EFFECTS OF CONTROL ALLOCATION ON PILOT-INDUCED OSCILLATION (PIO) SUSCEPTIBILITY OF AIRCRAFTS WITH MULTIPLE CONTROL EFFECTORS

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Keywords: *multiple control effectors; control allocation; PIO susceptibility; time domain Neal-Smith; sequential quadratic programming*

Abstract

The relationships between control allocation and PIO susceptibility of aircrafts with multiple control effectors are analyzed theoretically. Control allocation principles are presented in term of preventing PIO events. Time domain Neal-Smith (TDNS) criterion is selected to evaluate the PIO susceptibility. An optimal algorithm based on sequential quadratic programming (SQP) is used to obtain the parameters of pilot model in TDNS criterion. Aero-Data Model in Research Environment (ADMIRE) developed by FOI is taken as example, five different control allocation configurations are presented based on the control allocation principles for ADMIRE. The category Ⅱ PIO susceptibility of ADMIRE with these five control allocation configurations is evaluated by TDNS criterion. The evaluation results indicate that the PIO susceptibility of aircrafts with multiple control effectors is affected by the control allocation configuration, the evaluation results accord with the control allocation principles, and the evaluation methodology can reflect the characteristics of PIO susceptibility of the controlled object.

1 Introduction

To obtain desired performance, modern aircrafts are usually equipped with multiple control surfaces. The number of control effectors is larger than the number of control parameters, and there are an infinite number of solutions. Hence, an appropriate method of control allocation must be chosen to control the aircraft. For aircrafts with multiple control effectors, the flight control system (FCS) consist of two parts: flight control law (FCL) which transfer pilot inceptor into pseudo control parameters and the control selector which transfer pseudo control parameter into deflections of control effectors.[1]

The pilot-induced oscillation (PIO) susceptibility of an aircraft increases with respect to the complexity of the FCS. Like FCL, in order to get optimal flight performance, the control allocation configuration is tailored by mission, which makes the FCS even more complex. Typical aircrafts with multiple control effectors, such as F-22, JAS-39, C-17, etc. all have experienced severe PIO [2]. Therefore, the effects of control allocation on PIO susceptibility of aircrafts with multiple control effectors must be taken into consideration.

Recent researches on control allocation concentrate on optimizing control and flight performances. Meanwhile, researches on PIO seldom relate to aircrafts with multiple control effectors. The relationships between control allocation and PIO susceptibility are rarely considered. This paper will discuss the relationships between control allocation and PIO susceptibility of aircrafts with multiple control effectors.

2 Control Allocation and PIO Susceptibility

2.1 Control Allocation

The goal of control allocation is to find a set of admissible control effectors deflections to generate desired control effects. The input is the total control effect to be produced, the virtual control input $\boldsymbol{v}(t) \in \boldsymbol{R}^k$. The output is the true control input $\boldsymbol{u}(t) \in \boldsymbol{R}^m$, where $m > k$ [3]. For linear systems,

$$\boldsymbol{Bu}(t) = \boldsymbol{v}(t) \tag{1}$$

where $\boldsymbol{B}$ is the control effectiveness matrix.

To incorporate actuator position and rate constraints, it is required that:

$$\boldsymbol{u}_{\min} \le \boldsymbol{u}(t) \le \boldsymbol{u}_{\max} \tag{2}$$

$$\boldsymbol{\rho}_{\min} \le \dot{\boldsymbol{u}}(t) \le \boldsymbol{\rho}_{\max} \tag{3}$$

2.2 The Category of PIOs

A PIO results from the interaction of the pilot and the dynamics of the vehicle being controlled. This interaction makes the closed-loop of pilot-vehicle unstable.

PIO is sorted to three categories.

Category I: Linear pilot-vehicle system oscillations. The vehicle is basically linear and the pilot has the characteristics of quasi-linear as well as time saturation. Category I PIO results from identifiable phenomena such as excessive time delay, excessive phase loss due to filters and improper control/response sensitivity, etc. This is the simplest kind of PIO that can be easily understood and prevented.

Category II: Quasi-linear events with some series of nonlinear contributions, such as rate or position limiting. Besides this limiting, other parts of the system can be treated as linear. And the nonlinear contributions could be processed separately. Currently most PIO events on modern aircrafts are category II.

Category III: Nonlinear PIO with transients. For both vehicle and pilot, the dynamics change transiently. This phenomenon may be caused by gain of control system, mode switching or task and environment changing [2].

2.3 The Relationships between Control Allocation and PIO susceptibility

If the control system is designed appropriately, category I PIO can be almost fully prevented.

The use of active control technology leads to smaller control surfaces. In order to get the same control power, smaller surfaces are required to move rapidly. This stresses the actuators, resulting in lags and rate limiting. [4]

The deflection position and rate of each control surface are different, under different control allocation configurations for certain control purpose. If the control quantities are concentrated on one or a few control effectors concentrate, the deflection position and rate of these control effectors would be over actuated, which will increase the working load of the actuators. When situation becomes worse, the saturation will happen and trigger category II PIO.

To prevent category II PIO, any position saturation and rate limit of control effectors should be prevented. From this point, control allocation should make the proportion of control effectors deflection equal to the proportion of actuators' saturation and rate limit,

$$\delta_1 : \cdots : \delta_n = \begin{array}{c} a_u(u_{\max 1} - u_{\min 1}) + a_\rho(\rho_{\max 1} - \rho_{\min 1}) : \\ \cdots : \\ a_u(u_{\max n} - u_{\min n}) + a_\rho(\rho_{\max n} - \rho_{\min n}) \end{array} \tag{4}$$

where $\boldsymbol{u} = [\delta_1, \cdots, \delta_n]^{\mathrm{T}}$, a_u and a_ρ are the weight parameter of the position and rate of the actuator, such that $0 \le a_u \le 1$, $0 \le a_\rho \le 1$, $a_u + a_\rho = 1$. For aircrafts with Fly-by-Wire (FBW) system, the possibility for actuator to get rate limit is much higher than that for it to get position saturation, hence the value of a_u should be much smaller less than the value of a_ρ, or even zero.

3 Evaluating the PIO Susceptibility

Since the PIO susceptibility is mostly caused by the saturation of control effectors' deflection position and rate, classical frequency domain methods are no longer sufficient. Time domain Neal-Smith (TDNS) criterion, which is developed based on frequency domain Neal-Smith (FDNS) criterion and Step Target Tracking (STT) criterion, is employed to evaluate PIO tendency in this paper.

TDNS criterion is based on time domain and independent from the nonlinear elements in a system. So this method is suitable for evaluating both category I and category II PIO. By analyzing flight test data, TDNS shows 85.87% precision in predicting PIO [5].

3.1 Theory of TDNS Criterion

As shown in Figure 1, longitudinal TDNS criterion evaluates flight quality and PIO tendency by pilot-vehicle closed-loop response parameters.

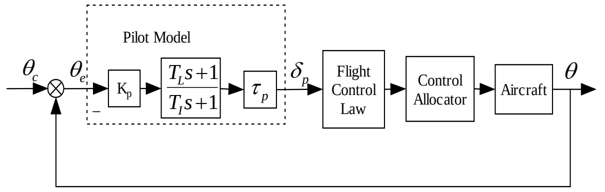

Fig.1 Closed-Loop Pitch Attitude Control Task Schematic Diagram of Aircraft with Multiple Control Effectors

A simple pilot model is adopted for the criterion:

$$\frac{\delta_p}{\theta_e}(s) = K_p \frac{T_L s+1}{T_I s+1} e^{-\tau_p s} \qquad (5)$$

Where K_p is pilot gain; τ_p is pilot time delay, usually equals to 0.25s; T_c is the pilot compensation time constant; K_p and T_c are independent variables.

Positive value of T_c corresponds to pilot lead compensation, using the pilot compensator model, where

$$T_L = T_c, T_I = 0$$
$$\Phi_p = 57.3\arctan(T_c \omega_{BW}) \qquad (6)$$

Negative value of T_c corresponds to pilot lead-lag compensation, where

$$T_I = 1/\omega_{BW} - T_c,\ T_L = 1/(T_I \omega_{BW}^2)$$
$$\Phi_p = 57.3[\arctan(T_L \omega_{BW}) - \arctan(T_I \omega_{BW})] \qquad (7)$$

The parameters of TDNS criterion include target acquisition time D, root mean square pitch tracking error after D $rms\theta_e|_{t>D}$ and phase angle compensation of pilot Φ_p . The acquisition time, D, is defined as the time at which the pitch attitude error first becomes less than the desired fine tracking standard (i.e. within a pipper diameter, Fig 2) after the pitch attitude command step input at 0.25 seconds.

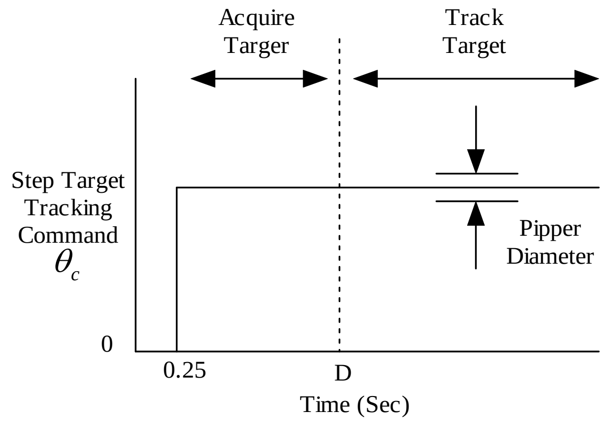

Fig.2 Step Target Tracking Task

Base on the definition of D, the task will be more difficult for the pilot if D is required to be smaller. D corresponds with the task bandwidth ω_{BW} of FDNS criterion:

$$\omega_{BW} = \frac{-1}{D-0.25}\ln\frac{1}{40} \qquad (8)$$

$rms\theta_e|_{t>D}$ is another important parameter which evaluates the tracking task with minimum overshoot and oscillation.

A good configuration - one with good flying qualities and without PIO tendencies - should not show significant variation in closed-loop response character as the task demands increase. Otherwise, a poor configuration will show significant degradation in closed-loop response character (oscillations, overshoots, even PIO) as the task demands or pilot aggressiveness in the task increase.

3.2 Quantitative PIO Criterion

PIO susceptibility is very sensitive to the task bandwidth. For an aircraft with PIO tendency, $rms\theta_e|_{t>D}$ will increase rapidly as D decreases; for a PIO-immune aircraft, $rms\theta_e|_{t>D}$ won't vary significantly when the D decreases. The best quantifying metric for PIO, found to date, is the local, second derivative of the $rms\theta_e|_{t>D}$ after D with respect to D.

If a configuration exhibits a local second derivative value of $rms\theta_e|_{t>D}$ with respect to D that is greater than 100, the configuration is predicted to be PIO-prone. Otherwise, the configuration is predicted to be PIO-immune.

The second derivative at a required task performance value of D (D_2 in this nomenclature) is computed by discrete approximation as:

$$\frac{\partial^2(rms\theta_e|_{D_2})}{\partial D^2} = \frac{rms\theta_e|_{D_1} + rms\theta_e|_{D_3} - 2rms\theta_e|_{D_2}}{\Delta T^2} \quad (9)$$

In which $\partial^2(rms\theta_e|_{D_2})/\partial D^2$ is the approximation local second derivative of the $rms\theta_e|_{t>D}$ with respect to D at the point of D_2 and $D_1 = D_2 - \Delta T, D_3 = D_2 + \Delta T$.

3.3 Calculating Process

The self-adaptability characteristics of pilots enable them to adjust control parameters according to the flying environment to obtain optimal operating performance.

The time domain response history under different D can be optimized: Adjust K_p and T_c to minimize $rms\theta_e|_{t>D}$. The outputs of optimization are the minimized $rms\theta_e|_{t>D}$, optimal pilot compensator K_p and T_c.

K_p and T_c will be solved by a time domain optimization method using the Simulink Response Optimization toolbox, which is suitable for linear and nonlinear system [6].

Calculating process:

1. Build simulation model in Simulink based on Figure 1;
2. Assign the initial values of optimization from control input and attach the Signal Constraint block to the output;
3. Write m-file to initialize controller and inputs;
4. Use “newsro” to build optimize model and set the bounds of desire response amplitudes according to the system requirements;
5. Use “findpar” and “set” to choose the parameters and bounds of the tuner;
6. Use “optimize” to run the model and get optimized parameters. If there is no solution, relax restriction of desire response and return to step 4.

4 Example and result

4.1 Example

This paper takes the ADMIRE (Aero-Data Model in Research Environment) developed by FOI as an example, [7] five different control allocation configurations are selected based on the control allocation principle of preventing PIO. The connection of control allocation and PIO susceptibility is validated by evaluating the PIO susceptibility in different control allocation configurations.

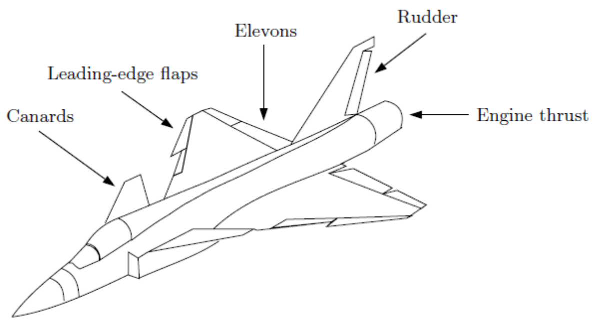

Fig. 3 Configuration Schematic of ADMIRE

ADMIRE, as shown in Fig 3, is developed based on the JAS-39. Because the flight control system (FCS) is designed appropriately, there is no category I PIO tendency. But the position and rate of actuator is limited, category II PIO might happen.

In normal range of angle of attack (AOA), the pitch control effectors of ADMIRE include closed-couple canard, inner elevon and outer elevon. The rate limits of these actuators are all 50 deg/sand the position limits are -55~25 deg, -25~25 deg and -25~25 respectively.

4.2 Control Allocation Configurations

Based on equation (4), the best control allocation configuration from the angle of preventing PIO is

$$\delta_n:\delta_{ie}:\delta_{oe}=(80a_u+100a_\rho):(50a_u+100a_\rho):(50a_u+100a_\rho) \qquad (10)$$

There are five control allocation configurations listed in table 1. Configuration 1 and 2 are based on equation (10), where a_u equals to 0 and 0.2 respectively. Configuration 3, 4 and 5 are based on generalized inverse with different weighting matrices.

Table 1 Parameters of 5 Control Allocation Configurations

Configuration	$\delta_n:\delta_{ie}:\delta_{oe}$	Weighting parameters
1	1: 1: 1	$a_u=0$
2	16: 15: 15	$a_u=0.2$
3	1.12: 1.57: 1	$W_u=\mathrm{diag}(1,1,1)$
4	1.125: 0.157: 1	$W_u=\mathrm{diag}(1,10,1)$
5	0.111: 0.157: 1	$W_u=\mathrm{diag}(10,1,1)$

The evaluating flight condition is H＝3Km， Ma＝0.4, the pitch angle command is 5° step signal from 0.25 second, pilot delay τ_p is 0.2 second.

The generalized inverse solves the equations in a manner which minimizes the 2-norm of the vector u. The bigger the weighting parameter is the smaller angle the actuator deflects. For the latter three configurations, configuration 3 is allocated more equally; the PIO tendency should be the lowest theoretically. By comparison in configuration 5, the control is allocated to the outer elevon which has lower control efficiency; the PIO tendency should be the highest theoretically. The PIO tendency of configuration 4 should be lower than configuration 5 and higher than configuration 3.

Besides, compared with configuration 1 and 2, the control of configuration 3 is relatively allocated to inner elevon.

4.3 Result and Analysis

Figure 4 to 6 show the evaluation results of the 5 configurations listed in table 1.

In Fig 5, the level boundaries, denotes lines of “constant” flying qualities equal to pilot rating of 3.5 (Level 1/Level 2 boundary) and 6.5 (Level 2/Level 3).

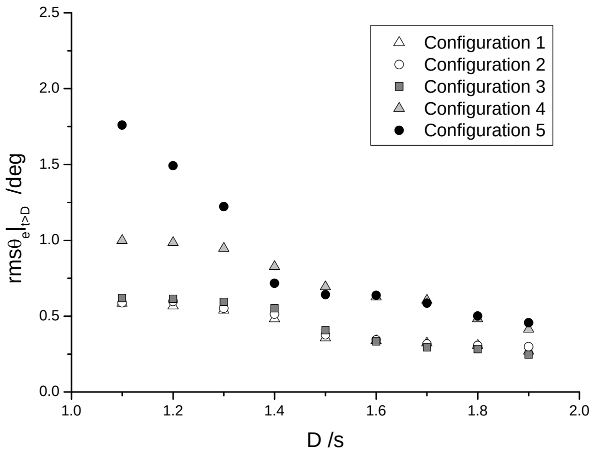

Fig.4 Root-Mean Squared Pitch Tracking Error after the Required Acquisition Time D

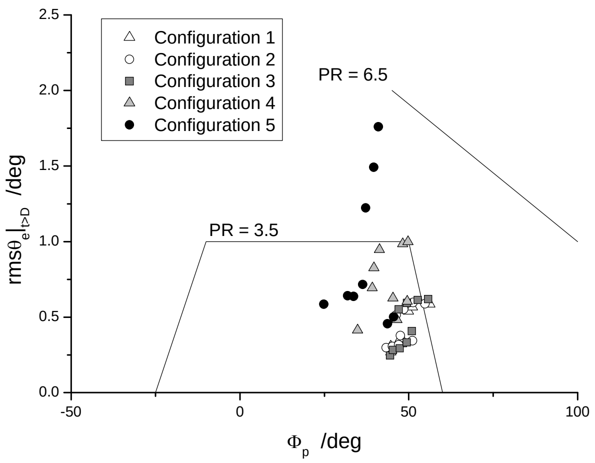

Fig.5 Flying Quality Evaluation Results

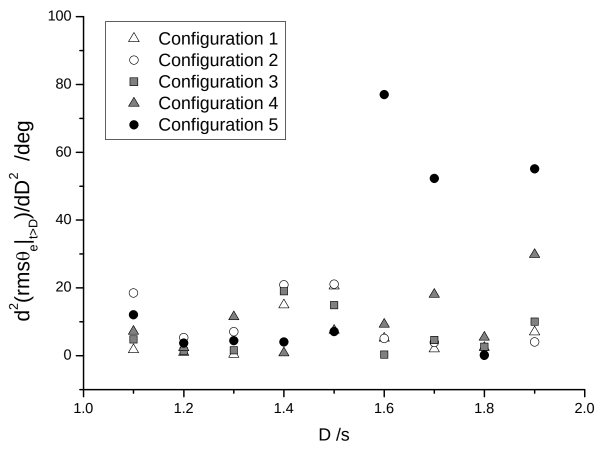

Fig 1 Quantitative PIO metric evaluation results

As shown in Fig 4~6, because of the high rate limit of the actuators, for all 5 different configurations, the flying quality is at least Level 2, the highest value of $\partial^2 (rms\theta_e|_{t>D})/\partial D^2$ is lower than 100. According to the TDNS criterion, aircraft with all 5 control allocation configurations is PIO-immune. However, the PIO tendencies of different configurations are still distinguishable.

The $rms\theta_e|_{t>D}$ of configuration 1, 2 and 3 are close, the tendency of configuration 3 is slightly larger than configuration 2, and he tendency of configuration 2 is slightly larger than configuration 1. The $rms\theta_e|_{t>D}$ of configuration 4 and 5 are apparently larger than the former 3 configurations, especially configuration 5. The highest $\partial^2 (rms\theta_e|_{t>D})/\partial D^2$ of configuration 1, 2 and 3 are about 20, of configuration 4 about 30, of configuration 5 about 80.

From above, we can indicate that the PIO tendency of configuration 5 is the highest, configuration 4 takes the second place. The PIO tendencies of configuration 1, 2 and 3 are relatively low. The evaluation results accord with the analysis results in Section 3.2.

5 Conclusion

This paper analyzes the effects of control allocation on PIO susceptibility of aircrafts with multiple control effectors and evaluates the PIO susceptibility of a prototype aircraft by using time domain Neal-Smith criterion.

The evaluation results indicate that control allocation will significantly influence the PIO susceptibility of aircrafts with multiple control effectors. From the angle of preventing PIO, control should not be allocated to one or several certain control effectors; the proportion of each control effectors should be proportional to the saturation rate of their actuators.

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