shown in Fig.5. The optimal solution for this problem gives the following values of varied parameters: $\bar{F}_{opt} = 1.1$, $b_a = L_I$, $l_a = 0$.

Conclusions

The decomposition of an aircraft multidisciplinary optimization problem on the basis of the maximum principle makes it possible to take into account the specific effect of each elementary trajectory section on the functional.

The local complex distributed criteria can be used for optimization within the framework of separate disciplines by aircraft mission efficiency criterion. They are determined as a by-product of the solution of the trajectory optimization problem.

The method is especially effective to optimize parameters of a multiregime aircraft when an arrangement of priorities between different flight regimes can be difficult or incorrect if they based on subjective experience.

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Appendix A

Task parameters

Two-stage space launcher [16] is chosen as primary.

Table A.1. Characteristics of the launcher

Ratio of vehicle length to diameter of I-st stage booster	14.3
Ratio of diameter of II-nd stage booster to diameter of I-st stage booster	1.24
Ratio of length of II-nd stage booster to total vehicle length	0.25
Reference area F_0	10.18 m^2
Initial thrust-to-weight ratio	1.24
Distance between vehicle mass center and launcher nose divided by reference length	1/3
Specific mass flow rate	$\mu g_i / T_i = 3.0 \cdot 10^{-3}$ c^{-1}

Aerodynamic characteristics of the launcher are determined by the technique described in [12].

Boundary conditions and constraint are presented in Table A.2.

Table A.2. Boundary conditions and constraint

Initial conditions	velocity $V_i = 0$ altitude $h_i = 0$ path angle $\gamma = 90°$ pitch angle $\theta = 90°$
Final orbit	circular, $h_{orb} = 200$ km
Maximum admissible transverse g-load	$n_{z\ \max} = 0.25$

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