

# The Development of 3 DoF Wing Section Model for Aeroelastic and Active Control Wind Tunnel Experimental Tests

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**Keywords:** *Aeroelasticity, wind-tunnel test, flutter*

## Abstract

*This paper discusses the development of a wind-tunnel testing equipment for investigating the dynamics behaviour of a 3 degree of freedom (DoF) aeroelastic system. It is intended to incorporate a mechanical platform into the test section of an open-loop wind-tunnel for this purpose. This platform must be designed such that the desired aeroelastic phenomena, such as instability (flutter), occurs and can be observed within the operation regime of the wind tunnel. Hence, a string of numerical calculations and analysis must be conducted to determine the platform parameters. Theoretical analysis needs to be carried out by first forming the system aeroelastic mathematical model. The model is derived from a 3 DoF mechanical system involving unsteady aerodynamics force and moment induced by the system dynamic response. The aerodynamic force is calculated using Doublet Point Method (DPM) by considering the wing section main modes. The obtained force then is combined with the system dynamic equations which further is transformed into a generalized coordinate system. By analyzing and simulating the mathematical model, the aeroelastic system parameters can be tuned to 'match' the wind-tunnel operation regime. Based on the obtained aeroelastic parameters, a platform configuration is designed and developed. The dynamic parameters of the platform must be adjusted such that they are equivalent to the mathematical model parameters. The dynamic characteristic of the platform then is evaluated and analyzed so that a compatible aeroelasticbehaviour can be observed during a wind-tunnel test.*

## 1. Introduction

Dynamic aeroelastic problems generally can be represented as interactions between unsteady aerodynamics forces/loads and the structural forces (inertia, damping, stiffness) of the structures [2],[5],[11]. In this interaction, the aerodynamic loads vary as functions of the structure dynamic response/deflections, hence they form such a closed loop system where there is a loop of influence from the structure response to the aerodynamic forces, and vice versa.

The study of aeroelasticity has been done in many researches, both theoretically and experimentally. It is obvious that experimental studies can be viewed as a way to validate the theoretical results, and as an effective way to observe and comprehend the physical phenomena in aeroelasticity problems.

Theoretical methods for analyzing and evaluating the characteristic of aeroelastic systems have been developed and presented in many literatures. In [2] and [5], 2 dimensional dynamic aeroelastic problem is described by analyzing 2 dimensional wing section system, where 2 degree of freedoms are involved, namely heaving (plunging) and pitching. Theodorsen unsteady aerodynamic coefficients, related to the heaving and pitching DoFs, are used for representing the aerodynamic force and moment in the 2D problem above. Based on this model, some methods for evaluating the characteristic of the aeroelastic system are discussed, for example the UG methods for determining the dynamic instability boundary (flutter speed). In [11], a case of dynamic aeroelasticity involving a lifting surface with pitching and flapping DoFs is discussed, while

in [4] a problem involving rigid body mode is investigated. Method for determining the instability boundary by evaluating the characteristic roots of aeroelastic equation is described, for example, in [6] and [11]. Algorithm for computing the unsteady aerodynamic coefficients in time domain is described in [3], while in [9] an algorithm called Doublet Point Method (DPM) is described for computing the unsteady aerodynamic pressure distribution over a lifting surface. In [10] an approximation technique for obtaining unsteady aerodynamic loads representation is presented. This approximation is used in [6] for forming an aeroelastic equation in time domain.

The experimental studies in aeroelasticity are also presented in many literatures and papers. In [8], some experimental studies in aeroelasticity conducted in NASA are presented, along with the description of some equipment used in the studies. The description of Benchmark Active Controls Technology (BACT) equipment for aeroelasticity wind tunnel testing is presented in [7] and [8]. Another test system is also discussed in [1] which is used for experimental study on the application of active control for nonlinear aeroelasticity.

In order to conduct comprehensive studies on aeroelasticity, an experimental test equipment is about to be designed and developed in ITB, Indonesia. This paper discusses the development of a wind-tunnel test system for the investigation of aeroelastic system with rigid body mode involved. In this paper the configuration of the system will be presented, along with several theoretical studies which need to be done for determining the parameters of the test system.

This paper will be presented in the following arrangement. The second section discuss the description of the existing wind tunnel system operated by Faculty of Mechanical and Aerospace Engineering, ITB, Indonesia, for aeroelasticity study. The intended modification will also be described in this section. The third section presents the first step of the theoretical studies for obtaining the parameters of the test system, which is the governing of the aeroelastic equations and the unsteady aerodynamic calculation. In the fourth

section, aeroelasticity analysis of the mathematical model is elaborated. Further, the determination of the test system parameters is presented in the fifth section, along with the description of a more detail configuration of the test system platform. The sixth section will conclude this paper by some analysis, evaluation, and proposition for further development.

## 2. Wind-Tunnel Test System

A low-speed open-loop wind tunnel system was developed in Faculty of Mechanical and Aerospace Engineering, Institut Teknologi Bandung (ITB), Indonesia, for aeroelasticity studies. This equipment has been used for several studies on aeroelasticity and active control implementation.



Fig.1. ITB open loop wind tunnel

It is intended to modify the equipment so that it can be used for further studies involving a 3 DoF system, i.e. by incorporating a mechanical linkage which can represent the effect of a rigid body mode. The main configuration of the wind-tunnel is Table 1.

Dimension	
Total length :	6 m
Test section length:	1.2 m
Test section width :	0.4 m
Test section height :	0.4 m
Max. flow speed :	20 m/s

Table 1. ITB wind tunnel specification

Currently, the wind tunnel is already equipped with a spring supported wing section model representing a 2 DoF aeroelastic system. Several studies in aeroelastic instability and active control implementation have been conducted using this model. Another experimental study on the aeroelasticity characteristic of a bridge structure section was also done in the test section of this wind-tunnel.

For designing the mechanical platform, besides the main system configuration, it is also required to determine suitable platform parameters such that the platform can exhibit particular aeroelastic behaviour in the wind-tunnel. To determine the parameters, theoretical analysis must be carried out via mathematical modeling and analysis of the aeroelastic system.

### 3. Aeroelastic Equation

#### 3.1 Equation of Motion

The mathematical model of a 3 DoF aeroelastic system can be obtained from a typical wing section model, as described in [2] and [5], with a rigid body mode added to its DoF, as depicted in the following figure :

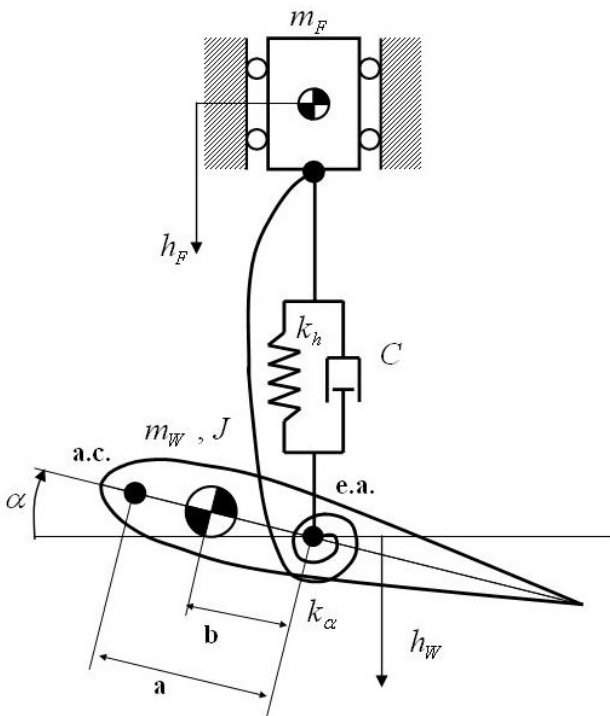


Fig.2 Diagram of a 3 DoF system

It can be seen from the diagram that the system has 2 variables related to flexible body DoFs, namely the wing heaving  $h_w$  and the wing pitching  $\alpha$ , while the rigid body DoF is represented by  $h_f$ .

Assuming that the structural damping is zero ( $C_s=0$ ), and taking the elastic axis and center of gravity of the wing section coincide at the midchord, and the aerodynamic center is at  $c/4$  from the leading edge, then the equation of motion for the system depicted in Fig.2 can be represented as follows:

$$\begin{bmatrix} m_w & 0 & 0 \\ 0 & J & 0 \\ 0 & 0 & m_f \end{bmatrix} \begin{Bmatrix} \ddot{h}_w \\ \ddot{\alpha} \\ \ddot{h}_f \end{Bmatrix} + \begin{bmatrix} k_h & 0 & -k_h \\ 0 & k_\alpha & 0 \\ -k_h & 0 & k_h \end{bmatrix} \begin{Bmatrix} h_w \\ \alpha \\ h_f \end{Bmatrix} = \begin{Bmatrix} -L \\ M \\ 0 \end{Bmatrix} \quad (1)$$

where  $m_w$  and  $J$  are the mass and moment of inertia of the wing structure,  $m_f$  is the mass of the rigid body,  $k_h$  is the stiffness for heave motion,  $k_\alpha$  is the stiffness of the pitch motion. While the unsteady aerodynamic force and moment are denoted by  $L$  and  $M$ , respectively.

#### 3.2 Doublet Point Method (DPM)

The aerodynamic loads ( $L$  and  $M$ ) are calculated by implementing Doublet Point method described in [9]. This approach principally computes the pressure distribution over a lifting surface area, by exploiting its relation to the downwash flow distribution over the area. Suppose the lifting surface area is discretized into elements distributed in chordwise and spanwise direction, and each element at location  $(x,y)$  has vertical displacement as a function of time, denoted by  $z(x,y,t)$ . Separating  $z(x,y,t)$  into the spatial part  $\Phi(x,y)$  and temporal part  $q(t)$ , we can express the element displacement as  $z = \Phi(x,y)q(t)$ . The spatial part can be composed from the structural modes representing the DoF of the system, in particular  $h_w$  and  $\alpha$ , hence it may be represented as  $\Phi(x,y) = [\phi_h(x,y) \ \phi_\alpha(x,y)]$ . Having the surface already discretized into elements, the relations between pressure distribution and the downwash is described by the following equation:

$$p(x, y) = D^{-1} \bar{w}(x, y) \quad (2)$$

where  $p(x, y)$  is the pressure distribution and  $D$  is the aerodynamics matrix, defined by the equation below :

$$D = [d_{ij}] = \frac{\Delta_j}{8\pi} K_A(x_i - \xi_j, y_i - \eta_j) \quad (3)$$

where  $\Delta_j$  is the  $j$ -th element surface area, and  $x_i, y_i, \xi_j, \eta_j$  are the element coordinates. The Kernel function  $K_A$  is described in [9], along with all the detailed explanation about the development of DPM method. Obviously, the aerodynamic matrix is determined by the geometry of the lifting surface.

The downwash distribution vector  $\bar{w}(x, y)$  is defined by the following relation:

$$\bar{w} = \frac{w}{U_\infty} = \frac{\partial z}{\partial x} + ik \frac{z}{b} = \left\{ \frac{\partial \Phi}{\partial x} + ik \frac{\Phi}{b} \right\} q \quad (4)$$

where  $k$  is the reduced frequency ( $k = ob/U_\infty$ ). The formulation above is called the aero-structure coupling, which denotes the relation between the downwash  $\bar{w}(x, y)$  and the vertical displacement distribution represented by the structural modeshape  $\Phi(x, y)$ .

Having the pressure distribution  $p(x, y)$  already computed via equation (2), the aerodynamic force and moment then can be computed by :

$$C_L = \sum_j^{N_y} \left\{ C_l(y_j) \sum_i^{N_x} \Delta_i \right\} / (S/2) \quad (5a)$$

$$C_M = \sum_j^{N_y} \left\{ C_m(y_j) \sum_i^{N_x} \Delta_i \right\} / (S/2) \quad (5b)$$

where, the coefficients  $C_l$  and  $C_m$  of each elements are obtained from:

$$C_l(y_j) = \sum_i^{N_x} \Delta p_i \Delta_i / \sum_i^{N_x} \Delta_i \quad (6a)$$

$$C_m(y_j) = \sum_i^{N_x} \Delta p_i \Delta_i (x_m - x_i) / \sum_i^{N_x} \Delta_i \quad (6b)$$

Note that the coefficients  $C_L$  and  $C_M$  above are computed from the pressure distribution related to each DoF of the system (1).

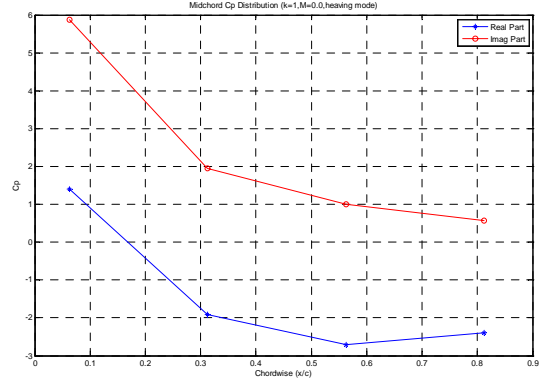


Fig. 3 Chordwise pressure distribution - midsection (heaving mode) for rectangular wing with  $c=0.25$  m,  $s=0.35$  m

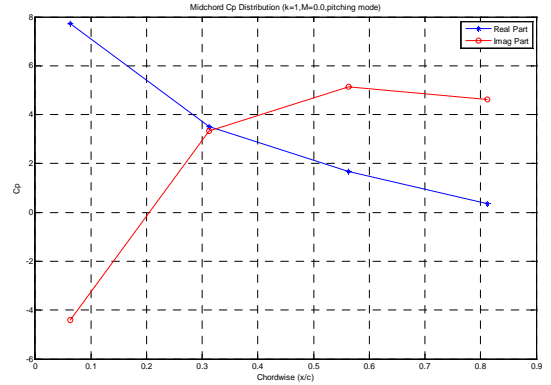


Fig. 4 Chordwise pressure distribution - midsection (pitching mode) for rectangular wing with  $c=0.25$  m,  $s=0.35$  m

### 3.3 Pade Approximation

Using DPM, the aerodynamic coefficients  $C_L$  and  $C_M$  in (5a) and (5b) are computed as functions of reduced frequency  $k$ . Further, these coefficients are approximated by functions in Laplace variable domain as defined below:

$$Q = [A_0] + [A_1] \left( \frac{b}{U_\infty} \right) s + [A_2] \left( \frac{b}{U_\infty} \right)^2 s^2 + \sum_{m=3}^6 \frac{[A_m] s}{\left( s + \frac{U_\infty}{b} \beta_{m-2} \right)} \quad (7)$$

where  $Q$  is the aerodynamics data ( $C_L$  and  $C_M$ ) obtained from DPM computation, and  $A_0, A_1, A_2, \dots$  are the approximated aerodynamics matrices obtained from curve fitting procedure. Time domain functions obtained via the inverse

of Laplace transformation of (7) then can be applied for representing the aerodynamic parts in (1). The procedure for fitting the form (7) to aerodynamic data is called Pade approximation algorithm described in [10]. For a rectangular wing (chord =0.25 m and span =0.35 m) having pitch and heave motions, the DPM computation and Pade approximation (with no aerodynamic lagging terms  $\beta$ 's involved) results are presented in the following figures.

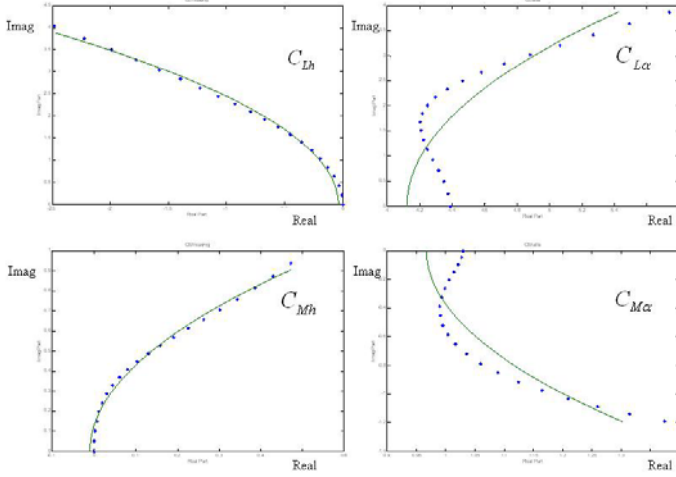


Fig. 5 DPM data (dots) and Pade approximation (line) of the unsteady aerodynamic coefficients

#### 4. Dynamic Stability Analysis

The aeroelastic equation (1), with aerodynamic parts represented by Pade approximation form, then can be rewritten in a state space form  $\dot{\bar{q}} = [A_s]\bar{q}$ , where the  $A_s$  matrix is defined as [6],[11]:

$$[A_s] = \begin{bmatrix} 0 & I & 0 & 0 \\ -[\bar{M}]^{-1}[\bar{K}] & -[\bar{M}]^{-1}[\bar{D}] & [\bar{M}]^{-1} & [\bar{M}]^{-1} \\ 0 & q_{dyn}[A_3] & \left(-\frac{U_\infty}{b}\beta_1\right)I & 0 \\ 0 & q_{dyn}[A_4] & 0 & \left(-\frac{U_\infty}{b}\beta_2\right)I \end{bmatrix} \quad (8)$$

where,

$$\begin{aligned} \bar{M} &= [M] - q_{dyn}[A_{2w}]\left(\frac{b}{U_\infty}\right)^2 \\ \bar{D} &= -q_{dyn}[A_{1w}]\left(\frac{b}{U_\infty}\right) \\ \bar{K} &= [K] - q_{dyn}[A_{0w}] \end{aligned} \quad (9)$$

where  $M$  and  $K$  are the matrices defined in the first and second terms of (1), and  $A_{0w}$ ,  $A_{1w}$ ,  $A_{2w}$  are obtained from Pade approximation form (7). Note that some elements of the aeroelastic model matrix (8) are determined by the structural properties, which are assumed to be constant, while others are functions of aerodynamics (flow) properties.

Based on the model (8), the aeroelastic analysis (instability and disturbance problems) is performed and discussed, and the results will be related to the determination of the parameters of the developed equipment.

It has been described that the developed equipment will be installed in a low-speed wind tunnel. By assuming that the flow is incompressible, and that the structural properties of the mechanical linkage are constant, hence the theoretical model (8) can be described as a function of flow speed  $U_\infty$ . Evaluating the characteristic roots (Eigenvalues) of (8), i.e. by solving  $\det[sI - A_s] = 0$ , at each incrementing value of  $U_\infty$ , gives the information about the change of the aeroelastic system stability. The flow speed at which the system becomes unstable is defined as the dynamic instability boundary speed, also called the flutter speed. The flutter speed can be determined by finding out the flow speed which makes the characteristic roots of (8) starting to cross the imaginary axis, from the left-half to the right-half, of the complex plane.

#### 4.1 Restrained $m_F$ (2-DoF) System

A 2 DoF system can be obtained by restraining the rigid body mass  $m_F$ , such that  $h_F = 0$ .

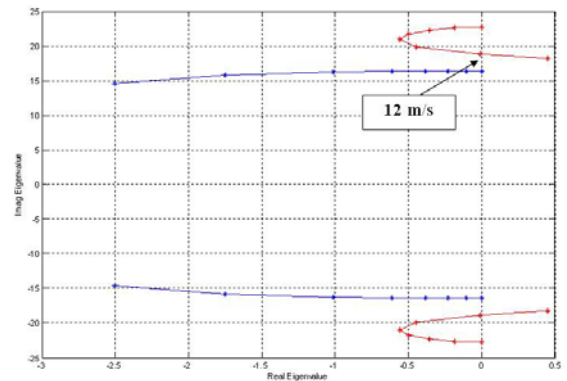


Fig. 6 Stability Analysis : Eigenvalues of (8) for restrained rigid body (2 DoF) system

Consider such system with a rectangular wing (chord = 0.25 m, span =0.35 m) which has flexural axis  $x_f$  at 0.5 chord,  $m_w = 2$  kg,  $J = 0.0104$   $\text{kgm}^2$ ,  $k_h = 540$  N/m,  $k_a = 5.4$  Nm/rad, and its aerodynamic loads coefficients are as depicted in Fig.5. The dynamic stability analysis by evaluating the Eigenvalues of (8), at each incrementing value of  $U_\infty$ , gives the system characteristic roots plot as shown in Fig. 6. The plot shows that the instability boundary speed (flutter speed) of this 2 DoF system is 12 m/s.

### 4.2 Unrestrained $m_F$ (3-DoF) System

When  $m_F$  is not restrained, a 3 DoF system with additional rigid body mass is obtained. The results of stability analysis for 3 DoF cases are presented below.

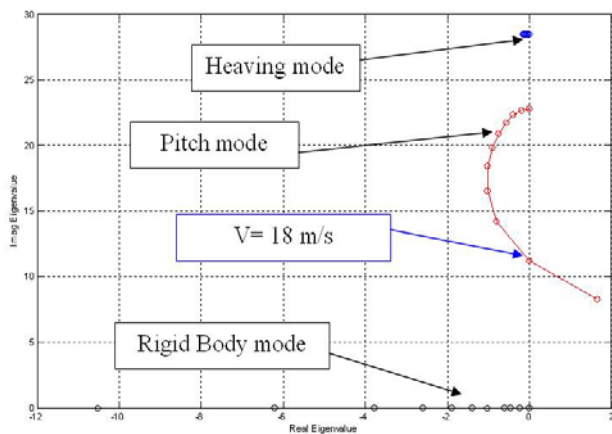


Fig.7 Unrestrained rigid body (3 DoF) problem with  $M_F = 0.5 M_w$

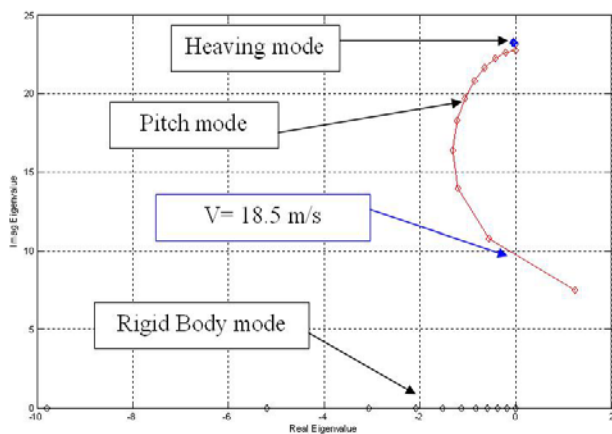


Fig.8 Unrestrained rigid body (3 DoF) problem with  $M_F = M_w$

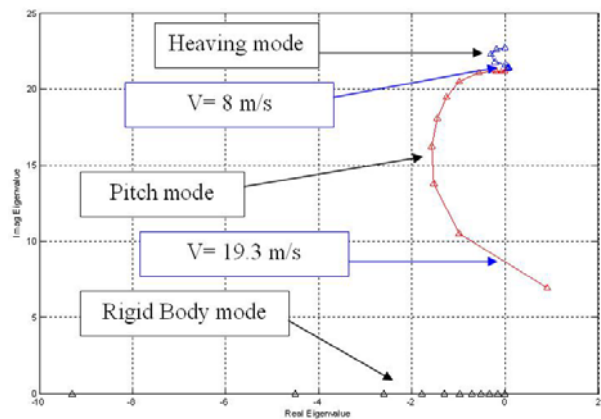


Fig.9 Unrestrained rigid body (3 DoF) problem with  $M_F = 1.5 M_w$

From the results shown in Fig. 7, 8, and 9, it can be seen that the rigid body mass  $m_F$  also affects the flexible body characteristics and the flutter speed of the system (8). In the cases presented here, the natural frequency of the heaving mode of the 3 DoF problems are higher than that of the 2 DoF problem (when  $m_F$  is restrained). For the cases of  $m_F = 0.5 m_w$  and  $m_F = m_w$ , instability occur when the pitch mode roots crossed the imaginary axis to the unstable area, while for the case of  $m_F = 1.5 m_w$ , it occurs when the heave mode roots move to the right-half plane at  $V=8$  m/s and followed by the pitch mode roots at  $V=19.3$  m/s. The results are summarized in the following table:

$m_F/m_w$	$V_f$	Flutter Mode
0.5	18 m/s	Wing Pitch Mode
1	18.5 m/s	Wing Pitch Mode
1.5	8 m/s (19.3 m/s)	Wing Heave Mode (Wing Pitch Mode)
2 DoF	12 m/s	Wing Pitch Mode

Table 2. Results of 3 DoF Flutter Analysis

The results discussed above also show that for the configuration of system as described in subsection 4.1, the introduction of rigid body mass with the ratio of  $(m_F/m_w)$  within 0.5 - 1.5, may increase (around 50 %) or decrease (around 30 %) the instability speed of the system, related to that of the 2 DoF system. This observation will be considered in the next subsection when an approach for determining the parameters of the developed test system is proposed.

## 5. Structural Parameters Determination

It is already showed that the stability analysis can be done by evaluating the system matrix (8). A systematic way for determining the parameters of the wind-tunnel test model is derived from the stability analysis. In this approach, the wind-tunnel flow speed limit is treated as a constraint, representing the speed at which the instability is expected to occur, while the aerodynamics coefficients are set as constants, since it is assumed to be only a function of lifting surface geometry. Then the structural parameters (mass and stiffness) can be calculated by examining the stability of (8) via Flutter Conic technique [11].

### 5.1 Flutter Conic

The Flutter Conic technique is usually used for determining the flutter speed of a binary (2 mode) system by examining the determinant of its equation matrix [11]. Consider the aeroelastic equation (1) with  $m_F$  restrained, hence  $h_F = 0$  and it becomes a 2 DoF system. Having the aerodynamics load represented as functions of structure response, then (1) can be rearranged and rewritten in a compact form as :

$$A\ddot{\bar{x}} + \rho U_\infty B \dot{\bar{x}} + (\rho U_\infty^2 C + E)\bar{x} = 0 \quad (10)$$

where A is the augmented inertial matrix which can be related to  $\bar{M}$  in (9), B is the aerodynamic damping matrix as described by  $\bar{D}$  in (9), C and D are the aerodynamic and structural stiffness matrices, related to  $\bar{K}$  in (9), and  $\bar{x} = [h_w \quad \alpha]^T$ . Assuming a solution of the form  $\bar{x} = \bar{x}_0 e^{i\omega t}$ , equation (10) can be expressed as:

$$[-A\omega^2 + i\omega\rho U_\infty B + \rho U_\infty^2 C + E]\bar{x} = 0 \quad (11)$$

where the non-trivial solution is defined by a complex valued polynomial obtained from:

$$\det [\tilde{A}_s(\omega)] = 0 \quad (12)$$

$$\det [-A\omega^2 + i\omega\rho U_\infty B + \rho U_\infty^2 C + E] = 0$$

The real part of (12) is a polynomial in  $\omega$  described by :

$$r_1\omega^4 + r_2U_\infty^2\omega^2 + r_3U_\infty^4 + r_4\omega^2 + \dots \quad (13)$$

$$\dots + r_5U_\infty^2 + r_6 = 0$$

where the coefficients  $r_1 \dots r_6$  are functions of the elements of A, B, C, E of (10).

The equation (13) forms a conic when it is plotted in  $(\omega^2 - U_\infty^2)$  plane.

The imaginary part of (12), given by

$$s_1\omega^2 + s_2U_\infty^2 + s_3 = 0 \quad (14)$$

is a linear relation between  $\omega^2$  and  $U_\infty^2$ . The coefficients of (14) are also functions of the elements of A,B,C,E of (10).

The instability boundary of the system  $(U_\infty^2)_f$  will be given by the value of  $U_\infty^2$  where the curves of (13) and (14) are intersected. The point of intersection also gives an information about the system frequency when instability starts to occur.

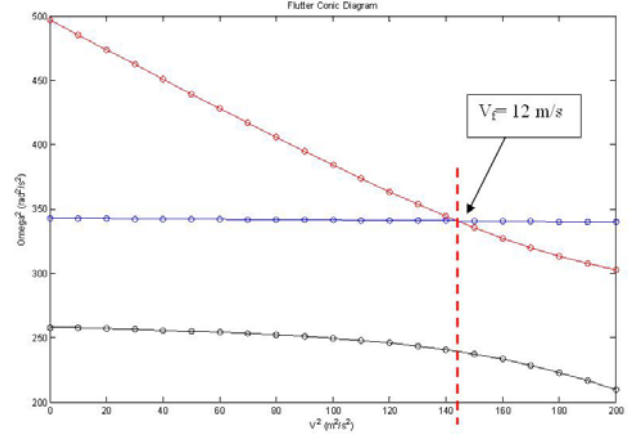


Fig.10 Flutter Conic of 2 DoF Problem

### 5.2. Parameter Determination Approach

The flutter conic technique is then be explored for determining the parameters of the developed test system. In the proposed approach, a particular 2 DoF flutter speed is set by considering the speed limitation of the wind-tunnel, and the relation between the flutter speed of a 2 DoF system and that of a 3 DoF system, which has been observed and discussed in Subsection 4.2. By setting the flutter speed,

then the equation (12), (13), and (14) become functions of structural parameters  $m_w$ ,  $J$ ,  $k_h$ , and  $k_a$ . Further, the moment of inertia  $J$  can be related to  $m_w$ , by considering a certain mass distribution of the wing structure, as described by the followings :

$$\begin{aligned} m_w &= m_d s c \\ J &= m_d s \left( \frac{c^3}{3} - c^2 x_f + c x_f^2 \right) \end{aligned} \quad (15)$$

where  $m_d$  is the weight/unit area,  $s$  is the wing span,  $c$  is the wing chord, and  $x_f$  is the flexural axis location measured from the leading edge. The stiffness for heave motion is also related to that of pitch motion by the following:

$$k_a = c_k k_h \quad (16)$$

where  $c_k$  is a constant which can be determined by considering the configuration used for applying the stiffness in the physical system. Applying the relations (15) and (16), the equations (12), (13), (14) can be further simplified as functions of  $m_w$  and  $k_h$ , which can be described by the following equation :

$$\begin{aligned} &\lambda_1 k_h^2 m_w^2 + \lambda_2 k_h^2 m_w + \lambda_3 k_h^2 + \dots \\ &\frac{\dots + \lambda_4 k_h m_w^2 + \lambda_5 k_h m_w + \lambda_6 k_h + \lambda_7 m_w}{\gamma_1 m_w^2 + \gamma_2 m_w + \gamma_3} = 0 \end{aligned} \quad (17)$$

where the coefficients  $\lambda_1, \lambda_2, \dots, \lambda_7, \gamma_1, \gamma_2, \gamma_3$  are obtained by considering the value of the elements of A,B,C,E in (10).

The procedure for computing the pair of ( $m_w, k_h$ ) which gives the expected flutter speed ( $U_\infty$ )<sub>f</sub> is described below:

- Set the expected flutter speed ( $U_\infty$ )<sub>f</sub> and the reference parameter (can be  $m_w$  or  $k_h$ )
- Suppose  $m_w$  is chosen to be the reference parameter, then set an arbitrary value of  $k_h > 0$ , and denote it as  $k_h(i)$
- Solve (14) using the pair ( $m_w, k_h(i)$ ), giving a solution  $\omega^2$
- Apply  $\omega^2$  from (c) into equation (13) to obtain a new value of  $k_h$ , and denote it as  $k_h(i+1)$

- Repeat (c) and (d) using the updated value of  $k_h(i+1)$  until it converges to the previous value  $k_h(i)$
- Set a new value of  $m_w$  and repeat the procedure from (a)

It should be noted that for a particular value of  $m_w$ , there may be 2 solutions of  $k_h$ , since the relation is in conic form. Hence another significantly different initial values of  $k_h$  must be chosen for each particular  $m_w$ . In addition, the value of  $m_w$  and  $k_h$  must also be positive. Suppose we want to obtain possible parameters for a system similar to the one defined in Subsection 4.1, such that it becomes unstable at  $U_\infty = 12$  m/s. Using the approach explained above, the obtained results are depicted in the following figures.

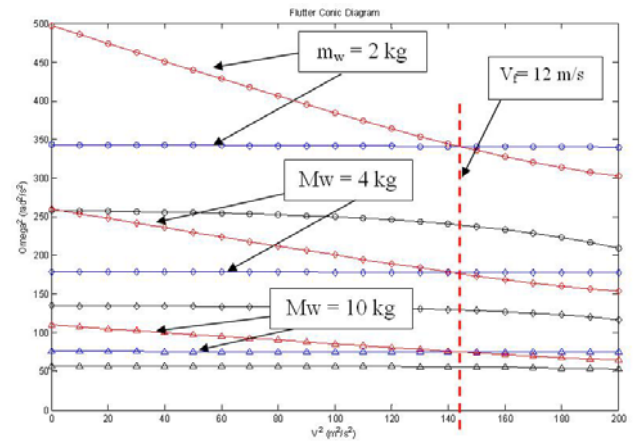


Fig.11 Flutter Conic plots for parameter determination of a 2 DoF problem (fixed parameter:  $k_h = 540$  N/m)

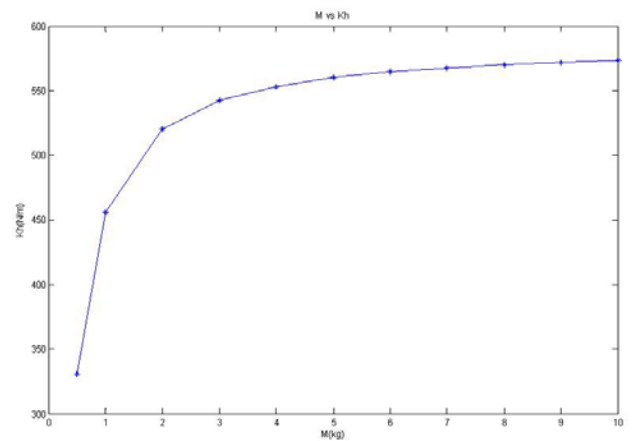


Fig.12 ( $m_w$ - $k_h$ ) parameter pairs for binary flutter at 12 m/s



In Fig. 11 it can be seen that the approach can be used for selecting the parameters of the test system by setting the expected flutter speed. This approach provides a set of parameter values resulting in particular stability characteristics, as reflected by the flutter speed and the flutter frequency. It should be noted that, as depicted in Fig.11, for each pair of parameters  $m_w$ - $k_h$ , flutter occurs at different frequency. This frequency information is important when we intend to equip the test system with sensors and actuators, so that we can select sensors and/or actuators with appropriate characteristics.

Fig. 12 shows the values of parameters  $m_w$  and  $k_h$  causing instability at a predefined flutter speed (12 m/s). This results shows that there are options in selecting system parameters for a particular characteristic (flutter speed), hence other factors (practical aspects) can be taken into consideration in the selection.

Having the parameters  $m_w$  and  $k_h$  obtained from the procedure, the other parameters, such as  $J$  and  $k_\alpha$  can be computed using the relations defined by (15) and (16).

The parameters provided by the proposed approach are determined based on a 2-DoF system instability problem. For extending the results to parameters determination of a 3-DoF system, the observation taken in Subsection 4.2 must be considered, so that the resulting 3-DoF flutter speed is still below the wind tunnel maximum speed.

### 5.3 Test System Configuration

The physical test system is designed such that it can represent the behaviour of a 3 DoF dynamic aeroelastic system as depicted in Fig. 2. It can be seen that the configuration must be arranged such that it allows the system to move following all of its DoFs, while it is still mechanically realistic and can be manufactured. The proposed configuration is depicted in Fig. 13.

Considering the equipment's configuration, its parameters are defined and related to the equivalent parameters of the theoretical model described in the previous subsections.

On the developed test system, the inertias of the flexible modes are provided by the physical

mass and moment of inertia of the main and additional parts of the wing structure, while the rigid body mass is provided by the mass of the swinging arm and the balance weight. The stiffness are produced using 2 cantilever arms and rotational spring supporting the wing structure. Some features for enabling parameters adjustment are also designed on the test system. Implementation of sensors and actuators on the test system is also considered in further development.

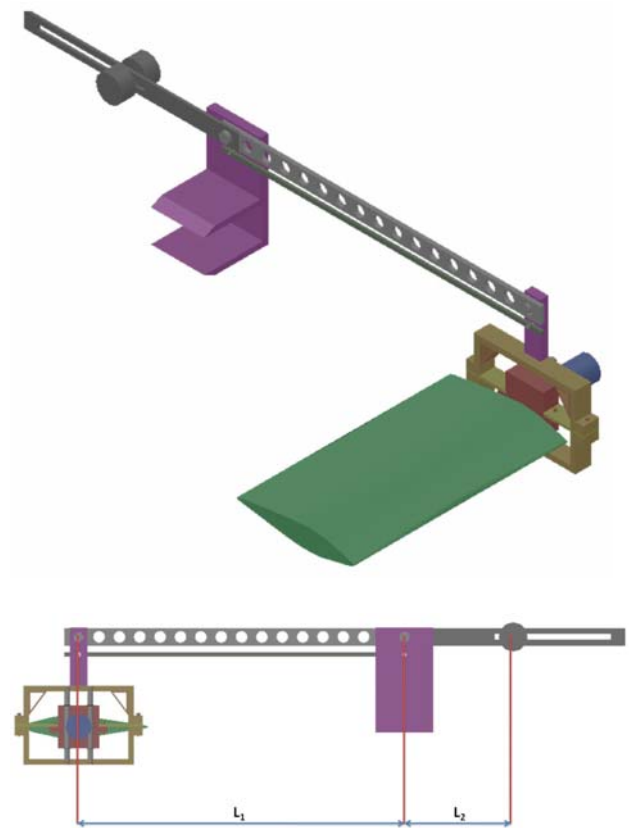


Fig.13 The configuration of the proposed wind tunnel equipment

## 6. Conclusion and Remarks

Aeroelastic analysis of a 3 DoF system has been presented and related to the development of a wind tunnel test system. An approach to determine the parameters of the test system, such that it can exhibit aeroelastic phenomena within the operation limit of the wind tunnel, is proposed. The approach is based on the Flutter Conic analysis for binary system. The approach

is able to provide a set of parameters which can be selected and applied to the test system. Since the approach stems from the analysis of a binary system, then its application to a 3-DoF system is done by considering the effect of the introduction of a rigid body dynamic to the system stability characteristics, which has been observed theoretically. In further development, this approach can be modified such that it includes all DoFs of the system in the theoretical analysis, hence a more accurate results can be obtained. A procedure for relating the parameters to the system's frequency response characteristic may also be developed to complement the proposed approach. The inclusion of frequency response analysis means that, besides system stability, the system dynamic response will also be considered in determining the parameters. Hence, the approach can also be employed for adjusting the equipment parameters to investigate problems such as the system dynamics response to gust disturbance.

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